CHAPTER 7

ALTERNATING CURRENT

AC Voltage and AC Current

- A voltage that varies like a sine function with time is called *alternating voltage (ac voltage).*
- The electric current whose magnitude changes with time and direction reverses periodically is called the *alternating current (ac* current).

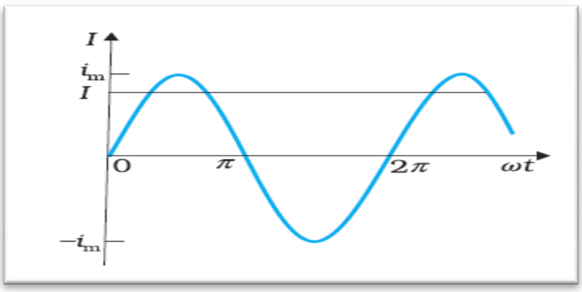
Advantages of AC:

- Easily stepped up or stepped down using transformer
- Can be regulated using choke coil without loss of energy
- Easily converted in to dc using rectifier (Pn - diode)
- Can be transmitted over distant places
- Production of ac is more economical

Disadvantages of ac

- Cannot used for electroplating - Polarity of ac changes
- ac is more dangerous
- It can't store for longer time

Representation of ac



- An ac voltage can be represented as

$$\boxed{v = v_m \sin \omega t}$$

- **v- instantaneous value of voltage , v_m- peak value of voltage,** ω - Angular frequency.

RMS Value (*effective current)*

- **r.m.s**. value of a.c. is the d.c. equivalent which produces the same amount of heat energy in same time as that of an a.c.
- It is denoted by ***I_{rms}* or *I*.**
- Relation between r.m.s. value and peak value is

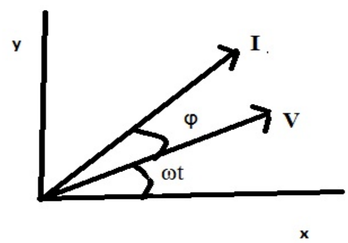
$$I_{rms} = \frac{i_m}{\sqrt{2}}$$

- The *r.m.s* voltage is given by

$$V_{rms} = \frac{v_m}{\sqrt{2}}$$

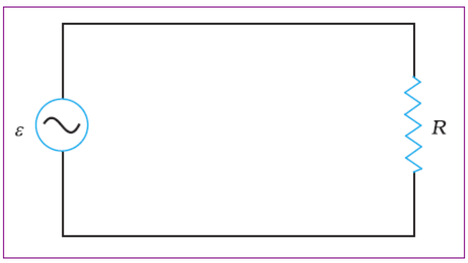
Phasors

- A phasor is a vector which rotates about the origin with angular speed ω.
- The vertical components of phasors **V** and **I** represent the sinusoidally varying quantities v and i.
- The magnitudes of phasors **V** and **I** represent the peak values v_m and i_m



- The diagram representing alternating voltage and current (phasors) as the rotating vectors along with the phase angle between them is called **phasor diagram.**

AC Voltage applied to a Resistor



- The ac voltage applied to the resistor is

$$\boxed{v = v_m \sin \omega t}$$

- Applying Kirchhoff's loop rule

$$v_m \sin \omega t = i R$$

$$i = \frac{v_m}{R} \sin \omega t$$

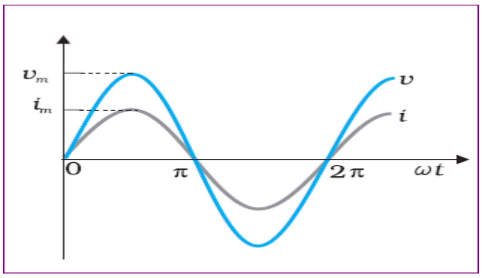
- Since R is a constant, we can write this equation as

$$\boxed{i = i_m \sin \omega t}$$

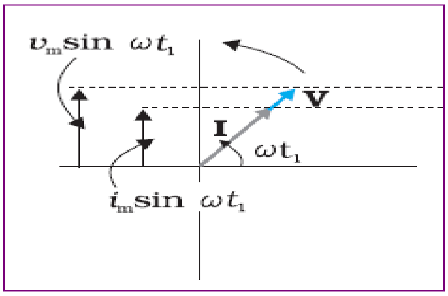
- Where peak value of current is

$$\boxed{i_m = \frac{v_m}{R}}$$

- Thus when ac is passed through a resistor *the **voltage and current are in phase*** with each other.



