## CHAPTER 2

## ELECTRICAL POTENTIAL \& CAPACITANCE

1. Define electric potential at a point.
Ans: Electric potential at a point is defined as the work done to bring a unit positive charge from infinity to that point.

$$
V=\frac{W}{q}
$$

2. Derive an expression for electric potential at a point due to a point charge.
Ans:


By definition electric potential at a point P is the work done to bring a +1 C charge from infinity to the point P.

Let the +1 C charge is at A ; The work done to move it from A to B
$\mathrm{dW}=\mathrm{E} .(-\mathrm{dx})$ [ Since displacement is against force]

$$
\begin{equation*}
=-E d x \tag{1}
\end{equation*}
$$

But $\mathrm{E}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{q}}{\mathrm{x}^{2}}$
Substituting (2) in (1)

$$
\mathrm{dW}=-\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{q}}{\mathrm{x}^{2}} \mathrm{dx}
$$

$\therefore$ The total work done to bring the +1 C charge from $\infty$ to P
$W=\int_{\infty}^{\mathrm{r}} \mathrm{d} W$
$=\int_{\infty}^{\mathrm{r}}-\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{q}}{\mathrm{x}^{2}} \mathrm{dx}$
$=\frac{-\mathrm{q}}{4 \pi \varepsilon_{0}} \int_{\infty}^{\mathrm{r}} \frac{1}{\mathrm{x}^{2}} \mathrm{dx}$
$=\frac{-\mathrm{q}}{4 \pi \varepsilon_{0}}\left[\frac{-1}{\mathrm{x}}\right]_{\infty}^{\mathrm{r}}$
$=\frac{\mathrm{q}}{4 \pi \varepsilon_{0}}\left[\frac{1}{\mathrm{x}}\right]_{\infty}^{\mathrm{r}}$
$=\frac{\mathrm{q}}{4 \pi \varepsilon_{0}}\left[\frac{1}{\mathrm{r}}-\frac{1}{\infty}\right]$
$=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{q}}{\mathrm{r}}$
ie, electric potential, $\mathrm{V}=\frac{\mathrm{q}}{4 \pi \varepsilon_{0}} \cdot \frac{1}{\mathrm{r}}$
Note: Potential is a scalar quantity.SI unit of electric potential is $\mathbf{J} / \mathbf{C}$ or volt (V)
3. Is electric potential a vector or a scalar?

Ans: Scalar
4. Draw the graph showing the variation of ' $V$ ' and ' $E$ ' with distance r.

Ans: $\mathrm{E} \alpha \frac{1}{\mathrm{r}}$ and $\mathrm{V} \alpha \frac{1}{\mathrm{r}^{2}}$ Since $E \propto \frac{1}{r^{2}}$, it decreases suddenly with distance.

$5[\mathrm{P}]$. Two charges $5 \times 10^{-8} \mathrm{C}$ and $3 \times 10^{-8} \mathrm{C}$ are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero.

6[P]. A regular hexagon of side
10 cm has a charge $5 \mu \mathrm{C}$ at each of its vertices. Calculate the potential at the centre of the hexagon.

7[P]. A cube of side ' $a$ ' has a charge ' $q$ ' at each of its vertices. Determine the potential and electric field at the centre of the cube?
8. Define electric potential difference.
Ans: Potential difference between two points is defined as the work done to bring a unit +ve charge from one point to another.


Potential at A,
$\mathrm{V}_{1}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{q}}{\mathrm{r}_{1}}$
Potential at B,
$\mathrm{V}_{2}=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{q}}{\mathrm{r}_{2}}$
$\therefore$ Potential difference between A and B $\quad=V_{1}-V_{2}$

$$
=\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{q}}{\mathrm{r}_{1}}-\frac{1}{4 \pi \varepsilon_{0}} \cdot \frac{\mathrm{q}}{\mathrm{r}_{2}}
$$

9. Derive the relation between electric field and potential.

Ans: We know that the potential difference between A and B is the work done, to move +1 C charge from A to B.

$\mathrm{dV}=\mathrm{dw}=-\mathrm{Edr}$
i.e, $d V=-E d r$
$\therefore \mathbf{E}=-\frac{\mathrm{dV}}{\mathrm{dr}}$
ie, electric field is the negative gradient of electrostatic potential.
10. Derive an expression for the potential due to an electric dipole.
Ans: Consider an electric dipole having a dipole moment $\vec{p}=q \times 2 a p$ .We have to find the electric potential at a point P , distant ' $r$ ' from the mid point of the dipole.


