

മലയാളത്തിലെ മനോരമ ഭാഗ്യ - 2018

ചോദ്യം : 12

പ്രശ്നം - മലയാള ഭാഗ്യം. വിഭാഗം - I

- 1)  $f(x) = x^2$     2) (6, 6)    3)  $k \neq -4$     4) ഗുരു ഗണി മലയാളം.
- 5) -1    6) 2    7)  $33\pi$     8)  $P \wedge (\sim P)$     9)  $\frac{1}{10} \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$     10) 40
- 11)  $\cos x$     12) 65    13)  $\frac{100}{3} \pi$     14)  $\frac{\pi}{3}$     15) 48    16)  $\frac{25}{51}$     17)  $-\tan x$
- 18) 3    19) (Z, .)    20) 256

21)  $A = \begin{pmatrix} 6 & 12 & 6 \\ 1 & 2 & 1 \\ 4 & 8 & 4 \end{pmatrix}$     വിഭാഗം - II

$\sim \begin{pmatrix} 1 & 2 & 1 \\ 6 & 12 & 6 \\ 4 & 8 & 4 \end{pmatrix} R_1 \leftrightarrow R_2$

$\sim \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} R_2 \rightarrow R_2 - 6R_1$   
 $R_3 \rightarrow R_3 - 4R_1$

$P(A) = 1$

മുഖ്യമന്ത്രി സെക്രട്ടേറിയത്തിൽ ഭൂമി  $x_1$  തിരിച്ചറിയുന്നതിനായി  $VF = a = 900$  മുഖ്യമന്ത്രി.

ഗുണമനുസരിച്ച്  $y^2 = 4ax$

$(x_1, 80)$  മൂലം  $80^2 = 4 \times 900 \times x_1$

മുഖ്യമന്ത്രി ഭൂമി  $x_1 = \frac{16}{9} 986$ .

22)  $\vec{a}, \vec{b}, \vec{c}$  മൂന്നും ഗുരു ഗണി മലയാളം.

$\therefore [\vec{a} \vec{b} \vec{c}] = 0$  - ①

$[\vec{a} + \vec{b} \vec{b} + \vec{c} \vec{c} + \vec{a}] \Leftrightarrow 2[\vec{a} \vec{b} \vec{c}]$

①  $\Leftrightarrow 0$

$\Leftrightarrow \vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$  ഗുരു ഗണി മലയാളം.

25)  $f(\theta) = \theta + \sin \theta$

ഗുണമനുസരിച്ച്  $f'(\theta) = 0$

$1 + \cos \theta = 0$

$\theta = \cos^{-1}(-1) = \pi$

ഗുണമനുസരിച്ച്  $\theta = \pi$

മൂലം  $f(\pi) = \pi + \sin \pi = \pi$

$(\pi, \pi)$

23)  $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1+2i+i^2}{2} = +i$

$\left(\frac{1+i}{1-i}\right)^n = 1 \Rightarrow (+i)^n = 1$

$\Rightarrow n = 4$ .

26)  $f(x) = 2x^3 + 5x^2 - 4x$

$f'(x) = 6x^2 + 10x - 4$

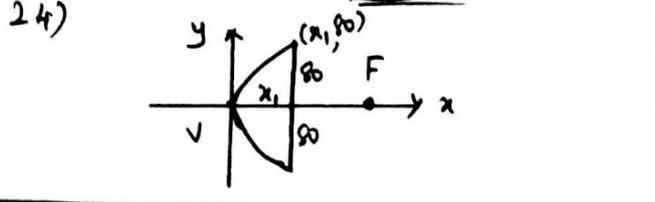
$f''(x) = 12x + 10$

മൂലം  $f''(x) = 0$

$12x + 10 = 0$

$x = -\frac{5}{6}$ .

