CHAPTER 1

RELATIONS AND FUNCTION

SAY 2018

1. Consider the relation R in the set N of natural numbers defined as $R=\{(a,b): ab \text{ is a factor of } 6\}$. Determine whether the relation is reflexive, symmetric or transitive. (3)

2. Consider the binary operation $*$ on the set R of real numbers, defined real numbers , defined by $a*b = \frac{ab}{4}$.
   a) Prove that $*$ is commutative and associative (2)
   b) Find the identity element of $*$ on R (1)
   c) Find the inverse of 5. (1)

3. Let $A=N\times N$ and $*$ be a binary operation on A defined by $(a,b)*(c,d)=(a+c,b+d)$
   a) Find $(1,2)*(2,3)$ (1)
   b) Prove that $*$ is commutative. (1)
   c) Prove that $*$ is commutative. (2)

MARCH 2018

4. If $f(x)=\frac{x}{x-1}, x\neq 1$
   a) Find $fof(x)$ (2)
   b) Find the inverse of f. (1)

5. Let $A=N\times N$ and $*$ be a binary operation on A defined by $(a,b)*(c,d)=(a+c,b+d)$
   a) Find $(1,2)*(2,3)$ (1)
   b) Prove that $*$ is commutative. (1)
   c) Prove that $*$ is commutative. (2)

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6. a) If $R=\{(x,y): x,y\in Z x-y\in Z\}$, then the relation R is
   a) Reflexive but not transitive
   b) Reflexive but not symmetric
   c) Symmetric but not transitive
   d) an Equivalence relation (1)

   b) Let * be a binary operation on the set Q of rational numbers by $a*b=2a+b$
   Find $2*(3*4) and (2*3)*4$ (2)

   c) Let $f:R\to R, g:R\to R$ be two one- one function. Check whether gof is one-one or not. (2)

MARCH 2017

7. a) Let R be a relation defined on $A=\{1,2,3\}$ by $R=\{(1,3),(3,1),(2,2)\}$. R is
   a) Reflexive b) Symmetric
   c) Transitive d) Reflexive but not transitive (1)

   b) Find fog and gof if $f(x)=|x+1|$ and $g(x)=2x-1$ (2)

   c) Let * be a binary operation defined on $N\times N$ by $(a,b)*(c,d)=(a+c,b+d)$.
   Find the identity element for * if it exists. (2)

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8. a) If $f:R\to R$ and $g:R\to R$ defined by $f(x)=x^2$ and $g(x)=x+1$, then $gof(x)$ is
   i) $(x+1)^2$ ii) x^3+1
   iii) x^2+1 iv) $x+1$ (1)

b) Consider the function $f: N \to N$, given by $f(x) = x^3$. Show that the function f is injective but not surjective. (2)

c) The given table shown an operation $*$ on $A = \{p, q\}$

$*$	p	q
p	p	q
q	p	q

i) Is $*$ a binary operation on A?
ii) Is $*$ commutative? Give reason. (2)

MARCH 2016

9. a) The function given $f: N \to N$, by $f(x) = 2x$ is (1)
i. one-one and onto
ii. one-one but not onto
iii. not one-one and not onto
iv. onto, but not one-one

b) Find $gof(x)$, if $f(x) = 8x^3$ and $g(x) = x^{1/3}$. (2)

c) Let * be an operation such that $a*b = LCM$ of a and b defined on the set $A = \{1,2,3,4,5\}$. Is $*$ binary operation? Justify your answer. (2)

SAY 2015

10. a) When a relation R on set A is said to be reflexive? (1)

b) Find gof and fog, if $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$. (2)

c) Show that $f:[-1,1] \to R$ is given by $f(x) = \dfrac{x}{x+2}$ is one-one. (2)

2015 MARCH

11. a) What is the minimum number of ordered pairs to form a non-zero reflexive relation on a set of n elements? (1)

b) On the set $\mathbb{R}$ of real numbers, S is a relation defined as $S = \{(x,y) \mid x \in \mathbb{R}, y \in \mathbb{R}, x + y = xy\}$. Find $a \in \mathbb{R}$ such that 'a' is never the first element of an ordered pair is S. Also find $b \in \mathbb{R}$ such that 'b' is never the second element of an ordered pair is S. (2)

c) Consider the function $f(x) = \dfrac{3x+4}{x-2}, x \neq 2$. Find a function $g(x)$ on a suitable domain such that $(gof)(x) = x = (fog)(x)$. (2)