<u>Phasor diagram</u>



<u>Instantaneous power</u>

- **The instantaneous power dissipated in the resistor is**

$$\boxed{p = i^2 R = i_m^2 R \sin^2 \omega t}$$

<u>Average power</u>

- The average value of *p over a cycle is*

$$\boxed{\bar{p} = \langle i^2 R \rangle = \langle i_m^2 R \sin^2 \omega t \rangle}$$

or

$$\boxed{\bar{p} = i_m^2 R \langle \sin^2 \omega t \rangle}$$

- Using the trigonometric identity,

$\sin^2 \omega t = 1/2\ (1 - \cos 2\omega t)$

$\langle \sin^2 \omega t \rangle = (1/2)\ (1 - \langle \cos 2\omega t \rangle)$

- Since $\langle \cos 2\omega t \rangle = 0$

$$\boxed{\langle \sin^2 \omega t \rangle = \frac{1}{2}}$$

- Thus

$$\boxed{\bar{p} = \frac{1}{2} i_m^2 R}$$

- In terms of r.m.s value

$$\boxed{P = \bar{p} = \frac{1}{2} i_m^2 R = I^2 R}$$

- Or

$$\boxed{P = V^2 / R = I V \quad (\text{since } V = I R)}$$

<u>AC VOLTAGE APPLIED TO AN INDUCTOR</u>

- Let the voltage across the source be

$$\boxed{v = v_m \sin \omega t}$$

- Using the Kirchhoff's loop rule

$$\boxed{v - L \frac{di}{dt} = 0}$$

- *Where L is the self-inductance*
- *Thus*

$$\boxed{\frac{di}{dt} = \frac{v}{L} = \frac{v_m}{L} \sin \omega t}$$

- Integrating

$$\boxed{\int \frac{di}{dt} dt = \frac{v_m}{L} \int \sin(\omega t) dt}$$

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$$i = -\frac{v_m}{\omega L}\cos(\omega t) + \text{constant}$$

- Since the current is oscillating , the constant of integration is zero.
- Using

$$-\cos(\omega t) = \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

- Where

$$i_m = \frac{v_m}{\omega L}$$

- Or

$$i_m = \frac{v_m}{X_L}$$

- Where X_L- inductive reactance

Inductive reactance (X_L)

- The resistance offered by the inductor to an ac through it is called inductive reactance.
- It is given by

$$X_L = \omega L$$

- The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω).

- The inductive reactance is directly proportional to the inductance and to the frequency of the current.

Phasor Diagram

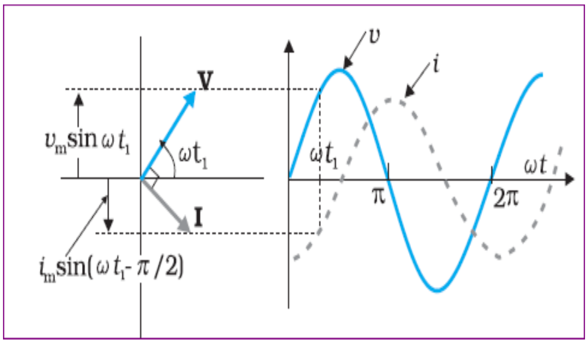
- We have the source voltage

$$v = v_m \sin\omega t$$

- The current

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

- Thus a comparison of equations for the source voltage and the current in an inductor shows that the **current lags the voltage by π/2** or one-quarter (1/4) cycle.



