SECOND YEAR HIGHER SECONDARY EXAMINATION (SAY) JUNE 2019

Subject: MATHEMATICS (SCIENCE) **Code No: SY 27**

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1	(i)	$\frac{x}{6}+\frac{y}{6}+\frac{z}{6}=1$ OR $x+y+z=6$	1	
	(ii)	Both are parallel planes, since their d.r's are proportional.	1	3
		Distance $= \left\lvert \frac{6-(-4)}{\sqrt{1+1+1}} \right\rvert = \frac{10}{\sqrt{3}}$	1	
		Remarks: for formula give 1 score.		
2.		$f(x) = x^2+2x-8$ is continuous in $[-4\ 2]$		
		$f(x)$ is differentiable in $(-4\ 2)$	1	
		$f(-4) = f(2) = 0$	1	3
		$f'(x) = 2x+2$	½	
		$f'(c) = 0$	½	
		$c = -1 \in (-4, 2)$		
		Hence Rolle's theorem is satisfied		
3		Equation of family of circles,		
		$x^2+(y-a)^2 = a^2$	1	
		$x^2+y^2 = 2ay$; $\frac{x^2+y^2}{y} = 2a$	1	
		Differentiating w.r.to x:		3
		$\frac{y(2x+2yy_1)-(x^2+y^2)y_1}{y^2} = 0$	1	
		$2xy - x^2y_1 + y^2y_1 = 0$		
		For any alternate method give full score		

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4		$L.H.S = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$	1	3
		$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$	1	
		$= (b-a)(c-a)(c+a-b-a)$		
		$= (a-b)(b-c)(c-a)$	1	
		$= R.H.S$		
5		$\int_0^2 e^x dx = \lim_{n\to\infty} \frac{2-0}{n}\left[e^0 + e^{2/n} + e^{4/n} + \dots + e^{\frac{2n-2}{n}}\right]$	1	3
		$= \lim_{n\to\infty} \frac{2}{n}\left(\frac{e^2-1}{e^{2/n}-1}\right)$	1	
		$= (e^2-1)\lim_{h\to 0}\left(\frac{h}{e^h-1}\right)$	1	
		$= e^2 - 1$		
		Remarks: For Formula give 1 score; for $S_n = \frac{a(r^n-1)}{r-1}$ - ½ score For direct answer give 1 score		
6	(i)	(b) $x = \tan^{-1}(3/4)$	1	3
	(ii)	$\tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$	½	
		$= \tan\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)$	1	
		Remarks $= \tan\left(\tan^{-1}\frac{17}{6}\right) = \frac{17}{6}$	½	
		For formula give ½ score		
7	(i)	(a) $x \in [-\pi/2, \pi/2]$	1	
	(ii)	$\sin^{-1}\left(\sin\frac{13\pi}{4}\right) = \sin^{-1}\left(\sin\frac{12\pi+\pi}{4}\right)$	1	
		$= \sin^{-1}(\sin(3\pi + \pi/4))$		

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		$= \sin^{-1}(-\sin \pi/4)$	1/2	3
		$= -\sin^{-1}(\sin \pi/4) = -\pi/4$	1/2	
8	(i)	$Fog(x) = f(g(x))$	1	4
		$= f(x^3)$	1	
		$= \sin(x^3)$		
	(ii)	$f(x) = \sin x$ is a trigonometric function $\therefore$ continuous		
		$g(x) = x^3$ is a polynomial function $\therefore$ continuous.	1	
		Hence $fog(x) = \sin(x^3)$ is continuous	1	
9		$\vec{a_1} = i + 2j + k \quad \vec{a_2} = 2i - j - k$		4
		$\vec{b_1} = i - j + k \quad \vec{b_2} = 2i + j + 2k$	1	
		$\vec{a_2} - \vec{a_1} = i - 3j - 2k$	1/2	
		$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3i + 3k$	1	
		Shortest distance $= \left\lvert \dfrac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{\lvert \vec{b_1} \times \vec{b_2} \rvert} \right\rvert$	1	
		$= \left\lvert \dfrac{-3 + 3(-2)}{\sqrt{9+9}} \right\rvert = 3/\sqrt{2}$	1/2	
10	(i)	local maximum at $x = 0$	1/2	4
		local minimum at $x = 2$	1/2	
	(ii)	$f'(x) < 0 \Rightarrow$ strictly decreasing on $(0\ 2)$	1	
	(iii)	$f(-1) = f(2) = -10$ absolute minimum	1	
		$f(4) = 10$ Absolute maximum	1	
		Remark: For decreasing function $f'(x) < 0$ ($\frac{1}{2}$ score)		

