ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENTS

**NB Electromagnetic induction

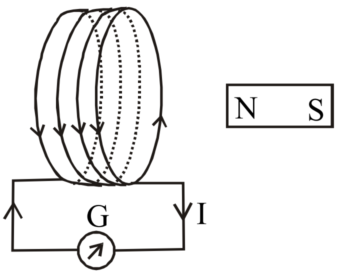
Oersted found that an electric current could produce a magnetic field. In 1831, Michael Faraday found that whenever the magnetic flux linked with a conductor changes, an emf is set up at the ends of the conductor. This emf lasts only as long as the magnetic flux associated with the conductor is changing. This phenomenon is called electromagnetic induction. The emf developed is called induced emf.

The phenomenon of production of emf in a conductor when the flux linked with the conductor changes is called electromagnetic induction.

Experiments to prove electromagnetic induction — Faraday's experiments.

1) Coil - magnet experiment.

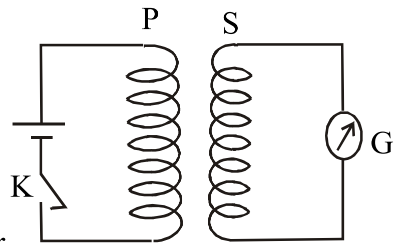
A magnet is moved towards a coil connected to a sensitive galvanometer.
The galvanometer shows a deflection. When magnet is moved away from the coil, the galvanometer gets deflected in the opposite direction. When the magnet is kept stationary, no deflection is observed. The same is the case when the coil is moved keeping the magnet stationary. Thus when there is relative motion between the magnet and the coil, the galvanometer shows deflection. This shows that an emf is induced in the coil.



2) Coil - Coil experiment.

Consider two coils P and S arranged as in figure. The coil P is connected to a battery through a key K. The coil S is connected to a sensitive galvanometer G.

Now when the key is pressed, the galvanometer shows a deflection and when key is released, the galvanometer shows deflection in the opposite direction. When the key is kept pressed, the galvanometer will show no (zero) deflection. Thus deflection is produced only at the make and break of the circuit. This shows that a magnetic flux change in the coil P produces an emf in coil S. This demonstrates electromagnetic induction.



**NB Laws of electromagnetic induction

Faraday's laws.

(1) Whenever the magnetic flux linked with a circuit changes, an emf is induced in it. The induced emf lasts only so long as the magnetic flux is changing.
(2) The magnitude of induced emf is equal to the rate of change of magnetic flux linked with the circuit.

If $d\phi$ is the change in magnetic flux in a time dt, then induced emf, $\varepsilon = \frac{d\phi}{dt}$

Lenz's law

The direction of induced emf is always so as to oppose the change in magnetic flux, which produces it.

ie induced emf, $\varepsilon = -\frac{d\phi}{dt}$

Lenz's law and conservation of energy.

In coil magnet experiment, when the north pole of the magnet is moved towards the coil, the direction of current induced in the coil will be such that the upper face of coil acquires north polarity. Since like poles repel, the coil opposes the motion of the magnet, which is the cause of induced current in the coil. Now, when the magnet is moved away from the coil, the direction of induced emf is reversed and the nearer end of the coil will become South Pole. Since unlike poles attract, the motion of the magnet is opposed.

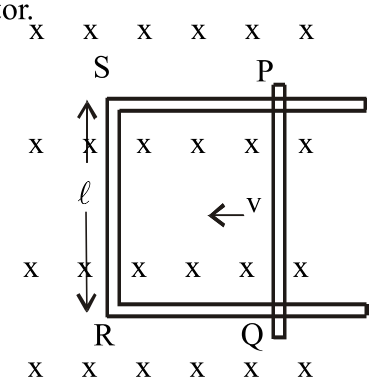
Thus, when a magnet is moved towards or away from a coil, the emf induced in the coil will oppose its motion. So we have to do work against this opposition. So here, the mechanical energy is converted into electrical energy. Thus Lenz's law is in accordance with the law of conservation of energy.

**NB Motional emf

The emf induced in a conductor or coil, due to the relative motion of the source of magnetic field and the conductor or coil is called motional emf.

Equation for motional emf in a conductor.

Consider a rectangular conductor PQRS in which the arm PQ is free to move. It is placed in a uniform magnetic field B that is perpendicular to the plane of the conductor.



Now the arm PQ is moved inwards through a distance x with a velocity v.

Then flux through the loop, $\phi = B\,\ell\,x$.

$$\therefore \text{Induced emf, } \varepsilon = -\frac{d\varphi}{dt} = \frac{d}{dt}\left(B\,\ell\,x\right) = -B\ell\frac{dx}{dt} = B\ell\,v$$

Here, $\frac{dx}{dt} = v$

This induced emf, $\varepsilon = B\ell\,v$ is called motional emf.

*** The direction of induced emf is give by Fleming's right hand rule.

Stretch the three fingers of the right hand so that they are mutually perpendicular. If forefinger represents the direction of the magnetic field and the thumb represents the direction of motion of the conductor, then the middle finger will represent the direction of the induced emf in the conductor.

Q1. A wire of length 0.1 mm moves with a speed of 10m/s perpendiuclar to a magnetic field of induction 1 wb/m^2. Calculate the motional emf. [1 V]

Q2. A small piece of metal wire is dragged across the gap between the pole pieces of a magnet in 0.5 s. the magnetic flux between the pole pieces is known to be 8 x 10^{-4} wb. Estimate the emf produced in the wire. [1.6 x 10^{-3} V]

Q3. An aircraft with a wing span of 40 m flies with a speed of 1080 km/h in the eastward direction at a constant altitude in the northern hemisphere, where the vertical component of earth's magnetif field is 1.75 x 10-5T. Find the emf that developed between the tips of the wings. [0.21 V]

**NB Eddy currents

Whenever the magnetic flux associated with a conductor changes, an emf and hence a current is induced in the conductor. It is not necessary that the conductor should be in the form of a wire or coil. Current will be induced even if the conductor is in the form of a sheet or block.

