CHAPTER 6

ELECTROMAGNETIC INDUCTION

INTRODUCTION

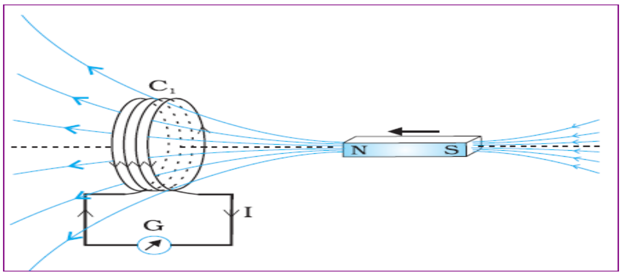
- experiments on electric current by Oersted, Ampere etc established the interrelationship between electricity and magnetism.
- The experiments of Michael Faraday in England and Joseph Henry in USA, demonstrated that electric currents were induced in closed coils when subjected to changing magnetic fields.

Electromagnetic induction

- The phenomenon in which electric current is generated by varying magnetic fields is called ***electromagnetic induction***.

The Experiments of Faraday and Henry

Experiment I

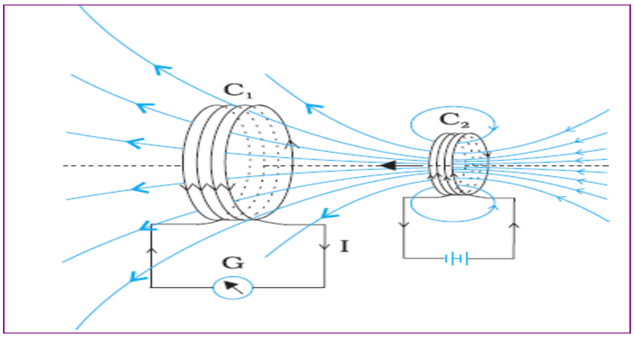


- When the North-pole of a bar magnet is pushed towards the coil, the galvanometer deflects.
- The deflection lasts as long as the bar magnet is in motion.
- When the magnet is pulled away from the coil, the galvanometer shows deflection in the opposite direction.
- When the South-pole of the bar magnet is moved towards or away from the coil, the deflections in the galvanometer are opposite to that observed with the North-pole for similar movements.
- The deflection (and hence current) is larger when the magnet is pushed towards or pulled away from the coil faster.
- When the bar magnet is held fixed and the coil C1 is moved towards or away from the magnet, the same effects are observed.

Conclusion

- *The **relative motion between the magnet and the coil** is responsible for generation (induction) of electric current in the coil.*

Experiment II

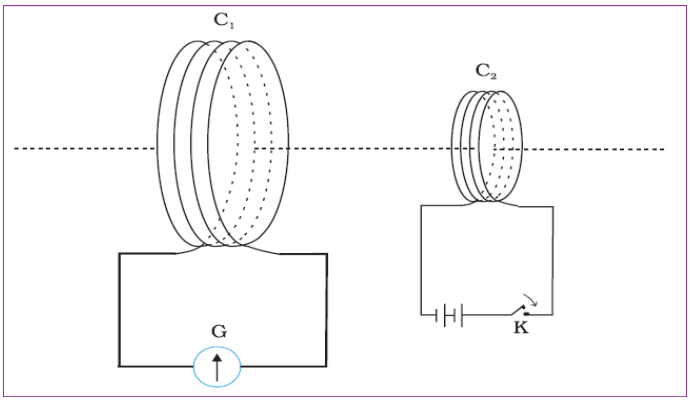


- The bar magnet is replaced by a second coil C_2 connected to a battery.
- If the coil C_2 is moved away or towards C_1 the galvanometer deflects.

Conclusion –

- *The **relative motion between the coils** induces the electric current.*

Experiment III

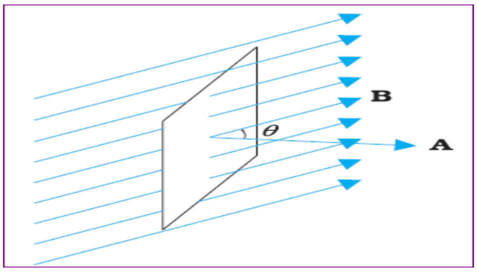


- The galvanometer shows a momentary deflection when the tapping key K is pressed.
- If the key is held pressed continuously, there is no deflection in the galvanometer.
- When the key is released, a momentory deflection is observed again, but in the opposite direction.
- The deflection increases dramatically when an iron rod is inserted into the coils along their axis.