Electric potential at P due to the +q charge
$\mathrm{V}_{+}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}}{\mathrm{r}_{1}}$

Electric potential at P due to the -q charge
$\mathrm{V}_{-}=\frac{1}{4 \pi \varepsilon_{0}} \frac{-\mathrm{q}}{\mathrm{r}_{2}}$
$\therefore$ Total electric potential at P ,
$\mathrm{V}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}}{\mathrm{r}_{1}}-\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}}{\mathrm{r}_{2}}$
From figure, $\mathrm{r}_{1}=\mathrm{r}-\mathrm{OC}$

$$
=r-a \cos \theta
$$

From figure, $r_{2}=r+O D$
$=r+a \cos \theta$
$\therefore$ Substituting in equation (1)
$V=\frac{q}{4 \pi \varepsilon_{0}}\left[\frac{1}{r-a \cos \theta}-\frac{1}{r+a \cos \theta}\right]$
$=\frac{\mathrm{q}}{4 \pi \varepsilon_{0}}\left[\frac{\mathrm{r}+\mathrm{a} \cos \theta-(\mathrm{r}-\mathrm{a} \cos \theta)}{\mathrm{r}^{2}-\mathrm{a}^{2} \cos ^{2} \theta}\right]$
$=\frac{\mathrm{q}}{4 \pi \varepsilon_{0}}\left[\frac{2 \mathrm{a} \cos \theta}{\mathrm{r}^{2}-\mathrm{a}^{2} \cos ^{2} \theta}\right]$
If $r^{2} \gg a^{2}, a^{2}$ can be neglected.

$$
\begin{array}{r}
\therefore \mathrm{V}=\frac{\mathrm{q}}{4 \pi \varepsilon_{\mathrm{o}}}\left[\frac{2 \mathrm{a} \cos \theta}{\mathrm{r}^{2}}\right] \\
\mathrm{V}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{P} \cos \theta}{\mathrm{r}^{2}}
\end{array}
$$

## Special cases

Potential at a point on the axial
line
Put $\theta=0^{0}$
$\mathrm{V}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{P} \cos 0}{\mathrm{r}^{2}}$
$=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{p} \times 1}{\mathrm{r}^{2}}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{p}}{\mathrm{r}^{2}}$
Potential at a point on the equatorial line
Put $\theta=90^{\circ}$
$\mathrm{V}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{P} \cos 90}{\mathrm{r}^{2}}$
$=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{p} \times 0}{\mathrm{r}^{2}}=0$
Note: The equatorial plane of the dipole is an equipotential surface having a potential zero.
11. Define one Electron Volt (eV). Give its relation with joule.
Ans: Electron volt is a smaller unit of energy.
1 eV is defined as the energy acquired by an electron, when it is accelerated through a p.d. of 1V.
$\mathrm{W}=\mathrm{qV} \Rightarrow 1 \mathrm{eV}=1.6 \times 10^{-19} \times 1 \mathrm{~J}$
$=1.6 \times 10^{-19} \mathrm{~J}$

$$
1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{Joule}
$$

12. Define potential energy of a system of charges.

Ans: Potential energy of a system of charges is the work done to bring the charges from infinity to their present positions.
13. Derive expressions for potential energy of (i) a single charge (ii) a two charge system in an external electric field.
Ans:
Potential energy of a single charge
$\qquad$
$\qquad$

Let $\mathrm{V}(\mathrm{r})$ the potential at a point due to an external e.f. $\overrightarrow{\mathbf{E}}$.
The potential energy of $q$ at that point, $\mathrm{PE}=\mathrm{W}=\mathrm{qV}(\mathrm{r})$

PE of a system of two charges in an external e.f.

$\qquad$

PE of the system of charges is the total work done to assemble the charges from infinity.
Work done to bring $\mathrm{q}_{1}=\mathrm{q}_{1} \mathrm{~V}\left(\mathrm{r}_{1}\right)$
Work done to bring $\mathrm{q}_{2}=\mathrm{q}_{2} \mathrm{~V}\left(\mathrm{r}_{2}\right)+$ $\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}_{12}}$
$\therefore \mathrm{PE}$ of the system $=\mathrm{q}_{1} \mathrm{~V}(\mathrm{r})+\mathrm{q}_{2} \mathrm{~V}\left(\mathrm{r}_{2}\right)+$ $\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}_{12}}$
14. Derive expressions for potential energy of a (i) two charges system (ii) three charge system, in the absence of external electric field.
Ans:
P.E. of a system of two charges


Work done to bring $\mathrm{q}_{1}=0$
The work done to bring $\mathrm{q}_{2}$ to the point $B$ from infinity in presence of $q_{1}$ is
$=$ Potential at $B$ due to $q_{1}$ charge $\times \mathrm{q}_{2}$
$=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathbf{q}_{1}}{\mathbf{r}} \times \mathbf{q}_{2}$
$\mathrm{W}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathbf{q}_{1} \mathbf{q}_{2}}{\mathrm{r}}$
P.E. $=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}}$

## P.E. of a system of three charges



Total potential energy of this system =
$\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}_{12}}+\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}_{2} \mathrm{q}_{3}}{\mathrm{r}_{23}}+\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}_{1} \mathrm{q}_{3}}{\mathrm{r}_{13}}$
$\mathrm{PE}=\frac{1}{4 \pi \varepsilon_{0}}\left[\frac{\mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}_{12}}+\frac{\mathrm{q}_{2} \mathrm{q}_{3}}{\mathrm{r}_{23}}+\frac{\mathrm{q}_{1} \mathrm{q}_{3}}{\mathrm{r}_{13}}\right]$
15. Derive an expression for the work done in rotating a dipole in an external electric field.