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11	(i)	$\frac{dy}{dx} = \frac{1}{\cos(e^x)} \times (-\sin(e^x))\, e^x$	2	
		$= -\tan(e^x) \times e^x$	1	
	(ii)	$y_1 = \frac{1}{\sqrt{1-x^2}}$	½	4
		$\sqrt{1-x^2}\, y_1 = 1$		
		$\sqrt{1-x^2}\cdot y_2 + y_1 \times \frac{1}{2\sqrt{1-x^2}}(-2x) = 0$	½	
		$(1-x^2)\, y_2 + -x y_1 = 0$		
12	(i)	$\int_0^{\pi/2} \cos 2x\, dx = \left(\frac{\sin 2x}{2}\right)_0^{\pi/2}$	1	
		$= \frac{1}{2}[\sin\pi - \sin 0]$	1	
		$= 0$	1	
	(ii)	Area $= 2 \times \int_0^{\pi/4} \cos 2x\, dx$		4
		$= 2 \times \left(\frac{\sin 2x}{2}\right)_0^{\pi/4}$	½	
		$= \sin \pi/2 - \sin 0$	½	
		$= 1$		
		Remarks (i) $\int \cos x\, dx = \sin x + c$ (give ½ score)		
		(ii) ..... (for fig: ½ score)		
		(iii) Area $= \int_0^{\pi/2} \cos 2x\, dx$ (give 1 score)		
13		$P(A) = 0.25$ $P(B) = 0.35$ $P(C) = 0.4$	½	
		E: getting a defective bolt		
		$P(E/A) = 0.05$, $P(E/B) = 0.04$,		
		$P(E/C) = 0.02$	1	
		$P(B/E) = \frac{P(E/B) \times P(B)}{P(E/A)\times P(A) + P(E/B)\times P(B) + P(E/C)\times P(C)}$	1	

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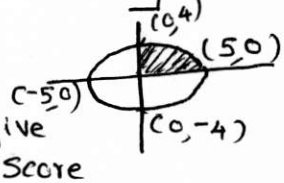
		$= \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02}$	1½	4
		$= \frac{28}{69}$		
14.	(i)	$X = \{0, 1, 2, 3\}$	½	
		$P(X=0) = 1/8$ $\because P(X=0) = P(\{TTT\})$		
		$P(X=1) = 3/8$ $P(X=1) = P(\{HTT\}, \{THT\}, \{TTH\})$		
		$P(X=2) = 3/8$ $\because P(X=2) = P(\{HHT, HTH, THH\})$		
		$P(X=3) = 1/8$ $\because P(X=3) = P(\{HHH\})$	1	
		Probability distribution		
		X: 0, 1, 2, 3 / P(X): 1/8, 3/8, 3/8, 1/8	½	4
	(ii)	Mean $= E(X) = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$	1	
		$= 3/2$		
		Variance $= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 4 \times \frac{3}{8} + 9 \times \frac{1}{8} - \left(\frac{3}{2}\right)^2$	1	
		$= 3/4$		
		Remarks: (i) for sample space (1 score) (ii) $\text{Var}(X) = E(X^2) - (E(X))^2$ (½ score) (iii) for $E(X) = \Sigma x_i P_i$ (½ score) or $\Sigma x_i P_i - \Sigma x_i^2 P_i$		
15.	(i)	$\frac{dy}{dx} = (x-1)(x-3)$	2	
		$= x^2 - 4x + 3$		
	(ii)	$dy = (x^2 - 4x + 3)\,dx$		
		$\int dy = \int (x^2 - 4x + 3)\,dx$	½	
		$y = \frac{x^3}{3} - 2x^2 + 3x + C$	1	
		passes through (3, 1)		
		$\therefore 1 = 9 - 18 + 9 + C$	½	
		$C = 1$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$\therefore$ Equation is $y = \frac{x^3}{3} - 2x^2 + 3x + 1$ Remarks: (i) For writing $f(x)$ as 3rd degree polynomial give ½ score (ii) For writing $f'(x)$ as 2nd degree poly. give ½ score. (iii) For analysing (ii) give 2 score.		4
16		$y^2 = 16\left(1 - \frac{x^2}{25}\right) = \frac{16}{25}(25 - x^2)$ $y = \frac{4}{5}\sqrt{25 - x^2}$ Area $= 4 \times \frac{4}{5}\int_0^5 \sqrt{25 - x^2}\, dx$ $= \frac{16}{5}\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_0^5$ $= \frac{16}{5}\left[0 + \frac{25}{2}\cdot\frac{\pi}{2} - 0 - 0\right]$ $= 20\pi$  Remarks: (i) For figure give ½ score (ii) for formula $\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ give 1 score (iii) For direct formula $\pi ab = 20\pi$ give 4 score	1 1 1 1	4
17		$\frac{dy}{dx} = \frac{2}{(x-3)^2}$ $\frac{2}{(x-3)^2} = 2$ $x = 2, 4$ when $x = 2$, $y = 2$	1 ½ ½ ½	

