#419776

Topic: Thermometry

The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.

Solution

The conversion formulae are

$t_c = t_k - 273.15$

$t_f = \frac{9}{5}t_c + 32$

Substituting the given values, we get

$\text{Neon: } 24.57\ K = -248.43\ ^oC = -415.44\ ^oF$

$\text{CO: } 216.55\ K = -56.45\ ^oC = -69.8\ ^oF$

#419783

Topic: Thermometry

Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B ?

Solution

Since triple point of water is constant, the relation is

$\frac{T_A}{200} = \frac{T_B}{350}$

or $T_A = \frac{4}{7}T_B$

#419787

Topic: Linear Expansion

The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law:

$R = R_o\left[1 + \alpha(T - T_o)\right]$

The resistance is 101.6 Ω at the triple-point of water 273.16K, and 165.5 Ω at the normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4 Ω

Solution

We first find α by putting the given values,

$165.5 = 101.6(1 + \alpha(600.5 - 273.16))$

$\therefore \alpha = 1.92 \times 10^{-3}$

Now, Temp when Resistance is 123.4 is

$123.4 = 101.6(1 + \alpha(T - 273.16))$

$T = 384.91\ K$

#419791

Topic: Thermometry

Answer the following:

(a) The triple-point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?

(b) There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0^oC and 100^oC respectively. On the absolute scale one of the fixed points is the triple-point of water, which on the kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (kelvin) scale?

(c) The absolute temperature (Kelvin scale) T is relate to the temperature t_c on the Celsius scale by $t_c = T - 273.15$ Why do we have 273.15 in this relation and not 273.16?

(d) What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale ?

Solution

(a) Triple-point has a unique temperature; fusion point and boiling point temperatures depend on pressure.

(b) The other fixed point is the absolute zero itself.

(c) Triple-point is 0.01^oC, not 0^oC.

(d) $T = 273.15 \times 9/5 = 491.69$

The temperature and pressure at which a substance can exist in equilibrium in the liquid, solid, and gaseous states.It is unique and constant. The problem with Celsius scale is that boiling and melting point change with atmospheric pressure at that place.

#420046

Topic: Linear Expansion

A steel tape $1m$ long is correctly calibrated for a temperature of 27.0^oC. The length of a steel rod measured by this tape is found to be $63.0cm$ on a hot day when the temperature is 45.0^oC. What is the actual length of the steel rod on that day? What is the length of the same steel road on a day when the temperature is 27.0^oC? Coefficient linear expansion of steel $= 1.20 \times 10^{-5} K^{-1}$.

Solution

Length of the steel tape at temperature $T = 27^oC, l = 1m = 100cm$

At temperature $T_1 = 45^oC$, the length of the steel rod, l_1 = 63 cm

Coefficient of linear expansion of steel, $\alpha = 1.2 \times 10^{-5}\ K$

Let l be the actual length of the steel rod and l' be the length of the steel tape at 45^oC.

$l' - l = l\alpha(T_1 - T)$

$l' = 100(1 + 1.20 \times 10^{-5} \times (45 - 27)) = 100.0216cm$

Hence, the actual length of the steel rod measured by the steel tape at 45^0C can be calculated as:

$l_2 = (100.0216/100)63 = 63.0136cm$

Therefore, the actual length of the rod at 45^oC is 63.0136 cm. Its length at 27^oC is $63.0cm.$

#420067

Topic: Linear Expansion

A large steel wheel is to be fitted on to a shaft of the same material. At 27^oC the outer diameter of the shaft is $8.70cm$ and the diameter of the central hole in the wheel is $8.69cm.$ The shaft is cooled using 'dry ice'. At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to the consta over the required temperature range : $\alpha_{steel} = 1.20 \times 10^{-5} K^{-1}$

Solution

The given temperature, T = 27^oC can be written in Kelvin as: 27 + 273 = 300 K

Outer diameter of the steel shaft at T, d_1 = 8.70 cm

Diameter of the central hole in the wheel at T, d_2 = 8.69 cm

Coefficient of linear expansion of steel, $\alpha_{steel} = 1.2 \times 10^5\ K^{-1}$

After the shaft is cooled using dry ice, its temperature becomes T_1.

