CHAPTER -3
CURRENT ELECTRICITY

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Electric current

- It is the rate of flow of electric charge.
- The instantaneous current is given by

$$I = \frac{dq}{dt}$$

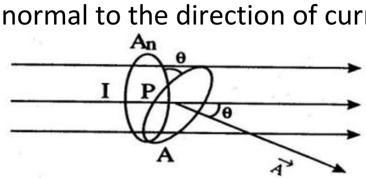
- Steady current is given by

$I = \frac{q}{t}$, q – charge, t- time

- Electric current is a scalar quantity.
- SI unit – ampere (A)
- 1A= 1C/s
- Other units are mA= 10^{-3} A, μA =10^{-6} A
- Lightning is an example of transient current.
- Current in a domestic appliance is of the order of 1A.
- Current carried by lightning is of the order of 10^4 A.
- Current in our nerves is of the order of 1 μA.
- By convention direction of motion of positive charges (direction opposite to the motion of electrons) is taken as the direction of current.

Current density (J)

- Current flowing through a unit area held normal to the direction of current.



- Current density is given by

$$J = \frac{I}{A\cos\theta}$$

- Where A –area of cross section, θ – angle between direction current and area.
- If the area is normal to the current flow, θ=0, thus $J = \frac{I}{A}$
- Unit – A/m^2 and dimensions are [AL^{-2}]
- Current density is a vector quantity.
- Also $I = JA\cos\theta = \vec{J}.\vec{A}$

Mechanism of current flow in conductors

- Metals have large number of free electrons nearly 10^{28}electrons / cm^3.
- In the absence of an electric field electrons are in random motion due to thermal energy.
- The average thermal velocity of electrons is zero.
- In the presence of an external electric field, electrons are accelerated and acquire an average velocity.
- During the random motion, electrons collide with each other or with positive metal ions.

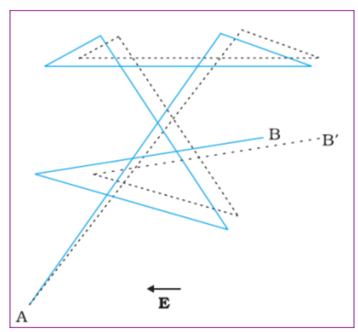
Drift velocity

- Average velocity acquired by an electron in the presence of an electric field.

Relaxation time

- Average time interval between two successive collisions.

Path of an electron



Relation connecting drift velocity and relaxation Time

- The force experienced by the electron in an electric field is
$F = -eE$, where E – electric field
- From Newton's second law $F = ma$,
a- acceleration, m- mass
- Thus, $ma = -eE$
- Therefore acceleration of electron is

$$a = \frac{-eE}{m}$$

- If an electron accelerates, the velocity attained is given by
$v_1 = u_1 + a\tau_1$, u_1- initial velocity, τ_1- time
- Similarly

$$v_2 = u_2 + a\tau_2$$
$$v_3 = u_3 + a\tau_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$v_N = u_N + a\tau_N$$

- Thus the average velocity (drift velocity) is given by

$$v = \frac{v_1 + v_2 + ... + v_N}{N}$$

$$v = \frac{u_1 + a\tau_1 + u_2 + a\tau_2 + ... + u_N + a\tau_N}{N}$$

$$v = \frac{u_1 + u_2 + ... + u_N}{N} + \frac{a(\tau_1 + \tau_2 + ... + \tau_N)}{N}$$

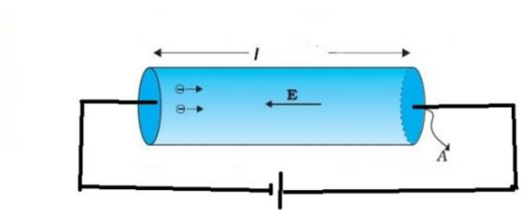
$$v = 0 + a\tau = a\tau$$

- Therefore the drift velocity is given by

$$v = -\frac{eE\tau}{m}$$

- Where , τ – relaxation time

Relation connecting drift velocity and current



- The number of electrons in the length l of the conductor = nAl
- Where n- electron density (number of electrons per unit volume) , A – area of cross section.
- Thus total charge $q = nAle$, e – charge of electron
- The electron which enter the conductor at the right end will pass through the conductor at left end in time