Thus, *whenever a sheet of metal is moved in a magnetic field or placed in a changing magnetic field, induced currents are produced in closed paths in it. Such currents induced in the mass of the metal are called* eddy currents or Foucault currents.

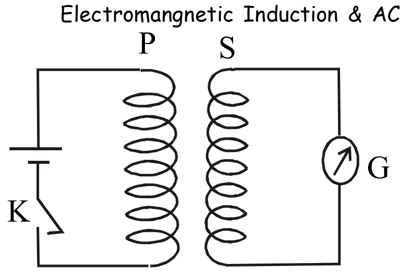
The direction of eddy current is so as to oppose the change in the magnetic flux associated with the metal sheet. *These currents are called eddy currents because they whirl around inside the metal like eddies in water.*

Note: Eddy currents cause wastage of energy in the form of heat. To minimize this loss, the cores of dynamos and transformers are made of thin sheets of lamination, insulated from one another. This increases the resistance of each sheet and reduces eddy currents thereby minimizing the loss of energy.

Applications of eddy currents.

1) Dead beat galvanometer. 2) Induction motors. 3) Induction furnaces. 4) Speedometers. 5) Energy meters. 6) Induction brakes.

*** NB **Mutual Induction.**



Consider two coils P (primary) and S (secondary) as in figure. A battery is connected in P through a key K, while the coil S is connected to a sensitive galvanometer.

On pressing key K, the galvanometer gives a deflection. . If the key is kept pressed, there is no deflection. When the key is released, the galvanometer again deflects in the opposite direction.

So an increase or decrease of current in coil P results in an increase or decrease of magnetic flux linked with the coil S. Hence an emf is induced in it.

The phenomenon of production of emf in the secondary with the change in the current or magnetic flux linked with the primary is known as mutual induction.

Let ϕ be the magnetic flux linked with the coil S due to a current I flowing through P.

Then, $\phi \propto I$

Or $\phi = MI$ .... (1)

Here, M is a constant called coefficient of mutual induction or mutual inductance of the two circuits.

Now, if $I = 1$; $M = \phi$.

Therefore, *coefficient of mutual induction or mutual inductance of two circuits is defined as the magnetic flux linked with the second circuit due to unit current flowing through the first.*

Again $\phi = MI$.

Differentiating w r t time, $\frac{d\phi}{dt} = M\frac{dI}{dt}$

But according to Faraday's law of electromagnetic induction, induced emf, $\varepsilon = -\frac{d\phi}{dt}$

$$\therefore \varepsilon = -M\frac{d\phi}{dt} \text{.......................(2)}$$

If $\frac{dI}{dt} = 1$, $M = \varepsilon$.

So mutual inductance of two circuits is defined as the emf induced in the second circuit due to current changing at a unit rate in the first.

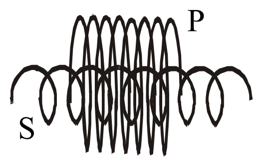
Now if $\frac{dI}{dt} = 1$ A/s; $\varepsilon = 1$ V, then M = 1 henry. So the SI unit of mutual inductance is henry (H).

Coefficient of mutual induction of two circuits is said to be 1 henry, if an emf of 1V is induced in the second circuit due to a current changing at a rate of 1 A/s in the first.

**NB Expression for Mutual Inductance.

Consider two coaxial solenoids P and S. The solenoid P is closely wound upon the solenoid S. P is the primary and S, the secondary.

Let ℓ be the length of the primary coil, N_1 the total number of turns of primary coil and N_2 the total number of turns of secondary coil. Let a current I flow through the primary.



The magnetic flux density at a point inside the primary, $B = \frac{\mu_0 N_1 I}{\ell}$

Magnetic flux linked with each turn of the secondary = $\frac{\mu_0 N_1 I A}{\ell}$; where A = area of cross-section of secondary.

$\therefore$ Total magnetic flux linked with N_2 turns of secondary, $\phi = \frac{\mu_0 N_1 N_2 I A}{\ell}$

But $\phi = MI$.

$$M = \frac{\mu_0 N_1 N_2 A}{\ell} \text{ henry.}$$

If the two coils are wound on a material of relative permeability μ_r then, $M = \dfrac{\mu_0 \mu_r N_1 N_2 A}{\ell}$ henry

If n is the number of turns of primary per unit length, then, $n = \dfrac{N_1}{\ell}$

$\therefore$ Mutual Inductance, $M = \mu_0 \mu_r n N_2 A$.

Q4. A current of 10 A in primary of a circuit is reduced to zero at a uniform rate in 10^{-3}s. If coefficient of mutual inductance is 3H, what is the induced emf in the stationary? [3×10^4 V]
Q5. What is the mutual inductance of a pair of coils, if a current change of six ampere in one coil causes the flux on the second coil of two thousand turns to change by 12×10^{-4} wb/turn? [0.4 H]
Q6. A solenoid of length 50 cm with 20 turns per cm and area of cross section 40 cm2 completely surrounds another co axial solenoid of the same length, area of cross section 25 cm2 with 25 turns per cm. Calculate the mutual induction of the system. [7.85×10^{-3} H]

**NB Self Induction.

In 1832, Joseph Henry discovered the phenomenon of self-induction. When a current flows through a coil or circuit, a magnetic field is produced in it. If the current through the coil is changed, the total magnetic flux associated with the coil changes and an emf is induced in it. The direction of induced emf is such as to oppose the change in current

i.e the induced emf is opposite to the direction of current, when the current is increasing and is in the direction of current when the current is decreasing. This induced emf is called back emf.

