NCERT SOLUTIONS
CLASS-XI PHYSICS
CHAPTER-7
SYSTEM PARTICLES AND ROTATIONAL MOTION

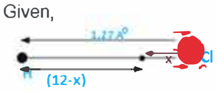
Q1. Where is the center of mass of the following structures located (a) ring, (b) cube, (c) sphere, and (d) cylinder? Is it necessary for a body's center of mass to lie inside it ?

Ans. All the given structures are symmetric bodies having a very uniform mass density Thus, for all the above bodies their center of mass will lie in their geometric centers

It is not always necessary for a body's center of mass to lie inside it , for example the center of mass of a circular ring is at its center.

Q2. In an HCL molecule the distance between the hydrogen and chlorine molecule is 1.27 Å (1Å = 10^{-10} m). If a chlorine atom is 35.5 times the size of a hydrogen atom and almost all its mass is concentrated in its nucleus, approximate where the molecule's (HCL) center of mass is located. **Ans.**

Given,



mass of hydrogen atom = 1 unit

mass of chlorine atom = 35.5 unit (As a chlorine atom is 35.5 times the size)

Let the center of mass lie at a distance x from the chlorine atom

Thus, the distance of center of mass from the hydrogen atom = 1.27 – x

Assuming that the center of mass lies of HCL lies at the origin, we get :

x = $\frac{m(1.27 - x) + 35.5mx}{m + 35.5m} = 0$

m(1.27 – x) + 35.5mx = 0

1.27 – x = -35.5x

Therefore, x = (-1 27)/35 5 -1

= -0.037 Å

The negative sign indicates that the center of mass lies at 0.037Å from the chlorine atom.

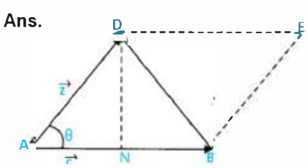
Q3. A 100 kg man is sitting in the corner of a small train moving at V velocity. If this man gets up and runs around inside the train will the speed of the center of mass of this system (train + man) change ?

Ans.

The man and the train constitute a single system and him moving inside the train is a purely internal motion. Since there is no external force on the system the velocity of the center of mass of the system will not change.

Q4. Prove that the triangle between the vectors $\vec{z}$ and $\vec{c}$ has an area equal to one half of the magnitude of $\vec{z} \times \vec{c}$.

Ans.



A.

let $\vec{c}$ be presented as $\overrightarrow{AB}$and $\vec{z}$ be represented as $\overrightarrow{AD}$, as represented in the above figure

Considering ΔADN

sinθ = DN/AD = DN / $\vec{z}$

DN = $\vec{z}$ sinθ

Now, by definition $|\vec{c} * \vec{z}|$ = $\vec{c}$ x $\vec{z}$ sinθ

= AB.DN x 2/2 = 2 x area of ABD

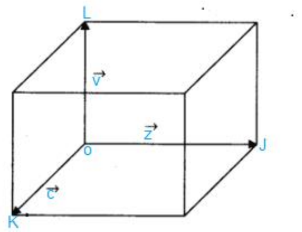
= AB.DN × 2/2 = 2 × area of ABD

Thus, area of ΔADB = ½ ×$|\vec{c} * \vec{z}|$

Q5. Prove that parallelepiped formed by the vectors z, c and v has a volume whose magnitude is equal to the product of z.(c * v)

Ans.

Let the parallelepiped formed be :



Here, $\overrightarrow{OJ} = \vec{z}$, $\overrightarrow{OL} = \vec{v}\, and\, \overrightarrow{OK} = \vec{c}$

Now, $\vec{v} * \vec{c}$ = vcsin90^0 $\hat{n}$

Where $\hat{n}$ is a unit vector along OJ perpendicular to the plane containing $\vec{v}\, and\, \vec{c}$

Now, $\vec{z}(\vec{v} * \vec{c}) = \vec{z}.v * c\hat{n}$

= z * v * c cos0

= z.(v * c) = Volume of the parallelopiped.

Q6. What are the components along x, z and y axes of angular momentum l of a body whose momentum is p with components p_X, p_Y and p_Z and position vector is r with components x, y, z. Prove that if this body's movement is only confined to the x-y its angular momentum will only have the z component.

Ans.