The wheel will slip on the shaft, if the change in diameter, $d = |8.69 - 8.7| = 0.01cm$

Temperature T_1, can be calculated from the relation:

$\Delta d = d_1\ \alpha_{steel}\ (T_1 - T)$

$0.01 = 8.7 \times 10^5 \times (T_1 - 300)$

$(T_1 - 300) = 95.78$

$T_1 = 204.21K$

$= 204.21273.16$

$= 68.95^oC$

Therefore, the wheel will slip on the shaft when the temperature of the shaft is 69^oC.

#420072

Topic: Linear Expansion

A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0^oC. What is the change in the diameter of the hole when the sheet is heated to 227^oC? Coefficient of the linear expansion of copper $= 1.70 \times 10^{-5} K^{-1}$

Solution

Initial temperature, $T_1 = 27.0^oC$

Diameter of the hole at $T_1, d_1 = 4.24cm$

Final temperature, $T_2 = 227^oC$

Diameter of the hole at $T_2 = D_2$

Co-efficient of linear expansion of copper, $\alpha_{Cu} = 1.70 \times 10^{-5} K^{-1}$

For co-efficient of superficial expansion β,and change in temperature $\triangle T$, we have the relation:

Change in area $(\triangle)$ / Original area (A) = $\beta \triangle T$

$[(\pi d_2^2/4) - (\pi d_2^2/4)]/(\pi d_1^1/4) = \triangle A/A$

$\therefore \triangle A/A = (d_2^2 - d_1^2)/d_1^2$

But $\beta = 2\alpha$

$\therefore (d_2^2 - d_1^2)/d_1^2 = 2\alpha \triangle T$

$(d_2^2/d_1^2) - 1 = 2\alpha(T_2 - T_1)$

$d_2^2/4.24^2 = 2 \times 1.7 \times 10^{-5}(227 - 27) + 1$

$d_2^2 = 17.98 \times 1.0068 = 18.1$

$\therefore d_2 = 4.2544cm$

Change in diameter $= d_2 - d_1 = 4.2544 - 4.24 = 0.0144cm$

Hence, the diameter increases by $1.44 \times 10^{-2}cm$

#420073

Topic: Thermal Stresses and Strain

A brass wire 1.8 m long at 27^0C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of -39^oC, what is the tension developed in tl wire, if its diameter is $2.0mm$? Co-efficient of linear expansion of brass $= 2.0 \times 10^{-5}K^{-1}$; Young's modulus of brass $= 0.91 \times 10^{11}$ Pa

Solution

Initial temperature, $T_1 = 27^oC$

Length of the brass wire at $T_1, l = 1.8m$

Final temperature, $T_2 = 39C$

Diameter of the wire, $d = 2.0\,mm = 2{\times}10^{-3}m$

Tension developed in the wire $= F$

Coefficient of linear expansion of brass, $= 2.0 \times 10^{-5}K^{-1}$

Youngs modulus of brass, $Y = 0.91 \times 10^{11}$ Pa

Youngs modulus is given by the relation:

Y= Stress / Strain

$$Y = \frac{F/A}{\Delta L/L}$$

$\Delta L = F \times L/(A \times Y)$ ......**(i)**

Where,

$F =$ Tension developed in the wire

$A =$ Area of cross-section of the wire.

$\Delta L =$ Change in the length, given by the relation:

$\Delta L = \alpha L(T_2$ -$T_1)$ .....**(ii)**

Equating equations **(i)** and **(ii)**, we get:

$$\alpha L(T_2 - T_1) = \frac{FL}{\pi(d/2)^2 Y}$$

$$F = \alpha(T_2 - T_1)Y\pi(d/2)^2$$

$$F = 2 \times 10^{-5} \times (-39 - 27) \times 3.14 \times 0.91 \times 10^{11} \times (2 \times 10^{-3}/2)^2$$

$$F = -3.8 \times 10^2 N$$

(The negative sign indicates that the tension is directed inward.)

Hence, the tension developed in the wire is 3.8 $\times 10^2$ N.

#420082

Topic: Thermal Stresses and Strain

A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250^oC, if the original lengths are at 40.0^oC? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand (Co-efficient of linear expansion of brass $= 2.0 \times 10^{-5}K^{-1}$, steel $= 1.2 \times 10^{-5}K^{-1}$)

Solution

Initial temperature, $T_1 = 40^oC$

Final temperature, $T_2 = 250^oC$

Change in temperature, $\triangle T = T_2 - T_1 = 210^oC$

Length of the brass rod at $T_1, l_1 = 50cm$

Diameter of the brass rod at $T_1, d_1 = 3.0cm$

Length of the steel rod at $T_2, l_2 = 50cm$

Diameter of the steel rod at $T_2, d_2 = 3.0mm$

Coefficient of linear expansion of brass, $\alpha_1 = 2.0 \times 10^{-5} K^{-1}$

Coefficient of linear expansion of steel, $\alpha_2 = 1.2 \times 10^{-5} K^{-1}$

For the expansion in the brass rod, we have:

Change in length $(\triangle l_1)$ / Original length $(l_1) = \alpha_1 \triangle T$

$\therefore \triangle l_1 = 50 \times (2.1 \times 10^{-5}) \times 210$

$= 0.2205\, cm$

For the expansion in the steel rod, we have:

Change in length $(\triangle l_2)$/ Original length $(l_2) = \alpha_2 \triangle T$

$\therefore \triangle l_1 = 50 \times (1.2 \times 10^{-5}) \times 210$

$= 0.126\, cm$

Total change in the lengths of brass and steel,

$\triangle l = \triangle l_1 + \triangle l_2$

$= 0.2205 + 0.126$

$= 0.346\, cm$

Total change in the length of the combined rod $= 0.346cm$

Since the rod expands freely from both ends, no thermal stress is developed at the junction.

#420083

Topic: Volume Expansion

The coefficient of volume expansion of glycerin is $49 \times 10^{-5} K^{-1}$. What is the fractional change in its density for a 30^oC rise in temperature?

Solution

Coefficient of volume expansion of glycerin, $\gamma = 49 \times 10^{-5} K^{-1}$

Rise in temperature, $T = 30^oC$

Fractional change in its volume $= \Delta V / V$

This change is related with the change in temperature as:

$$\frac{\Delta V}{V} = \gamma \Delta T$$

$$\frac{\Delta V}{V} = 49 \times 10^{-5} \times 30 = 0.0147$$

Let V : volume, m : mass, ρ : density

m is constant by law of conservation of mass.

$m = \rho V$

$0 = \rho\, dV + V\, d\rho$

$$\frac{d\rho}{\rho} = -\frac{dV}{V} = -0.0147$$

#420085

Topic: Calorimetry

A 10 kw drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium $= 0.91 J\, g^{-1} K^{-1}$

Solution

Power of the drilling machine, $P = 10kW$

Mass of the aluminum block, $m = 8.0kg$

Time for which the machine is used,$t = 2.5\,min = 2.5 \times 60 = 150s$

Specific heat of aluminium,$c = 0.91\ J/gK$

Rise in the temperature of the block after drilling$= T$

Total energy of the drilling machine$= Pt = 10 \times 10^3 \times 150 = 1.5 \times 10^6 J$

It is given that only 50% of the power is useful.

Useful energy, $Q = (50/100) \times 1.5 \times 10^6 = 7.5 \times 10^5 J$

BUT $Q = mc\Delta T$

$\Delta T = Q/mc = (7.5 \times 10^5)/(8 \times 10^3 \times 0.91) = 103^oC$

Therefore, in 2.5 minutes of drilling, the rise in the temperature of the block is 103^oC.

#420092

Topic: Calorimetry

A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500^oC and then placed on a large ice block. What is the maximum amount of ice that can melt ?

(Specific heat of copper$= 0.39\,J\,g^{-1}\,K^{-1};$ heat of fusion of water $= 335\,J\,g^{-1})$

Solution

Mass of the copper block, $m = 2.5\,kg = 2500\,g$

Rise in the temperature of the copper block, $= 500^oC$

Specific heat of copper, $C = 0.39\ J/g^oC$

Heat of fusion of water, $L = 335\ J/g$

The maximum heat the copper block can lose,

$Q = mC$

$= 2500 \times 0.39 \times 500$

$= 487500\,J$

Let m_1 g be the amount of ice that melts when the copper block is placed on the ice block.

The heat gained by the melted ice,$Q = m_1 L$

$m_1 = Q/L = 487500/335 = 1455.22\,g$

Hence, the maximum amount of ice that can melt is 1.45 kg.

#420096

Topic: Calorimetry

In an experiment on the specific heat of a metal a 0.20 kg block of the metal at 150^oC is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing $150\ cm^3$ o

water at 27^oC. The final temperature is 40^oC. Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller th

the actual value for specific heat of the metal ?