Instantaneous power

- The instantaneous power supplied to the inductor is

$$p_L = i v = i_m \sin\left(\omega t - \frac{\pi}{2}\right) \times v_m \sin(\omega t)$$
$$= -i_m v_m \cos(\omega t)\sin(\omega t)$$
$$= -\frac{i_m v_m}{2}\sin(2\omega t)$$

Average power

- The average power over a complete cycle in an inductor is

$$P_L = \left\langle -\frac{i_m v_m}{2}\sin(2\omega t)\right\rangle$$
$$= -\frac{i_m v_m}{2}\langle \sin(2\omega t)\rangle = 0,$$

- since the average of sin (2ωt) over a complete cycle is zero.

- Thus, the ***average power supplied to an inductor over one complete cycle is zero.***

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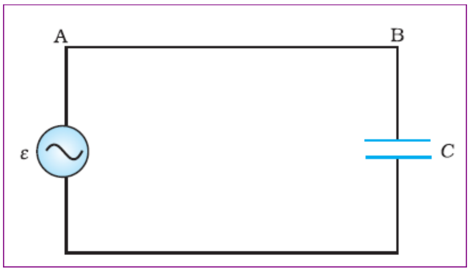
AC VOLTAGE APPLIED TO A CAPACITOR



- A capacitor in a dc circuit will limit or oppose the current as it charges.
- When the capacitor is connected to an ac source, it limits or regulates the current, but does not completely prevent the flow of charge.
- Let the applied voltage be

$$v = v_m \sin\omega t$$

- *The instantaneous voltage v across* the capacitor is

$$v = \frac{q}{C}$$

- Where q is the charge on the capacitor.
- Using the Kirchhoff's loop rule

$$v_m \sin \omega t = \frac{q}{C}$$

- Therefore

$$i = \frac{d}{dt}(v_m C \sin \omega t) = \omega C v_m \cos(\omega t)$$

- Using the relation

$$\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

- Where

$$i_m = \frac{v_m}{(1/\omega C)}$$

- Or

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$$i_m = \frac{v_m}{X_C}$$

- Where X_C – capacitive reactance

Capacitive Reactance

- It is the resistance offered by the capacitor to an ac current through it.
- The dimension of capacitive reactance is the same as that of resistance and its SI unit is ohm (Ω).

Phasor Diagram

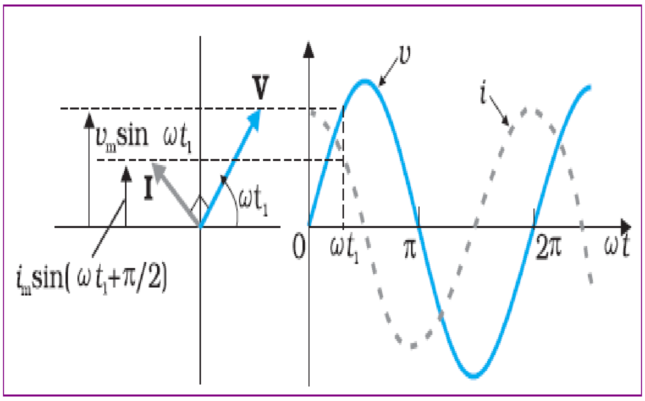
- The applied voltage is

$$v = v_m \sin\omega t$$

- The current is

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

- Thus the **current leads voltage by π/2.**



Instantaneous power

- The instantaneous power supplied to the capacitor is

$$p_c = i\,v = i_m \cos(\omega t) v_m \sin(\omega t)$$
$$= i_m v_m \cos(\omega t) \sin(\omega t)$$
$$= \frac{i_m v_m}{2} \sin(2\omega t)$$