6/11

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$\therefore$ Equation is $y-2 = 2(x-2)$	½	
		$2x - y = 2$		
		when $x = 4$, $y = -2$	½	4
		$\therefore$ Equation is $y+2 = 2(x-4)$	½	
		$2x - y = 10$		
		Remarks: (i) Slope $= \frac{dy}{dx}$ (give ½ score)		
		(ii) Writing equation of tangent as $y - y_1 = f'(x)(x - x_1)$ give ½ score		
		(iii) Writing equation of one line give 2 score		
18	(i)			

*	1	2	4	8
1	1	2	4	8
2	2	2	4	8
4	4	4	4	8
8	8	8	8	8

		Every element in the operation table are elements in A. Hence * is a binary operation.	3	6
	(ii)	For analysing the problem give 2 score	2	
	(iii)	For analysing the problem give 1 score	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
19	(I)	$A \times adj(A) = \begin{bmatrix} 4 & 3 & 2 \\ 1 & p & 0 \\ 1 & q & 2 \end{bmatrix} \begin{bmatrix} 4 & -4 & r \\ -2 & 6 & 2 \\ -1 & s & 2 \end{bmatrix}$		
		$= \begin{bmatrix} 8 & 2+2s & 4r+10 \\ 4-2p & -4+6p & r+2p \\ 2-2q & -4+6q+2s & r+2q+4 \end{bmatrix}$	(2)	
		$\|A\| = 8$ or $\|A\| = 6p+2q-6$	(1)	(6)
	(II)	For Analysing the problem give 3 score [value of 'r' is not unique)	3	
20	1	$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$	2	
	II	$A^T = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ -1 & 1 & 3 \end{bmatrix}$	1	
		$\frac{1}{2}[A+A^T] = \frac{1}{2}\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$ - symmetric matrix	1	6
		$\frac{1}{2}[A-A^T] = \frac{1}{2}\begin{bmatrix} 0 & -3 & -6 \\ 3 & 0 & -3 \\ 6 & 3 & 0 \end{bmatrix}$ → Skew symmetric matrix	1	
		$A = \frac{1}{2}[A+A^T] + \frac{1}{2}[A-A^T]$	1	