The phenomenon of production of an induced emf in a circuit when the current through it changes is known as self-induction.
Thus self-induction opposes the growth of current in a circuit when the current is switched on and opposes the decay of current when it is switched off.

The magnetic flux ϕ linked with a coil due to a current I is proportional to the strength of current.

ie $\phi \propto I$

Or $\phi = L I$ .... (1) ; where L is a constant called the self inductance of the coil.

When I = 1, ϕ = L.

Hence self-inductance of a coil is defined as the magnetic flux linked with the coil when a unit current passes through it

But induced emf, $\varepsilon = -\dfrac{d\phi}{dt}$

From (1), $\dfrac{d\phi}{dt} = L\dfrac{dI}{dt}$

$$\therefore \ \varepsilon = -L\frac{dI}{dt}$$

When $\dfrac{dI}{dt} = 1$, $\varepsilon = L$. numerically. *So self inductance is numerically equal to the emf induced in the coil when the rate of change of current in it is unity.*

When = 1 A/s and ε = 1V, L = 1 henry (H).

Self-inductance in a circuit is said to be one henry if an emf of 1volt is induced in it due to a current changing at the rate of 1 ampere/ second in itself.

**Note: When a circuit is broken, the current decreases at a more rapid rate than the rate of increase of current at the make of the circuit. Hence a large emf is induced at the break of the circuit. This produced a large spark at the switches.

**NB Expression for self-inductance.

Consider a long solenoid of length , radius r (ℓ >>> r) , having n turns per unit length. If I is the current through the solenoid, the magnetic field well inside it is given by $B = \mu_0 n I$.

Magnetic flux through the solenoid per turn, $\phi = \mu_0 n I A$; where $A = \pi r^2$, the area of cross-section of solenoid.

Total number of turns = $n\ell$.

$\therefore$ Total flux linked with the solenoid, $\phi = \mu_0 n I A n \ell$.

But $\phi = L I$.

$\therefore$ Self inductance, $L = \mu_0 n^2 A$

If N is the total number of turns of solenoid, then $L = \dfrac{\mu_0 N^2 A}{\ell}$

Also $L = \dfrac{\mu_0 \mu_r N^2 A}{\ell}$; μ_r = relative permeability of core.

****Note**: If M is the mutual inductance between two coils having self inductances L_1 and L_2;

$M = K\sqrt{L_1 L_2}$; where K = coefficient of coupling.

If the whole magnetic flux linked with one coil links with the other coil also, then coupling is called tight coupling.

For tight coupling, K = 1. Then $M = \sqrt{L_1 L_2}$

For loose coupling, $K < 1$.

Energy stored in an inductor.

When a current is passed through a coil, back emf is developed in the coil, which opposes the growth of current. So to maintain the flow of current, work has to be done against back emf.

Power $P = W/t$
$W = Pt = VIt$
$W = \varepsilon I\, dt$.

The back emf at any instant, $\varepsilon = L\dfrac{dI}{dt}$

Work done against back emf in a short time dt, $dw = \varepsilon I\, dt = L\dfrac{dI}{dt} I\, dt = L I\, dI$.

Hence the total work done to increase the current from 0 to I_0, $W = \int dw = \int_0^{I_0} L I\, dI = L\left[\dfrac{I^2}{2}\right]_0^{I_0}$

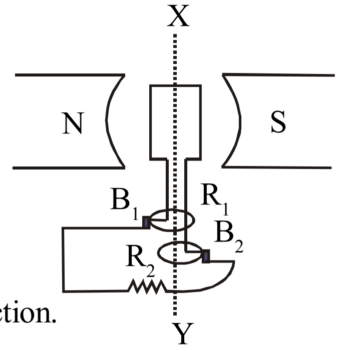
$$\therefore W = \frac{1}{2} L I_0^2$$

This work done is stored in the magnetic field around the inductor.

Q7. What emf will be induced in a 10 H inductor in which current changes from 10 A to 7 A in 9×10^{-2}s? [333.3 V]
Q8. A coil of inductance 0.5 H is connected to 18 V battery. Calculate the rate of growth of current? [36 A/s]
Q9. Find the change in current in an inductor of 10 H in which the emf induced is 300 V in 10^{-2} s. Also find the change in magnetic flux. [0.3 A; 3 wb]
Q10. The self inductance of a coil having 200 turns in 10 mH. Compute the total flux linked with the coil. Also determine the magnetif flux through the cross section of the coil corresponding to current of 4 mA. [4×10^{-5} wb; 2×10^{-7} wb]
Q11. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside, normal to the axis of the solenoid. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0.1 s, what is the induced voltage in the loop, while the current is changing? [7.5 x 10-6 V]

****NB AC generator or AC dynamo.**

AC generator is a device, which converts mechanical energy into electrical energy. It is based on the principle of electromagnetic induction.



AC generator consists of a coil of wire called armature, capable of rotating about an axis XY. The coil is placed between the pole pieces of a strong horseshoe magnet so that the field is radial. The ends of coil are connected to two slip rings. Two graphite or carbon brushes B_1 and B_2 pushes against the rings. These brushes are connected to an external circuit, which draws the alternating current.

Now let the coil be rotating with a constant angular velocity ω. At anyinstant 't', let the normal to the coil make an angle θ with the field direction.

Then $\theta = \omega t$.

The component of magnetic field normal to the plane of the coil = B $\cos\omega t$.

$\therefore$ Magnetic flux linked with the coil at instant 't', $\phi = BAN\cos\omega t$; where N = total number of turns and A = area of the coil.

As the coil rotates, the flux linked with it changes and an emf is induced in the coil.

$\therefore$ Induced emf, $\varepsilon = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(BAN\cos\omega t)$

ie $\varepsilon = BAN\omega \sin\omega t$.

Also, $\varepsilon = \varepsilon_0 \sin\omega t$; where $\varepsilon_0 = BAN\omega$, the maximum value of induced emf.