## Ans:

Consider a dipole placed in a uniform electric field at an angle ' $\theta$ ' with the electric field.


Work done in rotating the dipole through an angle $d \theta$
$\mathrm{dW}=\tau \mathrm{d} \theta$
But $\tau=\operatorname{PES} \operatorname{Sin} \theta$
$\mathrm{dw}=\mathrm{PE} \sin \theta \cdot \mathrm{d} \theta$
$\therefore$ The work done for rotating the dipole from an angle $\theta_{1}$ to an angle $\theta_{2}$.
$\mathrm{w}=\int_{\theta_{1}}^{\theta_{2}} \tau d \theta$
$=\int_{\theta_{1}}^{\theta_{2}} \mathrm{PE} \sin \theta \mathrm{d} \theta$
$=\mathrm{PE} \int_{\theta_{1}}^{\theta_{2}} \sin \theta \mathrm{~d} \theta=\mathrm{PE}[-\cos \theta]_{\theta_{1}}^{\theta_{2}}$
$=-\operatorname{PE}[\cos \theta]_{\theta_{1}}^{\theta_{2}}$
$=-\mathrm{PE}\left[\cos \theta_{2}-\cos \theta_{1}\right]$
$=\mathrm{PE}\left[\cos \theta_{1}-\cos \theta_{2}\right]$
If the dipole is rotated by an angle $\theta$ from stable equilibrium position, $\theta_{1}=$ 0 and $\theta_{2}=\theta$

$$
W=P E[1-\cos \theta]
$$

## Special cases:

Case I :- when $\theta=0^{0}$
Work done $\mathrm{w}=\mathrm{PE}[1-\cos 0]$
$=\mathrm{PE}[1-1]$
$=\mathrm{PE} \times 0=0$

Case II:- When $\theta=90^{\circ}$
Work done $\mathrm{W}=\mathrm{PE}[1-\cos 90]$
$=\mathrm{PE}[1-0]$
$\mathrm{W}=\mathrm{PE}$

Case III:- When $\theta=180^{\circ}$
Work done $\mathrm{W}=\mathrm{PE}\left[1-\cos 180^{\circ}\right]$
$=\mathrm{PE}[1-(-1)]$
= PE [2]
$\mathrm{W}=\mathbf{2 P E}$
This is the maximum work done and also the maximum potential energy.
16. Derive an expression for the potential energy of an electric dipole in an electric field.
Ans: Let - PE be the initial potential energy of the dipole when it is in stable equilibrium (for convenience).
$\therefore$ The total potential energy, when the dipole is rotated by an angle $\theta^{0}$.
$\mathrm{U}=\mathrm{U}_{0}+\mathrm{W}$
$=-\mathrm{PE}+\mathrm{PE}(1-\cos \theta)$
$=-\mathrm{PE}+\mathrm{PE}-\mathrm{PE} \cos \theta$
$=-\mathrm{PE} \cos \theta$
$=-\overrightarrow{\mathrm{P}} \cdot \overrightarrow{\mathrm{E}}$
$\mathrm{U}=-\mathrm{PE} \cos \theta$

## Case I

When $\theta=0^{0}$ (Stable equilibrium)
$\mathrm{U}=-\mathrm{PE} \cos 0=-\mathrm{PE}$
$\mathrm{U}=-\mathrm{PE}$ (minimum)

## Case II

When $\theta=90^{\circ}$
$\mathrm{U}=-\mathrm{PE} \cos 90^{\circ}=-\mathrm{PE} \times 0$
$\mathrm{U}=0$
Case III
When $\theta=180^{\circ}$
$=-\mathrm{PE} \times-1$
$\mathrm{U}=\mathrm{PE}$
[Maximum potential energy]
Therefore, unstable equilibrium.

## CAPACITORS

17. What is the use capacitor? Define capacitance.
Ans: It is a device used to store electric charge.
Capacitance or capacity (C)
It is the ability to capacitance to store electric charge
Capacitance $C=\frac{Q}{V}$
Q - charge
V - potential
18. What is the SI unit capacitance?

Ans: SI unit of capacitance is $\mathbf{C / V}$ or farad (F)

## 19. Define one farad

Ans: Capacitance of a capacitor is said to be one farad if one coulomb of charge raises its potential by one volt.
20. Explain the principle of a parallel plate capacitor.
Ans:


Consider a positively charged plate $\mathrm{P}_{1}$. If another plate $\mathrm{P}_{2}$ with no charge, is brought near to $P_{1}$ (and placed without touching), then on the inside of the plate negative charges are induced and on the outside positive charges are induced. If the second plate is earthed all the positive charges, will flow to earth. Now due to the presence of negative charges on the plate $\mathrm{P}_{2}$, the potential (V) of $\mathrm{P}_{1}$ decreases. Therefore, by equation $C=\frac{Q}{V}$
When potential decreases capacitance increases. This is the principle of capacitor.
21. Derive an expression for the capacitance of a parallel plate capacitor.
Ans:
Consider a parallel plate air capacitor having plate area ' $A$ ' and charge density $\sigma$.
Charge on a plate, $\mathrm{Q}=\sigma \mathrm{A}$

$$
\mathrm{V}=\mathrm{Ed}
$$



We know, capacitance $\mathrm{C}=\frac{\mathrm{Q}}{\mathrm{V}}$

$$
\begin{gathered}
=\frac{\mathrm{A} \sigma}{\left[\frac{\sigma \mathrm{~d}}{\varepsilon_{0}}\right]}=\mathrm{A} \sigma \times \frac{\varepsilon_{0}}{\sigma \mathrm{~d}}=\frac{\varepsilon_{0} \mathrm{~A}}{\mathrm{~d}} \\
\mathrm{C}_{\mathbf{a i r}}=\frac{\varepsilon_{0} \mathrm{~A}}{\mathrm{~d}}
\end{gathered}
$$

This is the expression for capacitance of a capacitor with air as the medium between the plates.
22. What happens if a dielectric material is introduced between the plates of the capacitor?
Ans:


If a dielectric material is introduced between the plates of the capacitor, the capacitance becomes
$\mathrm{C}_{\text {dielectric }}=\frac{\mathrm{k} . \varepsilon_{0} \mathrm{~A}}{\mathrm{~d}} \quad \mathrm{k}$ - dielectric const.

When the dielectric is introduced in the region between the plates, the capacitance increases $\mathbf{k}$ times
$\mathbf{C}_{\text {dielectric }}=\mathbf{k} . \mathbf{C}_{\text {air }} \quad \mathrm{k}=\varepsilon_{\mathrm{r}}$

23[P]. A parallel plate capacitor with air between the plates has a capacitance of $8 \mathrm{pF}\left(1 \mathrm{pF}=10^{-12} \mathrm{~F}\right)$. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant $6 ?$
24. Give the expression for capacitance of a parallel plate capacitor partially filled with a dielectric slab.


## Ans:

When a dielectric of relative permittivity $\quad \boldsymbol{\varepsilon}_{\mathbf{r}} \quad$ of thickness 't'is introduced partially between the plates of the capacitor.
Then capacity, $\mathbf{C}=\frac{\varepsilon_{0} \mathrm{~A}}{(\mathrm{~d}-\mathrm{t})+\frac{\mathrm{t}}{\varepsilon_{\mathrm{r}}}}$
$t=$ thickness of dielectric slab
$\mathrm{d}=$ distance between the plates of capacitor
25. What are the advantages of introducing dielectric slab between the plates of a capacitor.
Ans:
(i) The capacitance increases $\varepsilon_{r}$ times
(ii) Dielectric medium avoids sparking between the plates.

## 26. What are the different uses of a capacitor?

Ans: (i) To store charge
(ii) To generate electromagnetic radiation
(iii) To tune radio circuits
(iv) To reduce voltage fluctuation in power supply
27. Derive expressions for effective capacitance when capacitors are connected in (i) series and (ii) parallel.
Ans:
(i) Series: Consider 3 capacitors of capacitances $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}$ connected in series with a voltage V .
In a series circuit the charge stored in each of the capacitors is the same but the voltages across them are different.


Applied voltage,
$V=V_{1}+V_{2}+V_{3}$
We have $\mathrm{C}=\frac{\mathrm{q}}{\mathrm{V}}$
$\therefore \mathrm{V}=\frac{\mathrm{q}}{\mathrm{C}}$
But $V=\frac{\mathrm{q}}{\mathrm{C}_{\mathrm{s}}}$
$\mathrm{C}_{\mathrm{s}} \rightarrow$ effective capacitance (in series)
$\mathrm{V}_{1}=\frac{\mathrm{q}}{\mathrm{C}_{1}}, \quad \mathrm{~V}_{2}=\frac{\mathrm{q}}{\mathrm{C}_{2}} \quad \mathrm{~V}_{3}=\frac{\mathrm{q}}{\mathrm{C}_{3}}$
Eqn. (1) gives
$\frac{\mathrm{q}}{\mathrm{C}_{\mathrm{s}}}=\frac{\mathrm{q}}{\mathrm{C}_{1}}+\frac{\mathrm{q}}{\mathrm{C}_{2}}+\frac{\mathrm{q}}{\mathrm{C}_{3}}$
$\frac{\mathrm{q}}{\mathrm{C}_{\mathrm{s}}}=\mathrm{q}\left[\frac{1}{\mathrm{C}_{1}}+\frac{1}{\mathrm{C}_{2}}+\frac{1}{\mathrm{C}_{3}}\right]$
$\frac{1}{\mathrm{C}_{\mathrm{s}}}=\frac{1}{\mathrm{C}_{1}}+\frac{1}{\mathrm{C}_{2}}+\frac{1}{\mathrm{C}_{3}}$
If there are ' $n$ ' capacitors connected in series
$\frac{1}{\mathrm{C}_{\mathrm{s}}}=\frac{1}{\mathrm{C}_{1}}+\frac{1}{\mathrm{C}_{2}}+\frac{1}{\mathrm{C}_{3}}+\ldots \ldots \ldots .+\frac{1}{\mathrm{C}_{\mathrm{n}}}$
In the case of two capacitors
$\frac{1}{\mathrm{C}_{\mathrm{s}}}=\frac{1}{\mathrm{C}_{1}}+\frac{1}{\mathrm{C}_{2}}=\frac{\mathrm{C}_{2}+\mathrm{C}_{1}}{\mathrm{C}_{1} \mathrm{C}_{2}}$
$\frac{1}{\mathrm{C}_{\mathrm{s}}}=\frac{\mathrm{C}_{1}+\mathrm{C}_{2}}{\mathrm{C}_{1} \mathrm{C}_{2}}$