മൂലം  $(-\infty, -\frac{5}{6})$   $(-\frac{5}{6}, \infty)$



$(-\infty, -\frac{5}{6})$   $f''(x) < 0$  കുറയുന്നതിൽ

$(-\frac{5}{6}, \infty)$   $f''(x) > 0$  വർദ്ധിക്കുന്നതിൽ

27)  $y = f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$   
 $dy = \frac{1}{3} x^{-\frac{2}{3}} dx$

$x = 1000 \quad dx = -1 \text{ units.}$

$dy = \frac{1}{3} (1000)^{-\frac{2}{3}} (-1) = \frac{-1}{300}$

$\sqrt[3]{999} = f(1000-1) = f(1000) + dy$   
 $= 10 - \frac{1}{300}$

$\approx 10 - 0.0033$

$\approx 9.996$

$\approx 9.996$

28)  $I = \int_0^1 \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx$  units

$t = \sin^{-1} x$  units

$dt = \frac{1}{\sqrt{1-x^2}} dx$

$I = \int_0^{\frac{\pi}{2}} t^3 dt = \left[ \frac{t^4}{4} \right]_0^{\frac{\pi}{2}} = \frac{\pi^4}{64}$

29)  $\sim (P \rightarrow q)$

P	q	$P \rightarrow q$	$\sim (P \rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

30)  $n=5$   
 $\{(2,6) (3,5) (4,4) (5,3) (6,2)\}$   
 $5 \text{ ways}$   $P = \frac{5}{36}, Q = \frac{31}{36}$   
 $P(X=x) = {}^n C_x p^x q^{n-x}$   
 $P(X=0) = {}^5 C_0 \left(\frac{5}{36}\right)^0 \left(\frac{31}{36}\right)^5$   
 $= \left(\frac{31}{36}\right)^5$

Q. No. 32

31)  $\text{adj} A = (\text{Cofactor matrix})^T$

$\therefore \text{Cofactor matrix}$   
 $(A_{ij}) = \begin{pmatrix} 2 & 21 & -18 \\ 2 & -7 & 6 \\ 4 & -8 & 4 \end{pmatrix}$

$|A_{ij}| = |A|^2$  (we know that)

$\begin{vmatrix} 2 & 21 & -18 \\ 2 & -7 & 6 \\ 4 & -8 & 4 \end{vmatrix} = (20)^2$

$2(-28+48) - 2(84-144) + 4(126-126) = 400$

$60d = 360$  Qn. No. 32

$d = 6$  P.S. See the Last Page.

33)  $a^4 + 4 = 0 \Rightarrow a^4 = -4$

$a = [4(-1)]^{\frac{1}{4}}$

$a = \sqrt{2} (-1)^{\frac{1}{4}}$

$a = \sqrt{2} (\cos \pi + i \sin \pi)^{\frac{1}{4}}$   
 $= \sqrt{2} [\cos (2k+1)\pi + i \sin (2k+1)\pi]^{\frac{1}{4}}$

$a = \sqrt{2} \left[ \cos (2k+1)\frac{\pi}{4} + i \sin (2k+1)\frac{\pi}{4} \right]$

$k = 0, 1, 2, 3$

By using  
 $a = \sqrt{2} \cos \frac{\pi}{4}, \sqrt{2} \cos 3\frac{\pi}{4}, \sqrt{2} \cos 5\frac{\pi}{4}, \sqrt{2} \cos 7\frac{\pi}{4}$

34)  $x = x+3 \quad y = y-5$  units

$\frac{x^2}{6} + \frac{y^2}{4} = 1 \quad a^2 = 6$   
 $b^2 = 4$

$e = \frac{1}{\sqrt{3}} \quad ae = \sqrt{2}$

Βαθμωτήρι (±ae, 0) = (±√2, 0)

(√2, 0) ⇒ x = √2    y = 0  
 x = -3 + √2    y = 5  
 F<sub>1</sub> (-3 + √2, 5)

(-√2, 0) ⇒ F<sub>2</sub> (-3 - √2, 5)

35)  $\lim_{x \rightarrow 0} \frac{\frac{1}{x^2} - 2 \tan^{-1} \frac{1}{x}}{\frac{1}{x}}$   
 y =  $\frac{1}{x}$  ορίζ.   
 x → ∞ ορίζ. y → 0.

