ELECTROSTATICS

Electrostatics deals with the behavior of electric charges at rest.

Frictional electricity

It has been found that any substance if rubbed with some other substance acquires an attractive property. The bodies are then said to have become *electrified* or they are said to *acquire electric charge*. For eg; when a glass rod is rubbed with silk or an ebonite rod is rubbed with fur, they acquire this attractive property.

The phenomenon of acquiring electric charges by friction is called frictional electrification and the bodies are said to possess frictional electricity.

For eg; Take a plastic ruler or comb and rub it with dry hair or a piece of wool and bring it near small pieces of paper. Then the paper pieces will get attracted towards the ruler or comb. This is because the plastic ruler and comb got electrified due to friction with dry hair.

 # Note: 1. Good conductors like copper cannot be charged by friction because any charge produced on it can easily flow through the rod through our body and to the ground.

2. Insulators like plastic, ebonite, glass etc can be easily charged by friction because the charges will stay on them.

3. Electrostatic experiments cannot be performed in moist climate because moist air is slightly conducting. So the static charges will get conducted away from the charged body.

NB: **How is frictional electrification caused?**

The number of protons inside the nucleus of an atom is equal to the number of electrons outside the nucleus. When a body is rubbed with another, due to friction, some electrons from one body gets transferred to the other body. The body, which loses electrons, will become positively charged and which gains electrons becomes negatively charged. The two bodies thus acquire opposite charges and they are equal in magnitude. This is the reason for frictional electricity.

Electric Charge

It is found experimentally that the charges are of two types: 1.Positive charge 2.Negative charge

The unit of charge is coulomb (C). The names of positive and negative charges are purely conventional.

Note:

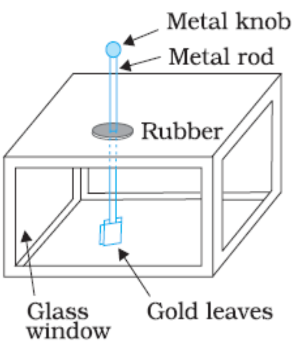
Positively charged body means deficiency of electrons in the body from its neutral state and a negatively charged body means excess of electrons.

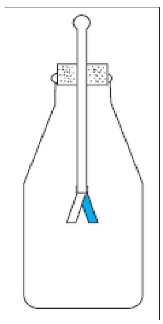
Gold-Leaf electroscope

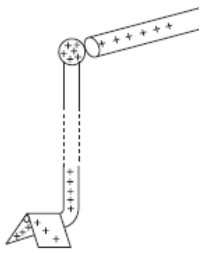
A simple apparatus to detect charge on a body is called a gold-leaf electroscope.

Apparatus

It consists of a vertical metal rod placed in a box. Two thin gold leaves are attached to its bottom end as shown in figure







Working

When a charged object touches the metal knob at the top of the rod, charge flows on to the leaves and they diverge. The degree of divergence is an indicator of the amount of charge.

Conductors and Insulators

Conductors are those substances which allow passage of electricity through them.

Insulators are those substances which do not allow passage of electricity through them.

Earthing (or) Grounding

When a charged body is brought in contact with earth, all the excess charge pass to the earth through the connecting conductor. This process of sharing the charges with the earth is called grounding or earthing.

Earthingprovides protection to electrical circuits and appliances.

Charging by Induction

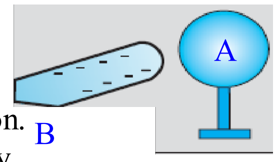
A body can be charged in different ways 1)Charging by friction 2)Charging by conduction 3)Charging by induction

Charging by friction

When two bodies are rubbed each other, electrons in one body (in which electrons are held lesstightly) transferred to second body (in which electrons are held more tightly)

Explanation

When a glass rod is rubbed with silk, some of the electrons from the glass are transferred to silk. Hence glass rod gets +ve charge and silk gets -ve charge.



Charging by conduction

Charging a body with actual contact of another body is called charging by conduction.

Explanation: If a neutral conducting body (A) is brought in contact with positively charged conducting body (B), the neutral body gets positively charged.

Charging by induction

The phenomenon by which a neutral body gets charged by the presence of neighboring charged body is called electrostatic induction.

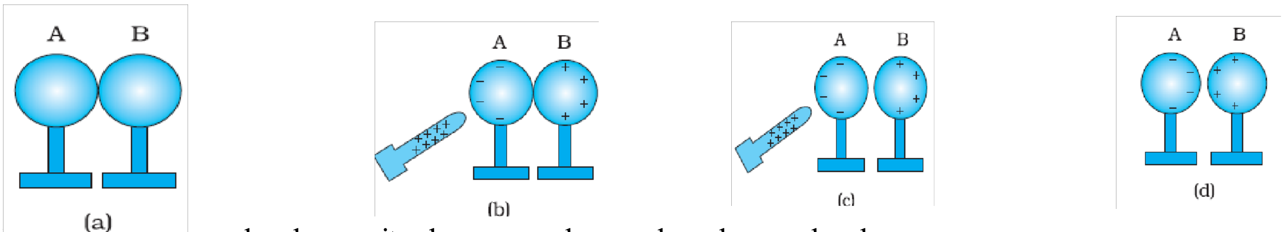
Explanation

Step I : Place two metal spheres on an insulating stand and bring in contact as shown in figure (a).

Step II: Bring a positively charged rod near to these spheres. The free electrons in the spheres are attracted towards the rod. Hence, one side of the sphere becomes negative and second side becomes positive as shown in the figure (b).

Step III: Separate the spheres by a small distance by keeping the rod near to sphere A. The two spheres are found to be oppositely charged as shown in figure (c).

Step IV: Remove the rod, the charge on spheres rearrange themselves as shown in figure (d).



In this process, equal and opposite charges are developed on each sphere.

How can you charge a metal sphere positively without touching it?

Place the uncharged metallic sphere on an insulating stand. Bring a negatively charged rod close to the metallicsphereas shown in figure (b). Due to the attraction of rod electrons are piling up at the rear end and positive charge at farther end. Connect the sphere to the earth (earthed) when the sphere is earthed, electrons flow from the ground to the sphere and neutralize the positive charge.

Disconnect the sphere from ground and then remove rod from it. The negative charge uniformly distribute over the sphere.

NB Properties of electric charges.

1. Electric charges are of two kinds – positive and negative.
2. Like charges repel and unlike charges attract each other.
3. The two kinds of charges ie +ve and – ve are really opposite. The combination of a charge +q with –q results in a net charge equal to zero.