$$
C_{s}=\frac{C_{1} C_{2}}{C_{1}+C_{2}}
$$

(ii) In Parallel


Consider 3 capacitors $\mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}$ connected in parallel with a voltage 'V'.

In a parallel circuit, the voltage is the same but the charges stored in the capacitors are different.

Here the total charge

$$
\begin{equation*}
\mathrm{q}=\mathrm{q}_{1}+\mathrm{q}_{2}+\mathrm{q}_{3} \tag{1}
\end{equation*}
$$

$$
\text { But } C=\frac{q}{V} \quad q=C V
$$

$\mathrm{q}=\mathrm{C}_{\mathrm{p}} \mathrm{V}$

Where $C_{p}$ is the effective capacitance when the three capacitors are connected in parallel.
$\mathrm{q}_{1}=\mathrm{C}_{1} \mathrm{~V} \quad \mathrm{q}_{2}=\mathrm{C}_{2} \mathrm{~V} \quad \mathrm{q}_{3}=\mathrm{C}_{3} \mathrm{~V}$
(1) $\rightarrow \mathrm{C}_{\mathrm{p}} \mathrm{V}=\mathrm{C}_{1} \mathrm{~V}+\mathrm{C}_{2} \mathrm{~V}+\mathrm{C}_{3} \mathrm{~V}$
$\mathrm{C}_{\mathrm{p}} \mathrm{V}=\mathrm{V}\left(\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}\right)$

$$
\mathrm{C}_{\mathrm{p}}=\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}
$$

If there are ' $n$ ' capacitors connected in parallel
$\mathrm{C}_{\mathrm{p}}=\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}+\ldots \ldots \ldots .+\mathrm{C}_{\mathrm{n}}$
$28[P]$. Three capacitors each of capacitance, 9 pF are connected in series.
a) What is the capacitance of the combination?
b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

29[P]. Three capacitors of capacitances $2 \mathrm{pF}, 3 \mathrm{pF}$ and 4 pF are connected in parallel.
a) What is the capacitance of the combination?
b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.
$30[P]$. Find the effective capacitance of the capacitors given in the network.


31[P]. You are given two capacitors of $2 \mu \mathrm{~F}$ and $3 \mu \mathrm{~F}$. What are the maximum and minimum values of capacitance that can be obtained by combining them?

32[P]. Calculate effective capacity of the capacitor combination given below.


33[P]. Calculate the effective capacity between A and B.

$\mathbf{3 4 [ P ]}$. Obtain the equivalent capacitance of the network in figure below. For a 300V supply, determine the charge and voltage across each capacitor.
35. Derive an expression for the

energy stored in a capacitor.
Ans: Consider a capacitor of a capacitance 'C'; it has given a voltage ' V '. Let at any instant the charge in the capacitor be ' $q$ '. Now the work done to increase the charge by an amount ' dq ' is given by
$\mathrm{d} w=\mathrm{Vdq}$

$$
\mathrm{V}=\frac{\mathrm{W}}{\mathrm{q}}
$$

But $V=\frac{\mathrm{q}}{\mathrm{C}} \quad \mathrm{W}=\mathrm{Vq}$
$\mathrm{dw}=\frac{\mathrm{q}}{\mathrm{C}} . \mathrm{dq}$
$\therefore$ the total work done to increase the charge from O to Q is given by
$\mathrm{W}=\int_{0}^{\mathrm{Q}} \mathrm{dW}$
$=\int_{0}^{\mathrm{Q}} \frac{\mathrm{q}}{\mathrm{C}} \mathrm{dq}$
$=\frac{1}{\mathrm{C}} \int_{0}^{\mathrm{Q}} \mathrm{q} \cdot \mathrm{dq}$
$=\frac{1}{\mathrm{C}}\left[\frac{\mathrm{q}^{2}}{2}\right]_{0}^{Q}$
$=\frac{1}{\mathrm{C}}\left[\frac{\mathrm{Q}^{2}}{2}-\frac{0^{2}}{2}\right]$
$=\frac{1}{\mathrm{C}}\left[\frac{\mathrm{Q}^{2}}{2}-0\right]$
$=\frac{1}{\mathrm{C}}\left[\frac{\mathrm{Q}^{2}}{2}\right]$
$\mathrm{W}=\frac{\mathrm{Q}^{2}}{2 \mathrm{C}}$
But $\mathrm{Q}=\mathrm{CV}$
$\mathrm{W}=\frac{(\mathrm{CV})^{2}}{2 \mathrm{C}}$
$=\frac{\mathrm{C}^{2} \mathrm{~V}^{2}}{2 \mathrm{C}}$
$=\frac{\mathrm{CV}^{2}}{2}$
$\mathrm{W}=1 / 2 \mathrm{CV}^{2}$, This work done is stored as the energy of the capacitor.