Magnetic Flux

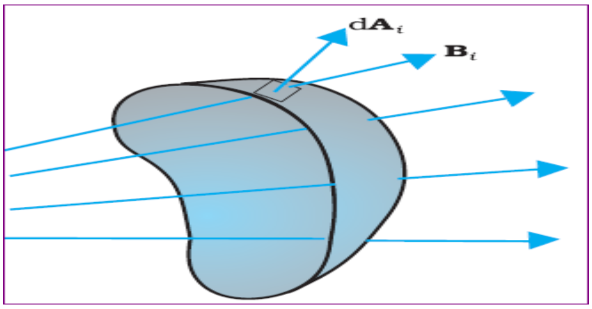
- Magnetic flux through a plane of area *A placed in a uniform magnetic field* ***B can*** be written as

$$\phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$$



- If the magnetic field has different magnitudes and directions at various parts of a surface then the magnetic flux through the surface is given by

$$\Phi_B = \mathbf{B}_1 \cdot d\mathbf{A}_1 + \mathbf{B}_2 \cdot d\mathbf{A}_2 + \cdots = \sum_{\text{all}} \mathbf{B}_i \cdot d\mathbf{A}_i$$



Faraday's Law of Electromagnetic Induction

- ***The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit.***
- Mathematically

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

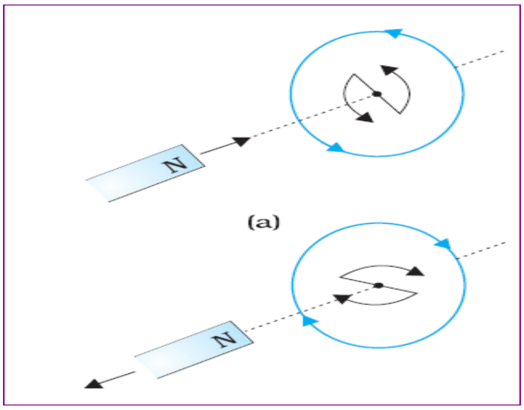
- If there are N turns

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

- The negative sign indicates the direction of emf.

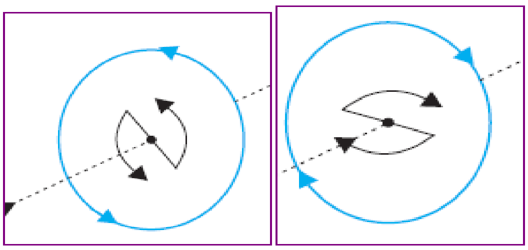
Lenz's Law

- **The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.**
- Lenz's law gives the direction of induced e.m.f.



Polarity of a current carrying loop

- **anticlockwise** current → **north pole.**
- **Clockwise** current → **south pole**



Lenz's law and Law of conservation of Energy

- Lenz's law is in accordance with the law of conservation of energy.
- In electromagnetic induction the mechanical energy is converted in to electrical energy.
- If the polarity of the coil is against Lenz's law, then the South-pole due to the induced current will face the approaching North-pole of the magnet and hence the bar magnet gains kinetic energy without expending any energy.
- This violates the law of conservation of energy and hence cannot happen.
- If the polarity is as per Lenz's law, the bar magnet experiences a repulsive force due to the induced current and hence a person has to do work in moving the magnet. The energy spent by the person is dissipated by Joule heating produced by the induced current.

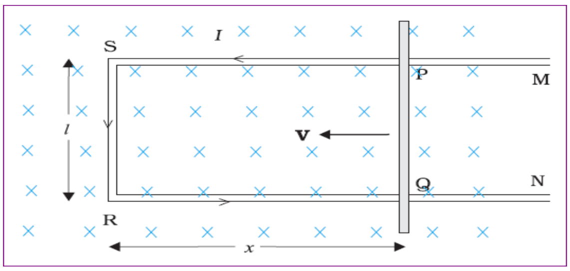
Ways to increase the induced emf

- By increasing the number of turns, N.
- By changing magnetic flux.
- The magnetic flux can be varied by
    - Changing magnetic field, B
    - Changing area, A.
    - Changing the angle, θ.
    - Rotating the coil in a magnetic field.
    - Shrinking or stretching the coil in a magnetic field.

Motional Electromotive Force

- The emf induced by the motion of a conductor in a magnetic field is called motional emf.