Solution

Mass of the metal, m = 0.20 kg = 200 g

Initial temperature of the metal, T_1 = 150 ^oC

Final temperature of the metal, T_2 = 40 ^oC

Calorimeter has water equivalent of mass, m = 0.025 kg = 25 g

Volume of water, V = 150 cm^3

Mass (M) of water at temperature T = 27^{o}C:

$150 \times 1 = 150\, g$

Fall in the temperature of the metal:

$\Delta T_m = T_1$-$T_2 = 150 - 40 = 110\, ^oC$

Specific heat of water, $C_w = 4.186\; J/g/K$

Specific heat of the metal $= C$

Heat lost by the metal, $= mCT$ .... **(I)**

Rise in the temperature of the water and calorimeter system: $T_1 - T = 40 - 27 = 13\, ^oC$

Heat gained by the water and calorimeter system: $= m_1 C_w T = (M+m) C_w T$....(ii)

Heat lost by the metal = Heat gained by the water and colorimeter system

$mC\Delta T_m = (M+m)C_w T_w$

$200 \times C \times 110 = (150 + 25) \times 4.186 \times 13$

$C = (175 \times 4.186 \times 13)/(110 \times 200) = 0.43\; Jg^{-1}k^{-1}$

If some heat is lost to the surroundings, then the value of C will be smaller than the actual value.

#420098
Topic: Calorimetry

A geyser heats water flowing at the rate of 3.0 litres per minute from 27^oC to 77^oC. If the geyser operates on a gas burner. What is the rate of consumption of the fuel if its hea of combustion is $4.0 \times 10^4\; J/g$?

Solution

Water is flowing at a rate of 3.0 litre/min.

The geyser heats the water, raising the temperature from 27^oC to 77^oC.

Initial temperature, $T_1 = 27\, ^oC$

Final temperature, $T_2 = 77\, ^oC$

Rise in temperature,$T = T_1 - T_2 = 77 - 27 = 50\, ^oC$

Heat of combustion $= 4 \times 10^4\; J/g$

Specific heat of water, $C = 4.2\; J/g\, ^oC$

Mass of flowing water, $m =$ 3.0 litre/min = 3000 g/min

Total heat used, $Q = mcT$

$= 3000 \times 4.2 \times 50$

$= 6.3 \times 10^5\; J/min$

Rate of consumption $= 6.3 \times 10^5/(4 \times 10^4)$ = 15.75 g/min.

#420103
Topic: Calorimetry

What amount of heat must be supplied to $2.0 \times 10^{-2} kg$ of nitrogen (at room temperature) to raise its temperature by 45^oC at constant pressure? (Molecular mass of $N_2 = 28 : R = 8.3\, mol^{-1}K^{-1}$)

Solution

Mass of nitrogen, $m = 2.0 \times 10^{-2} kg = 20g$

Rise in temperature, $\Delta T = 45^o C$

Molecular mass of $N2, M = 28$

Universal gas constant, $R = 8.3\ J/mol\ K$

Number of moles, $n = m/M$

$= (2 \times 10^{-2} \times 10^3)$ / 28

$= 0.714$

Molar specific heat at constant pressure for nitrogen, Cp = (7/2)R

$= (7/2) \times 8.3$

$= 29.05\ J\ mol^{-1}\ K^{-1}$

The total amount of heat to be supplied is given by the relation:

$Q = nC_P\Delta T$

$= 0.714 \times 29.05 \times 45$

$= 933.38\ J$

Therefore, the amount of heat to be supplied is 933.38 J.

#420225

Topic: Calorimetry

A child running a temperature of $101^0 F$ is given an antipyrin (i.e. a medicine that lowers fever) which causes an increases in the rate of evaporation of sweat from his body. If the fever is brought down to $98^o F$ in 20 min, what is the average rate of extra evaporation caused, by the drug. Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water and latent heat of evaporation of water at that temperature is about $580\ cal\ g^{-1}$.

Solution

Initial temperature of the body of the child, $T_1 = 101^o F$

Final temperature of the body of the child, $T_2 = 98^o F$

Change in temperature, $\triangle T = [(101 - 98) \times 5/9 = 5/3^o C$

Time taken to reduce the temperature, t = 20 min

Mass of the child, $m = 30\ kg = 30 \times 10^3 g$

Specific heat of the human body = Specific heat of water = c

$= 1000 cal/kg/^o C$

Latent heat of evaporation of water, $L = 580\ cal\ g^{-1}$

The heat lost by the child is given as:

$\triangle\theta = mc\triangle T$

$= 30 \times 1000 \times (101 - 98) \times (5/9)$

$= 50000\ cal$

Let m_1 be the mass of the water evaporated from the child's body in 20 min.