  $t = \frac{l}{v}$, v- drift velocity of electrons

- Thus the current, $I = \frac{q}{t} = \frac{nAle}{l/v} = nAve$
- That is

$$I = nAve$$

n-electron density, A –area, v- drift velocity, e- electron charge

- The current density $J = \frac{I}{A} = \frac{nAve}{A} = nve$

Mobility (μ)

- Ratio of magnitude of drift velocity to the electric field.

$$\mu = \frac{\frac{eE\tau}{m}}{E} = \frac{e\tau}{m}$$

- SI unit of mobility is $CmN^{-1}s^{-1}$

Difference between emf and potential difference

emf	Potential Difference
The difference in potential between the terminals of a cell, when no current is drawn from it.	The difference in potential between the terminals of a cell or between any two points in a circuit when current is drawn from the cell.
Exists only between the terminals of the cell.	Exists throughout the circuit.
It is the cause	It is the after effect.
Always greater than potential difference	Always less than emf

Ohm's law

- At constant temperature the current flowing through a conductor is directly proportional to potential difference between the ends of the conductor.
- Thus $V = IR$,

  V- potential difference, I – current, R- resistance

Resistance

- Ability of conductor to oppose electric current.

$$R = \frac{V}{I}$$

- SI unit – **ohm (Ω)**

Factors affecting resistance of a conductor

- Nature of material
- Proportional to length of the conductor
- Inversely proportional to area of cross section.
- Proportional to temperature

Relation connecting resistance and resistivity

$$R = \frac{\rho l}{A}$$

Where ρ- resistivity, A – area, l- length

Resistivity (specific resistance)

- Resistivity of the material of a conductor is defined as the resistance of the conductor having unit length and unit area of cross section.

$$\rho = \frac{RA}{l}$$

- Unit – **ohm meter (Ωm)**
- Resistivity of conductor depends on **nature of material** and **Temperature**

Conductance (G)

- Reciprocal of resistance

$$G = \frac{1}{R}$$

- Unit- **Ω^{-1}, or mho or siemens (S)**

Conductivity (σ)

- Reciprocal of resistivity

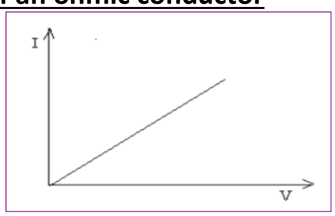
$$\sigma = \frac{1}{\rho}$$

- Unit- **$\Omega^{-1}m^{-1}$, or mho m^{-1}, or S m^{-1}**

Ohmic conductor

- A conductor which obeys ohm's law.
- Eg:- metals

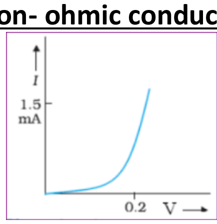
V-I graph of an ohmic conductor



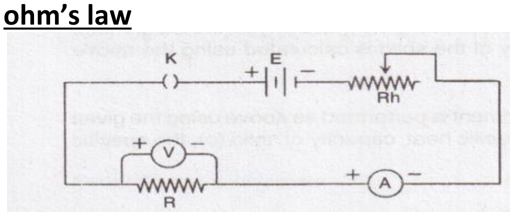
Non ohmic conductors

- Conductor which does not obey ohm's law.
- Eg :- diode, transistors, electrolytes etc.

V-I graph of a non- ohmic conductor (Diode)

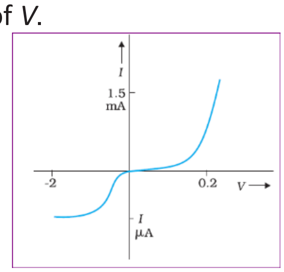


Circuit diagram for the experimental study of ohm's law

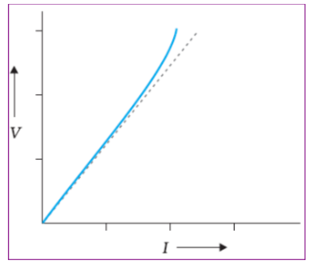


Limitations of ohm's law

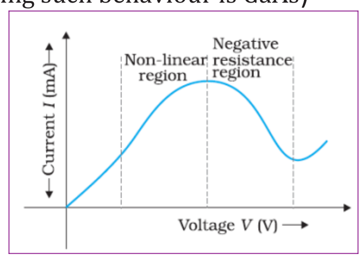
- The relation between *V* and *I* depends on the sign of *V*.