Expression of motional emf



- The magnetic flux Φ_B enclosed by the loop PQRS is

  Φ_B = Blx , where B – magnetic field

- Since *x is changing with time, the rate of change* of flux Φ_B will induce an emf given by

$$\varepsilon = \frac{-d\Phi_B}{dt} = -\frac{d}{dt}(Blx)$$

$$= -Bl\frac{dx}{dt} = Blv$$

- The induced emf **Blv** is called motional emf.

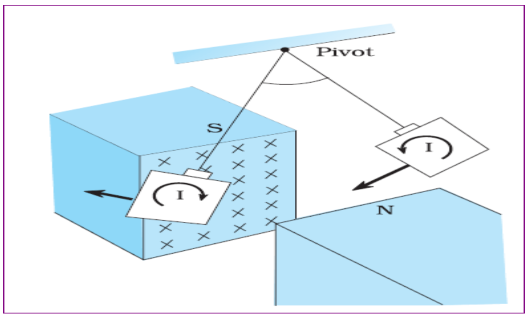
Eddy Currents

- Eddy currents are the **surface currents** produced when **bulk pieces of conductors** are subjected to **changing magnetic field**.
- Eddy currents flow in closed loops within conductors, in planes perpendicular to the magnetic field.

- This effect was discovered by physicist Foucault, and hence this current is also known as **Foucault current.**
- The direction of eddy currents is given by Lenz's law.

Demonstration of eddy currents

Experiment 1

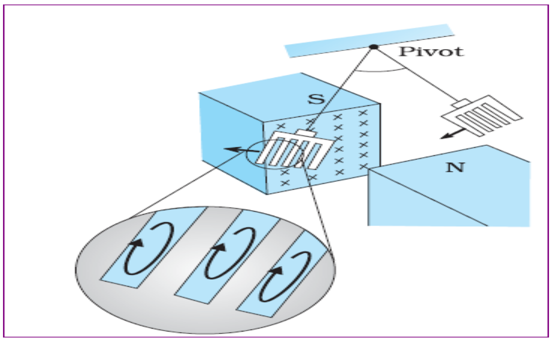


- When a copper plate is allowed to swing like a simple pendulum between the pole pieces of a strong magnet, it is found that the motion is damped and the plate comes to rest in the magnetic field.

Reason :

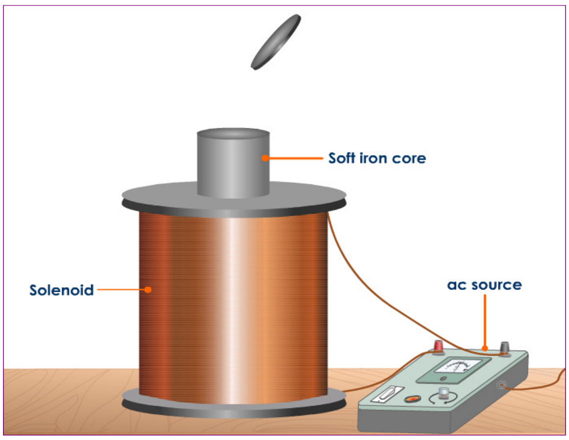
- As the plates moves the magnetic flux associated with it changes and eddy currents are induced on its surface.
- Directions of eddy currents are opposite when the plate swings into the region between the poles and when it swings out of the region.
- Hence the plate comes to rest.

Experiment II



- If rectangular slots are made in the copper plate the area available to the flow of eddy currents is less.
- The pendulum plate with holes or slots reduces electromagnetic damping and the plate swings more freely.

Experiment III



- When a metallic disc is placed on one end of a solenoid connected to an ac source and with a soft iron core in it, the disc is thrown up into air.

Reason

- The disc is subjected to a changing magnetic field and eddy currents are formed on it.
- The direction of the induced currents is as per Lenz's law and hence the disc is thrown up into air.

Disadvantages of eddy currents

- The eddy currents dissipate energy in the form of heat.
- Eddy currents are minimized by using laminations.
- Eddy currents are undesirable, in most of the electrical devices like transformer, induction coil, choke coil etc. Eddy currents produce heating in these devices, which is wastage of energy.

Applications of Eddy currents

- Magnetic braking in trains
- Electromagnetic damping in galvanometers.
- Induction furnace
- Electric power meters
- Metal detectors
- Induction cookers
- Speedometer
- Induction motors

Magnetic braking in trains

- Strong electromagnets are situated above the rails in some electrically powered trains.
- When the electromagnets are activated, the eddy currents induced

in the rails oppose the motion of the train.