4. **Charge is quantised: Millikan showed that all electric charges in nature exist either as a basic chargeor as some integral multiple of this charge. The basic charge is equal to electronic charge $e = 1.602 \times 10^{-19}$ C. Thus charge exists in discrete packets or charge is said to be quantised. *According to quantisation of electric charge, charge of a body is an integral multiple of a basic charge, which is the electronic charge.*

ie charge on a body, $q = \pm ne$; where, n is an integer and e is the electronic charge.

5.Charge is conserved**: It means that total charge of an isolated system remains constant. It also implies that electric charges can neither be created nor destroyed. If an object loses some charge, an equal amount of charge appears somewhere else.

6.**Charge is a scalar quantity**.

7. **Additivity of charge:** The total charge on a surface is the algebraic sum of individual charges present on that surface. If q_1, q_2, q_3 ...., q_n are the charges on a surface, then total or net charge,
$q = q_1 + q_2 + q_3 +$ ....$+ q_n$

Q1 A conductor has a negative charge of 4.8×10^{-17}C. Determine the number of excess electrons in the conductor. [300]
Q2. A polythylene piece rubbed with wool is found to have a negative charge of 3×10^{-7}C(a) Estimate the number of electrons transferred from which to which? (b) Is there a transfer of mass from wool to polethylene?

NB Point charges.

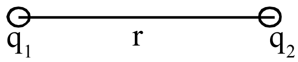
If an electric charge is confined to an extremely small volume, it is called a point charge. Physically, all charged bodies whose dimensions are very small compared to the distance between them are referred to as point charges. *Any charge whose dimensions are very small compared to its distance from a point where its effect are to be analysed is also called a point charge.*

NB Coulomb's law or inverse square law.

Coulomb's law or inverse square law states that *the force between two stationary electric charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.*

Consider two point charges q_1 and q_2 separated by a distance r, then the force between them,

Then $F \propto \frac{q_1 q_2}{r^2}$



$\therefore F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$; where ε_0 is the permittivity of free space, whose value $= 8.854 \times 10^{-12}\ C^2/Nm^2$. ε_r is the permittivity of the medium in which the charges are placed, relative to the permittivity of free space and is called **relative permittivity or dielectric constant** whose value is 1 for free space.

Now, value of $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2$.

Therefore; Force between two charges in free space $= 9 \times 10^9 \frac{q_1 q_2}{r^2}$ N.

If $q_1 = q_2 = 1C$ and r = 1m, then $F = 9 \times 10^9$ N. So we can define the unit of charge – coulomb as follows:

One coulomb is that charge which when placed in free space at a distance of one metre from an equal and similar charge repels with a force of 9×10^9 N.

Note: 1.Charge on one electron, $e = -1.6 \times 10^{-19}$ C . $\therefore 1C = e / (1.6 \times 10^{-19}) = 6.2 \times 10^{18}$ e.
ie 1C= charge on 6.2×10^{18} electrons.
2.Coulomb's law holds good only for point charges in free space.
3.The direction of force is always along the line joining the two-point charges q_1 and q_2. Hence this force is called central force.

Coulomb's law in vector form.

If $\vec{F}_{12}$ is the force on q_1 due to q_2 and r_{21} is the unit vector pointing from q_2 towards q_1,then

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\vec{r}_{21} \text{ ....(1)}$$

Similarly $\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}$ ....(2) ; where $\vec{F}_{21}$ is the force on q_2 by charge q_1 and r_{12} is the unit vector pointing from q_1 to q_2. Since $\hat{r}_{12} = -\hat{r}_{21}$ $\underline{\underline{\vec{F}_{12} = -\vec{F}_{21}}}$

(1) and (2) represents Coulomb's law in vector from.

Q3. What is the force between two small charged spheres having charges of 2×10^{-7}C and 2×10^{-7}C placed 30cm apart in air? [6×10^{-3}N]

Q4. Suppose you have two small point objects separated by a distance of 1 cm. Each object has a diameter of 1×10^{-3} cm. One object has an excess of 3×10^{10} electrons and other has an excess of 2×10^{10} electrons on it. What is the electrostatic force that they exert on each other? [1.38×10^{-3}N]

Relative permittivity

Relative permittivity of a medium is defined as the ratio between permittivity ε of the medium to the permittivity ε_0 of free space.

ie $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$ or permittivity of medium, $\varepsilon = \varepsilon_0 \varepsilon_r$

Comparison of electric force with gravitational force.

1.Gravitational force is always attractive whereas electric force is attractive as well as repulsive.

2.Electric force is much stronger than gravitational force.

Eg. The ratio of electric and gravitational force between electron and a proton separated by a distance r is given by

$$\frac{F_e}{F_g} = \frac{\frac{G\, m_p\, m_e}{r^2}}{\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}$$

Substuting values of ε_0, G, m_e and m_p, $F_e/F_g = 2.4 \times 10^{39}$. $\therefore$ $\mathbf{F_e >>> F_g}$

Superposition principle.

Superposition principle states that if a number of charges are interacting, the total force on a charge is the vector sum of the individual forces exerted on it by all other charges.

If F_1, F_2, F_3...... are the forces acting on a charge q due to charges q_1, q_2, q_3........., then the net force F acting on q due to all the charges, $F = F_1 + F_2 + F_3 + \ldots\ldots\ldots\ldots$.

Electric Field

The space surrounding a charge where its effect can be felt is called electric field.

An electric charge placed at any point in an electric field will experience a force of electrical origin.

Suppose a test charge q placed at a point in an electric field experiences an electric force $\vec{F}$, then electric field strength or intensity at that point is given by, $\vec{E} = \underset{q \to 0}{Lt} \frac{\vec{F}}{q}$ ....(1)

Thus *electric field intensity can be defined as the force per unit positive charge placed at a given point.*

Now (1) can be written as $E = \frac{F}{q}$

Note: 1. The electric field intensity E is a vector whose direction is the direction of the force F experienced by the +ve charge placed at that point.

2.The unit of electric field is N/C or V/m. [newton/coulomb or volt/metre.]

3.Dimension of electric field $= \dfrac{MLT^{-2}}{IT} = MLT^{-3}I^{-1}$

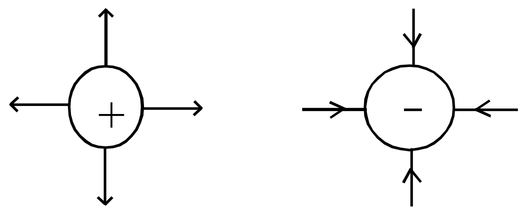
Electric field due to a point charge.

