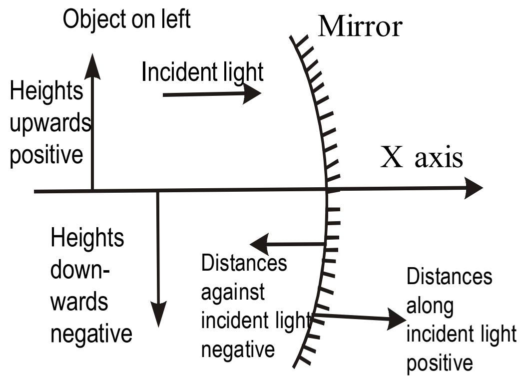
RAY OPTICS AND OPTICAL INSTRUMENTS

Reflection of light by spherical mirrors.

Laws of reflection

1.According to the first law of reflection, the angle of reflection is equal to the angle of incidence.
2.According to the second law of reflection, the incident ray, reflected ray and the normal to the point of incidence all lie in the same plane.



Sign convention

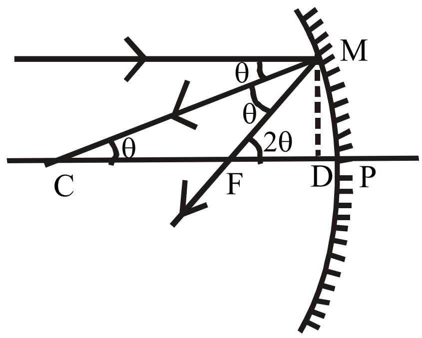
1) According to this convention, all distances are measured from the pole of the mirror or the optical centre of the lens.
2) The distances measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of incident light are taken as negative .
3) The heights measured upwards are taken as positive . The heights measured downwards are taken as negative.

Focal length of spherical mirrors

The geometric centre of a spherical mirror is called its **pole** while that of a spherical lens is called its **optical centre**. *The line joining the pole and the centre of curvature of the spherical mirror is known as the* **principal axis**. The rays coming from infinity parallel and closer to the principal axis are called **paraxial rays**.The reflected rays converges to a point F on the principal axis of a concave mirror. For a convex mirror, the reflected rays appear to diverge from a point Fon its principal axis . The point F is called the **principal focus** of the mirror. *The distance between the focus F and the pole P of the mirror is called the* **focal length** *of the mirror, denoted by f.*

Relation connecting focal length (f) and radius of curvature (R)

Consider a mirror having centre of curvature C. Let a ray parallel to the principal axis strike the mirror at M. Then CM will be perpendicular to the mirror at M. Let q be the angle of incidence, and MD be the perpendicular from M on the principal axis. Then,



$$MCP = \theta \text{ and } MFP = 2\theta$$

Now, $\tan\theta = \frac{MD}{CD}$; $\tan 2\theta = \frac{MD}{FD}$

For small values of θ, $\tan\theta \approx \theta$ and $\tan 2\theta \approx 2\theta$

Therefore, $\frac{MD}{FD} = 2\frac{MD}{CD}$; $FD = \frac{CD}{2}$

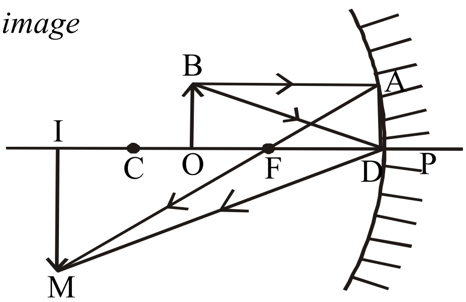
Now, for small θ, the point D is very close to the point P.

Therefore, FD = f and CD = R. then $f = \frac{R}{2}$

Mirror equation

Mirror equation is the relation connecting object distance u, image distance v and focal length f.

Let OB be an object kept at a distance u from the pole of a concave mirror of focal length f. Its image IM is formed at a distance v from the pole. Draw AD perpendicular to the principal axis.



Considering similar triangles POB and PIM,

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$$\frac{IM}{OB} = \frac{PI}{PO} \quad ....(1)$$

Now from similar triangles, FAD and FIM,

$$\frac{IM}{AD} = \frac{FI}{FO} \qquad \text{But } AD = OB \qquad \therefore \frac{IM}{OB} = \frac{FI}{FD} \quad ....(2)$$

From (1) and (2), $\frac{FI}{FD} = \frac{PI}{PO}$. Now FD $\approx$ FP = f, PI = v; PO = u ; FI = PI - PF = v - f.

$$\therefore \frac{v-f}{f} = \frac{v}{u} \qquad \text{i.e, } uv - uf = vf$$

Dividing throughout by uvf, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

This is the **mirror equation**.

Linear magnification (m) is the ratio of the height of the image (h_i) to the height of the object (h_o):

$$m = \frac{\text{hight of the image}}{\text{hight of the object}} = \frac{h_i}{h_o}$$

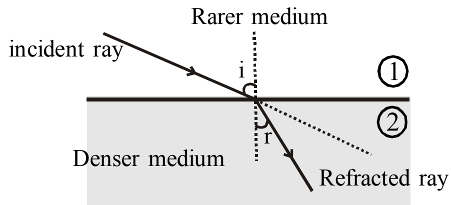
With the sign convention, this becomes

$$\frac{-h_i}{h_o} = \frac{-v}{-u} \quad ; \quad \frac{h_i}{h_o} = -\frac{v}{u}$$

Q1. The radius of curvature of a concave mirror is 30 cm. Find its focal length. [-0.15m]
Q2. An object is placed at a distance 6 cm from a concave mirror of focal length 12 cm. Find the position, nature and magnification of the image. [12 cm; virtual; 2]
Q3. The focal length of a concave mirror is 10 cm. At what distance must an object be placed so that a real image of magnification 2 is obtained. [- 15 cm]

Refraction

The bending of light (deviation of path of light) when light travells from one medium to another medium is called refraction.

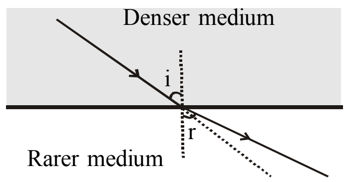


Light from rarer to denser medium

When light travels from a rarer medium to a denser medium, it deviates towards the normal.

Light from denser to rarer medium

When light travels from a denser medium to a rarer medium, it deviates away from the normal.



<u>Laws of refraction</u>

First law

The incident ray, the refracted ray and the normal at the point of incidence are all in the same plane.

Second law (Snell's law)

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media and for the given colour of light used.
This constant is known as the refractive index of second medium w.r. t. the first medium.

Explanation

If 'i' is the angle of incidence in the first medium and 'r' is the angle of refraction in the second medium,

then by Snell's law, $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = {}_1n_2 = \text{a constant}$ Where ${}_1n_2$ is the refractive index of the second medium with respect to the first medium.

If the first medium is air, then (sin*i*/sin*r*) is known as absolute refractive index of the second medium.

i.e., $\frac{\sin i}{\sin r} = n$; where 'n' is the refractive index of the second medium.

Refractive index and velocity

The constant refractive index is not simply a number obtained as the ratio of the sines of two angles It has important physical significance. Absolute refractive index represents the speed with which light travels in a medium when compared to vacuum. We have seen earlier (in the chapter wave optics) that absolute refractive index of a medium (n) $n = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}}$

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Relation between refractive indices

n_{21} is the refractive index of medium-2 with respect to medium-1.

$$n_{21} = \frac{n_2}{n_1}$$

n_1 and n_2 are the absolute refractive indices of medium-1 and medium-2.

Note: *There is no refraction when light falls normally on the surface of separation*

Q4. A ray of light is incident at an angle 30^0 on the surface of water. find the angle of refraction, if the refractive index os water is 4/3? [$22^0 1^1$]

Q5. Green light of mercury has wavelength 5.5×10^{-7}m. What is the frequency? If refractive index of glass is 1.5, what is the wavelength in glass? $C = 3 \times 10^8$ m/s. [5.45×10^{14} Hz; 366.9×10^{-9}m]

Q6. A light ray of wavelength 500 nm gets refracted through a glass slab. Calculate the speed, frequency and wavelength of refracted light. {$n_g = 1.5$} [2×10^8 m/s; 6×10^{14} Hz; 3.33×10^{-7}m]

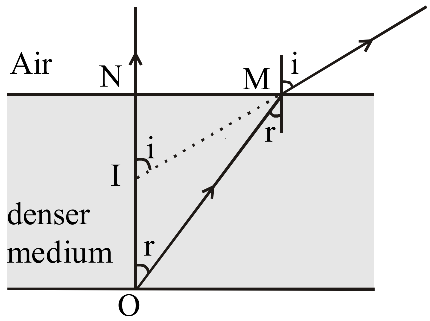
Some examples of refraction

(a) **Apparent depth and real depth**

When an object (in a denser medium) is viewed from a rarer medium, it seems to be raised towards the surface. This is called apparent depth.