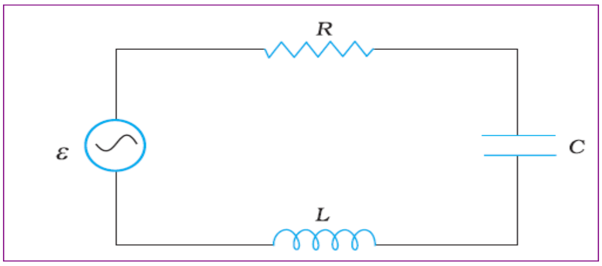
Average power

- The average power is given by

$$P_C = \left\langle \frac{i_m v_m}{2} \sin(2\omega t) \right\rangle = \frac{i_m v_m}{2} \langle \sin(2\omega t) \rangle = 0$$

- Thus **the average power over a cycle when an ac passed through a capacitor is zero**.

AC VOLTAGE APPLIED TO A SERIES LCR CIRCUIT



- Let the voltage of the source to be

v = v

$$v = v_m \sin \omega t$$

- *From Kirchhoff's loop* rule:

$$L\frac{di}{dt} + iR + \frac{q}{C} = v$$

- Where , q - the charge on the capacitor
  i – current
- Using, i=dq/dt

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = v_m \sin \omega t$$

Phasor-diagram solution

- Since the current through resistor, inductor and capacitor is the same as they are in series.
- If ϕ is the phase difference between the voltage across the source and the current in the circuit,

$$i = i_m \sin(\omega t + \phi)$$

- Let **I be the phasor representing the current in the circuit and VL, VR, VC, and V represent the voltage across the** inductor, resistor, capacitor and the source, respectively.
- The phasor diagram is

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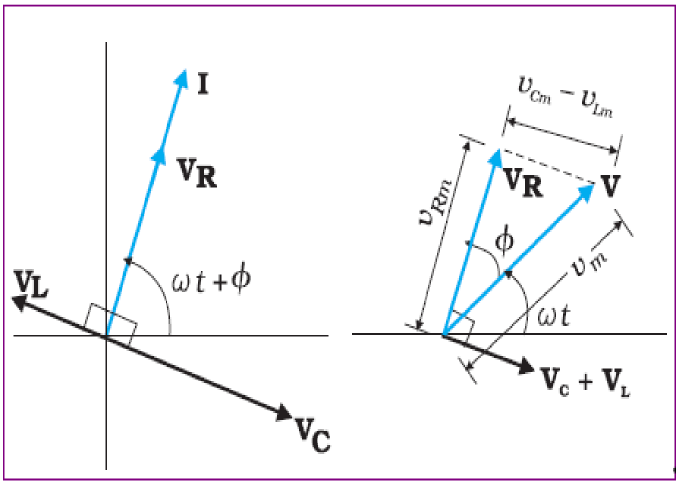


- The length of these phasors **V_R, V_C and V_L are:**

$$v_{Rm} = i_m R,\ v_{Cm} = i_m X_C,\ v_{Lm} = i_m X_L$$

- Also

$$\mathbf{V_L + V_R + V_C = V}$$

- Since **V_C and V_L are always along the same line and in** opposite directions, they can be combined into a single phasor (**VC + VL) which has a magnitude $|v_{Cm} - v_{Lm}|$**
- Since **V is represented as the** hypotenuse of a right-traingle whose sides are **V_R and ($V_C + V_L$),** the pythagorean theorem gives:

$$v_m^2 = v_{Rm}^2 + \left(v_{Cm} - v_{Lm}\right)^2$$

- Thus

$$v_m^2 = (i_m R)^2 + (i_m X_C - i_m X_L)^2$$
$$= i_m^2 \left[R^2 + (X_C - X_L)^2\right]$$