Loss of heat through water is given by:

$\triangle\theta = m_1 L$

$\therefore m_1 = \triangle\theta/L$

$= (50000/580) = 86.2g$

$\therefore$ Average rate of extra evaporation caused by the drug $= m_1/t$

$= 86.2/200$

$= 4.3g/min.$

#420235

Topic: Calorimetry

A 'thermacole' icebox is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is $45^o C$, and co-efficient of thermal conductivity of thermacole is $0.01\ J\ s^{-1} m^{-1} K^{-1}$. [Heat of fusion of water $= 335 \times 10^3\ J\ kg^{-1}$]

Solution

Side of the given cubical ice box, $s = 30\,cm = 0.3\,m$

Thickness of the ice box, $l = 5.0\,cm = 0.05\,m$

Mass of ice kept in the ice box, $m = 4\,kg$

Time gap, $t = 6\,h = 6 \times 60 \times 60s$

Outside temperature, $T = 45^oC$

Coefficient of thermal conductivity of thermacole, $K = 0.01\,Js^{-1}m^{-1}k^{-1}$

Heat of fusion of water, $L = 335 \times 10^3\,J\,kg^{-1}$

Let m be the total amount of ice that melts in 6 h.

The amount of heat lost by the food:

$\theta = KA(T-0)t/l$

Where,

$A =$ Surface area of the box $= 6s^2 = 6 \times (0.3)^2 = 0.54m^3$

$\theta = 0.01 \times 0.54 \times 45 \times 6 \times 60/0.05 = 104976J$

But $\theta = m'L$

$\therefore m' = \theta/L$

$= 104976/(335 \times 10^3) = 0.313\,kg$

Mass of ice left $= 4 - 0.313 = 3.687kg$

Hence, the amount of ice remaining after 6 h is 3.687 kg.

#420247

Topic: Phase Change

A brass boiler has a base area of $0.15\,m^2$ and thickness 1.0 cm. It boils water at the rate of 6.0 kg / min when placed on a gas stove. Estimate the temperature of the part of th flame in contact with the boiler. Thermal conductivity of brass $= 109\,J\,s^{-1}m^{-1}K^{-1}$; Heat of vaporisation of water $= 2256 \times 10^3 Jkg^{-1}$.

Solution

Base area of the boiler, $A = 0.15m^2$

Thickness of the boiler, $l = 1.0cm = 0.01m$

Boiling rate of water, $R = 6.0kg/min$

Mass, $m = 6\,kg$

Time, t = 1 min = 60 s

Thermal conductivity of brass, $K = 109\,J\,s^{-1}m^{-1}k^{-1}$

Heat of vaporisation, $L = 2256 \times 10^3\,J\,kg^{-1}$

The amount of heat flowing into water through the brass base of the boiler is given by:

$\theta = KA(T_1 - T_2)t/l$.... (i)

where,

$T_1 =$ Temperature of the flame in contact with the boiler

$T_2 =$ Boiling point of water $= 100^oC$

Heat required for boiling the water:

$\theta = mL$ .... (ii)

Equating equations **(i)** and **(ii)**, we get:

$\therefore mL = KA(T_1 - T_2)t/l$

$T_1 - T_2 = mLl/KAt$

$= 6 \times 2256 \times 10^3 \times 0.01/(109 \times 0.15 \times 60)$

$= 137.98^oC$

Therefore, the temperature of the part of the flame in contact with the boiler is 237.98^oC.

#420264

Topic: Radiation

Explain why :

(a) A body with large reflectivity is a poor emitter

(b) A brass tumbler feels much colder than a wooden tray on a chilly day

(c) An optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open but gives a correct value for the temperature when the same piece is in the furnace

(d) The earth without its atmosphere would be inhospitably cold

(e) Heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water

Solution

(a) A body with a large reflectivity is a poor absorber of light radiations. A poor absorber will in turn be a poor emitter of radiations. Hence, a body with a large reflectivity is a poor emitter.

(b) Brass is a good conductor of heat. When one touches a brass tumbler, heat is conducted from the body to the brass tumbler easily. Hence, the temperature of the body reduces to a lower value and one feels cooler.

Wood is a poor conductor of heat. When one touches a wooden tray, very little heat is conducted from the body to the wooden tray. Hence, there is only a negligible drop in th temperature of the body and one does not feel cool.

Thus, a brass tumbler feels colder than a wooden tray on a chilly day.