- As there are no mechanical linkages, the braking effect is smooth.

Electromagnetic damping in galvanometers

- Certain galvanometers have a fixed core made of nonmagnetic metallic material.
- When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest quickly.

Induction furnace

- Induction furnace can be used to produce high temperatures and can be utilized to prepare alloys, by melting the constituent metals.
- A high frequency alternating current is passed through a coil which surrounds the metals to be melted.
- The eddy currents generated in the metals produce high temperatures sufficient to melt it.

Electric power meters

- The shiny metal disc in the electric power meter (analogue type) rotates due to the eddy currents.
- Electric currents are induced in the disc by magnetic fields produced by sinusoidally varying currents in a coil.

Metal detectors

- The simplest form of a metal detector consists of an oscillator producing an alternating current that passes through a coil producing an alternating magnetic field.
- If a piece of electrically conductive metal is close to the coil, eddy currents will be induced in the metal, and this produces a magnetic field of its own.

Speedometer

- In the speedometer an aluminum drum rotates according to the speed of the vehicle.
- The aluminum drum is carefully pivoted and a magnet is placed inside it.
- As the vehicle moves the magnet rotates, the eddy currents are produced in the aluminum drum.
- These eddy currents try to reduce the relative motion and hence the cylinder also rotates with the magnet.
- The pointer of the speedometer moves according to the rotation of the drum.

Inductance

- **Inductance** is the property of a coil by which a change in current through it induces an e m f in both the conductor itself and in any nearby coil by mutual inductance.
- The flux through a coil is proportional to the current.

$$\Phi_B \propto I.$$

- For a closely wound coil of *N turns, **the same magnetic flux is linked*** with all the turns.
- The product of magnetic flux and number of turns of a coil is called **flux linkage.**

$$\textbf{Flux linkage} = N\Phi_B$$

- Thus

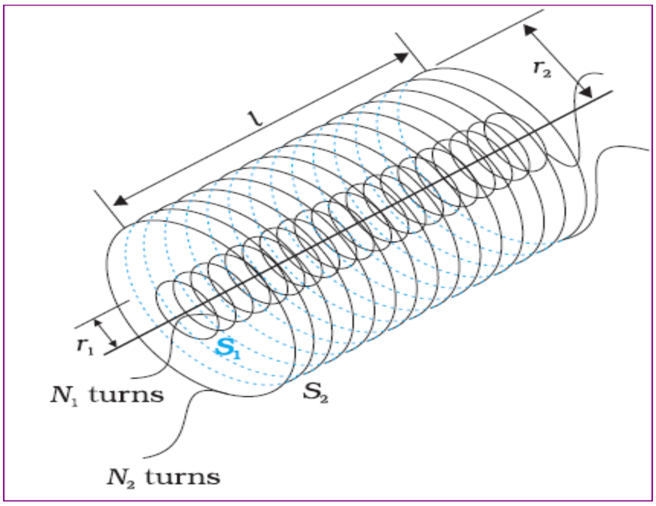
$$N\Phi_B \propto I$$

- The constant of proportionality, in this relation, is called ***inductance.***
- Inductance depends only on the geometry of the coil and intrinsic material properties.
- Inductance is a scalar quantity.
- The dimensions of inductance are $[M L^2 T^{-2} A^{-2}]$.

- SI unit of inductance is ***henry*** *and* is denoted by **H**.
- The two types of inductance are
  1) Mutual inductance 2) Self inductance

Mutual induction

- When current through a coil changes an e.m.f. is induced in the neighboring coil. This is called mutual induction.



- **For the inner solenoid S_1**

  Radius - r_1

  Number of turns per unit length- n_1

  Total number of turns – N_1

- **For the outer solenoid S_2**

  Radius – r_2

  Number of turns per unit length- n_2

  Total number of turns-N_2

- Flux linkage with solenoid S_1 *is*

$$N_1 \Phi_1 = M_{12} I_2$$

- Where Φ_1 is the magnetic flux through S_1 due to the current I_2 through S_2 and M_{12} is called the ***mutual inductance (coefficient of mutual induction)*** of solenoid S_1 with respect to solenoid S_2.