Explanation

'I' is the refracted image of the object 'O' kept in medium(say water) of refractive index 'n'. ON is the real depth. IN is known as apparent depth.



$$n_{\text{water air}} = \frac{\sin i}{\sin r}$$

$$\sin i = \frac{NM}{IM} \quad ; \quad \sin r = \frac{NM}{OM}$$

$$\frac{\sin i}{\sin r} = \frac{NM}{IM} \times \frac{OM}{NM} = \frac{OM}{IM}$$

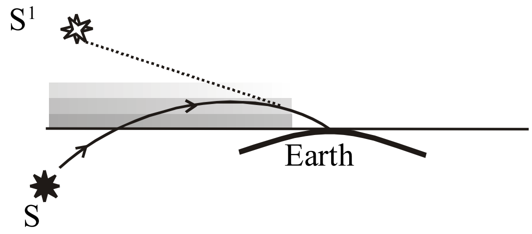
Since N is very close to M, IM≈IN, $n = \frac{\sin i}{\sin r} = \frac{ON}{IN}$

i.e. $n = \frac{\text{Real depth}}{\text{apparent depth}}$

(b) **Twinkling of stars**

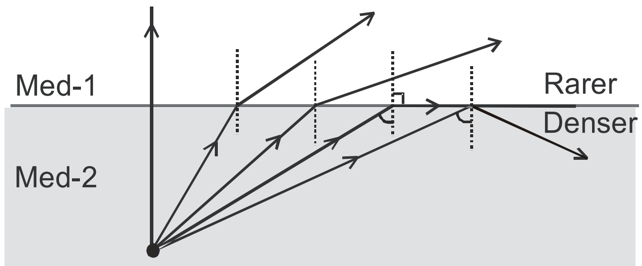
Light from a star travelled hundreds of light-years to reach earth. Twinkling of stars is due to the refraction of star light at different layers of the atmosphere. The temperature, density of the layers and the refractive indices of different media are continuously changing. i.e., the light from star may travel from denser to rarer and in the next moment from rarer to denser. So, the light miss from our eye sight many times. Thus the star appears to be twinkling.

(c) Apparent shift in the position of the sun at sunrise and sunset.



Sun is visible before sunrise and after sunset because of atmospheric refraction. The density of atmospheric air decreases as we go up. So the rays coming from the sun deviates towards the normal. Due to this, the apparent shift in the direction of the sun is by about half a degree and the corresponding time difference between actual sunset and apparent sunset is about 2 minutes . So the sun at 'S' appears to come from 'S[1]'. Thus an observer on earth can see the sun before sunrise and after sunset. The apparent flattening (oval shape) of the sun at sunset and sunrise is also due to the same phenomenon.

Total Internal Reflection



When light travels from an optically denser medium to a rarer medium, at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. This reflection is called the *internal reflection.* When a ray of light passes from a denser to rarer medium, after refraction the ray bends away from the normal. As we increase the angle of incidence the angle of refraction increases. For a particular angle of incidence the angle of reflection becomes 90^o or refracted ray travels along the surface of separation. If we further increase angle of incidednce there will be no refracted ray, the entire light will be reflected and total internal reflection will takes place inthe denser medium itself. The angle of incidence in the denser medium for which angle of refraction 90^o is called the **critical angle(C).** If the angle of incidence in the denser medium is greater than critical angle, the ray gets totally, internally reflected. This phenomenon is called **total internal reflection**.

Conditions for total internal reflection

(i) Light should travel from denser to rarer medium.
(ii) Angle of incidence should be greater than the critical angle.

Let the refractive index of medium-1 w.r.t. medium-2 is given by

$$\frac{n_1}{n_2} = n_{12} = \frac{\sin i}{\sin r}$$

when angle of incidence i= C, the critical angle then the angle of refraction = 90^o

$$n_{12} = \frac{\sin C}{\sin 90} = \sin C$$

$$n_{12} = \frac{1}{n_{21}}$$

$$n_{12} = \frac{1}{\sin C}$$

If rarer medium is air, $n_{12} = n$. Then $n = \dfrac{1}{\sin C}$

Critical angle of a medium depends upon wavelength of light. Greater the wavelength, greater will be critical angle.

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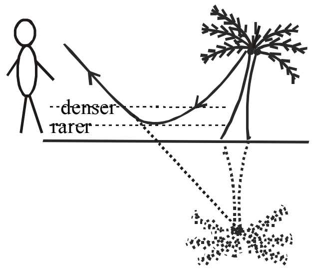
Some phenomena due to Total Internal Reflection

(1) Brilliance of diamond

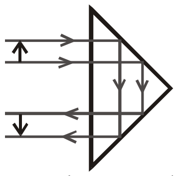
Refractive index of diamond is high (n=2.5) and the critical angle is very small ($C=24^0$). More over the faces of the diamond are cut in such a way that a ray of light entering the crystal undergoes multiple total reflections. This multiple reflected light come out through one or two faces. So these faces appears sparkling.

(2) Mirage

Mirage is an optical illusion. On hot summer days the layer of air in contact with the sand becomes hot and rarer. The upper layers are comparatively cooler and denser. When light rays travel from denser to rarer, they undergo total internal reflection. Thus image of the distant object is seen inverted. This phenomenon is known as mirage.



(3) **Total reflection prisms**



Ray is turned through 180^0

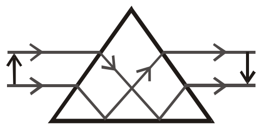
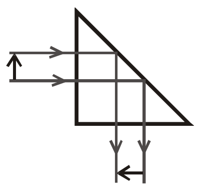


Image is inverted without deviation

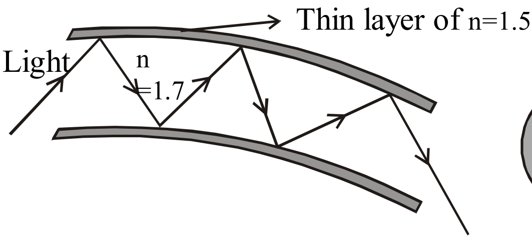


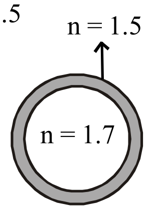
Ray is turned through 90^0

A right-angled isosceles prism is called a total reflecting prism. Total reflecting prisms based on the principle of total internal reflection. With the help of these prisms, the direction of the incident ray can be changed. The refractive index for glass is 1.5 and its critical angle is 42^o. When a ray of light makes an angle of incident more than 42^o (within the glass) the ray undergoes total internal reflection.

(4) **Optical Fibres**

Optical Fibre Cable (OFC) consist of a number of long fibres made of glass or quartz (n=1.7). Theyare coated with a layer of a material of lower refractive index (n=1.5). When light incident on the optical fibre at angle greater than the critical angle, it undergoes total internal reflection. Due to this total internal reflection, a ray of light can travel through OFC without any appreciable loss of energy.





Uses of OFC

(1) Used as a light pipe in medical and optical diagnosis.
(2) It can be used for optical signal transmissions.
(3) Used to carry telephone, television and computer signals as pulses of light.
(4) Used for the transmission and reception of electrical signals which are converted into light signals.

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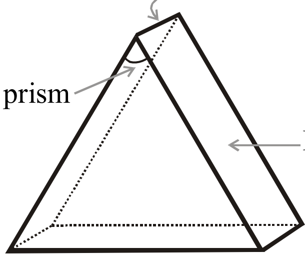
Looming (superior mirage)

Due to the mist and fog in cold countries, distant ship cannot be seen clearly. But due to the total internal reflection, the image of the ship appears hanging in air. This illusion is known as looming.

Q4. A ray of light is incident at an angle 30^0 on the surface of water. Find the angle of refraction, if the refractive index of water is 4/3. [$22^0 1^I$]
Q5. Green light of mercury has wavelength 5.5×10^{-7} m. What is its frequency? If refractive index of glass is 1.5, what is the wavelength in glass? Given $c = 3 \times 10^8$ m/s. [5.45×10^{14} Hz, 366.9×10^{-9} m]
Q6. Calculate the critical angle of diamond in air if its refractive index is 2.42. [$24^0 24^I$]
Q7. A fish is at a depth of 2m from the surface of water. Calculate the radius of the base of the cone of vision. Given $n_w = 4/3$. [2.267 m]
Q8. The velocity of light in medium 1 is 2 x 108m/s and in medium 2 is 2.5 x 108m/s. Find the critical angle at the interface of the two media. [53.13^0]

PRISM

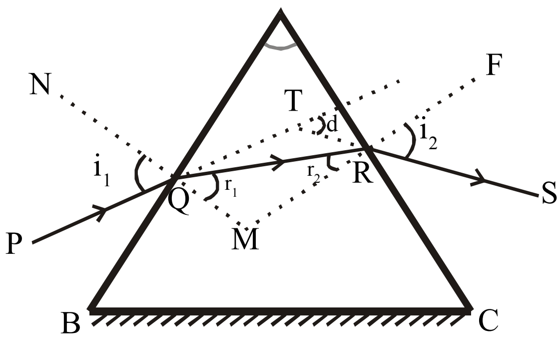


A prism is a portion of a transparent medium bounded by two inclined plane surfaces. The angle between the two inclined plane surfaces is called angle of the prism(A) and the line of intersection of these planes is . known as the refracting edge of the prism.

Refraction through a prism

Consider a prism ABC of angle A. A ray PQ incidents on the face AB at an angle i_1. QR is the refracted ray inside the prism, which makes two angles r_1 and r_2 (inside the prism). RS is the emergent ray at angle i_2.

The angle between the emergent ray and incident ray is called the angle of deviation 'd'.