$$
\mathrm{U}=1 / 2 \mathrm{CV}^{2}
$$

$$
\begin{aligned}
\mathrm{U} & =\frac{\mathrm{Q}^{2}}{2 \mathrm{C}} \\
\mathrm{U} & =\frac{1}{2} \mathrm{CV}^{2} \\
\mathrm{U} & =\frac{1}{2} \mathrm{QV}
\end{aligned}
$$

36. Derive an expression for energy density of a parallel plate capacitor. Ans: We have the expression for energy of a capacitor as, $U=1 / 2 \mathrm{CV}^{2}$
$=\frac{1}{2}\left(\frac{\varepsilon_{0} \mathrm{~A}}{\mathrm{~d}}\right)(\mathrm{Ed})^{2}=\frac{1}{2} \frac{\varepsilon_{0} \mathrm{AE}^{2} \mathrm{~d}^{2}}{\mathrm{~d}}=\frac{1}{2} \varepsilon_{0} \mathrm{AE}^{2} \mathrm{~d}$ Energy Density $(u)=\frac{\text { Energy }}{\text { Volume }}$

$$
\begin{aligned}
& =\frac{\frac{1}{2} \varepsilon_{0} \mathrm{AE}^{2} \mathrm{~d}}{\mathrm{Ad}} \\
& =\frac{1}{2} \varepsilon_{0} \mathrm{E}^{2}
\end{aligned}
$$

$$
\mathrm{u}=\frac{1}{2} \varepsilon_{0} \mathrm{E}^{2}
$$

37. If you connect the plates of a parallel plate capacitor by a copper wire, what happens to the capacitor? Justify your answer.
Ans: Sparking is produced. A part of the energy in the capacitor is wasted in the form of heat, sound and electromagnetic radiations.
$\mathbf{3 8}[\mathbf{P}]$. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

39[P]. In an experiment with a capacitor, the charge which was stored is measured for different values of p.d. The results are tabulated as follows:

| Charge <br> stored $/ \mu \mathrm{C}$ | 7.5 | 30 | 60 | 75 | 90 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{pd} / \mathrm{V}$ | 1 | 4 | 8 | 10 | 12 |

a) Plot a graph with charge on y-axis and p.d on x -axis
b) Using the graph, calculate the capacitance of the capacitor.
c) Determine the energy stored in the capacitor.
40. Derive an expression for the lost energy due to sharing of Capacitors. Ans: Let two capacitors $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ having charged to potentials $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$, connected in parallel. Let V be the common potential.
Now we have $\left(\mathrm{C}_{1}+\mathrm{C}_{2}\right) \mathrm{V}=\mathrm{C}_{1} \mathrm{~V}_{1}+$ $\mathrm{C}_{2} \mathrm{~V}_{2}$
Common potential $V=\frac{C_{1} V_{1}+C_{2} V_{2}}{C_{1}+C_{2}}$

$$
\mathrm{C}_{1}+\mathrm{C}_{2}=\text { total capacitance }
$$

Energy after sharing,
$\left.\begin{aligned} \mathrm{U} & =1 / 2 \mathrm{CV}^{2} \\ & =\frac{1}{2}\left(\mathrm{C}_{1}+\mathrm{C}_{2}\right)\left[\frac{\mathrm{C}_{1} \mathrm{~V}_{1}+\mathrm{C}_{2} \mathrm{~V}_{2}}{\mathrm{C}_{1}+\mathrm{C}_{2}}\right]^{2}\end{aligned} \right\rvert\, \begin{aligned} & \mathrm{C}=\mathrm{C}_{1}+\mathrm{C}_{2} \\ & \mathrm{~V}=\frac{\mathrm{CV}+\mathrm{C}^{2} V_{2}}{\mathrm{C}_{1}+\mathrm{C}_{2}}\end{aligned}$
$=\frac{1}{2}\left(\mathrm{C}_{1}+\mathrm{C}_{2}\right) \frac{\left[\mathrm{C}_{1} \mathrm{~V}_{1}+\mathrm{C}_{2} \mathrm{~V}_{2}\right]^{2}}{\left[\mathrm{C}_{1}+\mathrm{C}_{2}\right]^{2}}$
$=\frac{1}{2} \frac{\left(\mathrm{C}_{1} \mathrm{~V}_{1}+\mathrm{C}_{2} \mathrm{~V}_{2}\right)^{2}}{\mathrm{C}_{1}+\mathrm{C}_{2}}$
Total energy before sharing $\mathrm{U}_{1}+\mathrm{U}_{2}=1 / 2 \mathrm{C}_{1} \mathrm{~V}_{1}{ }^{2}+1 / 2 \mathrm{C}_{2} \mathrm{~V}_{2}^{2}$
Loss of energy $=\left(\mathrm{U}_{1}+\mathrm{U}_{2}\right)-\mathrm{U}$
$=1 / 2 C_{1} V_{1}^{2}+1 / 2 C_{2} V_{2}^{2}-\frac{1}{2} \frac{\left(C_{1} V_{1}+C_{2} V_{2}\right)^{2}}{C_{1}+C_{2}}$
On simplification we get
Loss of energy $\Delta \mathrm{U}=\frac{1}{2}\left[\frac{\mathrm{C}_{1} \mathrm{C}_{2}}{\mathrm{C}_{1}+\mathrm{C}_{2}}\right]\left(\mathrm{V}_{1}-\mathrm{V}_{2}\right)^{2}$