To find the electric field E at a point P due to a charge q, let us assume a unit positive test charge at P.

Then the force experienced by the test charge, $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q\times 1}{r^2}$; where r is the distance of point p from charge q.

Then electric field, $E = \dfrac{F}{q} = \dfrac{F}{1} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q\times 1}{r^2}$ $\quad\therefore E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

In vector form, $\vec{E} = \dfrac{q\ \vec{r}}{4\pi\varepsilon_0 r^2}$ where $\vec{r}$ is a unit vector.

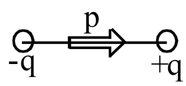


Note: If q is +ve, the electric field points radially outwards and if q is –ve, it points radially inwards.
The field is spherically symmetric as it has same magnitude in all directions for same value of r.

Q5. At a certain point in space an electron which has a mass of 9.11×10^{-31} kg, experiences a force of 3.2×10^{-19} N in the + X direction. What is the electric field at this point.? [2 N/C]
Q6.Calculate the electric field intensity due to a point charge 20 μC at a point distant 40 cm from the charge? [$1.125 \times 10^6 NC^{-1}$.]
Q7.Two spheres having charges +10 μC and +40 μC are placed 12 cm apart. Find the position of the point where the intensity is zero? [0.08m from +40 μC]

Electric dipole

An electric dipole is a system consisting of two equal and opposite charges separated by a small distance.



The strength of the dipole is expressed in terms of a quantity known as **dipole moment** $\vec{p}$.

Dipole moment is defined as the product of one of the charges and the distance between the charges(dipole length).

Its direction is from negative charge to positive charge. $\therefore$ Dipole moment $p = q(2\ell)$; where 2ℓ is the dipole length. The unit of dipole moment is coulomb – metre [C m]

Torque acting on a dipole placed in a uniform electric field.

Consider an electric dipole AB of moment $p = q(2\ell)$ placed in a uniform electric field E. Let the dipole make an angle θ with the field direction. Then the two charges will experience equal and opposite forces qE and thus form a couple.

The torque τ on the dipole will try to make the dipole in a line with the field.

Thus torque τ = one of the forces x perpendicular distance between them.

ie τ = qE x AC

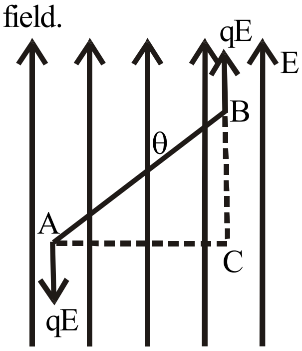
But from figure; $AC = 2\ell\sin\theta$

$\therefore \tau = qE \times 2\ell\sin\theta$

ie $\tau = q(2\ell)E\sin\theta$

But $q(2\ell) = p$; the dipole moment.

$\therefore \tau = pE\sin\theta$ OR $\vec{\tau} = \vec{p}\times\vec{E}$



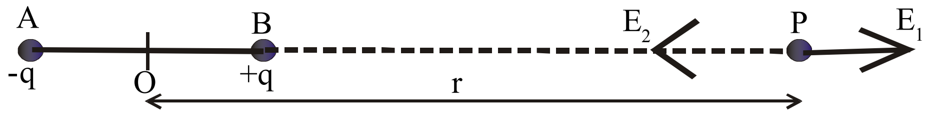
Now if $\theta = 90°$, torque = maximum

$\therefore$ Maximum torque $\tau_{max} = pE$.

****Note:** *If the dipole is placed in a uniform electric field, it will experience a torque. Here the net transalatory force acting on the dipole is zero. But if the dipole is placed in a non – uniform electric field, the forces acting on the charges +q and –q will be different. Hence the dipole will experience both translatory force and torque and hence will move sideway in addition to rotational motion .*

NB: **Electric field at a point on the axis of an electric dipole. (End on position)**

Consider a point P on the axial line of an electric dipole of moment $p = q(2\ell)$ at a distance **r** from the center of the dipole AB.



Then electric field at P due to +q; $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-\ell)^2}$ along BP produced.

The electric field at P due to –q; $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+\ell)^2}$ along PB.

Then the resultant field at P is given by $E = E_1 - E_2$ along AP produced.

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-\ell)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+\ell)^2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right]$$

i.e. $E = \frac{q(2\ell)2r}{4\pi\varepsilon_0(r^2-\ell^2)^2}$ i.e. $E = \frac{2pr}{4\pi\varepsilon_0(r^2-\ell^2)^2}$; along AP. {here $q(2\ell) = p$,dipole moment }

Now for a short dipole, $r >>> \ell$ Therefore ℓ^2 can be neglected being small.

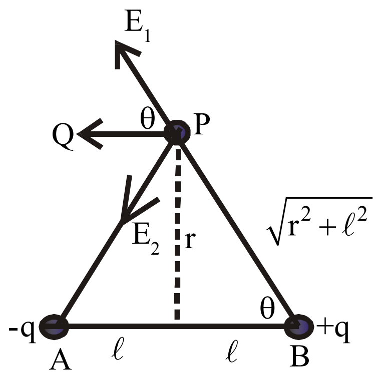
$\therefore E = \frac{2pr}{4\pi\varepsilon_0 r^4}$ i.e. $E = \frac{2p}{4\pi\varepsilon_0 r^3}$ acting along AP.

This is the expression for the electric field at a point on the axis of a short dipole.

NB **Electric field at a point on the perpendicular bisector of an electric dipole** [equatorial line or broad side on position]

A line perpendicular to the axis of the dipole and passing through its center is called the equatorial line or perpendicular bisector of an electric dipole.

Let P be a point on the perpendicular bisector of a dipole of moment $p = q(2\ell)$ at a distance **r** from its center.



Then the electric field at P due to +q, $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+\ell^2)}$ along BP

Electric field at P due to –q , $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+\ell^2)}$ along PA.

Here magnitudes of E_1 and E_2 are same. Resolving E_1 and E_2 into components, the Y components will cancel each other while the X components will add to get the resultant electric field E.

