

INVERSE T FUNCTIONS

1. Match the following:

A	B
(a) $\sin^{-1} x + \cos^{-1} x, x \in [-1, 1]$	$\frac{\pi}{4}$
(b) $\sin^{-1}\left(\sin \frac{4\pi}{5}\right)$	$\frac{5\pi}{6}$
(c) $\cot^{-1}(-\sqrt{3})$	$\frac{\pi}{2}$
(d) $\tan^{-1}\left(\frac{1-\sqrt{2}}{1-2\sqrt{2}}\right) + \tan^{-1}\left(\frac{1+\sqrt{2}}{1+2\sqrt{2}}\right)$	$\frac{\pi}{5}$
	$\frac{\pi}{3}$

- Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$
- Show that $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \frac{\pi}{4} + \frac{x}{2}$.
- Given an expression for $\tan(x+y)$
- Prove that for $xy < 1$, $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$.
- Using the above result Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$.
- The principal value of $\cot^{-1}(-1)$ is
- Write the function $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \pi$ in the simplest form.
- Prove that: $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$
- Solve the following for x $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}, x > 0$
- Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$
- Prove that $3\sin^{-1} x = \sin^{-1}(3x - 4x^3)$
- Evaluate $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$
- Find the value of $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$

15. Prove that $\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{3}{5} - \tan^{-1} \frac{8}{19} = \frac{\pi}{4}$
16. Prove that $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$
17. Prove that $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$
18. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$
19. Solve $\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) = \frac{\pi}{4}$
20. Solve $\tan^{-1} \left(\frac{1-x}{1+x} \right) = \frac{1}{2} \tan^{-1} x, x > 0$
21. Prove that $\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right) = \frac{\pi}{4} - x$
22. Prove that $\tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right) = \frac{\pi}{4} - \frac{x}{2}$
23. Prove that $\tan^{-1} \left(\frac{a \cos x - b \sin x}{b \cos x + a \sin x} \right) = \tan^{-1} \left(\frac{a}{b} \right) - x$
24. Prove that $\cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \frac{x}{2}$
25. Prove that $\tan^{-1} \left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$
26. Prove that $\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x} \right)$
27. Prove that $\tan^{-1} \left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right] = \frac{\pi}{4} - \frac{1}{2} \cos^{-1}(x^2)$

IV. Write the simplest form of the following functions:

- | | |
|---|---|
| <p>1. $\tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right)$</p> | <p>11. $\tan^{-1} \left(\frac{\sqrt{x} - x}{\frac{3}{1+x^2}} \right)$</p> |
| <p>2. $\tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right)$</p> | <p>12. $\tan^{-1} \left(\frac{3-2x}{1+6x} \right)$</p> |

$$3. \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

$$4. \tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$$

$$5. \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$$

$$6. \sin^{-1}\left(\frac{2\sqrt{x}}{1+x}\right)$$

$$7. \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$8. \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

$$9. \sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right)$$

$$10. \sin^{-1}\left(\frac{x^2}{\sqrt{x^4+a^4}}\right)$$

$$13. \sin\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$$

$$14. \cos^{-1}\left(\frac{1-x^{2n}}{1+x^{2n}}\right)$$

$$15. \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$$

$$16. \sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$$

$$17. \tan^{-1}\left(x+\sqrt{1+x^2}\right)$$

$$18. \sin^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{2}\right)$$

$$19. \sin^{-1}\left(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right)$$

$$20. \tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$$

Answer:

$$1. \frac{1}{2}$$

$$5. \frac{1}{2}\sin^{-1}x$$

$$8. \sin^{-1}\left(\frac{x}{a}\right)$$

$$11. \tan^{-1}(\sqrt{x})-\tan^{-1}x$$

$$14. \tan^{-1}(x^n)$$

$$17. \frac{\pi}{2}-\frac{1}{2}\cot^{-1}x$$

$$20. \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$$

$$2. \frac{\pi}{4}+\frac{x}{2}$$

$$6. 2\tan^{-1}\sqrt{x}$$

$$9. 2\sin^{-1}(ax)$$

$$12. \tan^{-1}3-\tan^{-1}2x$$

$$15. 2\tan^{-1}(2^x)$$

$$18. \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

$$3. \frac{1}{2}\sin^{-1}x \quad 4. \frac{\pi}{2}-\sec^{-1}x$$

$$7. \frac{x}{2}$$

$$10. \tan^{-1}\left(\frac{x^2}{a^2}\right)$$

$$13. \sin(\cos^{-1}x)$$

$$16. \sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}2x$$

$$19. \sin^{-1}x-\sin^{-1}\sqrt{x}$$