**#419429**

**Topic:** Pressure in Static Fluid

Explain why

(a) The blood pressure in humans is greater at the feet than at the brain

(b) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km

(c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area

**Solution**

(a) The height of the blood column in the human body is more at feet than at the brain. That is why, the blood exerts more pressure at the feet than at the brain.

(Pressure $= \rho g h$)

where $h =$ height, $\rho =$ density of liquid and $g =$ accleration due to gravity)

(b) Density of air decreases very rapidly with increase in height and reduces to nearly half its value at the sea level at a height of about 6 km. After 6 km, the density of air decreases but rather very slowly.

(c) When force is applied on a liquid, then according to Pascal's Law, the pressure in the liquid is transmitted in all directions.

Hence, hydrostatic pressure does not have a fixed direction and it is a scalar physical quantity.

**#419436**

**Topic:** Angle of Contact

Explain why

(a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.

(b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not).

(c) Surface tension of a liquid is independent of the area of the surface.

(d) Water with detergent dissolved in it should have small angles of contact.

(e) A drop of liquid under no external forces is always spherical in shape.

**Solution**

(a) When a liquid comes in contact with solid, three surfaces are formed namely liquid-air, solid-air and solid-liquid. The surface tensions corresponding to these interfaces are $T_{LA}$, $T_{SA}$ *and* $T_{SL}$ respectively

Now angle of contact of solid with liquid is

$$\theta = \frac{T_{SA} - T_{SL}}{T_{LA}}$$

In case of mercury-glass $T_{SA} < T_{SL}$ ,making $cos\theta$ negative and the angle of contact obtuse.

The angle between the tangent to the liquid surface at the point of contact and the surface inside the liquid is called the angle of contact $\theta$ , as shown in the given figure.

$S_{SL}$, $S_{LA}$, $S_{SA}$ are the respective interfacial tensions between the liquid-air, solid-air, and solid-liquid interfaces. At the line of contact, the surface forces between the three media must be in equilibrium,

The angle of contact $\theta$ , is obtuse if $S_{SA} < S_{LA}$ (as in the case of mercury on glass). This angle is acute if $S_{SL} < S_{LA}$ (as in the case of water on glass).

(b) $T_{LA}cos\theta + T_{SL} = T_{SA}$

When $\theta$ is an obtuse angle then molecules of liquids are attracted strongly to themselves and weakly to those of solid, it costs a lot of energy to create a liquid-solid surface, and liquid then does not wet the solid. This is what happens with mercury on a glass surface. On the other hand,

if the molecules of the liquid are strongly attracted to those of the solid, this will reduce $T_{SL}$ and therefore, cos$\theta$ may increase or may decrease. In the case of water, it is an acute angle and hence water

wets the glass surface.

(c) Surface tension is defined as the force acting per unit length on either side of a imaginary line drawn tangentially on the surface of a liquid. As the force is independent of area of liquid surface taken, so is the surface tension.

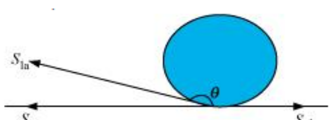
(d) The cloth has narrow spaces in form of fine capillaries.The rise of liquid in a capillary tube is given by

$$h = \frac{2Tcos\theta}{\rho rg}$$

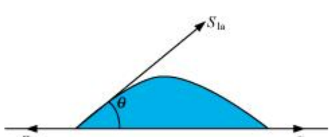
$h \propto cos\theta$

this implies that if detergent has a small angle of contact it will have a greater value of h, implying that detergent penetrates more in the cloth to remove dirt

(e) A liquid tends to acquire the minimum surface area to minimise the energy by the virtue of surface tension. For a given volume, the sphere has the least surface area.




Mercury on glass




Water on glass

**#419443**

**Topic:** Surface Tension

Fill in the blanks using the word(s) from the list appended with each statement.

(a) Surface tension of liquids generally ....... with temperatures. (increases / decreases)

(b) Viscosity of gases ........ with temperature, whereas viscosity of liquids ....... with temperature (increases / decreases)

(c) For solids with elastic modulus of rigidity, the shearing force is proportional to ....... , while for fluids it is proportional to ....... (shear strain / rate of shear strain)

(d) For a fluid in a steady flow, the increase in flow speed at a constriction follows (conservation of mass / Bernoullis principle)

(e) For the model of a plane in a wind tunnel, turbulence occurs at a ....... speed for turbulence for an actual plane (greater / smaller)

**Solution**

(a) Surface tension of liquids generally decreases with temperatures. As temperature increases, the kinetic energy of molecules increases and they overcome the attractive forces.

(b) $\eta$ of gases increase and $\eta$ of liquids decrease with an increases in temperature.

As temperature increases, the average speed of molecules in a liquid also increase and as a result, they spend less time with their "neighbors." Therefore, as temperature increases, the average intermolecular forces decrease and the molecules are able to interact without being "weighed down" by one another. However, the viscosity of a gas increases as temperature increases because there is an increase in frequency of intermolecular collisions at higher temperatures. Since the molecules are flying around in the void most of the time, any increase in the contact they have with one another will increase the intermolecular force which will ultimately lead to a disability for the whole substance to move.