- It is not a universal law.
- The relation between V and I is not linear.



- The relation between *V* and *I* is not unique, i.e., there is more than one value of *V* for the same current I (A material exhibiting such behaviour is GaAs)



Vector form of ohm's law

- We have $V = El$
- From ohm's law, $V = IR = \frac{I\rho l}{A}$
- Thus $El = \frac{I\rho l}{A}$
- That is $E = \frac{I\rho}{A} = \rho J$
- Therefore $\vec{E} = \rho \vec{J}$ or $\vec{J} = \sigma \vec{E}$

Classification of materials in terms of resistivity

- **Conductors**
  Resistivity between 10^{-8} Ωm and 10^{-6} Ωm
- **Semiconductors**
  Resistivity between 10^{-6} Ωm and 10^{4} Ωm
- **Insulators**
  Resistivity > 10^{4} Ωm

Relation connecting resistivity and relaxation time

- We have the drift velocity $v = \frac{eE\tau}{m}$
- Using $E = \frac{V}{l}$, we get

$$v = \frac{eV\tau}{ml}$$

- Substituting v in $I = nAve$

$$I = \frac{nAe^2\tau V}{ml}$$

- That is $\frac{V}{I} = \frac{ml}{nAe^2\tau}$

- From ohm's law $R = \frac{V}{I}$
- Therefore $R = \frac{ml}{nAe^2\tau}$
- Comparing with the equation $R = \frac{\rho l}{A}$
- Resistivity , $\rho = \frac{m}{ne^2\tau}$

Copper is used as for making connecting wires

- Copper has low resistivity.

Nichrome is used as heating element of electrical devices

- Nichrome has High resistivity
- High melting point.

Why materials like constantan and manganin are used to make standard resistances?

- Resistance does not change with temperature.
- Material has high resistivity.

Resistors

- The **resistor** is a passive electrical component to create resistance in the flow of electric current.

Symbol

Constant resisstance **Variable resistance**

Commercial resistors

Wire bound resistors

- Made by winding the wires of an alloy, like, manganin , constantan, nichrome or similar ones, around a ceramic, plastic, or fiberglass core.
- They are relatively insensitive to temperature.
- Large length is required to make high resistance.

Carbon resistors

- Made from a mixture of carbon black, clay and resin binder.
- Are enclosed in a ceramic or plastic jacket.
- Carbon resistors are small in size , and inexpensive.

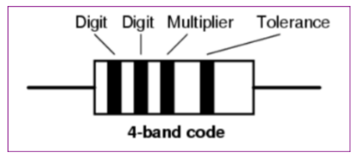
Colour code of resistors

Colour	Digit	Multiplier	Tolerance
Black	0	10^0	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold		10^{-1}	±5%
Silver		10^{-2}	±10%
No color			±20%

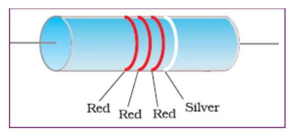
0 1 2 3 4 5 6
B B ROY of Great Britain

7 8 9
has a Very Good Wife

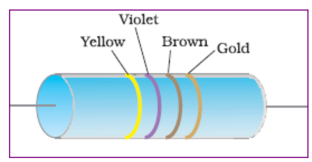
4 - Band code



- **Example**



Resistance = **(22 × 10^2 Ω) ± 10%**



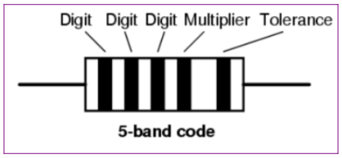
Resistance = **(47 × 10 Ω) ± 5%**

5 - Band code

Color	Digit	Multiplier	Tolerance (%)
Black	0	10^0 (1)	
Brown	1	10^1	1
Red	2	10^2	2
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	0.5
Blue	6	10^6	0.25
Violet	7	10^7	0.1
Grey	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
(none)			20

- The colors brown, red, green, blue, and violet are used as tolerance codes on 5-band resistors only.

- All 5-band resistors use a colored tolerance band.



Temperature dependence of resistivity

- We have $\rho = \dfrac{m}{ne^2\tau}$
- When temperature is increased, average speed of the electrons increases and hence number of collision increases.
- Thus the average time of collisions τ, decreases with temperature.