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21	I	$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$	1/2	
		$A = 1 \quad B = -5 \quad C = 4$	1/2	
		$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \left[\frac{1}{x-1} - \frac{5}{x-2} + \frac{4}{x-3}\right] dx$	(1)	
		$= \log\lvert x-1\rvert - 5\log\lvert x-2\rvert + 4\log\lvert x-3\rvert + C$	(1)	
	II	$I = \int e^x \sin x \, dx$		
		$= e^x(-\cos x) - \int e^x(-\cos x)\, dx$	1	
		$= -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$	1	
		$2I = e^x[\sin x - \cos x]$		
		$I = \frac{e^x}{2}[\sin x - \cos x) + C$	1	(6)
		Remark: Formula for integration by parts 1/2 score		
22	I	$\bar{a} \cdot \bar{b} = 0$	1/2	
		$2+m+3 = 0, \quad m = -5$	1/2	
	II	$\bar{c} = \bar{a} \times \bar{b}$	1	
		$= \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & -5 & 3 \end{vmatrix} = 8i - j - 7k$	1	
		Remark: $\bar{c} = (3-m)i - j + (m-2)k$ give 2 score.		

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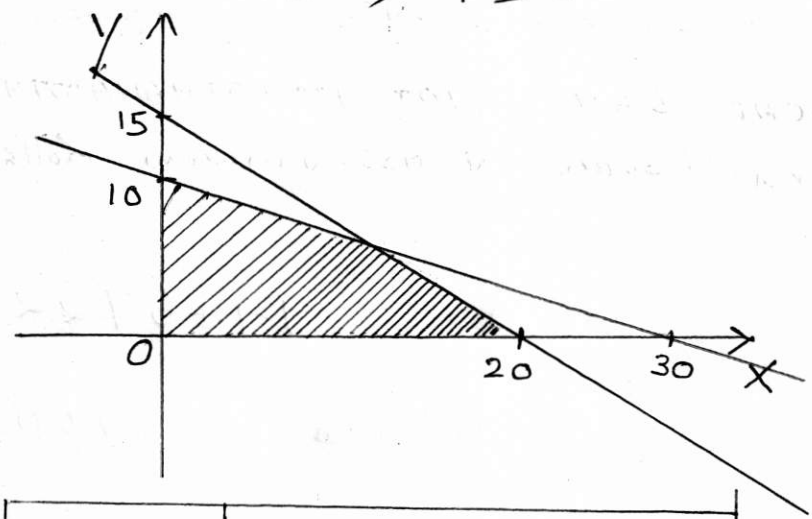
	III	$[\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -5 & 3 \\ 8 & -1 & -7 \end{vmatrix} = 114$	1½	
		$\lvert\bar{a}\rvert\lvert\bar{b}\rvert\lvert\bar{c}\rvert = \sqrt{3}\cdot\sqrt{38}\cdot\sqrt{114} = 114$	1½	
		Hence verified.		6
		Remarks: Attempting with 'm' give 3 score		
23	I	$(-3\ 5, -6)$	(1)	
	II	Equation of the plane $2(x+3)+1(y-6)+2(z+6)=0$ $2x+y+2z+13=0$	2	
	III	Any pt on the line is $(2\lambda-3, \lambda+5, 2\lambda-6)$	1	
		Distance between $(2\lambda-3, \lambda+5, 2\lambda-6)$ and $(-3, 5, -6)$ is 3	½	
		$(2\lambda)^2+(\lambda)^2+(2\lambda)^2 = 9$	½	(6)
		$9\lambda^2 = 9$		
		$\lambda = -1, 1$	½	
		The point is $(-1, 6, -4)$ or $(-5, 4, -8)$	½	

11/11

SECOND YEAR HIGHER SECONDARY EXAMINATION (SAY) JUNE 2019

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
24		Maximise : $Z = 8000x + 12000y$		
		constraints : $9x + 12y \leq 180$		
		$3x + 4y \leq 60$		
		$x + 3y \leq 30$		
		$x \geq 0, \quad y \geq 0$	2	
			2	6
		Corner points | $Z = 8000x + 12000y$ $O(0,0)$ | $Z = 0$ $A(20,0)$ | $Z = 160000$ $B(12,6)$ | $Z = 168000$ ← maximum $C(0,10)$ | $Z = 120000$	2	
		model A – 12 pieces		
		model B – 6 pieces.		
		maximum profit – Rs. 1,68,000		

11/11