(c) For solids with elastic modulus of rigidity, the shearing force is proportional to Shear Strain

$F = G\epsilon A$

For fluids, it is proportional to the rate of shear strain

$F = \mu \frac{du}{dy}$

(d) Conservation of mass will always be followed and since the flow is steady, even Bernoullis principle would be followed.

(e) Greater

Since in an actual environment, the external disturbances also come into picture that cannot be introduced in a lab.

**#419450**

**Topic:** Fluid Dynamics

Explain why

(a) To keep a piece of paper horizontal, you should blow over, not under, it .

(b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers.

(c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.

(d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.

(e) A spinning cricket ball in air does not follow a parabolic trajectory.

**Solution**

a)-When air is blown under a paper, the velocity of air is greater under the paper than it is above it. As per Bernoulli’s principle, atmospheric pressure reduces under the makes the paper fall. To keep a piece of paper horizontal, one should blow over it. This increases the velocity of air above the paper. As per Bernoulli’s principle, atmospheric pressure reduces above the paper and the paper remains horizontal.

b)According to the equation of continuity:

$Area \times velocity = C$ , as we decrease the area ,velocity increases .

For a smaller opening, the velocity of flow of a fluid is greater than it is when the opening is bigger. When we try to close a tap of water with our fingers, fast jets of water gush through the openings between our fingers. This is because very small openings are left for the water to flow out of the pipe. Hence, area and velocity are inversely proportional to each other .

c)- Small opening of a syringe needle controls the velocity of the blood flowing out. This is because of the equation of continuity. At the constriction point of the syringe system, the flow rate suddenly increases to a high value for a constant thumb pressure applied.

d)-When a fluid flows out from a small hole in a vessel, the vessel receives a backward. thrust. A fluid flowing out from a small hole has a large velocity according to the equation of continuity:

$Area \times velocity = C$

According to the law of conservation of momentum, the vessel attains a backward velocity because there are no external forces acting on the system

e)- spinning cricket ball has two simultaneous motions-rotary and linear. These two types of motion oppose the effect of each other. This decreases the velocity of air flowing below the ball. Hence, the pressure on the upper side of the ball becomes lesser than that on the lower side. An upward force acts upon the ball. Therefore, the ball takes a curved path. It does not follow a parabolic path.

**#419455**

**Topic:** Pressure in Static Fluid

A $50kg$ girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter $1.0cm$. What is the pressure exerted by the heel on the horizontal floor ?

**Solution**

Mass of the girl, $m = 50kg$

Diameter of the heel, $d = 1cm = 0.01m$

Radius of the heel, $r = \dfrac{d}{2} = 0.005m$

Area of the heel $= \pi r^2$

$= \pi \times (0.005)^2$

$= 7.85 \times 10^{-5} m^2$

Force exerted by the heel on the floor:

$F = mg$

$= 50 \times 9.8$

$= 490N$

Pressure exerted by the heel on the floor:

$P = \dfrac{Force}{Area}$

$= 490/(7.85 \times 10^{-5}) = 6.24 \times 10^6 N/m^2$

Therefore, the pressure exerted by the heel on the horizontal floor is $6.24 \times 10^6 N/m^2$.

**#419456**

**Topic:** Pressure Measurement

Toricelli's barometer used mercury. Pascal duplicated it using French wine of density 984 kg m$^{-3}$. Determine the height of the wine column for normal atmospheric pressure.

**Solution**

Density of mercury, $\rho_1 = 13.6 \times 10^3 kg/m3$

Height of the mercury column, $h_1 = 0.76m$

Density of French wine, $\rho_2 = 984\ kg/m3$

Height of the French wine column $= h_2$

Acceleration due to gravity, $g = 9.8m/s^2$

The pressure in both the columns is equal, i.e.,

Pressure in the mercury column $=$ Pressure in the French wine column

$\rho_1 h_1 g = \rho_2 h_2 g$

$h_2 = \rho_1 h_1 / \rho_2$

$= 13.6 \times 10^3 \times 0.76/984 = 10.5\ m$

Hence, the height of the French wine column for normal atmospheric pressure is 10.5 m.

**#419457**

**Topic:** Pressure in Static Fluid

A vertical off-shore structure is built to withstand a maximum stress of $10^9$ Pa. Is the structure suitable for putting up on top of an oil well in the ocean ? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.

**Solution**

Depth of sea $h = 3 \times 10^3 m$

Maximum stress $= 10^9 Pa$

Density of water $\rho = 10^3 kg/m^3$

Pressure exerted by a water column of depth 3 km

$P = h\rho g$

$= 3 \times 10^3 \times 10^3 \times 9.8$

$= 2.94 \times 10^7 Pa$

This pressure is less than the maximum stress of $10^9 Pa$; so the structure is suitable.

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**#419458**

**Topic:** Pressure in Static Fluid

A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is 425 $cm^2$. What maximum pressure would the smaller piston have to bear ?