- Therefore

$$i_m = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

- Or

$$i_m = \frac{v_m}{Z}$$

$$\text{where } Z = \sqrt{R^2 + (X_C - X_L)^2}$$

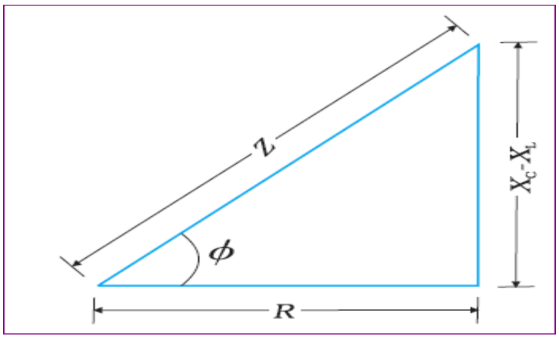
- Z is called ***impedance .***

Impedance

- It is the effective resistance offered by the inductor, capacitor and resistor in an LCR circuit.
- Impedance is given by

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

Impedance Triangle



- Since phasor **I is always parallel to phasor V_R, the phase angle ϕ** is the angle between **V_R and V.**

- **Thus**

$$\tan\phi = \frac{v_{Cm} - v_{Lm}}{v_{Rm}}$$

- Or

$$\tan\phi = \frac{X_C - X_L}{R}$$

- Thus

$$\phi = \tan^{-1}\frac{X_C - X_L}{R}$$

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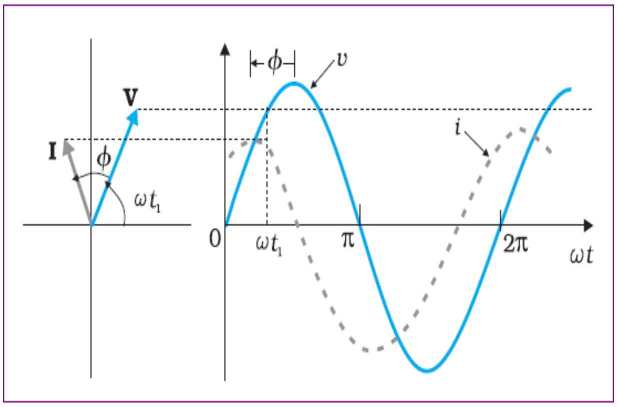
- If $X_C > X_L$, ϕ is positive and the circuit is predominantly capacitive.

- If $X_C < X_L$, ϕ is negative and the circuit is predominantly inductive.



L C R RESONANCE

- For an LCR circuit the impedance

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$i_m = \frac{v_m}{Z} = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

- If **$X_C = X_L$**, then **Z = R,** the impedance is minimum and the current in the circuit is maximum – **LCR Resonance**
- A series LCR circuit, which admits maximum current corresponding to a particular angular frequency of the ac source is called a **series resonant circuit**.
- Resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit.

Resonant frequency

- The angular frequency at which the current is maximum in an LCR circuit is called **resonant frequency (ω_0).**
- **That is**

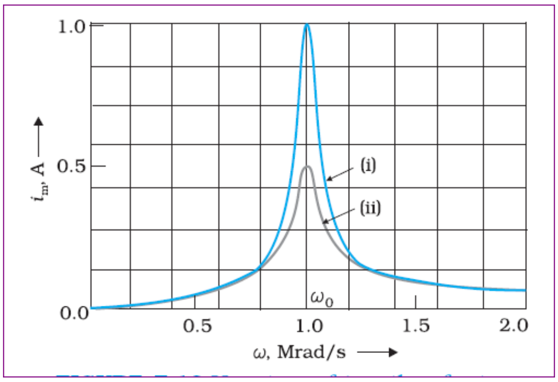
$$X_c = X_L \text{ or } \frac{1}{\omega_0 C} = \omega_0 L$$

$$\text{or} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

- At resonant frequency**,** the maximum current is given by

$$\boxed{i_m^{\max} = v_m / R\cdot}$$

<u>Graph showing variation of i_m with ω</u>



<u>Applications of resonance</u>

- In the tuning mechanism of a radio or a TV set
- In metal detectors.

<u>Tuning of radio or TV</u>

- In tuning, the capacitance of a capacitor in the tuning circuit is varied such that the resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal received.
- At this frequency , the amplitude of the current with the frequency of the signal of the particular radio station in the circuit is maximum.