In the quadrilateral AQMR,

$$\angle Q + \angle R = 180^0$$

[since FM and NM are normal]

Therefore, $\angle A + \angle M = 180^0$ ....(1)

Also from Δ QMR,

$$r_1 + r_2 + \angle M = 180^0 \quad ....(2)$$

Comparing (1) and (2)

$$r_1 + r_2 = A \quad \text{--- ----(3)}$$

From the Δ QRT,

Total deviation, d = deviation at the first face + deviation at the second face

$$d = (i_1 - r_1) + (i_2 - r_2)$$

$$d = (i_1 + i_2) - (r_1 + r_2)$$

but, $r_1 + r_2 = A$

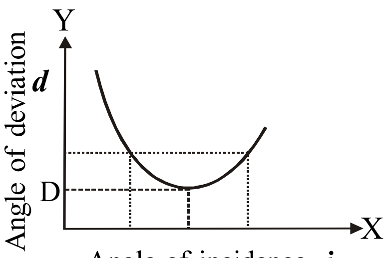
$$\therefore\ d = (i_1 + i_2) - A \quad ....(4)$$

While we are doing this experiment in the lab, as the angle of incidence (*i*) increased, the angle of deviation(*d*) decreases, reaches a minimum value and then increases. The graph obtained is as shown:

This angle of deviatiation which has minimum value is called **angle of minimum deviation D**. Here the deviation is found to be minimum.

At minimum deviation d = D or the incident ray and the emergent ray symmetrical w.r.t. the refracting faces and the refracted ray in the prism is parallel to the base.

***i - d* curve**



At the minimum deviation position,

$i_1 = i_2 = i$, $r_1 = r_2 = r$ and $d = D$

Hence (3) can be written as, $r + r = A$

or $$r = \frac{A}{2} \quad \text{---------(5)}$$

Similarly (4) can be written as, $i + i = A + D$ $$i = \frac{A + D}{2} \quad \text{--------(6)}$$

Let n be the absolute refractive index of the prism, then we can write,

$$n = \frac{\sin i}{\sin r} \quad \text{---------(7)}$$

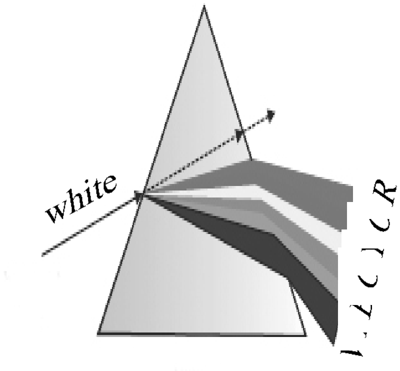
Substituting (5) and (6) in (7),

the refractive index of the material of the prism, $$n = \frac{\sin\frac{A + D}{2}}{\sin\frac{A}{2}}$$

Dispersion

The splitting of a beam of white light intodifferent component colours when it passes through a transparent medium is called dispersion. The refractive index is different for different colours. The variation of the refractive index of the medium with

the wavelength causes dispersion. $n \propto \frac{1}{\lambda^2}$. i.e., while dispersionred c



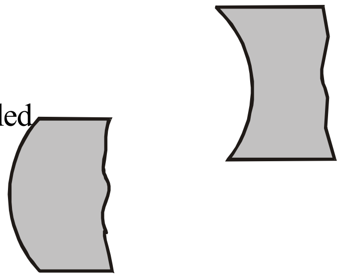
deviates most. The variation of refractive index with wavelength may be more prominent in some media than the other. In vacuum, of course, the speed of light is independent of wavelength.

Thus, vacuum (or air) is a non-dispersive medium in which all colours travel with the same speed. But glass is a dispersive medium.

Q9. The angle of minimum deviation for an equilateral glass prism is $48^0 30^I$. Find the refractive index of glass? [1.623]
Q10. A ray of light is incident normally on one face of a prism of refractive index 1.5 and emerges from the other face just grasing the surface. Find the angle of prism? [$41^0 48^I$]
Q11. A ray of light is incident at an angle of 500 on one face of an equilateral prism of refractive index 1.5. Find the deviation produced by the prism. [$37^0 12^I$.]

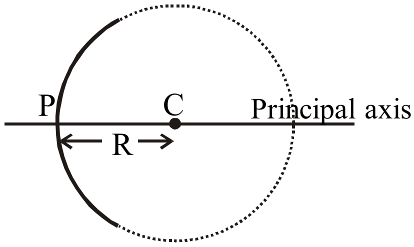
Spherical Refracting Surfaces

There are two types of spherical surfaces, convex and concave .



i) **Concave spherical refracting surface**
A spherical refracting surface which is concave towards rarer medium is called concave refracting spherical surface
ii) **Convex spherical refracting surface**
A spherical refracting surface which is convex towards rarer medium is called convex refracting spherical surface

Terms related to Spherical Refracting Surface



i) Pole : The mid point of the spherical refracting surface is called its pole. It is denoted by **P**.

ii)**Centre of curvature:** The centre of the sphere of which the curved refracting surface forms apart is called its centre of curvature. It is denoted by **C**

iii) **Radius of curvature:** It is the distance between the pole and Centre of curvature of the spherical refracting surface. It is denoted by **R**.

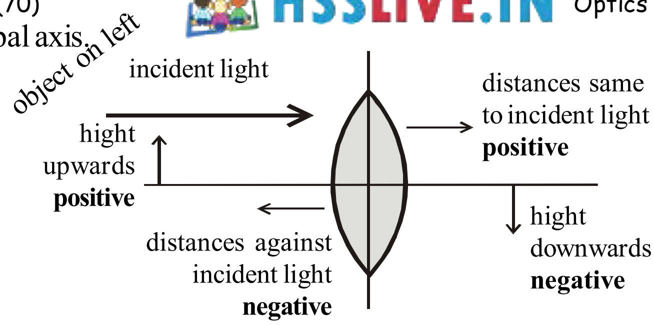
iv) **Principal axis:** A straight line passing through the Centre of curvature and the pole of the spherical refracting surface is called the principal axis .

v) **Aperture :** It is the effective diameter of the light refracting spherical surface.

In order to provide information regarding the position of the image formed by a lens certain relations involving object and image distance are developed. In these relations the object distance is represented by the symbol '**u**' and image distance by '**v**'.

Assumptions:
i) The aperture of spherical refracting surface is small.

ii) The object is a point object and lies on the principal axis.
iii) The angle of incidence and angle of refraction are suppose to be small.



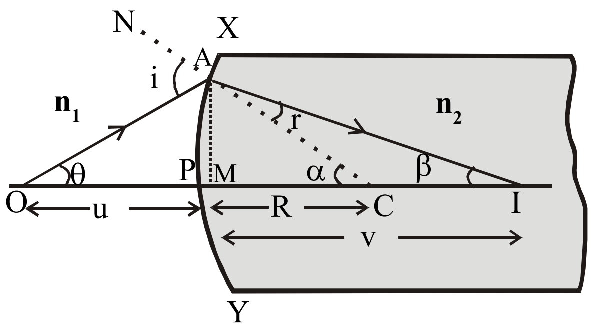
We shall adopt the following conventions called **New Cartesian Sign Conventions.**

i) All the distances are measured from the pole of the spherical refracting surface.
ii) The light rays are assumed to be incident from left to right.
iii) The distances measured in the direction of propagation of incident light are taken as positive.
iv) The distances measured in the direction opposite to the direction of propagation of incident light are taken as negative.
v) Measurements above the principal axis taken as positive and below the principal axis taken as negative.

Refraction at a convex spherical surface

Consider a convex surface XY , which separates two media having refractive indices n_1 and n_2 . ($n_2 > n_1$).

Let C be the centre of curvature and P be the pole. Let 'O' be a point object placed at a distance 'u' from the pole. I is the real image of the object at a distance 'v' from the surface. OA is the incident ray at angle 'i'.



Let us derive an equation connecting u , v, and R.

From Snell's law,

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

If 'i'and 'r' are small, then $\sin i \approx i$ and $\sin r \approx r$.

$$\text{i.e.,} \quad \frac{i}{r} = \frac{n_2}{n_1}$$

$$n_1 i = n_2 r \qquad \text{--------- (1)}$$

From the Δ OAC,

exterior angle = sum of the interior opposite angles i.e., $i = \alpha + \theta$ ----------(2)

Similarly, from Δ IAC,

$$\alpha = r + \beta$$

$$r = \alpha - \beta \qquad \text{-------------(3)}$$

Substituting the values of eq(2) and eq(3)in (1),we get,

$$n_1(\alpha + \theta) = n_2(\alpha - \beta)$$

$$n_1\alpha + n_1\theta = n_2\alpha - n_2\beta$$

$$n_1\alpha + n_1\theta = n_2\alpha - n_2\beta \qquad\qquad n_1\theta + n_2\beta = n_2\alpha - n_1\alpha$$

$$n_1\theta + n_2\beta = (n_1 - n_2)\alpha \quad \text{..........(4)}$$

Since AM is normal to principal axis

From Δ OAM, we can write,

$$\tan\theta \approx \theta = \frac{AM}{MO}$$

From Δ IAM, $\tan\beta \approx \beta = \dfrac{AM}{MI}$, From Δ CAM, $\tan\alpha \approx \alpha = \dfrac{AM}{MI}$

Now, $PI \approx MI, MO \approx PO, PC \approx MC$

Substituting θ, β and α in equation (4) we get,

$$n_1 \frac{AP}{PO} + n_2 \frac{AP}{PI} = (n_2 - n_1)\frac{AP}{PC}$$

$$\frac{n_1}{PO} + \frac{n_2}{PI} = \frac{(n_2 - n_1)}{PC} \quad \text{i.e,} \quad \frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

Applying sign convection, $PO = -u$, $PI = +v$ and $PC = R$

Substituting these values, we get

$$\frac{-n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

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$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

This formula is general for both concave and convex spherical surfaces.