**Solution**

Pressure on the piston $P = \dfrac{F}{A}$

Force $F = m \times a$

$= 3000 \times 9.8$

$= 29400\ N$

Area of cross section $A = 425 \times 10^{-4} sq\,m$

Therefore the pressure $P = \dfrac{3000 \times 9.8}{425 \times 10^{-4}} = 6.92 \times 10^5 Pa$.

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**#419459**

**Topic:** Pressure in Static Fluid

A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit ?

**Solution**

Height of the spirit column, $h_1 = 12.5 cm = 0.125 m$

Height of the water column, $h_2 = 10 cm = 0.1 m$

$P_o$ = Atmospheric pressure

$\rho_1$ = Density of spirit

$\rho_2$ = Density of water

Pressure at point B = $P_0 + \rho_1 h_1 g$

Pressure at point D =$P_0 + \rho_2 h_2 g$

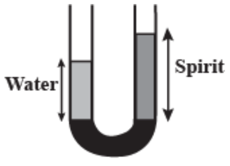
Pressure at points B and D is the same.

$P_o + \rho_1 h_1 g = P_o + \rho_2 h_2 g$

$\rho_1 / \rho_2 = h_2 / h_1$

$= 10/12.5 = 0.8$

Therefore, the specific gravity of spirit is $0.8$.

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**#419460**

**Topic:** Pressure in Static Fluid

In the below given problem, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms ? (Specific gravity of mercury = 13.6)

Problem:

[A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other ]

**Solution**

Height of the water column, $h_1 = 10 + 15 = 25cm$

Height of the spirit column, $h_2 = 12.5 + 15 = 27.5cm$

Density of water, $\rho_1 = 1g/cm^3$

Density of spirit, $\rho_2 = 0.8g/cm^3$

Density of mercury $\rho = 13.6g/cm^3$

Let h be the difference between the levels of mercury in the two arms.

Pressure exerted by height h, of the mercury column:

$= \rho hg$

$= h \times 13.6 \times g$ **(i)**

Difference between the pressures exerted by water and spirit:

$= \rho_1 h_1 g - \rho_2 h_2 g$

$= g(25 \times 1 - 27.5 \times 0.8)$

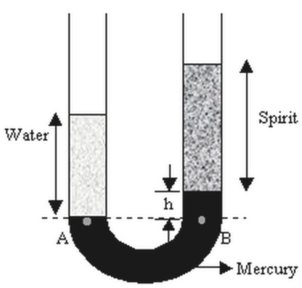
$= 3g$ **(ii)**

Equating equations **(i)** and **(ii)**, we get:

13.6 hg = 3g

$h = 0.221cm$

Hence, the difference between the levels of mercury in the two arms is 0.221 cm.




**#419461**

**Topic:** Bernoulli's Equation

Can Bernoulli's equation be used to describe the flow of water through a rapid in a river ? Explain.

**Solution**

Bernoullis equation cannot be used to describe the flow of water through a rapid in a river because of the turbulent flow of water. This principle can only be applied to a streamline flow.

**#419462**

**Topic:** Bernoulli's Equation

Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation ? Explain.

**Solution**

Using Bernoulli's theorem : $P + \rho gh + \frac{1}{2}\rho v^2 =$ constant

where $P$ is the absolute pressure at a point, $\rho$ is the density of the fluid, $h$ is the height of that point above a reference point and $v$ is the velocity of fluid at that point.

Thus $P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$

Subtracting atmospheric pressure from both sides.

We get $P_1 - P_o + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 - P_o + \rho gh_2 + \frac{1}{2}\rho v_2^2$

Gauge pressure $P_i' = P_i - P_o \quad (i = 1, 2)$

$\implies P_1' + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2' + \rho gh_2 + \frac{1}{2}\rho v_2^2$

Thus the Bernoulli's equation remains in the same form.

Hence, it does not matter if one uses gauge instead of absolute pressure in applying Bernoulli's equation.

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**#419463**

**Topic:** Stoke's Law and Terminal Velocity

Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is $4.0 \times 10^{-3} kgs^{-1}$, what is the pressure difference between the two ends of the tube ? (Density of glycerine $= 1.3 \times 10^3 kgm^{-3}$ and viscosity of glycerine = 0.83 Pa s). [You may also like to check if the assumption of laminar flow in the tube is correct].

**Solution**

Length of the horizontal tube, $l = 1.5m$

Radius of the tube, $r = 1cm = 0.01m$

Diameter of the tube, $d = 2r = 0.02m$

Glycerine is flowing at a rate of $4 \times 10^{-3}$ kg/ s.

$M = 4.0 \times 10^{-3}$ kg/s

Density of glycerine, $\rho = 1.3 \times 10^3 kg/m^3$

Viscosity of glycerine, $\eta = 0.83\ Pas$

Volume of glycerine flowing per sec:

$= 4 \times 10^{-3}/(1.3 \times 10^3)$

$= 3.08 \times 10^{-6} m^3/s$

According to Poisevilles formula, we have the relation for the rate of flow:

$V = \pi p r^4/8\eta l$

where $p$ is the pressure difference between the two ends of the tube

$p = V 8\eta l/\pi r^4$

$= 3.08 \times 10^{-6} \times 8 \times 0.83 \times 1.5/[\pi \times 0.01^4]$

$= 9.8 \times 10^2$ Pa

Reynolds number is given by the relation:

$R = 4\rho v/\pi d\eta$

$= 4 \times 1.3 \times 10^3 \times 3.08 \times 10^{-6}/(0.02 \times 0.83)$

$= 0.3$

Reynolds number is about 0.3. Hence, the flow is

laminar.