<u>Sharpness of resonance</u>

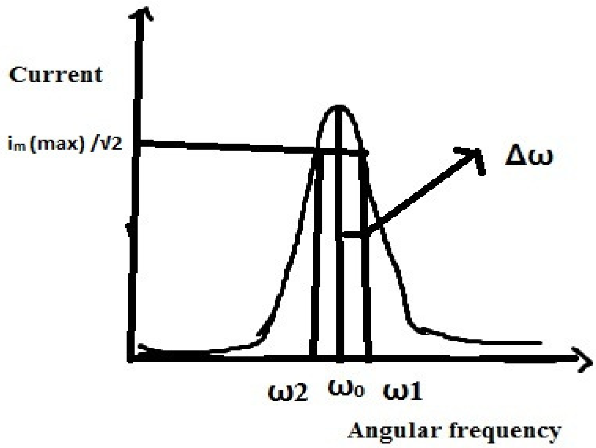
- The amplitude of the current in the series *LCR circuit is given by*

$$\boxed{i_m = \frac{v_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}}$$

- The maximum value is

$$\boxed{i_m^{\max} = v_m / R\cdot}$$

- We choose a value of ω for which the current amplitude is 1/ √2 times its maximum value.



- At this value, the power dissipated by the circuit becomes half.
- Thus

$$\boxed{\begin{aligned}\omega_1 &= \omega_0 + \Delta\omega \\ \omega_2 &= \omega_0 - \Delta\omega\end{aligned}}$$

- The difference **$\omega_1 - \omega_2 = 2\Delta\omega$** is called the ***<u>bandwidth</u>*** of the circuit.
- The quantity **(ω_0 / 2Δω)** is regarded as a measure of **the sharpness** of resonance.
- The smaller the Δω, the sharper or narrower is the resonance.

$$\text{at } \omega_1, \quad i_m = \frac{v_m}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}}$$

$$= \frac{i_m^{\max}}{\sqrt{2}} = \frac{v_m}{R\sqrt{2}}$$

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$$\text{or} \quad \sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} = R\sqrt{2}$$

$$\text{or} \quad R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = 2R^2$$

$$\omega_1 L - \frac{1}{\omega_1 C} = R$$

which may be written as,

$$(\omega_0 + \Delta\omega) L - \frac{1}{(\omega_0 + \Delta\omega) C} = R$$

$$\omega_0 L \left(1 + \frac{\Delta\omega}{\omega_0}\right) - \frac{1}{\omega_0 C \left(1 + \frac{\Delta\omega}{\omega_0}\right)} = R$$

- Using

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 L \left(1 + \frac{\Delta\omega}{\omega_0}\right) - \frac{\omega_0 L}{\left(1 + \frac{\Delta\omega}{\omega_0}\right)} = R$$

We can approximate $\left(1 + \frac{\Delta\omega}{\omega_0}\right)^{-1}$ as $\left(1 - \frac{\Delta\omega}{\omega_0}\right)$ since $\frac{\Delta\omega}{\omega_0} << 1$

$$\omega_0 L \left(1 + \frac{\Delta\omega}{\omega_0}\right) - \omega_0 L \left(1 - \frac{\Delta\omega}{\omega_0}\right) = R$$

$$\text{or} \quad \omega_0 L \frac{2\Delta\omega}{\omega_0} = R$$

$$\Delta\omega = \frac{R}{2L}$$

The sharpness of resonance is given by,

$$\frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$$

The ratio $\frac{\omega_0 L}{R}$ is also called the *quality factor*, Q of the circuit.

$$Q = \frac{\omega_0 L}{R}$$

- Thus

$$2\Delta\omega = \frac{\omega_0}{Q}$$

- So, larger the value of Q, the smaller is the value of 2Δω or the bandwidth and sharper is the resonance.
- Also

$$Q = 1/\omega_0 CR.$$

- If the resonance is less sharp, not only is the maximum current less, the circuit is close to resonance for a larger range Δω of frequencies and the tuning of the circuit will not be good.
- So, less sharp the resonance, less is the **selectivity of the** circuit or vice versa.