$\lim_{y \rightarrow 0} \frac{y^2 - 2 \tan^{-1}(y)}{y} = \frac{0}{0}$

=  $\lim_{y \rightarrow 0} \frac{2y - 2 \frac{1}{1+y^2}}{1}$

= 0 - 2(1) = -2

3b)  $\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt}$

$\frac{\partial w}{\partial x} = 1$  ,  $\frac{dx}{dt} = -\sin t$

$\frac{\partial w}{\partial y} = 2$      $\frac{dy}{dt} = \cos t$

$\frac{\partial w}{\partial z} = 2z$      $\frac{dz}{dt} = 1$

$\frac{dw}{dt} = 1(-\sin t) + 2 \cos t + 2z$   
 =  $-\sin t + 2 \cos t + 2t$

37)  $(D^2 - 2D - 3)y = \sin x \cos x$   
 =  $\frac{1}{2} \sin 2x$

Αντικαθιστώντας

$p^2 - 2p - 3 = 0$

p = 3, -1

CF =  $Ae^{3x} + Be^{-x}$

PI =  $\frac{1}{2} \frac{1}{D^2 - 2D - 3} \sin 2x$

=  $-\frac{1}{2} \frac{1}{2D + 7} \sin 2x$

=  $-\frac{1}{2} \frac{2D - 7}{4D^2 - 49} \sin 2x$

=  $-\frac{1}{2} \frac{4 \cos 2x - 7 \sin 2x}{-65}$

PI =  $\frac{4 \cos 2x - 7 \sin 2x}{130}$

ολική λύση y = CF + PI

y =  $Ae^{3x} + Be^{-x} + \frac{1}{130} (4 \cos 2x - 7 \sin 2x)$

38) Δίνεται ομάδα πηδ:

G ούς G ούς. a, b ∈ G ορίζ.

$(a * b)^{-1} = b^{-1} * a^{-1}$

Απόδειξη

$(a * b) * (b^{-1} * a^{-1}) = a * (b * b^{-1}) * a^{-1}$   
 =  $a * e * a^{-1}$

=  $a * a^{-1} = e$  — ①

$(b^{-1} * a^{-1}) * (a * b) = b^{-1} * (a^{-1} * a) * b$

=  $b^{-1} * e * b$

=  $b^{-1} * b = e$  — ②

①, ② από

$(a * b)^{-1} = b^{-1} * a^{-1}$

39) Ορισμός υπενθύμιση, x

P(x=2) =  $\frac{e^{-\lambda} \lambda^x}{x!}$

P(x=2) =  $\frac{e^{-5} 5^2}{2!}$

=  $\frac{0.0067 \times 25}{2}$

= 0.0838

$$40) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x| dx$$

$$f(x) = |\sin x| \text{ சின்ன}$$

$$f(-x) = |\sin(-x)| = |-\sin x| = |\sin x| = f(x)$$

$\therefore f$  - ஒரேமுகம் சரிய.

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x| dx = 2 \int_0^{\frac{\pi}{2}} |\sin x| dx$$

$$= 2 \int_0^{\frac{\pi}{2}} \sin x dx$$

$$= 2 [-\cos x]_0^{\frac{\pi}{2}}$$

$$= 2 [(-0) - (-1)]$$

$$= 2$$

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 0$$

41) 2)  $\therefore$  சமன்பாடுகளின் சீர்மை 2-வது.

2x2 சீர்மைக்கான  $\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} = -1 \neq 0$

$\therefore z = k$  சின்ன.  $k \in \mathbb{R}$ .