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**#419464**

**Topic:** Bernoulli's Equation

In a test experiment on a model aeroplane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are $70\ ms^{-1}$ and $63\ ms^{-1}$ respectively. What is the lift on the wing if its area is 2.5 m$^2$ ? Take the density of air to be $1.3\ kgm^{-3}$.

**Solution**

Speed of wind on the upper surface of the wing, $V_1 = 70\ m/s$

Speed of wind on the lower surface of the wing, $V_2 = 63\ m/s$

Area of the wing, $A = 2.5\ m^2$

Densityof air, $\rho = 1.3\ kg/m^3$

According to Bernoullis theorem, we have the relation:

$P1 + (1/2)\rho(V_1^2) = P2 + (1/2)\rho(V_2^2)$

$P2 - P1 = (1/2)\rho(V_1^2 - V_2^2)$

Where,

$P1 =$ Pressure on the upper surface of the wing

$P2 =$ Pressure on the lower surface of the wing

The pressure difference between the upper and lower surfaces of the wing provides lift to the aeroplane.

Lift on the wing $= (P2 - P1)A$

$= (1/2)\rho(V_1^2 - V_2^2)A$

$= (1/2)1.3[70^2 - 63^2]2.5$

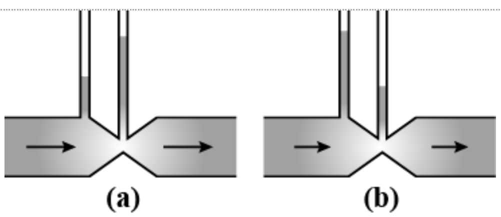
$= 1512.87$

$N = 1.51 \times 10^3\ N$

Therefore, the lift on the wing of the aeroplane is $1.51 \times 10^3\ N$.

**#419465**

**Topic:** Bernoulli's Equation



Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect ? Why ?

**Solution**

Fig. (a) is incorrect. Accoridng to equation of continuity, i.e., $av$ = Constant, where area of cross-section of tube is less, the velcoity of liquid flow is more. So the velocity of liquid flow at a constriction of tube is more than the other portion of tube.

Accroding to Bernoulli's Theorem,

$P + \frac{1}{2}\rho v^2$ = Constant,

where $v$ is more, $P$ is less and vice versa.

**#419466**

**Topic:** Fluid Dynamics

The cylindrical tube of a spray pump has a cross-section of $8.0\ cm^2$ one end of which has 40 fine holes each of diameter 1.0 mm. If the liquid flow inside the tube is $1.5\ m\ min^{-1}$, what is the speed of ejection of the liquid through the holes ?

**Solution**

The cross-section area of the cylindrical tube $A_1 = 8cm^2$

$= 8 \times 10^{-4} m^2$

The speed of the liquid flow inside the tube $V_1 = 1.5\ m/\ minute$

$= \frac{1.5}{60} ms^{-1}$

Area of each hole $= \pi(0.5 \times 10^{-3})^2\ m^2$

Area of 40 holes $A_2 = 40\pi(0.5 \times 10^{-3})^2 m^2$

$A_1 V_1 = A_2 V_2$

$V_2 = \frac{A_1 V_1}{A_2}$

$= \frac{8 \times 10^{-4} \times 1.5}{40\pi \times (0.5 \times 10^{-3})^2 \times 60} = 0.636\ ms^{-1}$.

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**#419467**

**Topic:** Surface Tension

A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of $1.5 \times 10^{-2}\ N$ (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film ?

**Solution**

Weight $mg = 1.5 \times 10^{-2} N$

Length of slider $l = 30 \times 10^{-2} m$

Force due to surface tension = weight

We know that $\sigma \times 2l = mg$

Therefore surface tension $\sigma = \frac{mg}{2l}$

$= \frac{1.5 \times 10^{-2}}{2 \times 30 \times 10^{-2}}$

$= 2.5 \times 10^{-2}\ Nm^{-1}$.

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**#419468**

**Topic:** Surface Tension

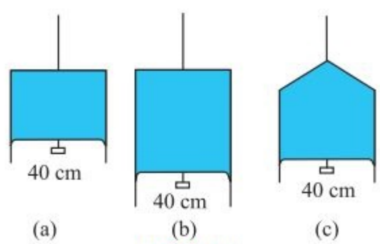


Figure (a) shows a thin liquid film supporting a small weight $= 4.5 \times 10^{-2}\ N$. What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically.

**Solution**

Take case (a):The length of the liquid film supported by the weight, l = 40 cm = 0.4 cm

The weight supported by the film, $W = 4.5 \times 10^{-2}\ N$

A liquid film has two free surfaces.