- Also flux linkage with solenoid S_2 when a current I_1 *is* passed through the solenoid S_1 **is**

$$N_2 \Phi_2 = M_{21} I_1$$

  - Where Φ_2 is the magnetic flux through S_2 due to the current I_1 through S_1 and M_{21} is called the mutual inductance of solenoid S_2 with respect to solenoid S_1

Equation of M_{12}

- The magnetic field due to the current I_2 *in* S_2 *is*

$$B_2 = \mu_0 n_2 I_2$$

- The resulting flux linkage with coil S_1 *is*

$$N_1 \Phi_1 = (n_1 l)(\pi r_1^2)(\mu_0 n_2 I_2)$$
$$= \mu_0 n_1 n_2 \pi r_1^2 l I_2$$

- We have

$$N_1 \Phi_1 = M_{12} I_2$$

- Comparing the two equations we get

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l$$

Equation of M_{21}

- The magnetic field due to the current I_1 *in* S_1 *is*

$$B_1 = \mu_0 n_1 I_1$$

- The resulting flux linkage with coil S_2 *is*

$$N_2 \Phi_2 = (n_2 l)(\pi r_1^2)(\mu_0 n_1 I_1)$$

- We have

$$N_2 \Phi_2 = M_{21} I_1$$

- Comparing two equations

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$$

General equation of mutual inductance

- We have

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l \qquad M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$$

- Thus

$$M_{12} = M_{21} = M \text{ (say)}$$

- Therefore the mutual inductance is

$$M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l$$

- The mutual inductance of a pair of coils, solenoids, etc., depends on their separation as well as their relative orientation.

Relation connecting induced emf and mutual inductance

- We have

$$N_1 \Phi_1 = M I_2$$

- For currents varying with time

$$\frac{d(N_1 \Phi_1)}{dt} = \frac{d(M I_2)}{dt}$$

- From Faraday's law

$$\varepsilon_1 = -\frac{d(N_1 \Phi_1)}{dt}$$

- Thus

$$\varepsilon_1 = -M \frac{dI_2}{dt}$$

- The magnitude of the induced emf depends upon the rate of change of current and mutual inductance of the two coils.

Self-inducction

- When the current through a coil changes, an e.m.f. is induced in it. This is called **self induction.**
- The flux linkage through a coil of *N turns is* proportional to the current through the coil.

$$N\Phi_B \propto I$$
$$N\Phi_B = L I$$

- Where constant of proportionality *L is called **self-inductance** of the coil. It* is also called the ***coefficient of self-induction*** *of the coil.*

Equation of self inductance

- The magnetic field due to a current *I flowing in the solenoid is* **B = μ_0n I.**

- The total flux linked with the solenoid is

$$N\Phi_B = (nl)(\mu_0 n I)(A)$$

$$= \mu_0 n^2 A l I$$

- We have **$N\phi_B = L I$**
- Thus

$$L = \frac{N\Phi_B}{I} = \mu_0 n^2 A l$$

- In general

$$L = \mu_r \mu_0 n^2 A l$$

- The self-inductance of the coil depends on its geometry and on the permeability of the medium.

- The self-induced emf is also called **the *back emf*** *as it opposes any* change in the current in a circuit.

- The *self-inductance plays the role of **inertia.***
- It is the electromagnetic analogue of mass in mechanics.

Relation connecting self inductance and induced emf

- We have

$$N\Phi_B = L I$$

- When the current is varied, the flux linked with the coil also changes and an emf is induced in the coil.
- Thus

$$\varepsilon = -\frac{d(N\Phi_B)}{dt}$$

$$\varepsilon = -L\frac{dI}{dt}$$

- The self-induced emf always opposes any change (increase or decrease) of current in the coil.

Energy stored in an inductor

- The work to be done against the back emf in an inductor is stored as **magnetic potential energy**.
- For the current *I at an instant in a circuit, the rate of work done is*

$$\frac{dW}{dt} = |\varepsilon| I$$

- But

$$\varepsilon = -L\frac{dI}{dt}$$

- Therefore

$$\frac{dW}{dt} = L\ I\frac{dI}{dt}$$

- Total amount of work done in establishing the current I is,

$$W = \int dW = \int_0^I L\, I\, dI$$

- Thus, the energy required to build up the current *I is*

$$W = \frac{1}{2}LI^2$$

Combined effect of self inductance and mutual inductance of coil

- When currents are flowing simultaneously through two coils we get

$$\varepsilon_1 = -L_1\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt}$$

- Where L_1 – self inductance of the first coil, M_{12} – mutual inductance of the first coil with respect to the second coil.