Here, as the coil rotates, the emf continuously changes. Such an emf is called alternating emf.

If E is the open circuit pd developed across the ends of coil, then $E = E_0 \sin\omega t$. .... (1) Where $E_0 = BAN\omega$.

If R is the resistance of external circuit, current, $I = \dfrac{V}{R} = \dfrac{V_0 \sin\omega t}{R}$

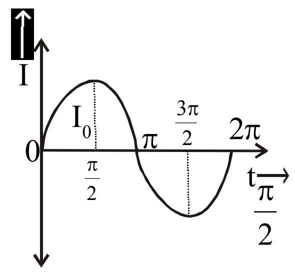
ie $I = I_0 \sin\omega t$.

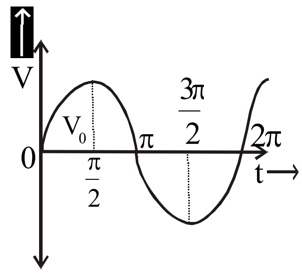
ie Alternating current, $I = I_0 \sin\omega t$. .... (2) Here $I_0 = \dfrac{V_0}{R} = \dfrac{BAN\omega}{R}$ is called peak value of alternating current

The graphical representation of alternating voltage and current is as shown in figures:

The alternating voltage and current is positive during one half cycle and negative during the next half. These two halves constitute a cycle. The time taken for one complete cycle is called period. The number of cycles per second is called frequency of AC.

In India, the household AC supply is 50 Hz.





*****NB Mean and rms value of alternating voltage and current.**

Since the alternating voltage and current reverses their direction, the average or mean value of emf and current for one complete cycle are zero. Therefore, direct current measuring instruments, if connected to an ac circuit will indicate zero deflection. So to measure alternating emf and current, we consider the root mean square (rms) value of voltage and current.

The rms value of voltage or current is the square root of the mean of the squares of emf's or currents in a complete cycle.

Let T be the period of AC then current $I = I_0 \sin\omega t$.

$\therefore \quad I^2 = I_0^2 \sin^2\omega t$.

The mean value of I^2 over a cycle = $\displaystyle\int_0^T \frac{I_0^2 \sin^2\omega t\, dt}{T}$

$$I_{rms}^2 = \frac{\int_0^{2\pi/\omega} I_0^2 \sin^2 \omega t \, dt}{2\pi/\omega} \qquad \left[\because T = \frac{2\pi}{\omega}\right]$$

$$\text{ie } I_{rms}^2 = \frac{I_0^2}{2} \qquad \because \frac{\int_0^{2\pi/\omega} I_0^2 \sin^2 \omega t \, dt}{2\pi/\omega} = \frac{1}{2}$$

Or $\quad I_{rms} = \frac{I_0}{\sqrt{2}}$; I_0 = peak or maximum value of current

I_{rms} is called rms value of current or virtual ampere.

Similarly, $E_{rms} = \frac{E_0}{\sqrt{2}}$; E_0 = peak or maximum value of voltage.

NB Note: When we measure dc voltage or current using a measuring instrument, we are measuring its actual value. But when we measure AC voltage or current using an instrument, we are measuring the rms value. So the peak value of current will be $I_0 = I_{rms} \times \sqrt{2}$ and voltage will be $E_{rms} \times \sqrt{2}$. Hence the actual voltage and current will be $\sqrt{2}$ times greater than measured value. **Hence AC is more dangerous than dc.**

Circuit Elements

(1) **Resistor** (R) : The resistance will act in the same manner for both the AC and dc offering the same resistance R.

(2) **Inductor**(L) : An inductor is a coil of wire wound over a suitable core. If a constant current flows through an inductor, it won't offer any opposition. {Because R = 0}. If an alternating current flows through it, an induced emf is set up in such a direction that it opposes the changing current. This opposition to the flow of electricity is called inductive reactance and is denoted by X_L. Now $X_L = \omega L$; where $\omega = 2\pi\nu$, ν = frequency of AC.

(3) **Capacitor** (C) : When dc is passed through a capacitor, it will offer infinite resistance to dc. When AC is passed through it, it will offer a resistance called capacitive reactance X_C. $\therefore X_C = \frac{1}{\omega C}$

The net opposition offered by all the circuit elements is called the impedance of the circuit (Z).

**Note: (1) The voltage current relationship for an ideal inductor is $V = L\frac{dI}{dt}$

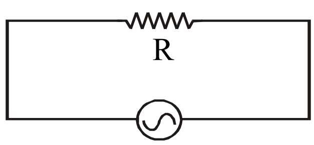
(2) The voltage current relationship for an ideal capacitor is $I = C\frac{dV}{dt}$

Phasors

A quantity, which varies sinusoidally with time and is represented as the projection of a rotating vector is called a phasor.

AC through a resistor.

Let an alternating emf, $E = E_0 \sin\omega t \rightarrow (1)$ be applied across a resistance R.

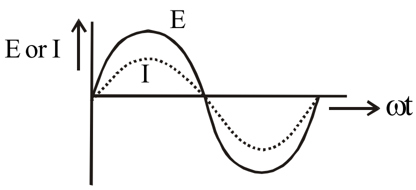


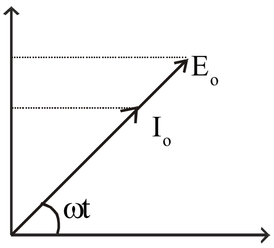
$E = E_0 \sin \omega t$

Then the current at any instant, $I = \frac{E}{R} = \frac{E_0 \sin\omega t}{R}$

i.e. $I = I_0 \sin \omega t \rightarrow (2)$ $\quad I_0 = \frac{E_0}{R}$, the peak or maximum value of current.

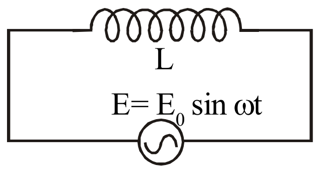
From (1) and (2), we can see that the current is in phase with the voltage or emf.