Metals (Conductors)

- In a metal number of free electrons per unit volume does not depend on temperature.
- When temperature is increased, relaxation time decreases and hence resistivity increases.
- The temperature dependence of resistivity of a metallic conductor is given by

$$\rho_T = \rho_0[1 + \alpha(T - T_0)]$$

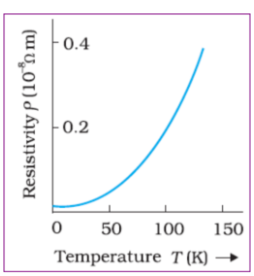
- Where **ρ_T** – resistivity at a temperature T, **ρ_0** - resistivity at a lower temperature T_0, α -temperature coefficient of resistivity.
- Thus for conductors resistivity increases with temperature.

Temperature coefficient of resistivity (α)

$$\alpha = \frac{(\rho_T - \rho_0)}{\rho_0(T - T_0)}$$

- Unit of α is $^0C^{-1}$.
- For metals α is positive.

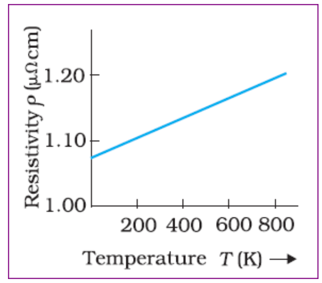
Temperature – resistivity graph (copper)



Alloys

- Resistivity of alloys like, Nichrome, Manganin, Constantan , is almost independent of temperature.
- Thus alloys are used to make wire bound resistors.

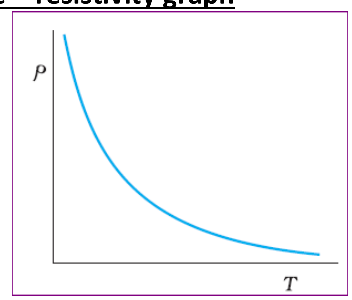
Temperature – resistivity graph (Nichrome)



Semiconductors

- The electron density (n) increases with temperature.
- Thus resistivity decreases with temperature.
- α is negative.

Temperature – resistivity graph

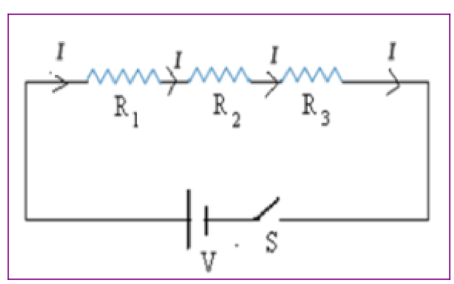


Insulators

- The electron density (n) increases with temperature.
- Resistivity decreases with temperature.

Combination of resistors

Resistors in Series



- In series connection same current pass through all resistors.
- The potential drop is different for each resistor.
- The applied potential is given by

$$V = V_1 + V_2 + V_3$$

- Where V_1, V_2 and V_3 are the potential drop across resistors R_1, R_2 and R_3 respectively.
- If all the resistors are replaced with a single effective resistance R_S , we get

$$V = IR_S$$

- Thus $IR_S = IR_1 + IR_2 + IR_3$
- Therefore the effective resistance is

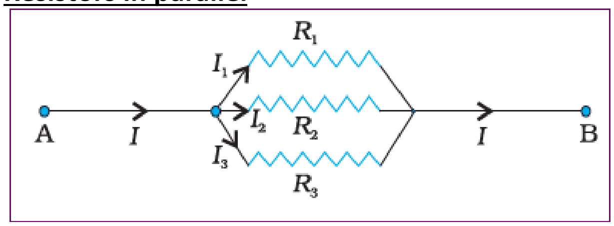
$$R_S = R_1 + R_2 + R_3$$

- For n resistors

$$R_S = R_1 + R_2 + R_3 + ...R_n$$

- Thus effective resistance increases in series combination.

Resistors in parallel



- In parallel connection current is different through each resistors.
- The potential drop is same for all resistors.
- The total current

$$I = I_1 + I_2 + I_3$$

- If all resistors are replaced with an effective resistor of resistance R_P, we get

$$I = \frac{V}{R_P}$$

- Thus

$$\frac{V}{R_P} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

- Therefore the effective resistance in parallel combination is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- For n resistors in parallel

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$$

- For two resistors

$$R_P = \frac{R_1 R_2}{R_1 + R_2}$$

- Thus effective resistance decreases in parallel combination.