Let θ be the angle between E and E_1, then $E = E_1\cos\theta + E_2\cos\theta$ i.e, $E = (E_1+E_2)\cos\theta$ ....(1)

But from figure, $\cos\theta = \frac{\ell}{\sqrt{r^2+\ell^2}}$

Substuting in (1), $\therefore E = \frac{2}{4\pi\varepsilon_0}\frac{q}{(r^2+\ell^2)}\frac{\ell}{(r^2+\ell^2)^{\frac{1}{2}}} = \frac{q\,(2\ell)}{4\pi\varepsilon_0(r^2+\ell^2)^{\frac{3}{2}}}$

$$\therefore E = \frac{p}{4\pi\varepsilon_0(r^2+\ell^2)^{\frac{3}{2}}} \quad \text{along PQ}$$

Now for a short dipole, r >>>> ℓ

$$\therefore E = \frac{p}{4\pi\varepsilon_0 r^3}$$

The direction of electric field is from positive charge to negative charge.

Q8. Two charges +15 μC and -15 μC are separated by a distance of 1 μm. What is the dipole moment? [15 x 10^{-12}Cm]

Q9. An electric dipole consists of two charges +10 μC and -10 μC separated by a distance of 5 mm. Calculate the electric field at a point on the axial line at a distance of 10 cm from the centre of the dipole. [9 x10^5N/C]

Q10. An electric dipole consists of two equal and opposite charges +150 μC and -150 μC separated by a distance of 10cm. Calculate the electric field due to the dipole at a distance of 15 cm from each charge. [4 x10^7N/C]

Electric lines of force OR Electric field lines. —— introduced by Michael Faraday.

An electric line of force is the path along which a unit positive charge would move, if it is free to do so. The tangent at any point on the line of force will give the direction of electric intensity at that point.

Properties:

1. A line of force will start from a positive charge and end in negative charge.
2. *Two lines of force will never intersect.* If they intersect, two tangents could be drawn to the lines of force at the common point. This means that there could be two directions for electric intensity at that point, which is impossible.
3. The density or closeness of lines of force gives the strength of electric field at various regions.
4. In a uniform electric field, lines of force are parallel to each other.

Work done in rotating a dipole in a uniform electric field. OR Potential energy of a dipole.

Consider a dipole of moment **p** placed in a uniform electric field of intensity **E** at an angle θ. Then,

Torque acting on the dipole, $\tau = pE\sin\theta$.

Now let the dipole be turned through an angle dθ against the torque. Then work done, $dw = pE\sin\theta\, d\theta$.

Therefore, work done in rotating the dipole from θ_1 to θ_2;

$$W = \int dw = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta = pE[-\cos\theta]_{\theta_1}^{\theta_2} \quad \text{i.e. } W = pE[\cos\theta_1 - \cos\theta_2]$$

This work done is stored in the dipole as potential energy.

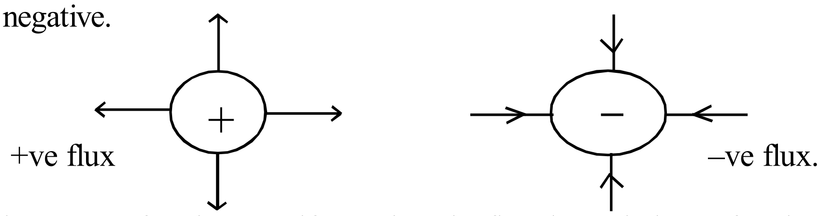
Electric flux. ϕ_E

Electric flux through a surface is the number of electric lines of force passing normally through the surface. The flux through an area is given by the product of the electric field and the component of area perpendicular to the field.

ie electric flux through a surface is given by $\phi_E = \vec{E}\cdot\vec{S}$; where S is the area of the surface and E is the electric field on the surface. S is a vector whose direction is normal to S. If θ is the angle between the direction of E and the normal to the surface, $\phi_E = E\,S\cos\theta$.

Electric flux is a scalar quantity. Unit: newton metre^2 / coulomb [Nm^2/C]

In case of a closed surface, if the lines of force are outward, the flux is taken as +ve and if lines of force are inward, the flux is negative.



If the electric field over a surface is not uniform, then the flux through the surface is given by $\phi_E = E \cdot ds$; where E is the electric field over an infinitesimal area ds of the surface.

Thus electric flux through a surface is the surface integral of the electric intensity over it.

The flux through a surface is maximum when surface is perpendicular to the field and minimum (zero) when the surface is parallel to the field.

Gauss's theorem.

Gauss's theorem states that " *the net electric flux over a closed surface is* $\frac{1}{\varepsilon_0}$ *times the net charge enclosed by the surface*". If S is a closed surface and q is the total charge enclosed by it, then Gauss's theorem may be expressed as

$$\phi_E = \oint E \bullet ds = \frac{q}{\varepsilon_0}$$

The surface over which we calculate the flux is called **Gaussian surface**.

APPLICATIONS OF GAUSS'S THEOREM

1. **Electric intensity or electric field due to an infinite plane sheet of charge**.

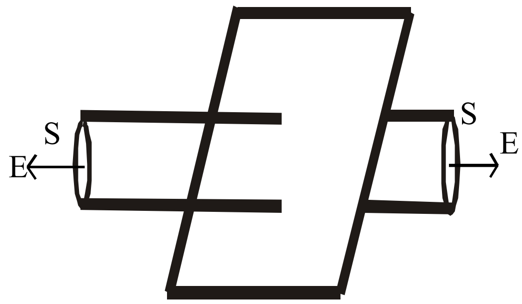
Consider a thin, infinite plane sheet of charge of charge density σ coulomb/m^2. The electric field E is perpendicular to the sheet everywhere. The electric field has same magnitude at equal distance from the surface but points in opposite directions on either side of the sheet. The field points away from the plane.

Imagine a cylindrical surface of cross-sectional area S passing normally through the sheet. Let P_1 and P_2 be two points on either side of the sheet and along the axis of the cylinder, at equal distances from the sheet. The electric fields at P_1 and P_2 are same in magnitude. Hence total area of Gaussian surface = 2S. [the curved surface area of the cylinder is not considered since E is parallel to the area]

$\therefore E \cdot ds = 2\,E\,S$ .... (1)

Also charge density $\sigma = q/S$

$\therefore q = \sigma\,S$ .... (2)

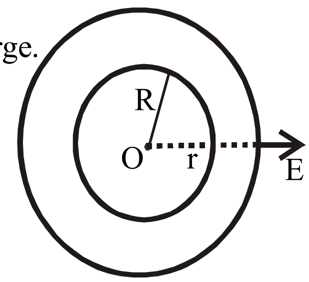


Now by Gauss's theorem, $\oint E \bullet ds = \frac{q}{\varepsilon_0}$

Substuting from (1) and (2), $2\,E\,S = \frac{\sigma S}{\varepsilon_0}$

$\therefore$ Field due to a plane sheet of charge, $E = \frac{\sigma S}{2\,S\varepsilon_0}$ i.e. $E = \frac{\sigma}{2\,\varepsilon_0}$

Hence E is independent of the distance of the point from the infinite sheet of charge.