AC through an inductor.

Let an alternating emf, $E = E_0 \sin \omega t \quad \rightarrow \quad (1)$ be applied across an inductor of inductance L.



Then the induced emf, $E = L\dfrac{dI}{dt}$

$$\therefore \quad L\frac{dI}{dt} = E_0 \sin \omega t$$

ie $\quad LdI = E_0 \sin \omega t \, dt$

or $\quad dI = \dfrac{E_0}{L} \sin \omega t \, dt$

Now current, $\quad I = \int dI = \int \dfrac{E_0}{L} \sin \omega t \, dt = \dfrac{E_0}{L} \int \sin \omega t \, dt$

ie $\quad I = \dfrac{E_0}{\omega L}(-\cos \omega t)$

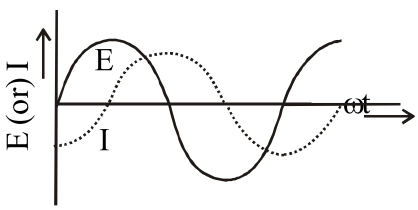
$I = I_0(-\cos \omega t) = I_0 \sin\left(\omega t - \dfrac{\pi}{2}\right) \rightarrow (2)$; where $I_0 = \dfrac{E_0}{\omega L}$; the peak value of current.

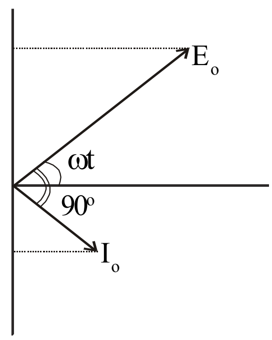
From (1) and (2), we can see that the current lags behind the emf by a phase angle of π/2 radians.

Now, . Comparing with $I = \dfrac{E}{R}$; we can see that ωL corresponds to resistance which opposes the flow of ac through the inductor. It is called *inductive reactance* (X_L) .

$\therefore$ Inductive reactance, $X_L = \omega L = 2\pi f L$; where f= frequency of AC.

The graphical representation, and phasor diagram are as shown below.





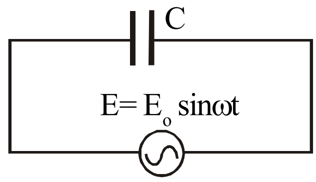
Can you say why the emf leads the current in case of an inductor?

In a resistive circuit, the opposition to the current is due to the obstruction to the passage of electrons through the resistor. In an inductive circuit, the self induced emf opposes the growth of current and hence emf leads and current lags.

AC through a capacitor.

Consider an ac emf $E = E_0 \sin \omega t \rightarrow$ (1) applied across a capacitor of capacitance C.

Then current through the capacitor,



$$I = C\frac{dE}{dt} = C\frac{d}{dt}(E_0 \sin \omega t)$$

$$I = CE_0 \frac{d}{dt}(\sin \omega t) = E_0 C\omega \cos \omega t$$

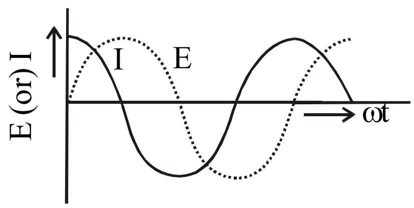
i.e $$I = \frac{E_0}{1/C\omega}\cos \omega t = \frac{E_0}{1/C\omega}\sin\left(\omega t + \frac{\pi}{2}\right)$$

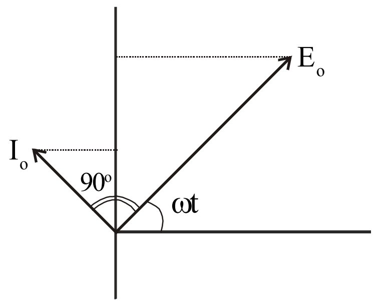
$$\therefore \quad I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \rightarrow (2) \quad \text{where } I_0 = \frac{E_0}{1/C\omega}, \text{ the peak value of current.}$$

Comparing with $I = \frac{E}{R}$, we can see that $\frac{1}{\omega C}$ is the resistance offered by the capacitor to the flow of current and is called *capacitive reactance (X_C).*

Now from (1) and (2), we can see that, **the current will lead the emf by a phase angle of π/2 radians. Or the emf lags the current by a phase π/2.**

The graphical representation and phasor diagram are as shown below.





Can you say why the emf lags the current in case of a capacitor?

In a resistive circuit, the opposition to the current is due to the obstruction to the passage of electrons through the resistor. But in a capacitive circuit, it is the current which charges the capacitor and rises its potential. Hence the current leads the emf.

Energy and Power associated with L, C and R in AC circuits.

(1) **Circuit containing an inductor.**

Consider an inductor of self-inductance L. When an alternating current flows through the inductor, the back emf produced in it opposes the growth of current. Then the total work done by the source when current increases from 0 to I_0 against back emf, $W = ½ L I_0^2$.

This work done is stored in the inductor as mechanical energy and is returned back to the source when the current returns from I_0 to zero. Hence the total work done or energy is zero. *Thus the power dissipated by a pure inductor in a circuit is zero.*

(2) **Circuit containing a capacitor**

In case of a capacitor, average power during one cycle,

P_{av} = work done in one cycle/ time

$$P_{av} = \int_0^T \frac{EI\,dt}{T} = \int_0^{2\pi/\omega} \frac{EI\,dt}{2\pi/\omega} = \int_0^{2\pi/\omega} \frac{E_0 \sin \omega t\, I_0 \cos \omega t\, dt}{2\pi/\omega} = 0.$$

Thus in case of a capacitor, when AC is passed, no power is dissipated. Hence the current through the capacitor is called **wattless current.**

3. Circuit containing a resistor.

Consider a resistance R connected to an alternating emf $E = E_0 \sin \omega t$. Then current $I = I_0 \sin \omega t$.