$l_X = yp_Z - zp_Y l_Y$

$= zp_X - xp_Z l_Z$

$= xp_Y - yp_X$

Linear momentum , $\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$

Position vector of the body , $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Angular momentum , $\vec{l} = \vec{r} * \vec{p}$

= $(x\hat{i} + y\hat{j} + z\hat{k}) * (p_x\hat{i} + p_y\hat{j} + p_z\hat{k})$

= $\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{bmatrix}$

$l_x\hat{i} + l_y\hat{j} + l_z\hat{k} = \hat{i}\,(yp_z - zp_y) + \hat{j}(zp_x - xp_z) + \hat{k}(xp_y - zp_x)$

From this we can conclude;

$l_X = yp_Z - zp_y$, $l_Y = zp_X - xp_Z$ and $l_Z = zp_Y - yp_X$

If the body only moves in the x-y plane then z = p_z = 0. Which means :

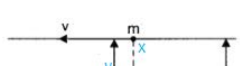
$l_X = l_Y = 0$

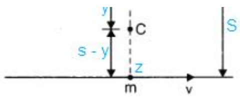
And hence only $l_Z = zp_Y - yp_X$, which is just the z component of angular momentum.

Q7. Two particles, moving at a speed v and each having a mass m, move parallel to each other but in opposite directions. If the distance between the two particle is s , prove that the vector angular momentum of this system is the same irrespective of the point about which the angular momentum is taken.

Ans.

Let us consider three points be Z , C and X :





Angular momentum at Z,

L_Z =mv x 0 + mv x d

=mvd

Angular momentum about B,

L_X = mv x d + mv x 0

= mvd

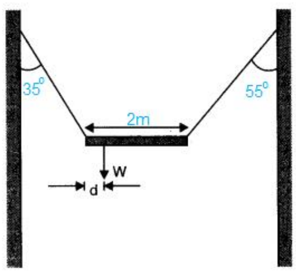
Angular momentum about C,

L_C = mv x y + mv x (s – y) = vdm

Thus we can see that ;

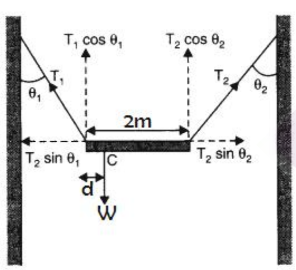
vec{L_Z} = vec{L_X} = vec{L_C} . This proves that the angular momentum of a system does not depend on the point about which its taken.

Q8. A 2m irregular plank weighing W kg is suspended in the manner shown below, by strings of negligible weight. If the strings make an angel of 35^0 and 55^0 respectively with the vertical , find the location of center of gravity of the plank from the left end.



Ans.

The free body diagram of the above figure is:



Given,

Length of the plank, l = 2 m

$\theta_1 = 35^0$ and $\theta_2 = 55^0$

Let T_1 and T_2 be the tensions produced in the left and right strings respectively.

So at translational equilibrium we have;

$T_1 \sin\theta_1 = T_2 \sin\theta_2$

$T_1 / T_2 = \sin\theta_2 / \sin\theta_1$ = sin55 / sin35

T_1 / T_2 = 0.819 / 0.573 = 1.42

$T_1 = 1.42T_2$

Let ‘d’ be the distance of center of gravity of the plank from the left.

For rotational equilibrium about the center of gravity :

T_1cos35 x d = T_2 cos 55 (2 – d)

(T_1 / T_2)x0.82d = (2 x 0.57 – 0.57d)

$1.42 \times 0.82\ d - 0.57\ d = 1.14$

$1.73d = 1.14$

Therefore $d = 0.65m$

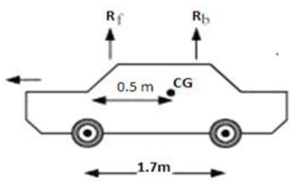
Q9. A small car weighs 1500 kg, its front and back axles have a distance of 1.7m between them. And its center of gravity is 0.5 m behind the front axle. What is the force exerted by the level ground on the front and back wheels ?

Ans.

Given,
Mass of the car, m= 1500 kg

Distance between the two axles, d = 1.7 m
Distance of the C.G. (centre of gravity) from the front axle = 0.5 m
The free body diagram of the car can be drawn as :

Let R_F and R_B be are the forces exerted by the level ground on the front and back wheels respectively.