Electric field due to a uniformly charged spherical shell.

A spherical shell is a hollow sphere of small thickness. Consider a spherical shell of radius R with its center at the origin. Let a charge q be given to the shell. It gets distributed over the shell uniformly. Let the charge density of the surface be σ.

Then total charge, $q = 4\pi R^2\sigma$. .... (1)

Here the electric field is acting radially outwards.

Case 1. Field at a point outside the shell.

Consider a point P outside the shell at a distance r form the center of the shell. Let E be the electric field at P. Now imagine a spherical surface of radius r and center coinciding with the center of the shell as the Gaussian surface.

$$\text{Then} \quad \oint \vec{E} \bullet d\vec{s} = \oint E \bullet ds = E\,ds.$$

But for Gaussian surface, $ds = 4\pi r^2$.

$\therefore \quad E \cdot ds = 4\pi r^2 E$

But by Gauss's theorem, $E \bullet ds = \dfrac{q}{\varepsilon_0}$

$$\therefore \; 4\pi r^2 E = \frac{q}{\varepsilon_0} \qquad \text{or } E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Substuting the value of q from (1),

$$E = \frac{4\pi R^2\sigma}{4\pi\varepsilon_0 r^2} \qquad \text{ie} \qquad E = \frac{R^2\sigma}{r^2\,\varepsilon_0}$$

Case 2. Field at a point on the surface of the shell.

For a point on the surface of the shell, r = R. Then field $E = \dfrac{\sigma}{\varepsilon_0}$

Case 3. Field at a point inside the shell.

For a point inside the shell, the Gaussian surface lies inside the shell and hence encloses no charge.

Therefore $E \cdot ds = 0$.

$\therefore$ E = 0 since $ds \neq 0$.

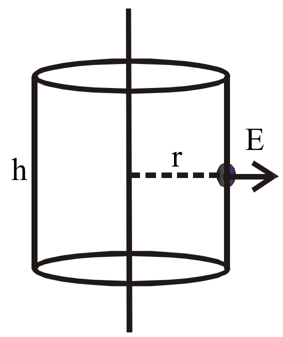
ie *the field inside a uniformly charged spherical shell is always zero.*

This is true for a shell of any shape or size. *This disappearance of electric field inside a cavity in a conductor is called* **electrostatic shielding**.

....

Electric field due to an infinitely long straight charged wire.

Consider an infinitely long straight charged wire of linear charge density (charge per unit length) λ. To find the electric field at a point P at a distance r from the wire, consider a Gaussian cylinder of radius r and length h coaxial with the wire. Now the point P is on the radial surface of the cylinder. The electric field E at P is radially outward.



Now the electric flux through the curved surface,

ϕ_E = E x area of the curved surface.

ie $\phi_E = E \times 2\pi rh$.

Flux through the two end surfaces = 0.

$\therefore$ Total flux through the Gaussian cylinder, $\phi_E = E \times 2\pi rh$.....................(1)

Charge enclosed by the Gaussian cylinder, $q = \lambda h$

Now by Gauss's theorem, $\phi_E = \dfrac{\lambda h}{\varepsilon_0}$ ....(2)

From (1) and (2), $E \times 2\pi rh. = \dfrac{\lambda h}{\varepsilon_0}$

$$\therefore E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Q11. A point charge 10-7C is situated at the centre of a cube of 1 m side. Calculate the electric flux through its surface. [$1.13 \times 10^4 Nm^2C^{-1}$]
Q12. A 20 cm diameter loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is found to be $1.3 \times 10^5\ Nm^2/C$. Calculate the field strength? [4.14×10^6 N/C]
Q13. The total electric flux through a closed surface in the shape of a cylinder is $5.60 \times 10^4\ Nm^2/C$. Calculate the net charge within the cylinder? [$4.96 \times 10^{-7}C$]

ELECTRIC POTENTIAL OR ELECTROSTATIC POTENTIAL

NB **Potential Difference (pd)

Consider an isolated point charge +q at O in free space. Let A and B be two points in the electric field due to this charge. Let a unit positive charge be placed at A. It experiences a force of repulsion.

Now let the unit positive charge be moved from A to B without acceleration against this force. For this, work has to be done against the force of repulsion. Let this work be denoted by W_{AB}. This is called the **potential difference** between the two points A and B.

Thus *potential difference between two points in an electric field may be defined as the amount of work done in moving a unit positive charge from one point to the other without acceleration against electrical forces.*

ie $V_B - V_A = W_{AB}$; where V_A and V_B are the potentials at A and B respectively. W_{AB} will be +ve, 0 or –ve according as the potential at B is greater than, equal to or less than the potential at A. The unit of pd is <u>*volt*</u>.

The pd between two points is one volt if one joule of work is done in moving one coulomb of charge from one point to another without acceleration against electrical forces.

NB **Electric potential (V)**

The potential at a point far away from the charge producing the field is taken as zero. So in the above figure, if A is at infinity, $V_A = 0$. $\therefore W_{AB} = V_B$.

Here W_{AB} is the work done in moving a unit positive charge from infinity to the point B and V_B is the potential at B. Thus *electric potential at a point is defined as the work done in moving a unit +ve charge from infinity to that point without acceleration against electric forces.*

This work done is stored in the unit +ve charge as the potential energy of the charge.

Note: 1.Electric potential is denoted by the letter V and is a scalar quantity.
2.The electric potential is measured by V = W/q ie work done per unit charge.
3.Unit of potential is J/C or volt (V)
4.Dimension of potential = $ML^2T^{-3}I^{-1}$.
5. Also, pd can be expressed as $V_B - V_A = W_{AB}/q$; if we are moving a charge q instead of unit charge.

Relation connecting electric field and electric potential.

Consider two close points A and B in the electric field due to a small charge +q, separated by a distance dr. Let dV be the pd between A and B. This means that dV is the work done in moving a unit positive charge from A to B.

ie $dV = E(-dr)$ {because W = F x S}

or $E = -\dfrac{dV}{dr}$ Here $\dfrac{dV}{dr}$ is called potential gradient, which shows the rate of change of potential with distance.

Thus electric field is equal to negative gradient of electric potential.

##Note: 1.If V is the pd between two plates kept at a distance d apart, then the electric field between them, E = V/d. or pd, V=Ed.
2.The work done in moving a charge q between two points in an electric field is given by W=qV;

where V is the pd between the two points. *This work done is stored as electric potential energy.*

3. If m is the mass of the particle or charge q and v is the velocity acquired by it in a pd V, then $qV = \frac{1}{2} mv^2$.