Case - I

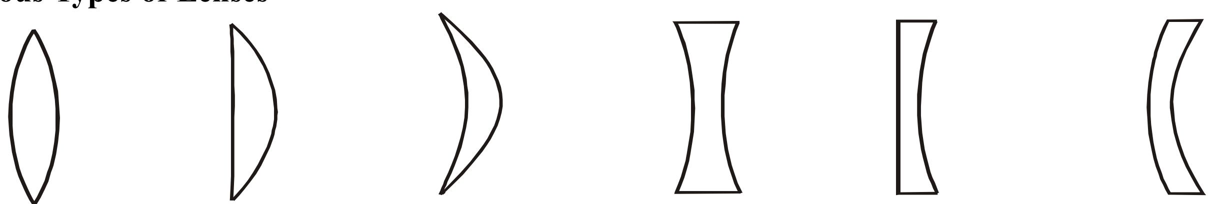
If the first medium is air, $n_1 = 1$, and $n_2 = n$, then, $\dfrac{n}{v} - \dfrac{1}{u} = \dfrac{n-1}{R}$

Lens

*A **lens** is a piece of transparent material bounded by two refracting surfaces out of which at least one is curved.*

If central portion of a lens is thicker than edges it behaves as **convergent** lens known as **convex** lens.
If central portion of a lens is thinner than edges it behaves as **divergent** lens known as **concave** lens.

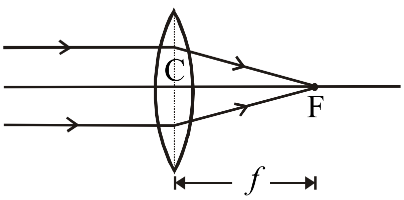
Various Types of Lenses

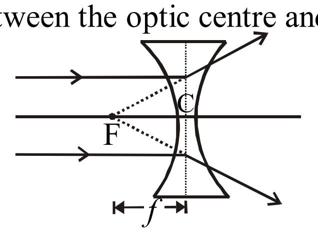


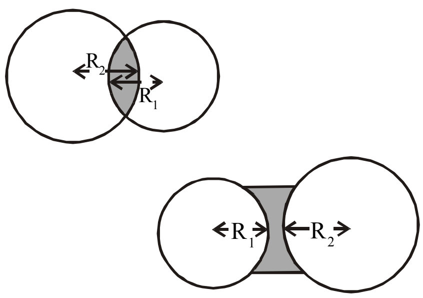
Bi-convex lens Plano-convex lens Concavo-convex Bi-concave lens Plano-concave Convexo-concave

Terms related to LENS

Optic centre: The point in the lens through which rays of light pass undeviated. It is denoted by **C**.
Principal focus. (F) The point on the pricipal axis where the incident ray parallel to principal axis meet or appear to meet after refracting through a lens is called **Principal focus.** It is denoted by **F.**
Focal length (f) It is distance between the optic centre and the principal focus. It is denoted by **f.**





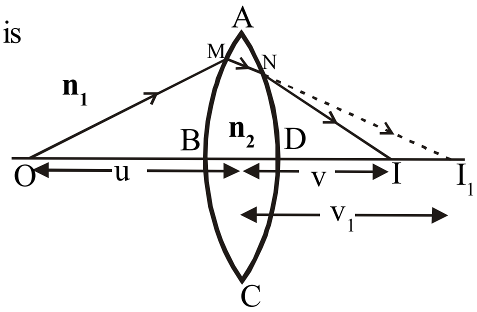


Note: A lens can have different radii of curvature on different surfaces

Lens Maker's Formula (for a thin lens)

The relation between the focal length of the lens(f), refractive index oft he material of the lens(n) and radii of curvature of its surfaces (R_1, R_2) is known as Lens maker's formula.

Consider a thin lens of refractive index n_2 formed by the spherical surfaces ABC and ADC. Let the lens is kept in a medium of refractive index n_1. Let an object 'O' is placed in the medium of refractive index n_1. Hence the incident ray OM is in the medium of refractive index n_1 and the refracted ray MN is in the medium of refractive index n_2.



The general equation for refraction at a spherical surface is

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

The spherical surface ABC (radius of curvature R_1) forms the image at I_1. Let 'u' be the object distance and 'v_1' be the image distance.

Then we can write,

$$\frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad \_\_\_\_\_\_\_(1)$$

This image I_1, will act as the virtual object for the surface ADC and forms the image at v.

Then we can write,

$$\frac{n_1}{v} - \frac{n_2}{v_1} = \frac{n_1 - n_2}{R_2} \quad \_\_\_\_\_\_\_\_(2)$$

Adding eq (1) and eq (2) we get

$$\frac{n_2}{v_1} - \frac{n_1}{u} + \frac{n_1}{v} - \frac{n_2}{v_1} = \frac{n_2 - n_1}{R_1} + \frac{n_1 - n_2}{R_2}$$

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Dividing throughout by n_1, we get

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

If the lens is kept in air, $\frac{n_2}{n_1} = n$

So the above equation can be written as,

$$\frac{1}{v} - \frac{1}{u} = (n - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{------}(4)$$

From the definition of the lens, we can take, when $u = \infty$, $f = v$

Substituting these values in the eq (3), we get

$$\frac{1}{f} - \frac{1}{\infty} = (n - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\therefore \frac{1}{f} = (n - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{----}(5)$$

This is **lens maker's formula**

By knowing the value of n, R_1 and R_2 we can make lens of any focal length.

For convex lens,

f = +ve, R_1 = +ve, R_2 = - ve

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

For concave lens,

f = -ve, R_1 = -ve, R_2 = +ve

$$\therefore \frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Thin Lens formula

From (4),

$$\frac{1}{v} - \frac{1}{u} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

From (5)

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

From these two equations, we get

$$\mathbf{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

This equation is called ***thin lens formula***.

Though we derived it for a real image formed by a convex lens, the formula is valid for both convex as well as concave lenses and for both real and virtual images.

Note: **A virtual image is that which cannot be caught on a screen.**

Linear magnification

Magnification (m) produced by a lens is defined as the ratio of the size of the image to that of the object.

for a lens $\mathbf{m = \frac{h'}{h} = \frac{v}{u}}$

When we apply the sign convention, we see that, for erect (and virtual) image formed by a convex or concave lens, **m** is positive, while for an inverted (and real) image, **m** is negative.

Power of a lens

Power can be defined as the reciprocal of focal length expressed in metre.

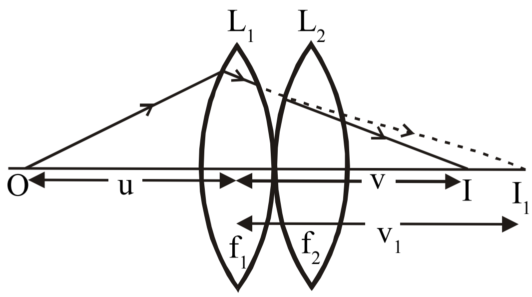
Power of a lens is a measure of the convergence or divergence, which a lens introduces in the light falling on it. For thin lenses

ie, Power, $P = \frac{1}{f \text{ (in metre)}}$

S.I. Unit of Power is **dioptre (D)** 1D = $1m^{-1}$

Combination of lenses

Consider two thin convex lenses of focal lengths f_1 and f_2 kept in contact. Let O be an object kept at a distance 'u' from the first lens L_1, I_1 is the image formed by the first lens at a distance v_1.



Then from the lens formula, we can write,

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u} \qquad \text{-----(1)}$$

This image will act as the virtual object for the second lens and the final image is formed at I (at a distance v). Then

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1} \qquad \text{----- (2)}$$

Adding eq(1) and eq(2),

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v_1} - \frac{1}{u} + \frac{1}{v} - \frac{1}{v_1}$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} \qquad \text{-------(3)}$$

If the two lenses are replaced by a single lens of focal length 'F' the image is formed at 'v'. Then we can write,

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u} \qquad \text{------ (4)}$$

from eq(3) and (4),

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{-----(3)}$$

or $P=P_1+P_2$; where P is the power of the combination, P_1 and P_2 are the powers of the individual lenses.

Magnification (combination of lenses)

If m_1, m_2, m_3,......are the magnification produced by each lens, then the net magnification,

$$m = m_1 \times m_2 \times m_3 ....$$

Relation connecting m,u and f

we know, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Multiplying throughout by u,

$$\frac{u}{v} - 1 = \frac{u}{f}$$

$$\frac{1}{m} - 1 = \frac{u}{f}$$

(since m = v/u) $\quad m = \frac{f}{f+u}$

Relation connecting m,v and f

we know, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Multiplying throughout by 'v',

$$1 - \frac{v}{u} = \frac{v}{f} \qquad 1 - m = \frac{v}{f} \qquad m = \frac{f - v}{f}$$

Scattering of Light

As sunlight travels through the earth's atmosphere, it gets *scattered* (changes its direction) by the atmospheric particles. Light of shorter wavelengths is scattered much more than light of longer wavelengths. According to *Rayleigh's scattering law, the intensity of the scattered light from a molecule is inversely proportional to the 4^th^ power of the wavelength.* $I \propto \frac{1}{\lambda^4}$

Hence, the bluish colour predominates in a clear sky, since blue has a shorter wavelength than red and is scattered much more strongly. In fact, violet gets scattered even more than blue, having a shorter wavelength.

But since our eyes are more sensitive to blue than violet, we see the sky blue.

Large particles like dust and water droplets present in the atmosphere behave differently. The relevant quantity here is the relative size of the wavelength of light λ, and the scatterer (of typical size, say, a). For $a \ll \lambda$, one has Rayleigh scattering which is proportional to $(1/\lambda)^4$. For $a \gg \lambda$, i.e., large scattering objects (for example, raindrops, large dust or ice particles) this is not true; all wavelengths are scattered nearly equally. Thus, clouds which have droplets of water with $a \gg \lambda$ are generally white.