SAY 2014

12. a) Let $f: R \to R$ be given by $f(x) = \dfrac{2x+1}{3}$. Find fof and show that f is invertible. (2)

b) Find the identity element of the binary operation $*$ on N defined by $a*b = ab^2$. (2)

MARCH 2014

13. a) Let R be the relation on the set N of the natural numbers given by $R = \{(a,b) : a - b > 2, b > 3\}$. Choose the correct answer. (1)

A) $(4,1) \in R$ B) $(5,8) \in R$
C) $(8,7) \in R$ D) $(10,6) \in R$

b) If $f(x) = 8x^3$ and $g(x) = x^{1/3}$, Find $g(f(x))$ and $f(g(x))$. (2)

c) Let $*$ be a binary operation on the set Q of rational numbers defined by $a*b = \dfrac{ab}{3}$. Check whether $*$ is commutative and associative? (2)

SAY 2013

14. Consider the set $A=\{1,2,3,4,5\}$, $B=\{1,4,9,16,25\}$ and a function $f:A\to B$ defined by $f(1)=1,\ f(3)=9,\ f(4)=16$ and $f(5)=25$

a) Show that f is one-to-one. (2)

b) Show that f is onto. (2)

c) Does f^{-1} exist? Explain. (1)

MARCH 2013

15. Consider $f:R\to R$ given by $f(x)=5x+2$.

a) Show that f is one-to-one. (1)

b) Is f invertible? Justify your answer. (2)

c) Let $*$ be a binary operation on N defined by $a*b=HCF\ of\ a\ and\ b$.

i) Is $*$ commutative? (1)

ii) Is $*$ associative? (1)

SAY 2012

16. i) $*:R\times R\to R$ is given by $a*b=3a^2-b$. Find the value of 2*3. Is * commutative? Justify your answer . (2)

ii) If $f:R\to R$ is defined by $f(x)=x^2-3x+2$, find $(fof)(x)$ and $(fof)(1)$. (3)

MARCH 2012

17. a) Give an example of a relation on a set $A=\{1,2,3,4\}$, which is reflexive, symmetric but not transitive. (1)

b) Show that $f:[-1,1]\to R$ is given by $f(x)=\dfrac{x}{x+2}$ is one-one. (2)

c) Let $*$ be a binary operation on Q^+ defined by $a*b=\dfrac{ab}{6}$. Find inverse of 9 with respect to $*$. (2)

SAY 2011

18. Let N be the set of natural numbers. Consider the function $f:N\to N$ defined by

$$f(x)=x+1,\ x\in N$$

a) Prove that f is not an onto function. (2)

b) If $g(x)=\begin{cases} x-1 & if \quad x>1 \\ 1 & if \quad x=1 \end{cases}$, then find gof. (2)

c) Check whether gof is an onto function. (1)

MARCH 2011

19. (a) i) $f:\{1,2,3,4\}\to\{5\}$ defined by $f=\{(1,5),(2,5),(3,5),(4,5)\}$.

Does the function f invertible? (1)

ii) $A=R-\left\{\dfrac{7}{5}\right\}$ $B=R-\left\{\dfrac{3}{5}\right\}$ $f:A\to B$ defined by $f(x)=\dfrac{3x+4}{5x-7}$. $g:B\to A$ defined by $g(y)=\dfrac{7y+4}{5y-3}$.

Find gof . (2)

(b) Let $A=N\times N$ (N-set of natural nos.) and $*$ be a binary operation on A defined by $(a,b)*(c,d)=(ac-bd,ad+bc)$. Show that $*$ is commutative on A. (2)

SAY 2010

20. Let $f(x)=\dfrac{x-1}{x-3}, x\neq 3$ and $g(x)=\dfrac{x-3}{x-1}, x\neq 1$ be two real valued functions defined on R.

a) Find $(fog)(x), x\neq 0$ (1)

b) Find $f^{-1}(x)$ and $g^{-1}(x), x\neq 1$ (2)

c) Find $(gof)^{-1}(x)$ (2)