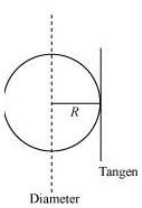
At translational equilibrium:
$R_F + R_B = mg$
$= 1500 \times 9.8$
$= 14700$ N .... (1)

For rotational equilibrium about the C.G., we have:
$R_F\ (0.5) = R_B\ (1.7 - 0.5)$
$R_B / R_F = 1.2 / 0.5$
$R_B = 2.4\ R_F$ .... (2)
Using value of equation (2) in equation (1), we get:
$2.4R_F + R_F = 14700$
$R_F = 4323.53$ N
$\therefore R_B = 14700 - 4323.53 = 10376.47$ N
Thus, the force exerted on each of the front wheel = 4323.53/ 2 = 2161.77 N, and
The force exerted on each back wheel = 10376.47 / 2 = 5188.24 N

Q10. (a) What is the moment of inertia of a sphere about a tangent to the sphere, if the sphere's moment of inertia about any of its radius is $2MR^2/10$, where R is the sphere's radius
and M is the sphere's mass. (b) If the moment of inertia of a disc, of radius R and mass m, is $MR^2/4$ about any of its axis. What is its moment of inertia about an axis going through a point on its circumference and normal to the disc ?

Ans.
Given,

(a) The moment of inertia (M.I.) of a sphere about its radius = $2MR^2/10$

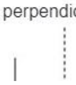


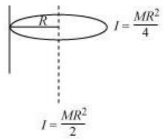
M.I. = $\frac{2}{5}MR^2$

According to the theorem of parallel axes, M.I of a sphere about a tangent to the sphere = $2MR^2/5 + MR^2 = (7MR^2)/5$

(b) Given, moment of inertia of a disc about its diameter =$(MR^2)1/4$

(i)According to the theorem of perpendicular axis, the moment of inertia of a planar body (lamina) about an axis passing through its center and perpendicular to the disc = $2 \times (1/4)MR^2 = MR^2/2$





The situation is shown in the given figure.

(ii) Using the theorem of parallel axes:

Moment of inertia about an axis normal to the disc and going through point on its circumference

$= MR^2 / 2 + MR^2$

$= (3MR^2) / 2$

Q11. A solid sphere and a hollow cylinder of the same mass and radius are subjected to torques of the same magnitude. The sphere is free to rotate about an axis and the cylinder is free to rotate about its axis. Find which of the two will gain a greater angular speed after a certain time ?

Ans.

let m be the mass and r be the radius of the solid sphere and also the hollow cylinder.

Moment of inertia of the solid sphere about an axis passing through its center,

$I_2 = (2mr^2) / 5$

The moment of inertia of the hollow cylinder about its standard axis, $I_1 = mr^2$

Let T be the magnitude of the torque being exerted on the two structures, producing angular accelerations of α_2 and α_1 in the sphere and the cylinder respectively.

Thus we have , $T = I_1\alpha_1 = I_2\alpha_2$
$\therefore \alpha_2 / \alpha_1 = I_1 / I_2 = \frac{mr^2}{\frac{2}{5}mr^2} = 5/2$
$\alpha_2 > \alpha_1$ . . . . . .(1)
Now, using the relation :
$\omega = \omega_0 + \alpha t$
Where,
ω_0 = Initial angular velocity
t = Time of rotation
ω = Final angular velocity
For equal ω_0 and t, we have:
$\omega \propto \alpha$ . . . . (2)
From equations (1) and (2), we can write:
$\omega_2 > \omega_1$
Thus from the above relation it is clear that the angular velocity of the solid sphere will be greater than that of the hollow cylinder.

Q12. A solid cylinder of mass 18 kg rotates about its axis at an angular speed of 100 rad / s. The radius of the cylinder is 0.20 m. Calculate (a)the magnitude of the kinetic energy associated with the rotation of the cylinder , and (b) the magnitude of angular momentum of the cylinder about its axis ?

Ans.
Given,

Mass of the cylinder, m = 18 kg
Angular speed, $\omega = 100 \text{ rad } s^{-1}$
Radius of the cylinder, r = 0.20 m
The moment of inertia of the solid cylinder:
$I = mr^2 / 2$
$= (1/2) \times 18 \times (0.20)^2$
$= 0.36 \text{ kg } m^2$
(a) $\therefore$ Kinetic energy $= (1/2) I \omega^2$
$= (1/2) \times 0.36 \times (100)^2 = 1800$ J
(b) $\therefore$ Angular momentum, $L = I\omega$
$= 0.36 \times 100$
= 36 Js

Q13. (i) A boy stands at the center of giant rotating disc with his arms stretched out. The disc is rotating at an angular speed of 42 rev / min. What will the angular speed of the boy be if he folds his hands inside and thus reduces his moment of inertia to 4/5 times the initial value? Neglect any friction in the disc while rotating.