41[P]. A 600 pF capacitor is charged by a 200V supply. It is then disconnected from the supply and then connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?
$42[\mathrm{P}]$. A $4 \mu \mathrm{~F}$ capacitor is charged by 200 V supply. It is then disconnected from the supply, and is connected to another uncharged $2 \mu \mathrm{~F}$ capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?
43. Write the expressions for the potential due to a shell.
Ans: Potential outside the shell ( $\mathrm{r}>\mathrm{R}$ )
$\mathrm{V}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}}{\mathrm{r}}$
Potential on the surface of the shell ( $\mathrm{r}=\mathrm{R}$ )
$\mathrm{V}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}}{\mathrm{R}}$
Potential inside the shell ( $\mathrm{r}<\mathrm{R}$ )
$\mathrm{V}=\frac{1}{4 \pi \varepsilon_{0}} \frac{\mathrm{q}}{\mathrm{R}}$, constant

## Van de Graaff Generator

44. What is the use of a Van De Graaff Generator? Give its principle. Explain its construction and working
Ans:
Use:- It is a device used to create very high electrostatic potential of the order of a few million volts.
This high voltage is used to supply the high energy needed for particle accelerators.

## Principle

Van de Graaff generator works on the following two principles.

1. Discharging action of sharp points:- electric discharge takes place in air or gases readily at pointed conductors.
2. If a charged conductor is brought into internal contact with a hollow conductor, all the charges are transferred to the surface of the hollow conductor irrespective of the potential of the hollow conductor.

## Explanation

Consider a large spherical shell of radius R and charge Q . Let us suppose we introduce a small sphere of radius
' $r$ ' carrying a charge $q$ into the large one, and place it at the centre.


Now the potential at the surface of the large sphere
$\mathrm{V}(\mathrm{R})=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{\mathrm{Q}}{\mathrm{R}}+\frac{\mathrm{q}}{\mathrm{R}}\right)$
Potential at the surface of the smaller sphere $V(r)=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{\mathrm{Q}}{\mathrm{R}}+\frac{\mathrm{q}}{\mathrm{r}}\right)$
$\mathrm{V}(\mathrm{r})-\mathrm{V}(\mathrm{R})=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{\mathrm{q}}{\mathrm{r}}-\frac{\mathrm{q}}{\mathrm{R}}\right)$
$=\frac{\mathrm{q}}{4 \pi \varepsilon_{0}}\left(\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{R}}\right)$, which is always positive (Assume that q is +ve ). Thus the smaller sphere is always at a higher potential. So charges are transferred from it to the larger sphere.

## Construction

It consists of a large conducting shell supported on an insulator column of several meters height. There is an insulating belt wound around two pulleys, moving continuously by a driven motor. The spray comb is connected to a high tension ( 10 kV ) battery. The collector comb is connected to the shell.

## Working

The high electric field applied to the spray comb ionizes the air near to it. The positive charges produced in air are repelled and get deposited on the moving belt, by a corona discharge. As

the belt moves up the charges reach the upper pulley. A similar discharge takes place at the collector comb and finally charges are transferred to the conducting shell, raising its potential to a few million volts.

## ELCTRIC AND DIELECTRIC POLARIZATIONS

## 45. Distinguish between polar and non-polar molecules.

Ans: In certain molecules, the centre of gravity of positive charges and centre of gravity of negative charges do not coincide. These molecules are called polar molecules.
Eg: $\mathrm{HCl}, \mathrm{H}_{2} \mathrm{O}, \mathrm{NH}_{3}$, etc.


But in some other molecules the centre of gravity of positive charges and centre of gravity of negative charges coincide. These molecules are called non-polar molecules.
$\mathrm{Eg}: \mathrm{O}_{2}, \mathrm{~N}_{2}, \mathrm{H}_{2}, \mathrm{CO}_{2}$, etc.