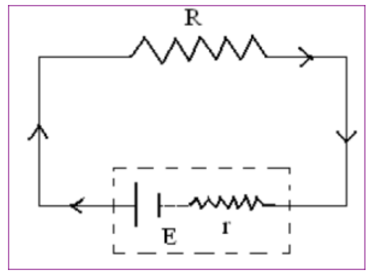
Internal resistance of a cell (r)

- Resistance offered by the electrolytes and electrodes of a cell.

Factors affecting internal resistance

- Nature of electrolytes
- Directly proportional to the distance between electrodes
- Directly proportional to the concentration of electrolytes.
- Inversely proportional to the area of the electrodes.
- Inversely proportional to the temperature of electrolyte.

Relation connecting emf and internal resistance



- Effective resistance = R+r
- Thus the current is $I = \frac{\varepsilon}{R + r}$
- Where ε –emf, R- external resistance, r- internal resistance.
- That is $I(R + r) = \varepsilon \Rightarrow IR + Ir = \varepsilon$
- From ohm's law, V=IR, therefore

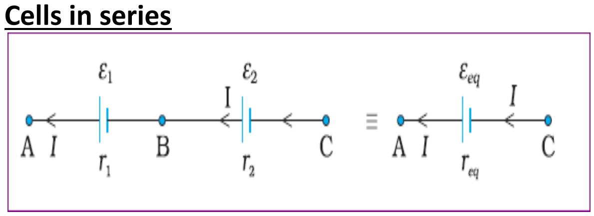
$$r = \frac{\varepsilon - V}{I}$$

- The potential is given by

$$V = \varepsilon - Ir$$

Combination of cells

Cells in series



- In series connection current is same; the potential difference across the cells is different.
- The potential difference across the first cell is $V_{AB} = \varepsilon_1 - Ir_1$
- Similarly $V_{BC} = \varepsilon_2 - Ir_2$
- Thus total potential across AC is

$$V_{AC} = V_{AB} + V_{BC}$$

- That is $V_{AC} = \varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2$

$$V_{AC} = (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)$$

- If the two cells are replaced with a single cell of emf ε_{eq} and internal resistance r_{eq}, we have , $V_{AC} = \varepsilon_{eq} - Ir_{eq}$
- Comparing the equations we get

$$\varepsilon_{eq} = (\varepsilon_1 + \varepsilon_2)$$

$$r_{eq} = (r_1 + r_2)$$

- For n cells in series

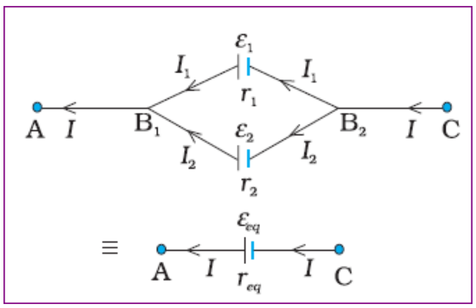
$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_n$$

$$r_{eq} = r_1 + r_2 + ... + r_n$$

- If the negatives of the cells are connected together

$$\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2 \quad (\varepsilon_1 > \varepsilon_2)$$

<u>Cells in parallel</u>



- In parallel connection current is different and potential is same.
- For the first cell , $V = \varepsilon_1 - I_1 r_1$
- Thus $I_1 = \dfrac{\varepsilon_1 - V}{r_1}$
- Similarly for the second cell, $V = \varepsilon_2 - I_2 r_2$

$$I_2 = \frac{\varepsilon_2 - V}{r_2}$$

- The total current is given by

$$I = I_1 + I_2$$

$$I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

- That is

$$I = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2}\right) - I$$

- Thus $V = \left(\dfrac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}\right) - I\left(\dfrac{r_1 r_2}{r_1 + r_2}\right)$
- Comparing this with the equation $V = \varepsilon_{eq} - Ir_{eq}$, we get

$$\varepsilon_{eq} = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}\right)$$

$$r_{eq} = \left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

- Or

$$\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} \qquad \frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}$$

- If the negative terminal of the second is connected to positive terminal of the first, the equations are valid with $(\varepsilon_2 \rightarrow -\varepsilon_2)$
- For n cells in parallel

$$\boxed{\frac{1}{r_{eq}} = \frac{1}{r_1} + \cdots + \frac{1}{r_n} \qquad \frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \cdots + \frac{\varepsilon_n}{r_n}}$$