$\therefore$ Average power $P_{av} = \dfrac{\int_0^T E\,I\,dt}{T}$ $\qquad P_{av} = \dfrac{\int_0^{2\pi/\omega} E_0 \sin \omega t\, I_0 \sin \omega t\, dt}{2\pi/\omega}$

$P_{av} = E_0 I_0 \dfrac{\int_0^{2\pi/\omega} \sin^2 \omega t\, dt}{2\pi/\omega}$ $\qquad$ But $\dfrac{\int_0^{2\pi/\omega} \sin^2 \omega t\, dt}{2\pi/\omega} = \dfrac{1}{2}$

$$\therefore \quad P_{av} = \frac{E_0 I_0}{2} = \frac{E_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}} = E_{rms} I_{rms}$$

ie $P_{av} = E_{rms} \times I_{rms}$.

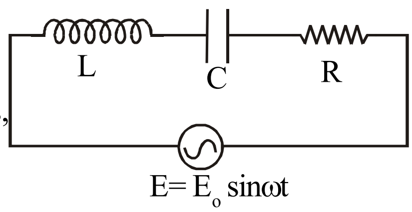
Wattless Current or idle current.

True power of an ac circuit is given by $P = E_{rms} I_{rms} \cos\phi$ where ϕ is the phase difference betweeen emf and current. If the ac circuit is purely inductive or purely capacitive ($R = 0$) the phase angle $\phi = \pm \dfrac{\pi}{2}$ so that $\cos\phi = 0$. Hence the power consumed in the circuit $P = 0$. The current in such a circuit does not perform any work and is therefore called **idle current or wattless current**. Even though the circuit does not consume any power, it offers an opposition to the flow of ac through it

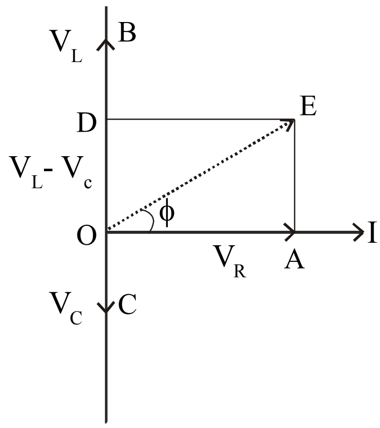
Thus when a current is passed through an inductor or capacitor, no power is consumed for performing work, such a current is called wattless current or idle current.

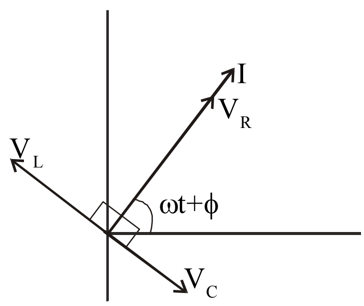
Q12. Find the inductive reactance of an inductor of inductance 5 H at 50 Hz ac. [1570 ohm]
Q13. Calculate the current through an inductance of 10 H, with negligible resistance in an ac source of 230 V, and frequency 50 Hz. [0.073 A]
Q14. Find the capacitive reactance of a capacitor of capacitance 10 μF to an ac of frequency 50 Hz. [318 ohm]
Q15. A capacitor of 10 μF is connected to 230 V ac mains at 50 Hz. Find the current and peak value of voltage in the circuit? [0.72 A; 325.27 V]

AC through a series L C R circuit.



Consider an inductance L, capacitor C and a resistance R connected in series to an AC supply of frequency f. Since they are connected in series, the current through L, C and R are same.





The potential difference across R is V_R. It is in phase with the current I and is represented by the vector OA. The potential difference across L is V_L which leads the current by $\pi/2$. It is represented by the vector OB. The pd across C is V_C, which lags the current by $\pi/2$ and is represented by the vector OC. The resultant of V_L and V_C is $V_L - V_C$ (assuming that $V_L > V_C$) because they differ in phase by 180° and is represented by the vector OD. The resultant of $(V_L - V_C)$ and V_R is represented by OE.

ie $OE = V$ (the applied voltage).

From right triangle OAE, $OE^2 = OA^2 + AE^2$

$$V^2 = V_R^2 + (V_L - V_C)^2$$

But $V_R = IR$, $V_L = IL\omega$, $V_C = I \times \frac{1}{\omega C}$

$$V^2 = I^2 R^2 + \left[I\omega L - \frac{I}{\omega C}\right]^2 = I^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]$$

ie $V = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$,

Now,

$$\frac{V}{I} = Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

ie $Z = R + X_L + X_C$

Where Z is called the impedance of the LCR circuit and is defined as the effective opposition offered by the inductance, capacitance and the resistance to the flow of ac.

If ϕ is the phase difference between voltage V and current I, then

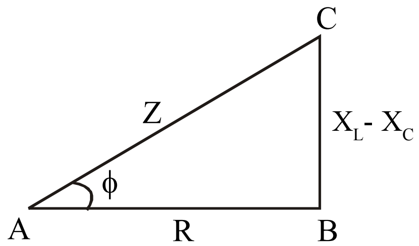
$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{I R}$$

The instantaneous voltage of the LCR circuit is given by $E = E_0 \sin \omega t \rightarrow (1)$

Then instantaneous current in the circuit $I = I_0 \sin(\omega t - \phi) \rightarrow (2)$; where $\phi = \tan^{-1}\frac{(X_L - X_C)}{R}$

The current lags the emf if $V_L > V_C$ i e $\omega L > \frac{1}{\omega C}$

The current leads the emf if $V_C > V_L$ ie $\omega L < \frac{1}{\omega C}$.



ABC gives the impedance triangle. AB represents the resistance R, BC represents the reactance X.

$$X = X_L - X_C = \left(\omega L - \frac{1}{\omega C}\right).$$

AC represents the impedance of the circuit.