Electron volt (eV)

One electron volt is the energy acquired be an electronic charge when it is accelerated through a pd of one volt. $1\text{eV} = 1.602 \times 10^{-19}$ J

NB **Electric potential due to a point charge.**

Consider two points A and B in the electric field E due to a point charge +q. Let a unit positive charge be moved from A to B. Since electric field is negative gradient of electric potential,

$E = -\frac{dV}{dr}$ $\therefore$ The workdone in moving the unit positive charge from A to B, $dV = -E\,dr$.

Therefore the total work done in moving the unit +ve charge from infinity toa point P, distant r from q , ie the potential at P is given by,

$$V = \int dV = \int_{\infty}^{r} -E\,dr \text{ ....(1)}$$

The electric field E at a distance r from a point charge +q is given by $E = \frac{q}{4\pi\varepsilon_0 r^2}$ ....(2)

From (1) and (2);

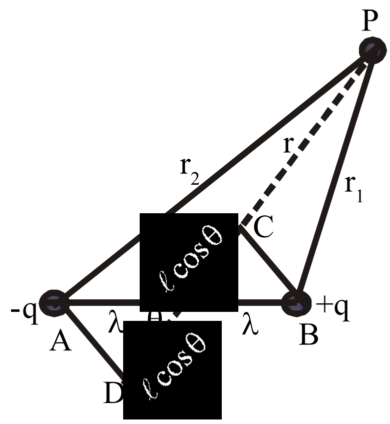
$$V = \int_{\infty}^{r} -\frac{q}{4\pi\varepsilon_0 r^2} dr = \frac{-q}{4\pi\varepsilon_0}\int_{\infty}^{r} r^{-2}\,dr$$

$$\text{i.e. } V = -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{\infty}^{r} = \frac{q}{4\pi\varepsilon_0 r}$$

So for any point at a distance r from a point charge q, the electric potential, $V = \frac{q}{4\pi\varepsilon_0 r}$

Thus the potential due to a point charge varies inversely as the distance.

Electric potential due to an electric dipole.



Let P be a point at a distance r form the center of a dipole AB of moment $p = q(2\ell)$. Let line OP makes an angle θ with the dipole.

Now potential at P due to +q, $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP}$

Potential at P due to –q $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{-q}{AP}$

$\therefore$ Net potential at P, $V = V_1 + V_2$

Therefore $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP} - \frac{1}{4\pi\varepsilon_0}\frac{q}{AP} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{BP} - \frac{1}{AP}\right]$

But $BP \approx CP = r - \ell\cos\theta$ and $AP \approx DP = r + \ell\cos\theta$

$$\therefore V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r - \ell\cos\theta)} - \frac{1}{(r + \ell\cos\theta)}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{r + \ell\cos\theta - r + \ell\cos\theta}{r^2 + \ell^2\cos^2\theta}\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q(2\ell)\cos\theta}{(r^2 - \ell^2\cos^2\theta)} \qquad \therefore\ V = \frac{p\cos\theta}{4\pi\varepsilon_0(r^2 - \ell^2\cos^2\theta)}$$

Since $p = q(2\ell)$ and $\ell^2 <<< r^2$; $V = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

Case 1. If point p is on the axis of dipole, then $\theta = 0$.

$$V = \frac{p\cos 0}{4\pi\varepsilon_0 r^2} = \frac{p}{4\pi\varepsilon_0 r^2}$$

Case 2. If A is on the equatorial line of dipole, then $\theta = 90°$.

$$V = \frac{p\cos 90}{4\pi\varepsilon_0 r^2} = 0$$

Equipotential surface.

If the potential remains same at every point on a surface, it is called an equipotential surface. No work is to be done in moving a charge from one point to another along the equipotential surface.

Electric field will be perpendicular to the equipotential surface.

Eg: 1. Surface of a charged conductor. 2. All points equidistant from a point charge.

Q14. The work done to carry $1\mu C$ from one point to another in an electric field is 1 mJ. What is the potential difference between the two points? [10^3V]

Q15. Calculate the work done in carrying a charge 3×10^{-6}C from a point at a distance 1.2 m from a charge of 0.48×10^{-6}C in air to a point 0.2 m from it? [5.4×10^{-2}J]

Q16. To carry a charge of 2 x 10-3 C from a point where the potential is - 5 V to another point where the potential is V, the workdone is 0.5 J. What is the value of V? [245 V]

Q17. Calculate the electric potential due to a point charge - 15 μC at a point distant 9 cm from the charge. [-1.5×10^5V]

Q18. Two point charges - 4 μC and 2 μC are separated by a distance of 1m. Find the point at which the electric potential due to the charges is zero. [0.667 m from -4 μC]

CAPACITANCE

Capacitance of a conductor is its ability to store electric charges. If electric charges are given to a conductor, its potential rises. The potential V of a conductor is directly proportional to its charge q.

ie $q \propto V$

Or $q = CV$; where C is called the **capacity or capacitance** of the conductor.

Thus the capacitance of a conductor is defined as the ratio of the charge to the potential through which the conductor rises.

ie $C = \dfrac{q}{V}$. If V = 1 volt, C = q. *Thus capacitance of a conductor may be defined as the charge required to raise its potential by unity.*

Unit of capacitance is **coulomb/ volt or farad (F)**

The capacitance of a conductor is one farad if a charge of one coulomb is sufficient to raise its potential through one volt.

Note: A farad is a large unit. So for convenience, we use micro farad (μF) ie 10^{-6} F, nano farad (nF) 10^{-9} F, pico farad (pF) 10^{-12} F etc.

Capacitance of a spherical conductor.

Consider a charged spherical conductor of radius r and carrying a charge q, kept isolated from the influence of other charged bodies.

Then the potential at any point on the surface of the spherical conductor, $V = \dfrac{q}{4\pi\varepsilon_0 r}$

$$\therefore \text{ Capacitance } C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\varepsilon_0 r}} = 4\pi\varepsilon_0 r$$

If the sphere is kept in a medium of dielectric constant ε_r, then $C = 4\pi\varepsilon_0\varepsilon_r r$.

Capacitor or condenser

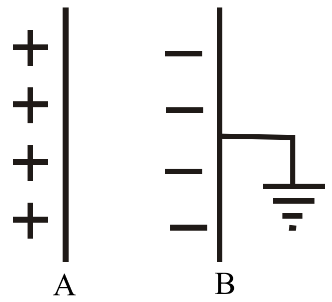
An arrangement by which the capacity of a conductor is increased is called a capacitor or electrical condenser.