<u>Joule's law of heating</u>

- The heat energy dissipated in a current flowing conductor is given by

$$H = I^2 Rt$$

- I- current, R –resistance, t –time

<u>Electric power</u>

- It is the energy dissipated per unit time.
- Power , $P = \dfrac{H}{t} = I^2 R$
- Also $P = VI = \dfrac{V^2}{R}$
- SI unit is **watt (W)**
- 1 kilo watt (1kW) = 1000W
- 1mega watt (MW) = 10^6W
- Another unit horse power (hp)
- 1 hp = 746 W

<u>Electrical energy</u>

- Electrical energy = electrical power X time
- SI unit – joule (J)
- Commercial unit – kilowatt hour (kWh)
- 1kWh = 3.6 x 10^6 J.

<u>Efficiency</u>

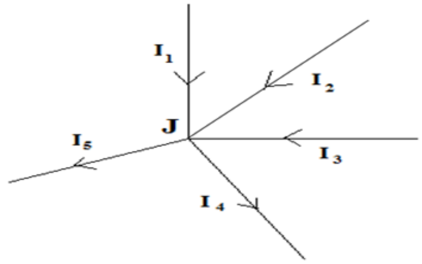
- The efficiency of an electrical device is

$$\eta = \frac{output\ power}{input\ power}$$

<u>Kirchhoff's rule</u>

First rule (junction rule or current rule)

- Algebraic sum of the current meeting at junction is zero.
- Thus , Current entering a junction = current leaving the junction



$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

Sign convention

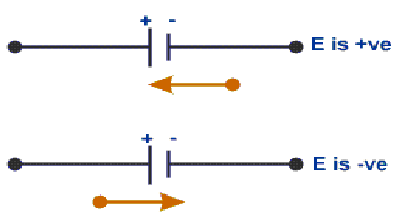
- Current entering the junction – positive
- Current leaving the junction - negative

Second rule (loop rule or voltage rule)

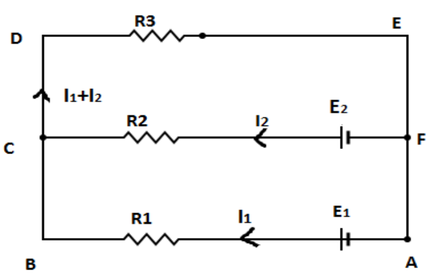
- Algebraic sum of the products of the current and resistance in a closed circuit is equal to the net emf in it.
- This rule is a statement of law of conservation of energy.

Sign convention

- Current in the direction of loop – positive
- Current opposite to loop - negative



Illustration



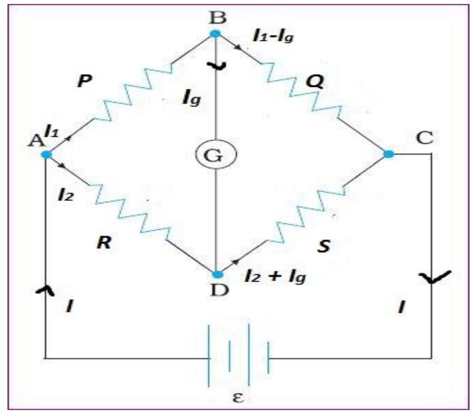
Loop ABCFA

$$I_1R_1 - I_2R_2 = E_1 - E_2$$

Loop CDEFC

$$I_2R_2 + (I_1 + I_2)R_3 = E_2$$

Wheatstone' s bridge



Wheatstone' s principle

- If galvanometer current is zero, $\frac{P}{Q} = \frac{R}{S}$

Derivation of balancing condition

- Applying voltage rule to the loop ABDA

$$I_1P + I_gG - I_2R = 0$$

- For the loop BCDB

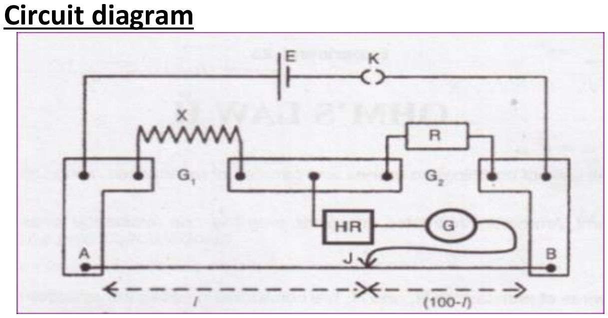
$$(I_1 - I_g)Q - (I_2 + I_g)S - I_gG = 0$$

- When the bridge is balanced I_g=0.
- Thus $I_1P - I_2R = 0$ and $I_1Q - I_2S = 0$
- Or , $I_1P = I_2R$ and $I_1Q = I_2S$
- Thus $\frac{P}{Q} = \frac{R}{S}$
- This is the balancing condition of a Wheatstone bridge.