Surface tension $= W/2l$

$= 4.5 \times 10^{-2}/(2 \times 0.4) = 5.625 \times 10^{-2}\ N/m$

In all the three figures, the liquid is the same. Temperature is also the same for each case. Hence, the surface tension in figure (b) and figure (c) is the same as in figure (a), i.e., $5.625 \times 10^{-2}\ N/m$.

Since the length of the film in all the cases is 40 cm, the weight supported in each case is $4.5 \times 10^{-2}\ N$.

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**#419469**

**Topic:** Liquid Drops and Bubbles

What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature ? Surface tension of mercury at that temperature ($20^o\ C$) is $4.65 \times 10^{-1}\ Nm^{-1}$. The atmospheric pressure is $1.01 \times 10^5 Pa$. Also give the excess pressure inside the drop.

**Solution**

Radius of the mercury drop, $r = 3.00 mm = 3 \times 10^{-3}\ m$

Surface tension of mercury, $S = 4.65 \times 10^{-1}\ N/m$

Atmospheric pressure, $P_o = 1.01 \times 10^5\ Pa$

Total pressure inside the mercury drop

= Excess pressure inside mercury + Atmospheric pressure

$= 2S/r + P_o$

$= [2 \times 4.65 \times 10^{-1}/(3 \times 10^{-3})] + 1.01 \times 10^5$

$= 1.0131 \times 10^5$

Excess pressure = 2S / r

$= [2 \times 4.65 \times 10^{-1}/(3 \times 10^{-3})] = 310\ Pa$

**#419472**

**Topic:** Liquid Drops and Bubbles

What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature ($20^oC$) is $2.50 \times 10^{-2}\ Nm^{-1}$ ? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble ? (1 atmospheric pressure is $1.01 \times 10^5\ Pa$)

**Solution**

Excess pressure inside the soap bubble is $20\ Pa$;

Pressure inside the air bubble is $1.06 \times 10^5 Pa$

Soap bubble is of radius, $r = 5.00\ mm = 5 \times 10^{-3} m$

Surface tension of the soap solution, $S = 2.50 \times 10^{-2} Nm^{-1}$

Relative density of the soap solution $= 1.20$

$\therefore$ Density of the soap solution, $\rho = 1.2 \times 10^3\ kg/m^3$

Air bubble formed at a depth, $h = 40 cm = 0.4 m$

Radius of the air bubble, $r = 5 mm = 5 \times 10^{-3} m$

1 atmospheric pressure $= 1.01 \times 10^5 Pa$

Acceleration due to gravity, $g = 9.8 m/s^2$

Hence, the excess pressure inside the soap bubble is given by the relation:

$$P = \frac{4S}{r}$$
$$= \frac{4 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}}$$
$$= 20\ Pa$$

Therefore, the excess pressure inside the soap bubble is 20 Pa.

The excess pressure inside the air bubble is given by the relation:

$$P' = \frac{2S}{r}$$
$$= \frac{2 \times 2.5 \times 10^{-2}}{(5 \times 10^{-3})}$$
$$= 10\ Pa$$

Therefore, the excess pressure inside the air bubble is 10 Pa.

At a depth of 0.4 m, the total pressure inside the air bubble

= Atmospheric pressure $+ h\rho g + P'$

$= 1.01 \times 10^5 + 0.4 \times 1.2 \times 10^3 \times 9.8 + 10$

$\approx 1.06 \times 10^5\ Pa$

Therefore, the pressure inside the air bubble is $1.06 \times 10^5\ Pa$

**#419474**

**Topic:** Pressure in Static Fluid

A tank with a square base of area 1.0 $m^2$ is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area 20 $cm^2$. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. compute the force necessary to keep the door close.

**Solution**

Base area of the given tank, $A = 1.0 m^2$

Area of the hinged door, $a = 20\ cm^2 = 20 \times 10^{-4} m^2$

Density of water, $\rho_1 = 10^3\ kg/m^3$

Density of acid, $\rho_2 = 1.7 \times 10^3\ kg/m^3$

Height of the water column, $h_1 = 4m$

Height of the acid column, $h_2 = 4m$

Acceleration due to gravity, $g = 9.8$

Pressure due to water is given as:

$P_1 = h_1\rho_1 g$

$= 4 \times 10^3 \times 9.8 = 3.92 \times 10^4\ Pa$

Pressure due to acid is given as:

$P_2 = h_2\rho_2 g$

$= 4 \times 1.7 \times 10^3 \times 9.8 = 6.664 \times 10^4\ Pa$

Pressure difference between the water and acid columns:

$\triangle P = P_2 - P_1$

$= 6.664 \times 10^4 - 3.92 \times 10^4$

$= 2.744 \times 10^4$

Hence, the force exerted on the door $= \triangle P \times a$

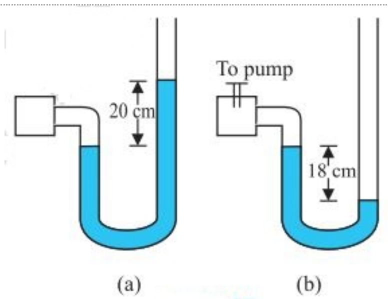
$= 2.744 \times 10^4 \times 20 \times 10^{-4}$

$= 54.88N$

Therefore, the force necessary to keep the door closed is $54.88N$.