The impedance triangle is helpful in finding (a) impedance of the ac circuit. (b) phase difference between current and emf (c) power factor.

Q16. A series LCR circuit with L = 10 H, C = 10 μF and R = 35 ohm is connected to an ac of frequency 50 Hz. Obtain the impedance of the circuit. [2822 ohm]

Q17. A series RC circuit with C = 100 μF and R = 10 ohm is connected to an ac source of frequency 50 Hz. Obtain the impedance of the circuit. [33.38 ohm]

Q18. In an ac circuit, an inductance coil of resistance 50 ohm at 50 Hz supply has a voltage leading the current by 30^0. Find the inductance of the coil? [0.092 H]

Resonance.

For an LCR circuit driven by a voltage E and frequency ω, the amplitude of current,

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + (X_L - X_C)^2}};$$

where $X_L = \omega L$ and $X_C = 1/\omega C$

Now if frequency ω is varied, for a particular frequency ω_r, $X_L = X_C$ and the impedance is minimum ie

$Z = \sqrt{R^2} = R$. This frequency is called *resonant frequency.*

The angular frequency of the applied AC voltage at which the impedance of an LCR circuit is minimum is called resonant frequency ω_r

At resonance, $\omega L = \frac{1}{\omega C}$ ie $\omega_r^2 = \frac{1}{LC}$

$$\therefore \quad \omega_r = \frac{1}{\sqrt{LC}}$$

If f_r is the frequency of applied voltage, then $f_r = \frac{1}{2\pi\sqrt{LC}}$ Because, $\omega_r = 2\pi f_r = \frac{1}{\sqrt{LC}}$

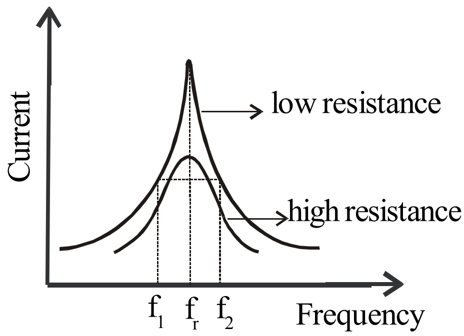
Now at resonance, the current is maximum

ie., $I_0 = \frac{E_0}{R}$

LCR circuit is called acceptor circuit because it accepts maximum current at the resonant frequency.

If resistance R is low, the peak of curve is sharp and if R is high, the peak is flat. *Thus sharpness of resonance depends on the resistance* of the circuit.

The variation of current with frequency is as shown in fig.



Q-factor

Q-factor is the quality or figure of merit of the resonant circuit and it determines the sharpness of resonsnce. The smaller the resistance of the circuit, sharper is the resonance curve. This fact is expressed by quality factor.

$$Q = \frac{I\,L\omega}{IR} = \frac{L\omega}{R} \quad \text{or} \quad Q = \frac{1/C\omega}{1/R} = \frac{1}{C\omega R}$$

In terms of resonance frequency $Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{f_r}{f_2 - f_1}$

Power in an LCR circuit.

The current in an LCR circuit when an alternating voltage, $E = E_0 \sin wt$ is applied is $I = I_0 \cos(wt - f)$.

$\therefore$ Average power consumed by the circuit in one cycle,

$$P_{av} = \int_0^T \frac{E\,I\,dt}{T} \qquad P_{av} = \int_0^{2\pi/\omega} \frac{E_0 \sin\omega t\, I_0 \sin(\omega t - \phi)dt}{2\pi/\omega} \qquad P_{av} = E_0 I_0 \int_0^{2\pi/\omega} \frac{\sin\omega t\,.\sin(\omega t - \phi)dt}{2\pi/\omega}$$

$$\text{But} \int_0^{2\pi/\omega} \frac{\sin\omega t\,.\sin(\omega t - \phi)dt}{2\pi/\omega} = \frac{1}{2}\cos\phi \qquad P_{av} = E_{rms}\, I_{rms} \cos\phi$$

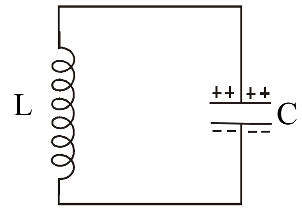
Average power is also called true power. The quantity, $E_{rms}\ I_{rms}$ is called virtual power or apparent power and $\cos\phi$ is called the power factor.

$\therefore$ True power = apparent power x power factor.

Thus the power factor of a circuit is defined as the ratio of true power to the apparent power.

LC Oscillations.

We know that a capacitor and inductor can store electrical and magnetic energy respectively. When a charged capacitor is connected to an inductor, the charge on the capacitor and the current in the circuit exhibit the phenomenon of electrical oscillations.



Consider a circuit containing a capacitor C and an inductor L. Let Q be the charge of the capacitor and no current flow through the inductor.

Then the energy of the capacitor, $U_E = Q^2/2C$.

The energy stored in the inductor = 0.

Now the capacitor begins to discharge through the inductor and a current will flow in the anti – clockwise direction. As the charge on the capacitor decreases, its energy U_E decreases. At the same time, the energy in the magnetic field of the inductor $U_B = \frac{1}{2} L I^2$ increases.

After some time, the capacitor gets fully discharged and the energy stored in the capacitor is transferred to the magnetic field of the inductor. At this stage, the current through the inductor is maximum. This current carries positive charge from the top plate of the capacitor to the bottom plate of the capacitor. The energy now flows from the inductor to the capacitor as the electric field builds up again. The capacitor will begin to discharge again. The current will now flow clockwise and the capacitor will return to its original state at the beginning. This process continues at a definite frequency f. The energy is continuously shuttled back and forth between the electric field in the capacitor and the magnetic field in the inductor.