Meter bridge (slide wire bridge)

- Works on Wheatstone's principle.
- Used to find resistance of a wire.

Circuit diagram



- Where k – key, X – unknown resistance, R- known resistance, HR- high resistance, G – Galvanometer, J – Jockey

Equation to find unknown resistance

- From wheatstone's principle

$$\frac{P}{Q} = \frac{R}{S}$$

- Here P – unknown resistance , Q- known resistance, R- resistance of the wire of length *l* , S - resistance of wire of length (100-*l*).

- The length l for which galvanometer shows zero deflection – balancing length.
- Thus

$$\frac{X}{R} = \frac{lr}{(100-l)r}$$

- Where r – resistance per unit length of the meterbridge wire.
- Therefore the unknown resistance is given by

$$X = \frac{Rl}{(100-l)}$$

- The resistivity of the resistance wire can be calculated using the formula

$$\rho = \frac{\pi r^2 X}{l}$$

Where r – radius of the wire, l –length of the wire.

Potentiometer

- A device used to measure an unknown emf or potential difference accurately.

Principle

- When a steady current (I) flows through a wire of uniform area of cross section, the potential difference between any two points of the wire is directly proportional to the length of the wire between the two points.
- From ohm's law , $V = IR$
- That is . $V = \frac{I\rho l}{A}$
- Therefore , $V \alpha l$ or $V = kl$
- Thus $\frac{V}{l} = k$, where k – constant.

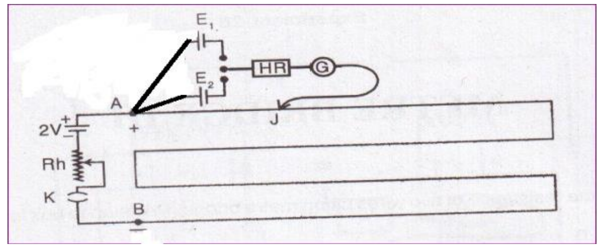
$\frac{V}{l}$ – potential gradient.

Uses of potentiometer

- To compare the emf of two cells
- To find the internal resistance of a cell

Comparison of emfs

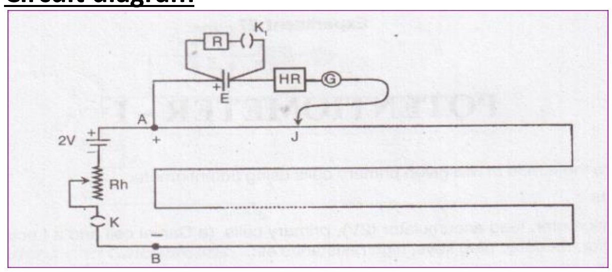
Circuit diagram



- We have, $E_1 \propto l_1$ and $E_2 \propto l_2$
- Thus $\frac{E_1}{E_2} = \frac{l_1}{l_2}$
- l_1- balancing length with cell E_1
- l_2- balancing length with cell E_2
- To get the balancing length $E_1 > E_2$

To find internal resistance

Circuit diagram



- when the key K_1 is open

$$\varepsilon \propto l_1$$

- when the key K_1 is closed

$$V \propto l_2$$

- Thus $\frac{\varepsilon}{V} = \frac{l_1}{l_2}$
- But we have

$$V = IR$$

$$\varepsilon = I(R+r).$$

r – internal resistance

- Therefore $\frac{\varepsilon}{V} = \frac{I(R+r)}{IR} = \frac{(R+r)}{R}$
- Thus $\frac{(R+r)}{R} = \frac{l_1}{l_2}$
- The internal resistance is given by

$$r = \frac{R(l_1 - l_2)}{l_2}$$

- Where l_1- balancing length, key K_1open, l_2- balancing length, key K_1 closed.

Why potentiometer is preferred over voltmeter for measuring emf of a cell?

- In potentiometer **null method** is used, so no energy loss in measurement.

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