$$x + y = -2k$$

$$3x + 2y = -k$$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} = -1$$

$$\Delta_x = \begin{vmatrix} -2k & 1 \\ -k & 2 \end{vmatrix} = -4k + k = -3k$$

$$\Delta_y = \begin{vmatrix} 1 & -2k \\ 3 & -k \end{vmatrix} = -k + 6k = 5k$$

அளவுகள் எடுத்து,

$$x = \frac{\Delta_x}{\Delta} = \frac{-3k}{-1} = 3k$$

$$y = \frac{\Delta_y}{\Delta} = \frac{5k}{-1} = -5k$$

$$\text{சீர்மை } (x, y, z) = (3k, -5k, k)$$

(சீர்மை)

$$41) 2) u = \tan^{-1} \left( \frac{x^3 + y^3}{x - y} \right)$$

$$f = \tan u = \frac{x^3 + y^3}{x - y}$$

$$f(tx, ty) = \frac{t^3(x^3 + y^3)}{t(x - y)}$$

$$= t^2 f(x, y)$$

$f$  2-வது  $x$  மற்றும்  $y$  2-வது சீர்மை.

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 2f$$

$$x \frac{\partial}{\partial x} (\tan u) + y \frac{\partial}{\partial y} (\tan u) = 2 \tan u$$

$$x \sec^2 u \frac{\partial u}{\partial x} + y \sec^2 u \frac{\partial u}{\partial y} = 2 \tan u$$

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

$$42) 2) \sin(A - B) = \sin A \cos B - \cos A \sin B$$

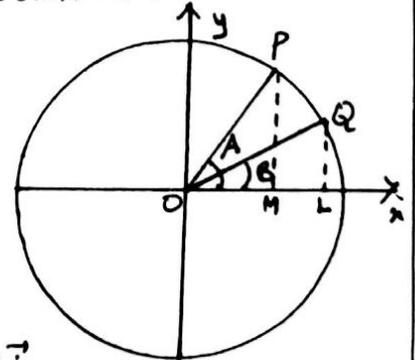
$$P(\cos A, \sin A)$$

$$Q(\cos B, \sin B)$$

$$\therefore \angle POQ = A - B.$$

$$\vec{OP} = \vec{OM} + \vec{MP} = \cos A \vec{i} + \sin A \vec{j}$$

$$\vec{OQ} = \vec{OL} + \vec{LQ} = \cos B \vec{i} + \sin B \vec{j}$$



$$\vec{OQ} \times \vec{OP} = |\vec{OQ}| |\vec{OP}| \sin(A-B) \vec{k}$$

$$= \sin(A-B) \vec{k} \quad \text{--- (1)}$$

$$\vec{OQ} \times \vec{OP} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos B & \sin B & 0 \\ \sin A & \cos A & 0 \end{vmatrix}$$

$$= \vec{k} [\sin A \cos B - \cos A \sin B] \quad \text{--- (2)}$$

(1), (2) ରୁ,

$$\sin(A-B) = \sin A \cos B - \cos A \sin B.$$

42) 25) ଦିଆଯାଇଥିବା ଦିଗରେ ଉପଲବ୍ଧ

$3\vec{i} - 2\vec{j} + 4\vec{k}$  ଧିରାଧିରା  $(1, 2, 3)$   
 $(2, 3, 1)$

ଓଜମାଧାରରେ ଉପଲବ୍ଧ ସମୀକରଣ

$$\vec{r} = \vec{a} + s(\vec{b} - \vec{a}) + t\vec{v}$$

$$\vec{r} = (\vec{i} + 2\vec{j} + 3\vec{k}) + s(\vec{i} + \vec{j} - 2\vec{k}) + t(3\vec{i} - 2\vec{j} + 4\vec{k})$$

ସମୀକରଣର ସମୀକରଣ

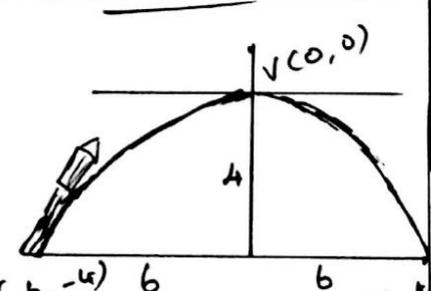
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 3 & -2 & 4 \end{vmatrix} = 0$$

$$2y + z - 7 = 0$$

43) 21)