**#419478**

**Topic:** Pressure Measurement




A manometer reads the pressure of a gas in an enclosure as shown in Fig (a). When a pump removes some of the gas, the manometer reads as in Fig (b). The liquid used in the manometers is mercury and the atmospheric pressure is $76\ cm$ of mercury.

(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of $cm$ of mercury.

(b) How would the levels change in case (b) if $13.6\ cm$ of water (immiscible with mercury) are poured into the right limb of the manometer? (Ignore the small change in the volume of the gas).

**Solution**

(a) For figure (a)

Atmospheric pressure, $P_0 =$ 76 cm of Hg

Difference between the levels of mercury in the two limbs gives gauge pressure

Hence, gauge pressure is 20 cm of Hg.

Absolute pressure = Atmospheric pressure + Gauge pressure

= 76 + 20 = 96 $cm$ of Hg

For figure (b)

Difference between the levels of mercury in the two limbs $= -18cm$

Hence, gauge pressure is –18 cm of Hg.

Absolute pressure = Atmospheric pressure + Gauge pressure

$= 76cm - 18cm = 58\ cm$

(b) 13.6 cm of water is poured into the right limb of figure (b).

Relative density of mercury = 13.6

Hence, a column of 13.6 cm of water is equivalent to 1 cm of mercury.

Let h be the difference between the levels of mercury in the two limbs.

The pressure in the right limb is given as:

$PR =$ Atmospheric pressure + 1 cm of Hg

= 76 + 1 = 77 $cm$ of Hg ..... **(i)**

The mercury column will rise in the left limb.

Hence, pressure in the left limb, $PL = 58 + h$ ..... **(ii)**

Equating equations **(i)** and **(ii)**, we get:

$77 = 58 + h$

$\therefore\ h = 19\ cm$

Hence, the difference between the levels of mercury in the two limbs will be 19 cm.

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**#419481**

**Topic:** Pressure in Static Fluid

Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill upto a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases ? If so, why do the vessels filled with water to that same height give different readings on a weighing scale ?

**Solution**

Pressure (and therefore force) on the two equal base areas are identical. But force is exerted by water on the sides of the vessels also, which has a nonzero vertical component when the sides of the vessel are not perfectly normal to the base. This net vertical component of force by water on sides of the vessel is greater for the first vessel than the second. Hence the vessels weigh different even when the force on the base is the same in the two cases.

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**#419483**

**Topic:** Pressure Measurement

During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein ? [Density of whole blood $= 1.06 \times 10^3 kgm^{-3}$].

**Solution**

Gauge pressure, $P = 2000Pa$

Density of whole blood, $\rho = 1.06 \times 10^3$ kg/ m$^3$

Acceleration due to gravity, $g = 9.8m/s^2$

Height of the blood container $= h$

Pressure of the blood container, $P = h\rho g$

$h = P/\rho g$

$= 2000/(1.06 \times 10^3 \times 9.8)$

$= 0.1925\, m$

The blood can enter the vein if the blood container is kept at a height higher than 0.1925 m, i.e., about 0.2 m

**#419484**

**Topic:** Stoke's Law and Terminal Velocity

In deriving Bernoullis equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy. (a) What is the largest average velocity of blood flow in an artery of diameter $2 \times 10^{-3}$ m if the flow must remain laminar ? (b) Do the dissipative forces become more important as the fluid velocity increases ? Discuss qualitatively.

**Solution**

(a)

Diameter, $D = 2 \times 10^{-3}m$

Viscosity of the blood $\eta = 2.084 \times 10^3 Pa$

Density of the blood $\rho = 1.06 \times 10^3 kg\, m^{-3}$

Maximum value of Reynolds number of flow to be laminar, $N_R = 2000$

Average velocity $V_c = \dfrac{N_R \eta}{\rho D}$

$$= \frac{2000 \times 2.084 \times 10^3}{1.06 \times 10^3 \times 2 \times 10^3}$$

$$= \frac{4.168}{2.12} = 1.96\ m/s$$

(b) The dissipative forces become more important with increasing flow velocity, because of turbulence.

**#419486**

**Topic:** Stoke's Law and Terminal Velocity

(a) What is the largest average velocity of blood flow in an artery of radius $2 \times 10^{-3}\ m$ if the flow must remain lanimar? (b) What is the corresponding flow rate ? (Take viscosity of blood to be $2.084 \times 10^{-3}\ Pas$).

**Solution**

(a) Radius of the artery $r = 2 \times 10^{-3} m$

Diameter, $D = 2 \times 2 \times 10^{-3} m = 4 \times 10^{-3} m$

Viscosity of the blood $\eta = 2.084 \times 10^{3} Pa$

Density of the blood $\rho = 1.06 \times 10^{3} kg\, m^{-3}$

Maximum valu of Reynold number of flow to be laminar, $N_R = 20N$

Average velocity $V_c = \dfrac{N_R \eta}{\rho D}$

$$= \frac{2000 \times 2.084 \times 10^3}{1.06 \times 10^3 \times 4 \times 10^3}$$

$$= \frac{4.168}{4.24} = 0.98 m/s$$

(b) Flow rate is given by the relation:

$R = \pi r^2 V(avg)$

$= 3.14 \times (2 \times 10^{-3})^2 \times 0.983$

$= 1.235 \times 10^{-5}\ m^3/s$

Therefore, the corresponding flow rate is $1.235 \times 10^{-5} m^3/s$.