At sunset or sunrise, the sun's rays have to pass through a larger distance in the atmosphere. Most of the blue and other shorter wavelengths are removed by scattering. The least scattered light reaching our eyes, therefore, the sun looks reddish. This explains the reddish appearance of the sun and full moon near the horizon.

Q12. A beam of light converges to a point P. A lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm (b) a concave lens of focal length 16 cm. [7.5 cm, 48 cm]

Q13. A real image is to be produced with image size 2 times that of the object by using a convex lens of focal length 20 cm. Find the object distance? [-30 cm]

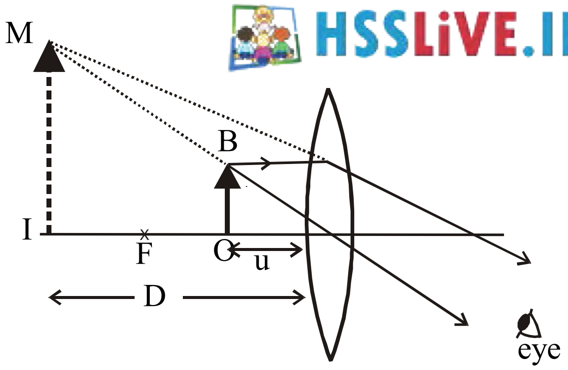
Q14. The radii of curvature of the two surfaces of a convex lens are 20 cm and 25 cm. Its focal length is 22 cm. Find the power of the lens and refractive index of material of the lens. [4.5 D; 1.505]

Q15. Calculate the power in air and in water of a convex lens of radii of curvature 50 cm and 40 cm. The refractive index of glass is 1.55 and that of water 1.33. [2.475 D; 0.744 D]

Q16. Find the power of the combination of a thin convex lens of focal length 10 cm and concave lens of focal length 20 cm. [5 D]

OPTICAL INSTRUMENTS

Simple Microscope



A simple microscope is a device used to get an enlarged image of a nearby object. Usually a converging lens (convex lens) of small focal length is used as a simple microscope.

Here an object is placed in between optic centre and principal focus of the lens. Then we will get an enlarged , erect and virtual image at the near point. {Near point is a point at which a person can see an object clearly. For a normal person it is 25 cm. This is also called least distance of distinct vision.}

Viewing image at near point causes some strain on the eye. Therefore, the image formed at infinity is often considered most suitable for viewing by the relaxed eye. For that the object should be at the focus.

The linear magnification m, for the image formed at the near point D, by a simple microscope can be obtained by using the relation

$$m = \frac{v}{u} \quad ....(1)$$

We also know lens formula

$$\frac{-1}{u} + \frac{1}{v} = \frac{1}{f}$$

Multiplying by v, we get

$$\frac{-v}{u} + \frac{v}{v} = \frac{v}{f}$$

$$\frac{v}{u} = 1 - \frac{v}{f}$$

$$m = 1 - \frac{v}{f} \quad [\text{since } m = \frac{v}{u}]$$

But the image is formed at the least distance of distinct vision (ie v = -D)

$$\therefore\ m = 1 + \frac{D}{f} \quad ....(2)$$

If image is formed at infinity the magnification, $m = \frac{D}{f}$

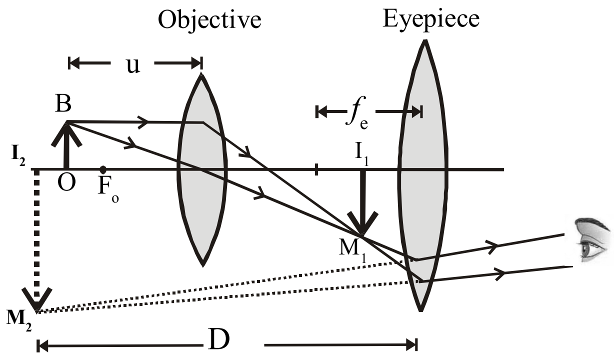
Magnification power of a microscope can also be represent as the ratio of the angle subtended by the image at the eye to that by the object at the least distance of distinct vision. This is known as angular magnification.

Compound microscope

Apparatus

A compound microscope consists of two convex lenses, one is called the objective and the other is called eye piece. The convex lens near to the object is called objective. It has smaller focal length. The lens near to the eye is called eye piece. It has larger focal length.

The object is placed in between F and 2F of objective lens. The objective lens forms real inverted and magnified image (I_1M_1) on the other side of the lens. If this image falls in the focal plane of the eye piece, Now the eyepiece will act as simple microscope and an enlarged, virtual and inverted image is formed at the near poing D.



Magnification

The magnification produced by the compound microscope

$$m = \frac{\text{size of image}}{\text{size of object}} \quad \text{ie,} \quad m = \frac{I_2M_2}{OB}$$

Multiplying and dividing by I_1M_1 we get,

$$m = \frac{I_1M_1}{OB} \times \frac{I_2M_2}{I_1M_1}$$

but we know, $m_o = \frac{I_1M_1}{OB}$ and $m_e = \frac{I_2M_2}{I_1M_1}$

Where m_0 & m_e are the magnifying power of objective lens and eyepiece lens

$$\therefore\ m = m_o \times m_e \quad —— (1)$$

Eyepiece acts as a simple microscope. Therefore $m_e = 1 + \frac{D}{f_e}$ —— (2)

We know magnification of objective lens $m_o = \frac{v_o}{u_o}$ —— (3)

Where v_0 and u_0 are the distance of the image and object from the eye piece.

Substituting (2) and (3) in (1), we get

$$m = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) \quad —— (4)$$

For compound micro scope $u_o \approx f_o$ (because the object of is placed very close to the principal focus of the objective) and $v_o \approx L$, length of microscope (because the first image is formed very close to the eye piece.) Substituting these values in eq(4) we get

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$$m = \frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$$

The total magnification when the image is formed at infinity, is $m = \frac{L}{f_o}\frac{D}{f_e}$

Note: To achieve large magnification of a small object, the objective and eyepiece of a compound microscope should have small focal lengths.

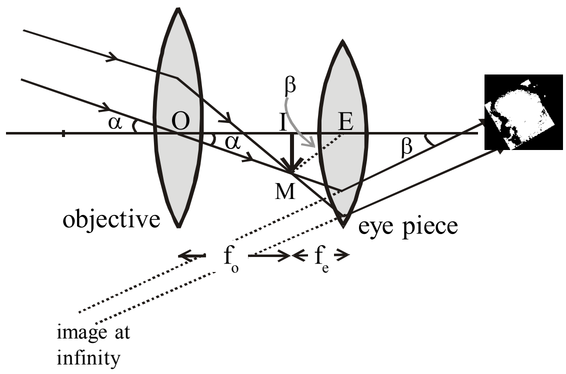
Telescope

The telescope is used to provide angular magnification of distant objects. It also has an objective and an eyepiece. But here, the objective has a large focal length and a much larger aperture than the eyepiece. Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image. The magnifying power *m* is the ratio of the angle b subtended at the eye by the final image to the angle a which the object subtends at the lens.

There are two types of telescopes: Refracting and Reflecting Telescope.

Refracting Telescope

The objective lens forms the image (IM) of a distant object at its focus .This image (formed by objective) is adjusted to be focus of the eyepiece. The magnifying power of a telescope is the ratio of the angle subtended by the image at the eye to the angle subtended by the object at the objective.



$$m = \frac{\text{angle subtented by the image at eye (eye piece)}}{\text{angle subtended by the object at the objective}}$$

i.e. $m = \frac{\beta}{\alpha}$ -----(1) [from figure]

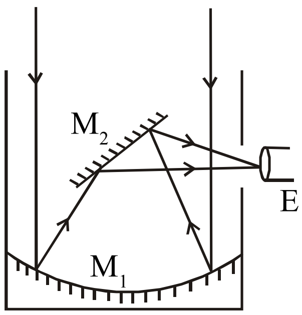
For small values of θ, $\tan\theta \approx \theta$

$$m \approx \frac{\beta}{\alpha} \approx \frac{IM}{IE} \Big/ \frac{IM}{IO}$$

$$m = \frac{IM}{IE} \cdot \frac{IO}{IM}$$

$$\therefore \quad m = \frac{f_o}{f_e}$$

Reflecting Telescope:- Newtonian type reflecting Telescope:-



The Newtonian reflector consists of a parabolic mirror made of an alloy of copper and tin. It is fixed at one end of a metal tube. The parallel rays from a distant stars incident on the mirror M_1 . After reflection from the mirror, the ray incident on a plane mirror M_2. The reflected ray from M_2 enter into eye piece E.

The eyepiece forms a magnified, virtual and erect image.Magnifying power of Newton Telescope $m = \frac{f_o}{f_e}$

where f_o is the focal length of concave mirror f_e is the focal length of eyepiece.

The eye

Our eye consists of a convex lens. It focusses the images on to our retina. The shape and focal length of the lens can be modified with the help of ciliary muscles. When the miscle is relaxed, the focal length is about 2.5 cm and the objects at infinity will be focussed on to the retina. When the object is brought close to the eye, to maintain the same focal length, the eye lens will become shorter by the action of ciliary muscles. This property of the eye is called *accomodation*.