If no resistance is present in the LC circuit, the LC oscillations will continue indefinitely. In actual practice, some resistance is always present and hence energy is lost in the form of heat. So LC oscillations will not continue indefinitely.

Equation of LC oscillation

Let q be the charge on the capacitor at a time t and $\frac{dI}{dt}$ the rate of change of current. Since no battery is connected in the circuit, $L\frac{dI}{dt} + \frac{q}{C} = 0$

ie $L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$

or $\frac{d^2q}{dt^2} + \frac{q}{LC} = 0$

$$\therefore \frac{d^2q}{dt^2} + \omega^2 q = 0 \quad \rightarrow (1);$$

This is the equation for simple harmonic electrical oscillations. The solution of this equation is $q = q_0 \cos\omega t$. This equation shows that the charge on the capacitor is oscillatory.

Now we have, $\omega^2 = \frac{1}{LC}$ Or $\omega = \frac{1}{\sqrt{LC}}$

Therefore, the frequency of LC oscillation is $f = \frac{1}{2\pi\sqrt{LC}}$

Transformer.

Transformer is a device to convert a given alternating voltage to one of higher or lower voltage. It is based on the principle of electromagnetic induction.

Transformer consists of a soft iron core, circular or rectangular in shape over which two insulated coils are wound. The core is laminated to minimise the loss of energy due to eddy currents. The coil to which voltage is supplied is called primary and that from which energy is delivered to the external circuit is called the secondary.

Theory:

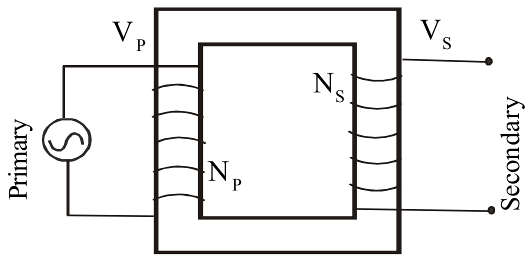
Let N_P and N_S be the number of turns of primary and secondary respectively. Let an alternating voltage V_P be applied to the primary. Then a current I_P will flow through the primary. This current magnetizes the core and causes flux ϕ to flow in the core. Assuming that there is no leakage of flux, the same flux will be linked with the secondary.

$\therefore$ Flux linked with primary = $N_P\phi$

Flux linked with secondary = $N_s\phi$

This flux linked will induce emf's E_P and E_S in primary and secondary.

$\therefore$ According to law of electromagnetic induction,



$$E_P = -N_P\frac{d\phi}{dt} \quad \text{and} \quad E_S = -N_S\frac{d\phi}{dt}$$

$$\therefore \frac{E_P}{N_P} = \frac{E_S}{N_S} \qquad or \qquad \frac{E_P}{E_S} = \frac{N_P}{N_S} = K$$ where K is called Transformer ratio or turns ratio of transformer

As the primary and secondary coils have very low resistances,

$E_P = -V_P$ and $E_S = -V_S$.

$$\therefore \frac{E_S}{E_P} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

For an ideal transformer, power output = power input.

ie $V_S I_S = V_P I_P$; where I_P and I_S are the rms values of primary and secondary currents.

$$\therefore \frac{V_S}{V_P} = \frac{I_P}{I_S}$$ Hence for a transformer, $$\frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P}$$

Case 1: If $N_S > N_P$; then $V_S > V_P$. Such a transformer is called step-up transformer.

Case 2: If $N_P > N_S$, then $V_S < V_P$. Such a transformer is called step-down transformer.

Efficiency of a transformer.

The ratio of output power to the input power is called efficiency of a transformer.

Efficiency, $\eta = \frac{V_S\,I_S}{V_P\,I_P}$ The efficiency decreases due to various losses.

Power losses in a transformer.

** *Copper loss or Joule loss*: When current flows, heat will be developed due to resistance offered by the primary and secondary coils. This is called copper loss or Joule loss. To reduce copper loss, thick wire should be used to wind the transformer.

** *Eddy current loss*: When current flows through transformer, the flux linked with the core changes. This generates eddy currents in the core. This eddy current generates heat in the core and a part

** *Hysteresis losses*: When current flows, the core is subjected to cycles of magnetisation. Due to this, some energy is lost and is called hysteresis loss. This loss depends on the area of the hysteresis loop. To reduce this loss, the hysteresis loop should be having small area. So we use soft iron as transformer core.

** *Magnetic flux leakage loss*: A part of the flux produced by the primary current goes into air without linking with the secondary. Thus some of the flux goes waste and this will cause loss of energy. If two coils are wound over the same iron core taken in the form of a ring, this energy loss can be minimised.

Q19. Calculate the resonant frequency and quality factor for a series LCR circuit having 5 H inductor, 500 μF capacitor and 10 ohm resistor. [3.185 Hz; 10]

Q20. Find the power of a series LCR circuit containing 0.1 H inductor, 200 mF capacitor and 50 ohm resistor and working on 240 V, 50 Hz ac mains. [1050.9W]

Q21. A transformer used in 240 V main has 4000 turns primary and 100 turns secondary. What is the output voltage? If 2 A current flows through the secondary, find the current through the primary. [6V; 0.05 A]

Q22. The voltage in primary and secondary of a step up transformer are 100 V and 1000 V. If the primary and secondary currents are 10 A and 0.7 A. find the efficiency of the transformer. [70%]

ANALOGIES BETWEEN MECHANICAL AND ELECTRICAL QUANTITIES

Mechanical system	Electrical system
Mass m	Inductance L
Force constant k	Reciprocal capacitance $1/C$
Displacement x	Charge q
Velocity $v=\frac{dx}{dt}$	Current $i=\frac{dq}{dt}$
Mechanical energy	Electromagnetic energy
$E=\frac{1}{2}kx^2+\frac{1}{2}mv^2$	$E=\frac{1}{2}\frac{q^2}{C}+\frac{1}{2}LI^2$