<u>POWER IN AC CIRCUIT: THE POWER FACTOR</u>

- We have

$$i = i_m \sin(\omega t + \phi)$$

- Where

$$i_m = \frac{v_m}{Z} \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

- Therefore, the instantaneous power *p supplied by the source is*

$$p = v\,i = (v_m \sin\omega t) \times [i_m \sin(\omega t + \phi)]$$

$$= \frac{v_m i_m}{2}[\cos\phi - \cos(2\omega t + \phi)]$$

- Therefore

$$P = \frac{v_m i_m}{2}\cos\phi = \frac{v_m}{\sqrt{2}}\frac{i_m}{\sqrt{2}}\cos\phi$$

$$= V\,I\cos\phi$$

This can also be written as,

$$P = I^2 Z\cos\phi$$

- The quantity cosϕ is called the ***power factor.***

<u>Special cases</u>

<u>Resistive circuit:</u>

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- If the circuit contains only pure *R*, it is called *resistive*. In that case ϕ= 0, cosϕ = 1. There is maximum power dissipation.

Purely inductive or capacitive circuit:

- If the circuit contains only an inductor or capacitor, the phase difference between voltage and current is $\pi/2$.
- Therefore, cos ϕ= 0, and no power is dissipated even though a current is flowing in the circuit. This current is sometimes referred to as **wattless current.**

LCR series circuit:

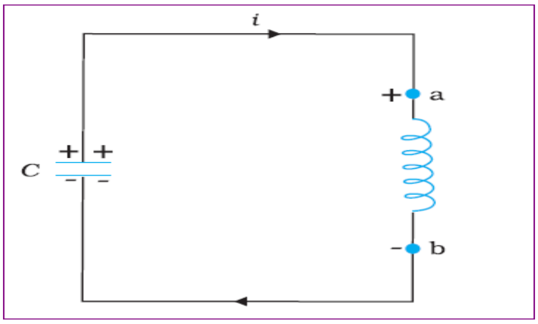
- In an *LCR* series circuit the power is dissipated only in the resistor.

Power dissipated at resonance in LCR circuit:

- At resonance $X_c - X_L$= 0, and ϕ = 0. Therefore, cos ϕ = 1 and $P = I^2Z = I^2 R$.
- That is, maximum power is dissipated in a circuit (through R) at resonance.

LC OSCILLATIONS

- When a capacitor (initially charged) is connected to an inductor, the charge on the capacitor and the current in the circuit exhibit the phenomenon of electrical oscillations similar to oscillations in mechanical systems.



- According to Kirchhoff's loop rule,

$$\frac{q}{C} - L\frac{di}{dt} = 0$$

- But $i = -(dq/dt)$
- Therefore

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

- This equation has the formfor a simple harmonic oscillator

$$\frac{d^2x}{dt^2} + \omega_0^2\, x = 0$$

- The charge, therefore, oscillates with a natural frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- The charge varies sinusoidally with time as

$$q = q_m \cos\left(\omega_0 t + \phi\right)$$

- Where q_m is the maximum value of q and ϕ is a phase constant.
- When ϕ = 0

$$q = q_m \cos(\omega_0 t)$$

- The current is

$$i = i_m \sin(\omega_0 t)$$

$$\text{where } \; i_m = \omega_0 q_m$$

- The LC oscillation is similar to the mechanical oscillation of a block attached to a spring.

Comparison between an electrical system and a mechanical system

Mechanical system	Slectrical system
Mass m	Inductance L
Force constant k	Reciprocal capacitance $1/C$
Displacement x	Charge q
Velocity $v = dx/dt$	Current $i = dq/dt$
Mechanical energy	Electromagnetic energy
$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$	$U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2$

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