AC Generator

- An ac generator converts mechanical energy into electrical energy.
- Nicola Tesla is credited with the development of an ac generator.
- Modern day generators produce electric power as high as 500 MW.
- The frequency of rotation is **50 Hz** in India. In certain countries such as USA, it is **60 Hz.**

Principle/Theory

- A.C. generator works on the principle of **electro-magnetic induction**.
- The rotation of the coil causes the magnetic flux through it to change, so an emf is induced in the coil.
- When the coil is rotated with a constant angular speed ω, the angle θ between the magnetic field vector **B** and the area vector **A** of the coil at any instant t is **θ = ωt**
- The flux at any time *t is*

$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

- From Faraday's law, the induced emf for the rotating coil of *N turns is* then,

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t)$$

- Thus, the instantaneous value of the emf is

$$\varepsilon = NBA\omega \sin\omega t$$

- where ***NBAω** is the **maximum value** of the emf, which occurs when* **sin ωt = ±1.**

- If we denote *NBAω as* ε_0*, then*

$$\varepsilon = \varepsilon_0 \sin \omega t$$

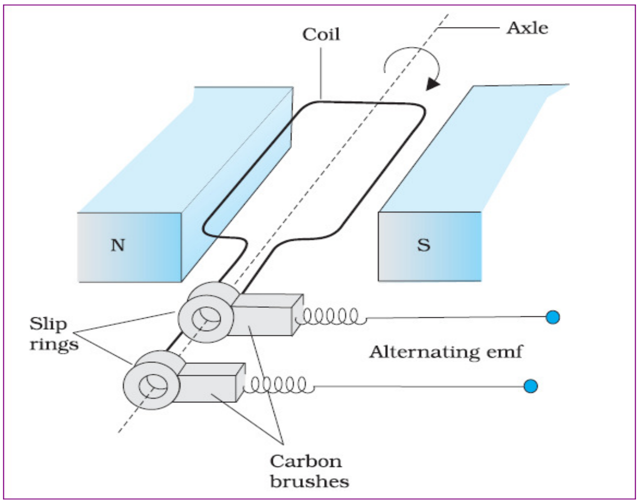
- The direction of the current changes periodically and therefore the current is called ***alternating current (ac).***
- Since ω = 2πν

$$\varepsilon = \varepsilon_0 \sin 2\pi \nu t$$

- Where ν is the frequency of revolution of the generator's coil.

Construction

- An AC Generator consists of a coil mounted on a rotor shaft.
- The axis of rotation of the coil is perpendicular to the direction of the magnetic field.
- The coil (called armature) is mechanically rotated in the uniform magnetic field by some external means.
- The ends of the coil are connected to an external circuit by means of **slip rings** and brushes.

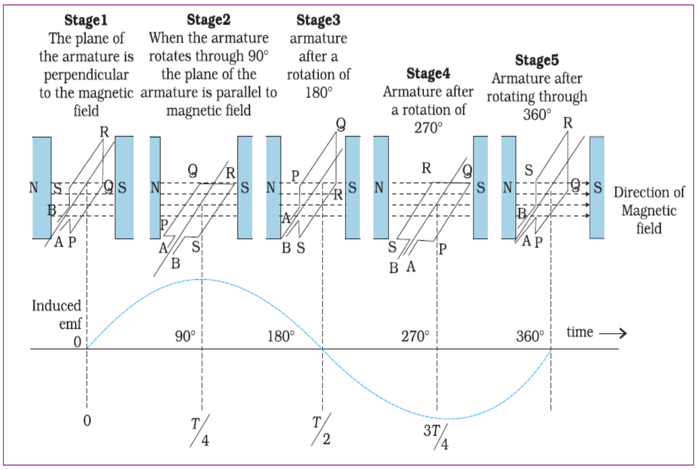


Working

- When the armature coil is mechanically rotated in a uniform magnetic field, the magnetic flux through the coil changes and hence an emf is induced in the coil.

$$\varepsilon = \varepsilon_0 \sin \omega t$$

- The ends of the coil are connected to external circuit by means of slip rings and brushes.



- In most generators, the coils are held stationary and it is the electromagnets which are rotated.

Hydro-electric generators.

- The mechanical energy required for rotation of the armature is provided by water falling from a height.

Thermal generators

- Water is heated to produce steam using coal or other sources.
- The steam at high pressure produces the rotation of the armature.

Nuclear power generators

- Nuclear fuel is used to heat water to produce steam.

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