Thus capacitor is an electrical device for storing quantities of electricity.

A capacitor consists of two conductors – one of them is charged and the other usually connected to earth. Actually, two conductors kept a distance apart with a dielectric or air in between them forms a capacitor. For a capacitor, C= q/V.

So *a capacitor has a capacity of* ***one farad*** *if a charge of one coulomb flows from one conductor to the other when the pd between the conductors is one volt.*

Principle of a capacitor.



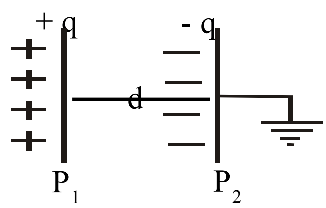
Imagine a plate A, which is +vely, charged. If charge on plate is q and its potential is V, then C = q/V.Now let another plate B be brought near A. Then –ve charge will be induced on that side of B which is near to A and +ve charge on the other side of B. If B is earthed, these +ve charges will flow to earth. Consequently the potential at A decreases and its capacitance increases.This is because, with the presence of B, the amount of work done in bringing a unit +ve charge from infinity to A decreases as there will be a force of repulsion due to A and a force of attraction due to B.

Thus, the resultant force of repulsion on a unit +ve charge is reduced. So the amount of work is less and potential at A decreases. Therefore, capacity of A increases.

Thus if an earthed conductor is placed near a charged conductor, the capacitance of the charged conductor is considerably increased. This is the principle of a capacitor.

Note: Commonly used capacitors: 1) Parallel plate capacitor, 2).Cylindrical capacitor and 3) Spherical capacitor.

Capacitance of a parallel plate capacitor.



A parallel plate capacitor consists of two similar, large, flat conducting plates arranged parallel to each other with a small distance d between them. Let A be the area of each plate. Let plate P_1, be charged +q and P_2 be charged –q. The electric field is uniform between the plates.

Its capacity, $C = \frac{q}{V}$; where V is the pd between the plates.

If σ is the charge density on one plate, $q = \sigma A$.

But $\sigma = \varepsilon_0 E$

$\therefore q = \varepsilon_0 E A$

Also V = E d; where E is the electric field between the plates.

$\sigma = q/A \quad \therefore q = \sigma A$
Also $E \cdot S = q/\varepsilon_0$
$\therefore \varepsilon_0 E = q/S = \sigma$

$$\therefore \quad C = \frac{q}{Ed} = \frac{\varepsilon_0 A}{d} \qquad \text{i.e} \quad C = \frac{\varepsilon_0 A}{d}$$

Hence the capacitance of a parallel plate capacitor increases with increase in area and decreases with separation between plate's increases.

If the space between the plates is filled with a medium of dielectric constant ε_r,

$$\text{Capacity} \quad C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

****Note:** Consider a parallel plate capacitor having n plates each of area A and arranged at equal distance d from one another. If alternate plates are connected together, then these n plates form (n –1) capacitors connected in parallel.

Hence effective capacitance $C = \dfrac{\varepsilon_0 (n-1) A}{d} = \dfrac{\varepsilon_0 \varepsilon_r (n-1) A}{d}$

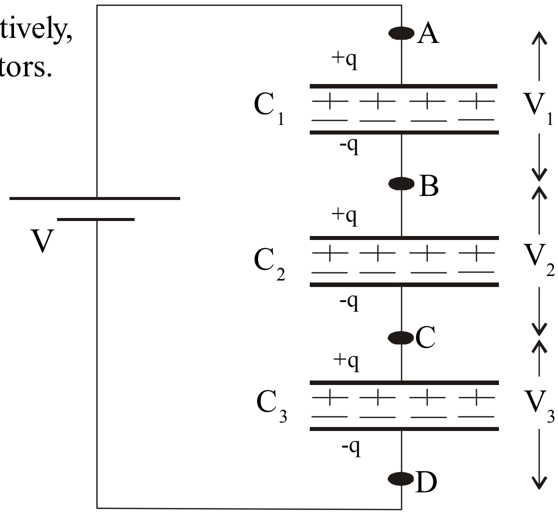
Effect of dielectric on capacitors.

1. A dielectric medium increases the capacitance by ε_r times.
2. It avoids the electric discharge between the plates.
3. The dielectric prevents the two plates coming into contact and keeps the plates very close together.

NB **Capacitors in Series.**

Consider three capacitors of capacitance C_1,C_2 and C_3 connected in series.In series combination, each capacitor is charged with the same charge while they will be raised through different potentials in accordance with their capacities.

If V_1,V_2 and V_3 are the pd's of capacitors respectively, $V= V_1+V_2+V_3$; where V = pd between the three capacitors.



Now $V_1 = \dfrac{q}{C_1}$; $V_2 = \dfrac{q}{C_2}$; $V_3 = \dfrac{q}{C_3}$

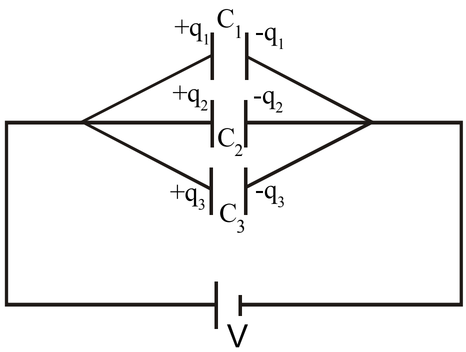
Also $V = \dfrac{q}{C}$; where C is called equivalent or effective capacitance.

$$\therefore \quad \frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\text{i.e} \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Here C = effective or equivalent capacitance of the combination.

NB **Capacitors in parallel**



Consider three capacitors of capacitance C_1,C_2 and C_3 connected in parallel. Let V be the pd applied. Then all the capacitors will have the same pd, V. Let Q_1, Q_2 and Q_3 be the charges acquired by the capacitors.

Then total charge, $Q = Q_1+Q_2 +Q_3$

$\therefore$ $Q = (C_1+C_2 +C_3)$ V.

Now if the three capacitors are replaced by a capacitor of capacitance C, then Q = C V.

$\therefore$ $C V = (C_1+C_2 +C_3)$ V.

or $C= C_1+ C_2 + C_3$.

Here, C is the effective or equivalent capacitance of the combination.

NB **Energy of a capacitor.**

The energy of a charged capacitor is the amount of work done in charging it. This work done will be stored in the capacitor as potential energy.