## 46. What are dielectrics?

Dielectrics are non-conducting substances or insulators. But they allow electric field to pass through them.
47. What is the difference in the behavior of a conductor and dielectric in an external electric field?
Ans:


When a conductor is placed in an external electric field ( $\overrightarrow{\mathrm{E}}_{0}$ ) the free charge carries (electrons) are redistributed in such a way that an equal and opposite electric field ( $\overrightarrow{\mathrm{E}}_{\text {in }}$ ) is set up inside the conductor. So net electrostatic field is zero inside the conductor. $\overrightarrow{\mathrm{E}}_{0}+\overrightarrow{\mathrm{E}}_{\text {in }}=0$


But when a dielectric is placed in an external electric field ( $\overrightarrow{\mathrm{E}}_{0}$ ), the molecular dipoles are arranged in such a way that an opposite electric field ( $\vec{E}_{\text {in }}$ ) is set up inside the dielectric. But this electric field is always less
than the external electric field. Thus the dielectric only reduces the external field. Here $\overrightarrow{\mathrm{E}}_{\mathrm{o}}+\overrightarrow{\mathrm{E}}_{\text {in }} \neq 0$

## 48. What is the value of dielectric constant for a metal?

Ans: Infinity

## 49. Explain the polarization in non-

 polar molecules.Ans: In the absence of external e.f., non-polar molecules have no permanent dipole moment. In an external e.f., the positive and negative centres of the non-polar molecule are displaced in the opposite directions. Thus the molecule develops an induced dipole moment. Then the dielectric is said to be polarized. The induced dipole moments of different molecules add up giving a net dipole moment of the dielectric in the presence of external electric field.

(a) Non-polar molecules
50. Explain the polarization in polar molecules.
Ans:

(b) Polar molecules

In a polar dielectric, each molecule has permanent dipole moment but in the absence of external e.f., the dipoles are arranged randomly due to thermal agitation; so the total dipole moment is zero.
When an external e.f. is applied, the individual dipoles tend to align with the field. Then a net dipole moment is developed.

## 51. Define Polarization and electric susceptibility.

Whether polar or non-polar, a dielectric develops a net dipole moment in the presence of an external electric field.
The dipole moment per unit volume of the dielectric is called polarization ( $\overrightarrow{\mathrm{P}}$ ). For linear isotropic dielectrics,

$$
\overrightarrow{\mathbf{P}}=\chi_{\mathrm{e}} \overrightarrow{\mathbf{E}}
$$

$\chi_{\mathrm{e}}$ is called electric susceptibility of the dielectric medium.
52. How does external electric field is reduced in a polarized dielectric?
Ans: Consider two parallel plates having charge densities $+\sigma$ and $-\sigma$ and a dielectric slab placed between them. Due to polarization of the dielectric in the external field $\left(\mathrm{E}_{0}\right)$, the charge densities of plates $P_{1}$ and $P_{2}$ are reduced to $\sigma-\sigma_{P}$ and $-\sigma+\sigma_{P}$. We know that, electric field $\mathrm{E}=\frac{\sigma}{\varepsilon_{0}}$ between two sheets of opposite charge densities $(+\sigma$ and $-\sigma)$. But because of the polarization of dielectric slab, charge densities are reduced so electric field is reduced to $E=\frac{\sigma-\sigma_{P}}{\varepsilon_{0}}$.

## ELECTROSTATICS OF CONDUCTORS

## 53. Explain the main points of electrostatics of conductors.

Ans: The following are the important results regarding the electrostatics of conductors:

## 1. Inside a conductor,

 electrostatic field is zero.Inside a conductor (neutral or charged) the electrostatic field is zero. This is true even in the presence of an external field.
Reason: In the static situation, the free charge carriers are so distributed themselves that the e.f is zero everywhere inside.
2. At the surface of a charged conductor, electric field must be normal to the surface at every point.
Reason: If E were not normal to the surface, it would have some non-zero component along the surface. Free charges on the surface of the conductor would then experience force and move.
3. The interior of a conductor can have no excess charge in static situation.
Reason: A neutral conductor has equal amounts of positive and negative charges. When the conductor is charged the excess charge can reside only on the surface in the static situation.
4. Electrostatic potential is constant through the volume of the conductor and has the same value (as inside) on its surface.

Reason: Since $E=0$ inside the conductor and has no work is done in moving a small test charge within the conductor and on its surface. That is there is no potential difference between any two points inside or on the surface of the conductor.

## 5. Electric field at the surface of

 a charged conductor $E=\frac{\sigma}{\varepsilon_{0}} \hat{n}$$\sigma$ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.
If $\sigma$ is -ve , electric field is normal to the surface inward.

## 6. Electrostatic shielding.

Electric field is zero inside the cavity of a conductor of any shape.

## EQUIPOTENTIAL SURFACE

54. What is an equipotential surface? Give examples. Write some properties of it.

Ans: It is a surface having same potential at all points.

Example 1: Concentric spheres with a point charge at the centre are equipotential surfaces.


Example 2: In a uniform e.f parallel planes perpendicular to the electric lines of force are equipotential surfaces.


Properties of equipotential surfaces:-

1. The work done to move a charge from one point to another on an equipotential surface is zero.
2. Two equipotential surfaces will never intersect.
3. Electric lines of force pass normal to an equipotential surface.

## More Examples

The figure below shows the equipotential surfaces due to
(i) a dipole

(ii) two positive charges

$55[\mathrm{P}]$. Two charges $2 \mu \mathrm{Cand}-2 \mu \mathrm{C}$ are placed at points A and B, 6 cm apart.
a) Identify the equipotential surface of the system.
b) What is the direction of the electric field at every point on this surface?
$\qquad$