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**#419489**

**Topic:** Bernoulli's Equation

A plane is in level flight at constant speed and each of its two wings has an area of 25 m$^2$. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. (Take air density to be $1\ kgm^{-3}$).

**Solution**

The area of the wings of the plane, $A = 225 = 50 sq - m$

Speed of air over the lower wing, $V_1 = 180 km/h = 50 m/s$

Speed of air over the upper wing, $V_2 = 234 km/h = 65 m/s$

Density of air, $= 1 kg/cu.\ m$

Pressure of air over the lower wing $= P_1$

Pressure of air over the upper wing $= P_2$

The upward force on the plane can be obtained using Bernoullis equation as:

$P_1 - P_2 = \dfrac{1}{2}\rho(V_2^2 - V_1^2)$

$F = (P_1 - P_2)A = \dfrac{1}{2}\rho(V_2^2 - V_1^2)A$

$F = (P_1 - P_2)A = \dfrac{1}{2} \times 1 \times (65^2 - 50^2) \times 50 = 43125N$

$\therefore\ m = \dfrac{F}{g} = 4400 kg$

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**#419492**

**Topic:** Stoke's Law and Terminal Velocity

In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius $2.0 \times 10^{-5} m$ and density $1.2 \times 10^3\ kg\, m^{-3}$. Take the viscosity of air at the temperature of the experiment to be $1.8 \times 10^{-5}\ Pa\, s$. How much is the viscous force on the drop at that speed ? Neglect buoyancy of the drop due to air.

**Solution**

Terminal speed $= 5.8\ cm/s$

Viscous force $= 3.9 \times 10^{-10} N$

Radius of the given uncharged drop, $r = 2.0 \times 10^{-5} m$

Density of the uncharged drop, $\rho = 1.2 \times 10^{-3} kg\, m^{-3}$

Viscosity of air, $\eta = 1.8 \times 10^{-5} Pa\, s$

Density of air ($\rho_0$) can be taken as zero in order to neglect buoyancy of air.

Acceleration due to gravity, $g = 9.8\ m/s^2$

Terminal velocity (v) is given by the relation:

$V = 2r^2 \times (\rho - \rho_0) g / 9\eta$

$= 2 \times (2 \times 10^{-5})^2 (1.2 \times 10^3 - 0) \times 9.8 / (9 \times 1.8 \times 10^{-5})$

$= 5.8 \times 10^{-2} m/s$

$= 5.8\ cm\, s^{-1}$

Hence, the terminal speed of the drop is $5.8\ cm\, s^{-1}$.

The viscous force on the drop is given by:

$F = 6\pi\eta r v$

$\therefore F = 6 \times 3.14 \times 1.8 \times 10^{-5} \times 2 \times 10^{-5} \times 5.8 \times 10^{-2}$

$= 3.9 \times 10^{-10} N$

Hence, the viscous force on the drop is $3.9 \times 10^{-10} N$.

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**#419493**

**Topic:** Capillarity

Mercury has an angle of contact equal to $140^o$ with soda lime glass. A narrow tube of radius $1.00\ mm$ made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside ? Surface tension of mercury at the temperature of the experiment is $0.465\ N\, m^{-1}$. Density of mercury $= 13.6 \times 10^3\ kg\, m^{-3}$.

**Solution**

Angle of contact between mercury and soda lime glass, $\theta = 140^o$

Radius of the narrow tube, $r = 1 mm = 1 \times 10^{-3} m$

Surface tension of mercury at the given temperature, $s = 0.465 N m^{-1}$

Density of mercury, $\rho = 13.6 \times 10^3 kg/m^3$

Dip in the height of mercury $= h$

Acceleration due to gravity, $g = 9.8 m/s^2$

Surface tension is related with the angle of contact and the dip in the height as:

$s = h\rho g r / 2 cos\theta$

$\therefore h = 2s\, cos\theta / \rho g r$

$= 2 \times 0.465 \times cos 140^o / (1 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8)$

$= -0.00534 m$

$= -5.34 m$

Here, the negative sign shows the decreasing level of mercury. Hence, the mercury level dips by 5.34 mm.

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**#419496**

**Topic:** Capillarity

Two narrow bores of diameters $3.0\ mm$ and $6.0\ mm$ are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube ? Surface tension of water at the temperature of the experiment is $7.3 \times 10^{-2}\ N\, m^{-1}$. Take the angle of contact to be zero and density of water to be $1.0 \times 10^3\ kg\, m^{-3}$ ($g = 9.8\ m\, s^{-2}$).