(c) An optical pyrometer calibrated for an ideal black body radiation gives too low a value for temperature of a red hot iron piece kept in the open.

Black body radiation equation is given by:

$E = \sigma(T^4 - T_0^4)$

Where,

E = Energy radiation

T = Temperature of optical pyrometer

$T_0 =$ Temperature of open space

$\sigma =$ Constant

Hence, an increase in the temperature of open space reduces the radiation energy.

When the same piece of iron is placed in a furnace, the radiation energy, $E = \sigma T^4$

(d) Without its atmosphere, earth would be inhospitably cold. In the absence of atmospheric gases, no extra heat will be trapped. All the heat would be radiated back from earth's surface.

(e) A heating system based on the circulation of steam is more efficient in warming a building than that based on the circulation of hot water. This is because steam contains surplus heat in the form of latent heat (540 cal/g).

#420267

Topic: Newton's Law of Cooling

A body cools from 80^oC to 50^oC in 5 minutes. Calculate the time it takes to cool from 60^oC to 30^oC. The temperature of the surroundings is 20^oC.

Solution

According to Newton's law of cooling, we have:

$(-dT/dt) = K(T - T_0)$

$dT/K(T - T_0) = -Kdt$.... (i)

Where,

Temperature of the body = T

Temperature of the surroundings $T_0 = 20^oC$

K is a constant

Temperature of the body falls from 80^oC to 50^oC in time, t = 5 min = 300 s

Integrating equation (i), we get:

$$\int_{50}^{80} \frac{dT}{K(T-T_0)} = -\int_0^{300} Kdt$$

$$log_e[T - T_0]_{50}^{80} = -K[t]_0^{300}$$

$$\frac{2.3026}{K} log_{10} \frac{80-20}{50-20} = -300$$

$$\frac{2.3026}{K} log_{10} 2 = -300$$

$\dfrac{-2.3026}{300} log_{10} 2 = K$ ... (ii)

The temperature of the body falls from 60^0C to 30^oC in time =t'

Hence, we get:

$$\frac{2.3026}{K} log_{10} \frac{60-20}{30-20} = t'$$

$\dfrac{-2.3026}{t'} log_{10} 4 = K$ ... (iii)

Equating (ii) and (iii), we get:

$$\frac{-2.3026}{t'} log_{10} 4 = \frac{-2.3026}{300} log_{10} 2$$

$\therefore t' = 300 \times 2 = 600s = 10min$

Therefore, the time taken to cool the body from 60^oC to 30^oC is 10 minutes

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#420400

Topic: Volume Expansion

A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for exampl does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so-called law of atmospheres

$n_2 = n_1 exp\left[-mg\left(h_2 - h_1\right)/k_BT\right]$

where n_2, n_1 refer to number density at heights h_2 and h_1 respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid colum

$n_2 = n_1 exp[-mg\,N_A(\rho - \rho')(h_2 - h_1)/(\rho\,RT)]$

where ρ is the density of the suspended particle and ρ' that of surrounding medium. [N_A is Avogadro's number, and R the universal gas constant.]

[Hint : Use Archimedes principle to find the apparent weight of the suspended particle.]

Solution

According to the law of atmospheres, we have:

$n_2 = n_1\ exp[-mg(h_2 - h_1)/k_BT]$...(i)

where,

n_1 is the number density at height h_1, and n_2 is the number density at height h_2

mg is the weight of the particle suspended in the gas column

Density of the medium $= \rho'$

Density of the suspended particle $= \rho$

Mass of one suspended particle $= m'$

Mass of the medium displaced $= m$

Volume of a suspended particle $= V$

According to Archimedes' principle for a particle suspended in a liquid column, the effective weight of the suspended particle is given as:

Weight of the medium displaced – Weight of the suspended particle

$= mg - m'g$

$= mg - V\rho'g = mg - (m/\rho)\rho'g$

$= mg(1 - (\rho'/\rho))$... (ii)

Gas constant, $R = k_BN$

$k_B = R/N$ ...(iii)

Substituting equation (ii) in place of mg in equation (i) and then using equation (iii), we get:

$n_2 = n_1\ exp[-mg(h_2 - h_1)/k_BT]$

$\quad = n_1\ exp[-mg(1 - \rho'/\rho))(h_2 - h_1)(N/RT)]$

$\quad = n_1\ exp[-mg(\rho - \rho')(h_2 - h_1)(N/RT\rho)]$