ଉପରୋକ୍ତ ଧିରାଧିରା ସମୀକରଣ

$$x^2 = -4ay$$


$(b, -4)$  ଉପରେ ଉଲ୍ଲେଖ କରାଯାଇଛି,  $3b = 16a$

$$\Rightarrow a = \frac{9}{4}$$

ସମୀକରଣ  $x^2 = -9y$

ଅତୀତରେ,

$$\frac{dy}{dx} = -\frac{2}{9}x$$

$$m = \left(\frac{dy}{dx}\right)_{(-6, -4)} = -\frac{2}{9}(-6) = \frac{4}{3}$$

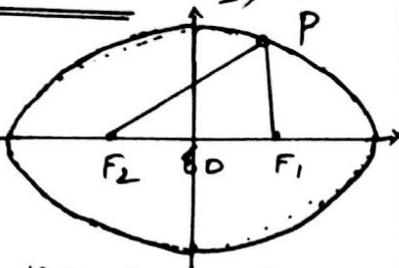
$$\therefore m = \tan \theta = \frac{4}{3}$$

ଠିକ୍ ସମୀକରଣ  $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

43) 25)

P - ଅକ୍ଷରରେ ଉପର ଧିରାଧିରା

$F_1, F_2$  - ଉପର ଧିରାଧିରା



$$F_1P + F_2P = 2a = 120 \Rightarrow a = 60.$$

$$F_1F_2 = 2ae = 60 \Rightarrow e = \frac{1}{2}$$

$$b^2 = a^2(1 - e^2) = 60^2(1 - \frac{1}{4})$$

$$= 3600 \times \frac{3}{4}$$

$$b^2 = 2700$$

ଠିକ୍ ସମୀକରଣ ସମୀକରଣ

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{3600} + \frac{y^2}{2700} = 1$$

44) 21)

ଧିରାଧିରା  $(a \cos^4 \theta, a \sin^4 \theta)$

$$\frac{dx}{d\theta} = -4a \cos^3 \theta \sin \theta$$

$$\frac{dy}{d\theta} = 4a \sin^3 \theta \cos \theta$$

$$\therefore \frac{dy}{dx} = -\frac{\sin^4 \theta}{\cos^4 \theta}$$

ଠିକ୍ ସମୀକରଣର ସମୀକରଣ

$$y - y_1 = m(x - x_1)$$

$$y - a \sin^4 \theta = -\frac{\sin^4 \theta}{\cos^4 \theta} (x - a \cos^4 \theta)$$

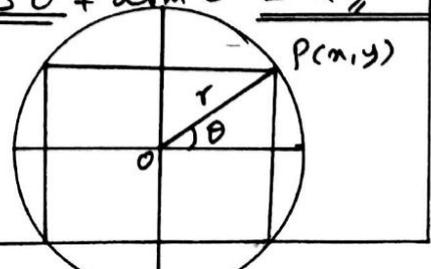
$$\frac{x}{a \cos^4 \theta} + \frac{y}{a \sin^4 \theta} = 1$$

ଠିକ୍ ସମୀକରଣର ସମୀକରଣ

$$-a \cos^2 \theta + a \sin^2 \theta = a$$

25)

$$x = r \cos \theta$$

$$y = r \sin \theta$$


P(m, y)

ඉඩය ත්‍රිකෝණයේ උස, එකතුව  
 $2x = 2r \cos \theta, 2y = 2r \sin \theta$   
 $0 \leq \theta \leq \frac{\pi}{2}$

ඉඩය ත්‍රිකෝණයේ වර්ග =  $4r^2 \sin \theta \cos \theta$

$$A(\theta) = 2r^2 \sin 2\theta$$

$$A'(\theta) = 4r^2 \cos 2\theta = 0$$

$$\theta = \frac{\pi}{4}$$

$$A''(\theta) = -8r^2 \sin 2\theta \quad A''\left(\frac{\pi}{4}\right) < 0$$

$\theta = \frac{\pi}{4}$  ට  $A$  ඉහළමය.

$$\therefore 2x = 2r \times \frac{1}{\sqrt{2}} = \sqrt{2}r$$

$$2y = 2r \times \frac{1}{\sqrt{2}} = \sqrt{2}r$$

ඉඩය ත්‍රිකෝණයේ උස, එකතුව ~~ඉහළම~~  
 $\sqrt{2}r, \sqrt{2}r$

$\therefore$  ඉඩයෙහි ඉහළ ස්ථරයටය.