The commonly seen eye defects are:

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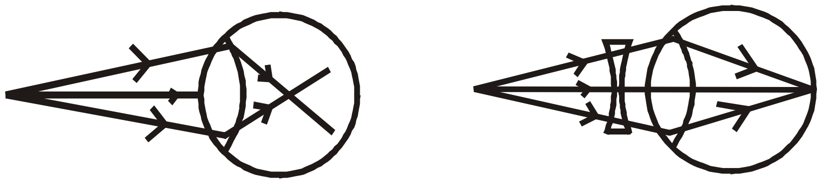
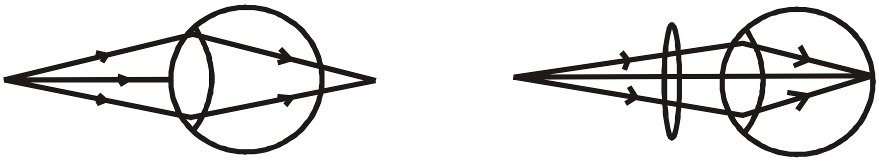
Myopia (Near sightedness or short sight)

Here the light from a distant object is converged by the eye at a point in front of the retina. It can be corrected by using a concave lens of suitable focal length.



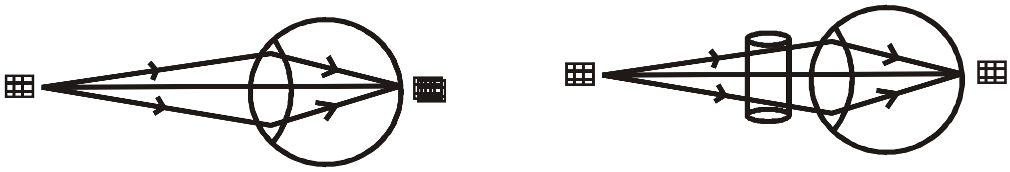
Hypermetropia (Far sightedness or long sight)

Here the light is focussed at a point behind the retina. It can be corrected using a convex lens of suitable focal length.



Astigmatism

This occurs when the cornea is not spherical in shape. It can be corrected using a cylindrical lens of desired radius of curvature with a properly directed axis.

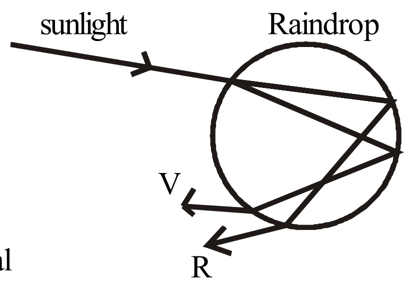


Presbyopia

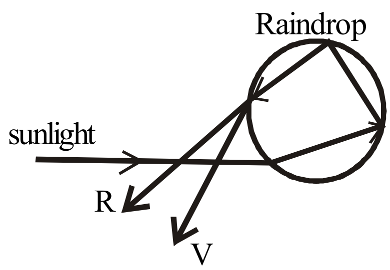
The closest distance for which the lens can focus light on the retina is called the least distance of distinct vision or the near point. It is taken as 25 cm represented by the symbol D. This least distance increases with age and if an elderly person tries to read a book about 25 cm from the eye, the image appears blurred. This defect of the eye is called presbyopia. It can be corrected by using a convex lens.

Formation of Rainbow

Rainbow is a phenomenon due to combined effect of dispersion, refraction and reflection of sunlight by water droplets of rain. The sunlight is frist refracted as it enters into rain drop. The colours are separated, violet bending the most and red the least. The refracted colors are totally reflected from the inner surface of the drop. These rays are further refrated and come out of the drop. The violet light emerges at an angle of 40^0 and red at 42^0 with the initial direction of sunlight. Thus red appears at the top and violet at bottom.



Secondary rainbow: This is formed, when light undergoes two total internal reflections inside the raindrop. Here violet emerges at 53^0 and red at 50^0. Hence violet comes at the top and red at bottom.



Q17. A simple microscope of focal length 5 cm forms the image of an object at the near point of eye. Find the magnification produced by the microscope. [6]

Q18. A compound microscope has an objective of focal length 1 cm and an eye piece of focal length 5 cm. The length of the compound microscpoe tube is 20 cm. Find the objects distance from the objective so that the final image is formed at the near point. [-1.1 cm]

Q19. The objective and eyepiece of a refracting telescope are separated by a distance of 20 cm. Its magnifying power in normal adjustment is 10. Find the focal length of eyepiece and objective. [1.82 cm ; 18.18 cm]

Q20. What focal length should the reading spectacles have for a person for whom the least distance of distinct vision is 50 cm? [+ 50cm]

Q21. The far point of a myopic person is 80 cm in front of the eye. What is the power of the lens required to enable him to see very distant objects clearly? [1.25 D]

WAVE OPTICS

Wave Theory of light

In 1678, a Dutch Physicist and astronomer, Christian Huygens put forward the wave theory of light. According to this theory, *light is propagated in the form of waves through an all pervading hypothetical medium called ether. These waves carry energy and produce the sensation of vision on falling on the eye*. The phenomenon like interference, diffraction and polarisation are well explained using this concept.

Wave front

According to Huygenes theory, light travels in the form of waves. During the wave propagation, all particles of the medium which are located at the same distance from the source receive the disturbance simultaneously and vibrate in the same phase.

The wavefront is defined as the locus of all points which have the same phase of vibration.

The common types of wave fronts are:

(i) **Spherical wavefront**: The wavefront near a point source si a spherical wavefront. This is because at a particular instant all the disturbances from a point source of light reach on the surface of a sphere and will be in the same phase of vibration.

(ii) **Cylindrical wavefront**: If a source of light is linear in shape (eg: slit) and is very near, the wavefront is cylindrical. This is because, all the points equidistant from a linear source lie on the surface of a cylinder.

(iii) **Plane wavefront** : If the source of light is at infinity, we will get plane wavefront.

Note: *Any line perpendicular to a wavefront is called ray of light.*

Huygen's Principle

Huygen's Principle states that:

(1) Every point on a given wavefront can be considered as a source of secondary waves, called wavelets.

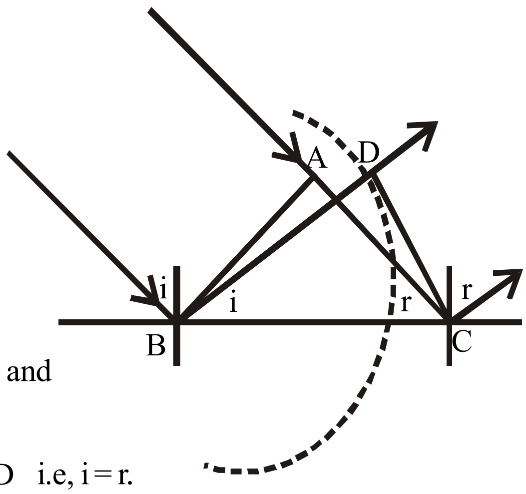
(2) The secondary wavelets spread in all directions with the velocity of light.

(3) The new wavefront at any instant is the envelop of these wavelets in the forward direction.

Reflection of a plane wavefront at a plane surface.

Let XY be a plane reflecting surface and AB be the incident plane wavefront. All the particles on AB will be vibrating in phase. Let **i** be the angle of incidence.

By the time the disturbance at A reaches C, the secondary waves from the point B will travel a distance BD = AC. With the point B as centre and AC as radius, construct a sphere. Draw tangent CD to the sphere. Then CD is the reflected wavefront.



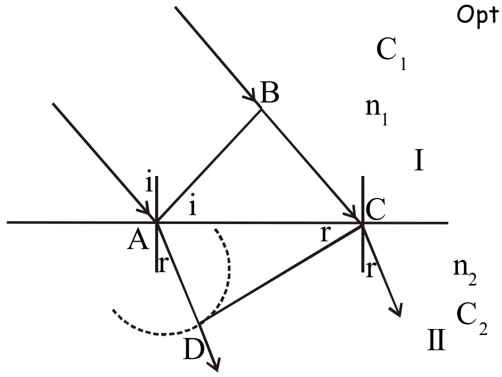
In the triangles BAC and BDC, BC is common. BD = AC and

$\angle BAC = \angle BDC = 90^{O}$

Therefore, the triangles are congruent. $\therefore \quad \angle ABC = \angle BCD$ i.e, i = r.

Thus the angle of incidence is equal to the angle of reflection. Also the incident ray, reflected ray and the normal to the surface at the point of incidence all lie in the same plane. These are the laws of reflection.

Refraction of a plane wavefront at a plane surface.



Let XY be a plane refracting surface separating two media having refractive indices n_1 and n_2 respectively ($n_2 > n_1$). Let C_1 and C_2 be the velocity of light in medium 1 and medium 2. Let AB be an incident plane wavefront. By the time the disturbance at B reaches C, secondary waves from A must have travelled a distance C_2t in medium 2, where t is the time taken by the wave to reach from B to C. Now with A as centre and C_2t as radius draw a sphere. Then draw tangent CD to the sphere. Then CD will give the refracted wavefront.

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Now in right triangle ABC, $\sin i = \frac{BC}{AC}$

From right Δ ADC, $\sin r = \frac{AD}{AC}$

$$\therefore \frac{\sin i}{\sin r} = \frac{BC}{AC} \times \frac{AC}{AD} = \frac{BC}{AD}$$ But $BC = C_1t$ and $AD = C_2t$

$$\therefore \frac{\sin i}{\sin r} = \frac{C_1 t}{C_2 t} = \frac{C_1}{C_2} = n_{21} = \text{constant}$$

i.e $\frac{\sin i}{\sin r} = \text{constant}$. This is snell's law in refraction. Now the incident ray, refracted ray and the normal are in the same plane. Thus both laws of refraction are proved.

Interference of light.