**Solution**

Diameter of the first bore, $d_1 = 3.0mm = 3 \times 10^{-3}m$

Hence, the radius of the first bore, $r_1 = d_1/2 = .5 \times 10^{-3}m$

Diameter of the first bore, $d_2 = 6.0mm = 6 \times 10^{-3}mm$

Hence, the radius of the first bore, $r_2 = d_2/2 = 3 \times 10^{-3}m$

Surface tension of water, $s = 7.3 \times 10^{-2}Nm^{-1}$

Angle of contact between the bore surface and water, $\theta = 0$

Density of water, $\rho = 1.0 \times 10^3 kg/m^{-3}$

Acceleration due to gravity, $g = 9.8m/s^2$

Let $h_1$ and $h_2$ be the heights to which water rises in the first and second tubes respectively. These heights are given by the relations:

$h_1 = 2s\cos\theta/r_1\rho g$ ..... (i)

$h_2 = 2s\cos\theta/r_2\rho g$ .... (ii)

The difference between the levels of water in the two limbs of the tube can be calculated as:

$$= \frac{2s\cos\theta}{r_1\rho g} - \frac{2s\cos\theta}{r_2\rho g}$$

$$= \frac{2\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2 \times 7.3 \times 10^{-2} \times 1}{1 \times 10^3 \times 9.8}\left[\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right]$$

$= 4.966 \times 10^{-3}m$

$= 4.97\ mm$

Hence, the difference between levels of water in the two bores is 4.97 mm.

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**#419498**

**Topic:** Pressure in Static Fluid

(a) It is known that density of air decreases with height y as

$\rho = \rho_0 e^{-y/y_0}$

where $\rho_0 = 1.25$ kg m$^{-3}$ is the density at sea level, and $y_0$ is constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of the atmosphere remains a constant (isothermal conditions). Also, assume that the value of g remains constant.

(b) A large He balloon of volume 1425 m$^3$ is used to lift a payload of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise?

[Take $y_0$ = 8000 m and $\rho_{He} = 0.18$ kg m$^{-3}$].

**Solution**

(a)

Consider an atmospheric layer of thickness dy at height y and cross section area A at static equilibrium.

Mass of the layer = density × volume=Number of atoms per unit volume × volume × mass of an atom

$M = \rho Ady = mANdy$

$\implies \rho = mN.......(i)$

For equilibrium, upward force = downward force + weight

$PA - (P + dP)A = mNAdy\ g$

where $m$: mass of an atom, $N$: Number of atoms per unit volume

$dP = -mANgdy$

Substituting from (i),

$dP = -A\rho gdy..........(ii)$

From gas law,

$P = NkT$

$P = \rho kT/m$

$dP = \frac{kT}{m} d\rho \ldots\ldots\ldots (iii)$

From (ii) and (iii),

$\frac{kT}{m} d\rho = -A\rho g dy$

$\frac{d\rho}{\rho} = -\alpha dy \quad where\ \alpha$ is a constant

$\int_{\rho_o}^{\rho} \frac{d\rho}{\rho} = \int_0^y -\alpha dy$

$ln\rho - ln\rho_o = -\alpha y$

$ln\frac{\rho}{\rho_o} = -\alpha y$

$\rho = \rho_o e^{-\alpha y} \ldots\ldots\ldots\ldots (iv)$

Putting $y = y_o, \rho = \rho_o$

$\alpha = 1/y_o \ldots\ldots\ldots\ldots (v)$

From (iv) and (v),

$\rho = \rho_o e^{-y/y_o}$

Hence proved.

(b) Density $\rho =$ Mass / Volume

$\rho =$ (Mass of the payload + Mass of helium) / Volume

$= (m + V\rho_{He})/V$

$= (400 + 1425 \times 0.18)/1425$

$= 0.46 kg\, m^{-3}$

From part (a)

$\rho = \rho_0 e^{-y/y_0}$

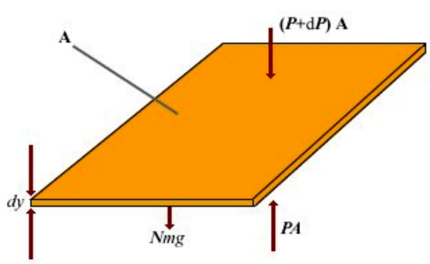
$log_e(\rho/\rho_0) = -y/y_0$

$\therefore y = -8000 \times log_e(0.46/1.25)$

$= -8000 \times (-1)$

$= 8000 m = 8 km$

Hence, the balloon will rise to a height of 8 km.




**#419525**

**Topic:** Density

In what range do you expect the density of the Sun to be, in the range of densities of solids and liquids or gases? Briefly explain with the help of the following data : mass of the Sun = $2.0 \times 10^{30} kg$, radius of the Sun $= 7.0 \times 10^8 m$

**Solution**

$$Density = \frac{mass\ of\ sun}{\frac{4}{3}\pi r^3}$$

$$= \frac{2 \times 10^{30}}{\frac{4}{3} \times \pi \times 7^3 \times 10^{24}} = 1392.0257 kg/m^3$$

The density of the Sun is in the range of density of liquids and solutions not gases.

This high density of the hot plasma arises due to inward gravitational attraction on outer layers due to inner layers of the Sun, and the gases inside the sun are subjected to enormous hydrostatic pressure of gases themselves.