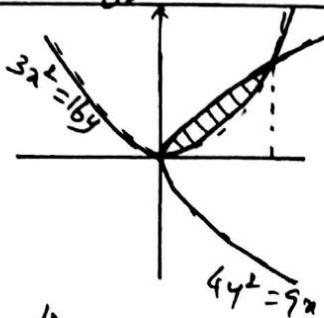
45) 2)

$$4y^2 = 9x$$

$$3x^2 = 16y$$

ඉහළම භාගයේ උස

$$(0,0) \quad (4,3)$$



$$\text{ඉහළම භාගයේ වර්ග} = \int_0^4 (y_1 - y_2) dx$$

$$= \int_0^4 \left( \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{16}x^2 \right) dx$$

$$= \left( x^{\frac{3}{2}} - \frac{x^3}{16} \right)_0^4 = 4 \text{ ඉඩ}$$

45) 2)

$$f(x) = ce^{-x^2+3x} = ce^{-(x-\frac{3}{2})^2 + \frac{9}{4}}$$

$$= ce^{-\frac{1}{2}\left(\frac{x-\frac{3}{2}}{\frac{1}{\sqrt{2}}}\right)^2 + \frac{9}{4}}$$

$$= ce^{\frac{9}{4}} \cdot e^{-\frac{1}{2}\left(\frac{x-\frac{3}{2}}{\frac{1}{\sqrt{2}}}\right)^2}$$

$$= ce^{\frac{9}{4}} \cdot e^{-\frac{1}{2}\left(\frac{x-\frac{3}{2}}{\frac{1}{\sqrt{2}}}\right)^2}$$

$$\text{ඉහළම } \mu = \frac{3}{2} \quad \text{ඉහළ } \sigma = \frac{1}{\sqrt{2}} \quad \sigma^2 = \frac{1}{2}$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\frac{1}{\sigma\sqrt{2\pi}} = ce^{\frac{9}{4}} \Rightarrow ce^{\frac{9}{4}} = \frac{\sqrt{2}}{\sqrt{2\pi}}$$

$$\Rightarrow c = \frac{e^{-\frac{9}{4}}}{\sqrt{\pi}}$$

46) 2)  $(x+y)^2 \frac{dy}{dx} = 1$  — ①

$$x+y = z \quad \text{එවිට} \quad 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{dz}{dx} - 1$$

$$\therefore \text{①} \Rightarrow z^2 \left( \frac{dz}{dx} - 1 \right) = 1$$

$$\Rightarrow z^2 \frac{dz}{dx} = 1 + z^2$$

$$\int \frac{1+z^2-1}{1+z^2} dz = \int dx$$

$$\int \left( 1 - \frac{1}{1+z^2} \right) dz = \int dx + c$$

$$z - \tan^{-1} z = x + c$$

$$y - \tan^{-1}(x+y) = c$$

46) 2)

උෂ්ණත්වයේ වෙනසට අනුපාතිකව  $T$  අඩුවීමේ වේගය  $\propto T-S$  බව දක්වයි.

$$\frac{dT}{dt} \propto T-S \Rightarrow T-S = ce^{kt}$$

$$T = 15 + ce^{kt} \quad \text{--- ①}$$

$$t=0 \text{ විට } T=100 \Rightarrow c=85$$

$$\text{①} \Rightarrow T = 15 + 85e^{kt}$$

$$t=5 \text{ විට } T=60 \Rightarrow e^{5k} = \frac{45}{85}$$

$$t=10 \text{ විට } T = 15 + 85e^{10k}$$

$$T = 15 + 85 \left( \frac{45}{85} \right)^2$$

$$T = 38.82^\circ \text{C}$$