When two light waves of same amplitude, same wavelength and in same phase or with a constant phase difference travel through a medium, the energy distribution doesn't remain uniform in all the directions. At certain points, the crest of one wave falls on the crest of the other. Then the resultant amplitude will be maximum and hence intensity of light will be maximum and such points appear bright. It is called **constructive interference**. At certain other points, the crest of one wave falls on the trough of the other. Therefore, the resultant amplitude becomes minimum (zero) and such points appear dark. It is called **destructive interference**.

The effect produced in a region by the superposition of two or more wave trains passing simultaneously through that region is called interference.

Ordinary light sources cannot produce interference. *Two light sources, which produce stationary interference pattern are said to be mutually* **coherent**.

Two sources emitting light waves of same wavelength and amplitude in the same phase or with constant phase difference are called coherent sources.

Note:

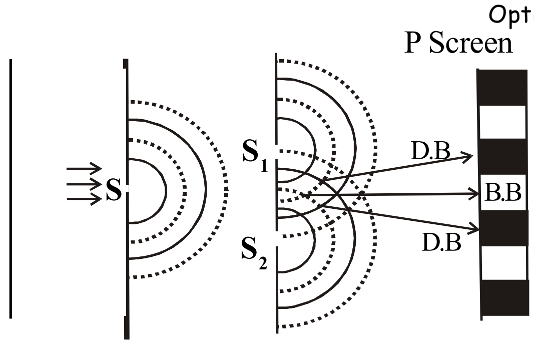
The condition for obtaining brightness (maxima) at a point is that the path difference between the two waves on reaching that point should be $n\lambda$ (even multiple of $\frac{\lambda}{2}$) ; where $n = 0,1,2,3,........$

The condition for obtaining darkness (minima) is that the path difference between the two rays on reaching that point should be $(2n+1)\frac{\lambda}{2}$; $n = 0,1,2,3,........$ (i.e. odd multiple of $\frac{\lambda}{2}$)

Conditions for sustained interference

(i) The sources must be coherent (ii) The coherent sources must be narrow and very close to each other. (iii) The screen must be comparatively at a large distance from the coherent source.

Interference of monochromatic light by double slit - Young's double slit experiment.



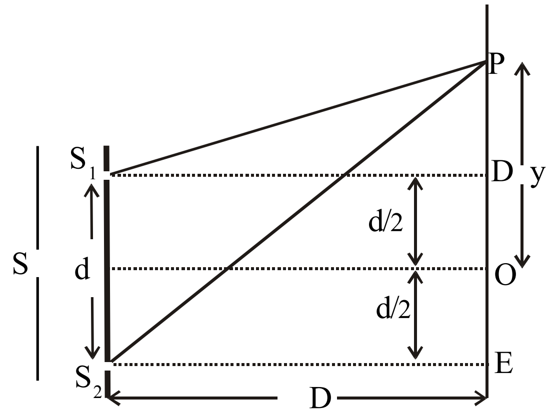
An English scientist Thomas Young in 1802 demonstrated the interference of light. Two identical slits S_1 and S_2 which are parallel and very close to each other are made on an opaque screen. These slits are illuminated by another narrow slit S which in turn is illuminated by a bright source.

Then S_1 and S_2 behave as coherent sources. P is a screen at a certain distance from S_1S_2. Light spreading out from the slits overlap in the region separating the double slit and the screen. As a result, a number of alternate bright and dark bands (fringes) parallel to the length of the slits are observed on the screen.

These bands are called **interference bands**. The bright bands are produced due to constuructive interference and dark bands due to destructive interference.

To find the positions of the dark and bright fringes, let us find the path difference between the light beams from the two slits.

Path difference.



Light from a slit S is made to fall on two slits S_1 and S_2 separated by a distance d. A screen is placed at a distance D from S_1 and S_2. Let P be a point at a distance y from the centre of the screen o. Two rays S_1P and S_2P starting from S_1 and S_2 interfear each other.

Now the path difference between the rays,

$x = S_2P - S_1P$.

From ΔS_1DP, $\quad S_1P^2 = S_1D^2 + PD^2$

$$= D^2 + \left(y - \frac{d}{2}\right)^2 \quad ....(1)$$

Similarly from ΔS_2EP, $\quad S_2P^2 = S_1E^2 + BP^2$

$$= D^2 + \left(y + \frac{d}{2}\right)^2 \quad ....(2)$$

$$S_2P^2 - S_1P^2 = D^2 + \left(y + \frac{d}{2}\right)^2 - \left[D^2 + \left(y - \frac{d}{2}\right)^2\right]$$

$$\text{i.e} \quad (S_2P + S_1P)\,(S_2P - S_1P) = y^2 + yd + \frac{d^2}{4} - y^2 + yd - \frac{d^2}{4}$$

$$\text{i.e.} \quad 2\,D\,x = 2\,y\,d \quad [\Theta\ S_2P \approx S_1P \approx D]$$

$$\therefore \text{ Path difference, } x = \underline{\underline{\frac{yd}{D}}}$$

Maxima

Point P will be a bright spot if the path difference x is an even multiple of $\lambda/2$.

$$\text{i.e. if } \frac{yd}{D} = 2n\frac{\lambda}{2}$$

$$\text{Or } y = \frac{n\lambda D}{d} \ ; \ n = 0,1,2,3,...........$$

Thus bright bands are obtained at distances, $0, \frac{\lambda D}{d}, \frac{2\lambda D}{d}, \frac{3\lambda D}{d}, \ldots\ldots\ldots\ldots$ from the centre O of the screen.

Minima

Point P will be a dark spot if the path difference x is an odd multiple of $\lambda/2$.

$$\text{i.e. if } \frac{y\,d}{D} = (2n+1)\frac{\lambda}{2}$$

$$\text{or } \quad y = (2n+1)\frac{\lambda D}{2d}\ ; \ n = 0,1,2,3,\ldots\ldots$$

Thus dark bands are obtained at distances, $\frac{\lambda D}{2d}, \frac{3\lambda D}{2d}, \frac{5\lambda D}{2d}, \ldots\ldots\ldots\ldots$ from the centre O of the screen.

From the positions of the dark and bright fringes, we can see that, alternate dark and bright bands are formed on the screen.

Fringe width or Band width (β)

Fringe width is the distance between two consecutive bright or dark fringes.

Let x_n and x_{n-1} be the distances of n^{th} and $(n-1)^{th}$ bright fringes respectively from O.

$$\text{Then, } \beta = x_n - x_{n-1} = \frac{n\lambda D}{d} - (n-1)\frac{\lambda D}{d}$$

$$\text{i.e. } \beta = \frac{\lambda D}{d}(n - n + 1) \qquad \text{i.e. } \beta = \frac{\lambda D}{d}$$

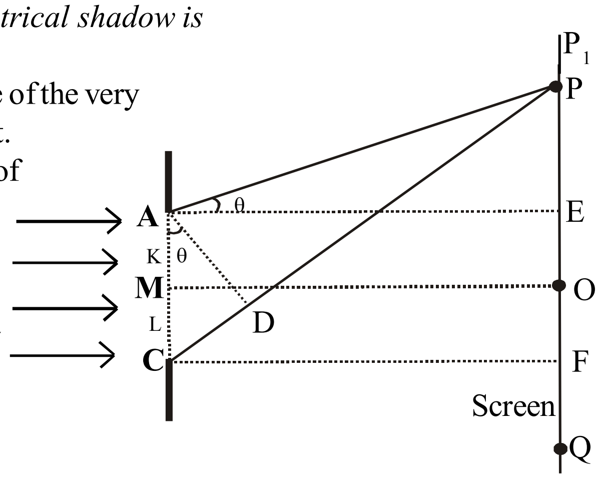
Similarly the distance between two consecutive dark fringes is also equal to $\frac{\lambda D}{d}$. Hence *the bright and dark fringes are equally spaced.*

Q22. Two slits are made 1 mm apart and a screen is placed 1 m away. What is the fringe separation when a light of wavelength 500 nm is used? [5×10^{-4}m]
Q23. In a Young's double slit experiment, the slits are 1.2 mm apart, screen is 1 m away from the slits. How many fringes can be observed on the screen in a space of 1 cm if the wavelength of light used is 546 nm.[22]
Q24. Two coherent sources of light of wavelength 6000A0 are placed at a certain distance from a screen. The band width of the interference bands formed on the screen is 0.020 cm. When the distance of the screen from the sources is increased by 20 cm, the band width becomes 0.024 cm. Calculate the distance between the two coherent sources. [3 mm]

Diffraction

The phenomenon of bending of light around the edges of an opaque obstacle or the encroachment of light into the geometrical shadow is called diffraction.

Diffraction of light is not commonly observed because of the very large size of obstacle compared to the wavelength of light. Diffraction of sound waves is easily observable because of comparatively long wavelength of sound waves.



Diffraction at a single slit.

Consider a parallel beam of light incident normally on a narrow slit AB of width a. A screen is placed at a suitable distance from the slit. Let M be the midpoint of the slit.

If light travels in straight lines, there would be uniform illumination in the region EF and complete darkness in the regions above E and below F. But actually this is not the case.

Consider a point P on the screen making an angle θ with the normal MO. The path difference between the rays leaving from the bottom and top of the slit, towards P is given by

BP - AP = BN.

But BN = AB sinθ = a sinθ . i.e. path difference = a sin$\theta \approx$ aθ ; the angle being small.

Similarly the path difference between the rays proceeding from any two points K and L of the slit, towards point P is LP - KP = y sinθ , where y = KL. i.e, path difference = yθ.

Case (1) At central point O on the screen, $\theta = 0$. The path difference between rays from points equidistant from the centre of the slits is zero, on reaching the point O. Hence all the rays reinforce each other at O. This gives maximum intensity to O. This is called **central maximum** or **principal maximum**.