Consider a capacitor of capacitance C, charged to a pd V by giving a charge Q to it from a battery. Let q be the charge at any instant and V be the potential.

Then $V = \dfrac{q}{C}$

Let an additional charge dq be supplied to the capacitor by the battery.

Then Work done, $dW = V\,dq$.

$\therefore$ The total work done in charging the capacitor from 0 to Q, $W = \int dw = \int_0^Q \frac{q}{C} dq = \frac{1}{C}\int_0^Q q\,dq$

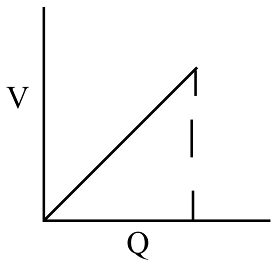
i.e $W = \frac{Q^2}{2C}$ This work done is stored as potential energy in the capacitor.

$\therefore$ Energy of a capacitor, $U = \frac{Q^2}{2C}$ ....(1)

But Q = C V. Therefore, $U = \frac{C^2\,V^2}{2C} = \frac{1}{2}CV^2$ ....(2)

Also C = Q/ V. Therefore $U = \frac{\frac{1}{2}QV^2}{V} = \frac{1}{2}QV$ ....(3)

Thus energy of a capacitor, $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$.



Note: **Energy of a capacitor (Graphical Method).**

In case of a capacitor, charge $Q \propto V$. So a graph drawn connencting Q and V is as shown.

The area under the curve give the energy of the capacitor.

Therefore, energy, $E = \frac{1}{2}\text{base} \times \text{altitude} = \frac{1}{2}Q\,V$

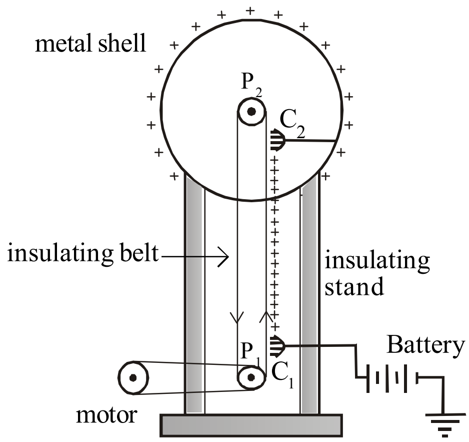
Energy density of a capacitor.

Energy stored per unit volume of the medium between the plates of a capacitor is called energy density of the capacitor.

Consider a parallel plate capacitor of charge density σ and capacitance C. Then charge, q = σ A and capacitance $C = \frac{\varepsilon_0 A}{d}$.

Therefore, Energy density $= \frac{E}{V} = \frac{E}{A\,d} = \frac{1}{2}\frac{CV^2}{A\,d} = \frac{1}{2}\frac{\varepsilon_0 A}{d}\frac{V^2}{A\,d} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$

NB **Van de Graff Generator.**



C_1 - lower comb
C_2 - upper comb
P_1 - first pulley
P_2 - second pulley

In 1931, Van de Graff designed an electrostatic generator capable of producing pd of 6 to 10 million volts. Such pd was then made use of to accelerate charged particles so as to carry out nuclear reactions.

The generator is based on the principle that *if a charged conductor is brought into internal contact with a hollow conductor, all of its charge gets transferred to the hollow conductor, irrespective of its own potential.*

The generator consists of a large spherical metal shell (a few metres in radius) mounted on a tall insulating stand above the ground. A long narrow belt made of an insulating material is wound around two pulleys P_1 and P_2. P_1 is at the center of the shell and P_2 is at the ground level. The belt is made to rotate using motor and continuously carries +ve charges. These +ve charges are sprayed out by a brush B_1 (spray brush). Another brush B_2 (collecting comb) at top carries these +ve charges from belt to the conducting shell, where they spread out uniformly. As the belt rotates, charge will accumulate on the shell. Hence the potential of the shell increases and a high voltage difference can be built up.

If potential goes beyond a maximum value, insulation of air breaks down and sphere gets discharged. To avoid this, Van de Graff generator is usually kept in an enclosure filled with nitrogen at high pressure.

Note: This generator does not generate charge but separates and accumulates charge.

Conductors and insulators.

Conductors allow flow of electric charges through them. Eg: copper, silver, iron etc.
Insulators do not allow flow of charges through them. Eg: paper, mica, wax, glass etc.

When a conductor is charged, the charges will be uniformly distributed over its surface.

Dielectrics.

Dielectrics are substances, which do not conduct electricity but transmits electrical influence.

In some dielectrics, in the absence of an external field, the centers of +ve and –ve charges of each molecule coincide. They are called non – polar dielectrics. They have zero dipole moment in their normal states.

Eg: H_2, N_2, O_2, CO_2 etc.

In some dielectrics, the two centers of charges do not coincide in normal state due to the asymmetric shape of the molecule. They are called polar dielectrics. They have permanent dipole moment.

Eg: H_2O, NH_3, HCl etc.

When a non – polar dielectric is placed in an external electric field, the centers of +ve and–ve charge of each atom gets slightly separated. Then each molecule is said to possess an induced dipole moment. This is known as **dielectric polarization.**

Behavior of conductor in an electric field.

When a conductor is placed in an electric field,

1) Inside the conductor or at the interior of the conductor, both electric field and net charge are zero.
2) Just outside the conductor, electric field is $\perp^r$ to the conductor surface.
3) Charges will reside only on the surface of the conductor and are uniformly distributed on its surface.

Q19. Two conductors have net charges of +20.0 μC. If the potential difference between the conductors is 20.0 V, calculate the capacitance of the system. [10^{-6}F]

Q20. A liquid drop has a capacitance of 2 pF. (i) Calculate its radius. (ii) If its potential is 100 V, what is the charge on the drop? [18×10^{-3} m, 2×10^{-10} C]

Q21. A parallel plate capacitor has an area of 4.00×10^{-4} m^2 and a plate separation of 2.00 mm. (i) Calculate its capacitance. (ii) If the plate separation is increased to 3.00 mm, find the capacitance. (iii) If the space between the plates is filled with a medium of dielectric constant 6.7, calculate the capacitance in the second case. [1.77×10^{-12}F; 1.18×10^{-12}F; 7.90×10^{-12}F]

Q22. The energy of a capacitor is 0.2 J. If its capacitance is 2 μF, calculate, (i) the charge on the vapacitor and (ii) the potential difference between the plates. [0.89×10^{-3} C; 449.4 V]