Case (2) Let the point P be such that the path difference between the rays BP and AP on reaching P be λ.

i. e BP - AP = λ .

i.e. $a\theta = \lambda$. Or $\theta = \frac{\lambda}{a}$

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Now let the slit be imagined to be split into two equal halves AM and MB each of width a/2. For every point K on the upper half, there is a corresponding point L on the lower half such that KL = a/2. The path difference between the rays proceeding from K and L on reaching P, is approximately $\frac{a}{2}\theta = \frac{a}{2}\frac{\lambda}{a} = \frac{\lambda}{2}$.

This means that the rays from K and L on reaching P are out of phase and hence annul each other.

Hence intensity at P is zero or point P appears dark. i.e at an angle of diffraction, $\theta = \frac{n\lambda}{a}$; the intensity falls to zero; where n = 1,2,3,................

Similarly on lower half of the screen, the intensity is zero at points for which $\theta = -\frac{n\lambda}{a}$

So condition for zero intensity is given by $\theta = \pm\frac{n\lambda}{a}$ or $\sin\theta = \pm\frac{n\lambda}{a}$; n = 1,2,3,4,.........

Thus corresponding to n = 1,2,3,..... the first, second, third,............... minima are formed on either side of the central maximum.

Case (3) Let P be a point on the screen such that BP - AP = $\frac{3}{2}\lambda$. i.e. $a\theta = \frac{3}{2}\lambda$ Or $\theta = \frac{3}{2}\frac{\lambda}{a}$

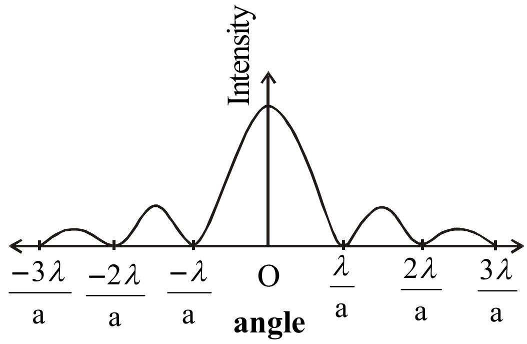
This angle is midway between the angles of diffraction corresponding to the first minimum ($\theta = \frac{\lambda}{a}$) and second minimum ($\theta = 2\frac{\lambda}{a}$). The slit can now be imagined to be made up of three parts of equal width. The rays from corresponding points on any two adjacent parts on reaching P differ in path by $\frac{\lambda}{2}$ and hence annul each other. Hence the resultant intensity at P is only due to the light coming from the remaining part of the slit. Hence this point though bright, appears less intense compared to the central maximum O. Similarly still weaker maxima are formed corresponding to $\theta = \frac{5}{2}\frac{\lambda}{a}, \frac{7}{2}\frac{\lambda}{a}$,...........

In general, condition for secondary maxima is $\theta = \pm\left(n + \frac{1}{2}\right)\frac{\lambda}{a}$ or $\sin\theta = \pm\left(n + \frac{1}{2}\right)\frac{\lambda}{a}$;

n = 1,2,3,4,.....

Thus between two consecutive minima, a secondary maximum is formed.

The variation of intensity on the screen with angle is as shown.



Interference and diffraction - a comparison

(i) Interference is due to superposition of waves coming from two wavefronts. Diffraction is due to the superposition of waves coming from different parts of the same wavefront.

(ii) Interference bands are of equal width. Diffraction bands are of un equal width.

(iii) Minimum intensity regions are perfectly dark. Minimum intensity regions are not perfectly dark.

(iv) All the bright bands are of equal intensity. All bright bands are not of same intensity.

Polarisation

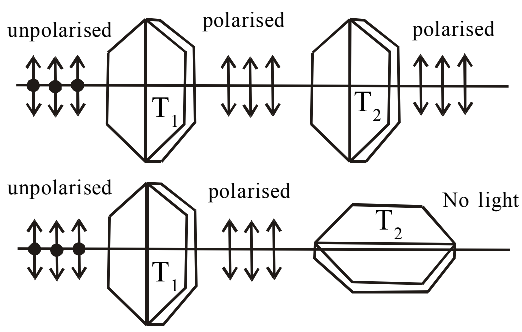
According to electromagnetic theory of light, is the propagation of mutually perpendicular vibrating electric and magnetic fields. The electric field functions as light vector. When light is passed through certain crystals like tourmaline, the vibrations of electric field vector are restricted. This property exhibited by light is known as **polarisation**.

Light having electric field vector vibrations confined to a single plane and in a particular direction is known as linearly polarised or plane polarised light.

Polarisation is exhibited by transverse waves only. Thus polarisation proves the light is transverse in nature.

Demonstration

Polarisation can be demonstrated using a tourmaline crystal.

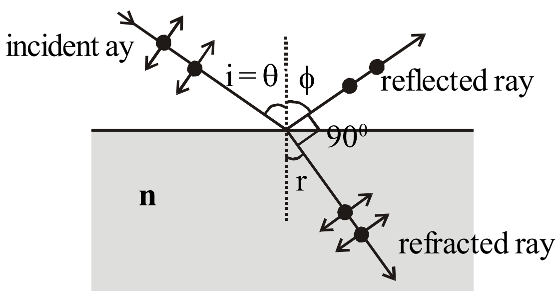


Light from a source is allowed to pass through a tourmaline crystal. The vibrations parallel to the optical axis will pass through it and other vibrations are absorbed by the crystal. The crystal is called **polarisor** or **polaroid,** since the emergent light is polarised. To test whether the emergent light is polarised or not, a second tourmaline crystal is used. Keeping the first crystal fixed, the second one is rotated about the incident ray as axis.

Then it is found that the intensity of emergent light from second crystal varies between maximum and minimum (zero). The intensity of transmitted light i maximum when the polarising directions are parallel and intensity falls to zero when the polarising directions become perpendicular to each other.

The light coming out of first polaroid has acquired a property which the incident light did not have (i.e. the intensity is reduced to half). This property is called polarisation of light. The second crystal is called **analyser or detector**. *The plane in which the vibrations of electric field vector are confined is known as plane of vibration and the plane perpendicular to the plane of vibration is known as plane of polarisation.*

Polarisation by reflection



Malus in 1808 found that when light gets reflected from a transparent medium, the reflected light is partially polarised. At a particular angle of incidence, the reflected light is fully polarised. This particular angle is known as **polarising angle** or **angle of polarisation** or **Brewster angle (θ)** for that medium. *When light is incident at polarising angle, the reflected and refracted rays are mutually perpendicular.*

i.e. $r + \theta = 90^0$; r = angle of refraction and θ = polarising angle.

Brewster's law

Brewster's law states that "*the tangent of the polarising angle is equal to the refractive index of the medium on which light is incident*"

If θ is the polarising angle and n is the refractive index of the medium, then $\mathbf{\tan\theta = n}$. This is Brewster's law.

Note: The polarised light has vibrations perpendicular to the plane of incidence.

For glass, n = 1.5, therefore, $\theta = 57^0$ (approximately).

Uses of plane polarised light

(i) To project stereoscopic pictures on a screen. (ii) If polarised light is used in optical instruments, it becomes polarised instrument. (iii) It is used in liquid crystal display (LCD). (iv) Sun glasses.

Polaroid

Polaroid is a polariser in the form of a large film (sheet). When unpolarised light is passes through a polaroid, we get polarised light.

Uses: (i) In polarising optical instruments. (ii) In order to improve colour contrast in old oil paintings, polaroids are used. (iii) Used to produce and view 3D films. (iv) In aeroplanes, polaroid glasses are used to control light coming in. (v) In sun glasses (vi) In wind glass of vehicles to avoid glare.

Diffraction and Optical Instruments

The objective lens of optical instruments like telescope or microscope etc. acts like a circular aperture. When light falls on it, a diffraction pattern consisting of circular rings is obtained.

The minimum radius of the image disc is

$r = 1.22 \dfrac{\lambda}{2a} f$ where , f = focal length of lens **,** 2a is the diameter of the lens (aperture), λ = wave length of light used

Resolving power

Diffraction limits the ability of optical instruments to form clear images when they are close to each other. The minimum distance of separation between two points so that they can be seen as separate (say *just resolved*) by the optical instrument is known as its limit of resolution.

The ability of an optical instrument to form distinctly separate images of the two closely placed points is called its resolving power. Resolving power is also defined as reciprocal of limit of resolution. i.e., *smaller is the limit of resolution of an optical instrument, larger is its resolving power.*

According to Rayleigh, two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation must then be at least

$\theta_R = 1.22 \dfrac{\lambda}{d}$ (Rayleigh cirtirian) where d is the diameter of aperture.

Solved Problems

1. What is the angular resolution of a 10cm diameter telescope at a wavelength of 0.5mm?

Answer: Given $= 0.5\times10^{-6}$m, a = 10 cm = 0.1m. The angular resolution is given by

$$\Delta\theta = \frac{1.22\lambda}{a} = \frac{1.22 \times 0.5 \times 10^{-6}}{0.1}$$

$= 6\times10^{-6}$rad $= 1.2$□□

2. Two coherent source of intensity ratio 81:64 interfere. Deduce the ratio of intensity between the maxima and minima in the interference pattern.

Answer: Let a_1 and a_2 be the amplitudes of the two interfering waves.

$I_1 : I_2 = 81.64$

$a_1^2 : a_2^2 = 81:64$ or $a_1 : a_2 = 9.8$

$$= \frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = [\frac{9+8}{9-8}]^2 = \frac{17^2}{1} = \frac{289}{1}$$ Imax : I max = 289.1