(ii) Is the boy's new kinetic energy of rotation more than his initial kinetic energy ?Explain

(ii) Is the boy's new kinetic energy of rotation more than his initial kinetic energy ? Explain.

Ans.

(a) Given,

Initial angular velocity, ω_1= 42 rev/min
let the final angular velocity = ω_2
Let the boy's moment of inertia with hands stretched out = I_1
Let the boy's moment of inertia with hands folded in = I_2
We know :
$I_2 = (4/5)\ I_1$
As no external forces are acting on the boy, the angular momentum L will be constant.
Thus, we can write:
$I_2\,\omega_2 = I_1\,\omega_1$
$\omega_2 = (I_1/I_2)\,\omega_1$
$= [\ I_1 / (4/5)I_1\ ] \times 44 = (\ 5/4\ ) \times 44 = 55$ rev/min

(b) Final kinetic energy of rotation, $E_F = (1/2)\ I_2\,\omega_2^2$
Initial kinetic energy of rotation, $E_I = (1/2)\ I_1\,\omega_1^2$
$E_F / E_I = (1/2)\ I_2\,\omega_2^2 / (1/2)\ I_1\,\omega_1^2$
$= (4/5)\ I_1\ (55)^2 / I_1\ (40)^2$
$= 1.5$
$\therefore E_F = 1.5\ E_1$
It is clear that there is an increase in the kinetic energy of rotation and it can be attributed to the internal energy used by the boy to fold his hands.

Q14 . A string of minimal mass is wound round a hollow cylinder of radius 30 cm and mass 2 kg. Find (a) the cylinder's angular acceleration when 40 newtons of force is used to pull the rope , and (b) the rope's linear acceleration.

Ans.

Given,

Mass of the hollow cylinder, m = 2 kg
Radius of the hollow cylinder, r = 30cm = 0.3 m
Force applied, F = 40 N
Moment of inertia of the hollow cylinder about its axis:
$I = mr^2$
$= 2 \times (0.3)^2 = 0.18$ kg m^2
Torque, $T = F \times r = 40 \times 0.3 = 12$ Nm
Also, we know that :
$T = I\alpha$
(a) Therefore, $\alpha = T / I = 12 / 0.18 = 66.66$ rad s^{-2}
(b) Linear acceleration = $R\alpha = 0.3 \times 66.66 = 20$ m s^{-2}

Q15. A car engine transmits 210 Nm of torque to rotate a wheel at a constant angular speed of 180 rad / s. Calculate the power the engine requires.
(Let the engine efficiency be 1)

Ans.

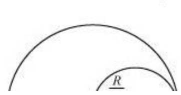
Given,

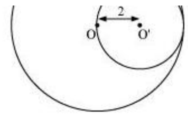
Angular speed of the rotor, ω = 180 rad/s
Torque, T = 210 Nm
Therefore, power of the rotor (P) :
$P = T\,\omega$
$= 210 \times 180$
= 37.8 kW
Therefore, the engine requires 37.8 kW of power.

Q.16. A circular hole of radius r is cut out from a circle of radius 2r. The center of the hole is located r /2 from the center of the original disc. Find the position of center of gravity in the resulting structure.

Ans.

Let the mass / unit area of the original disc = σ
Radius of the original disc =$2r$
Mass of the original disc, $m = \pi(2r^2)\sigma = 4\,\pi r^2\sigma$ . . . . (i)
The disc with the cut portion is shown in the following figure:





Radius of the smaller disc = r

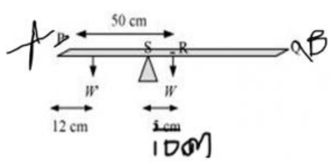
Mass of the smaller disc, $m' = \pi r^2\sigma$

=> m' = m/4 [From equation (i)]
Let O' and O be the respective centers of the disc cut off from the original and the original disc. According to the definition of center of mass, the center of mass of the original disc is concentrated at O, while that of the smaller disc is supposed at O'.
We know that :
OO'= $R/2$ = r2.
After the smaller circle has been cut out, we are left with two systems whose masses are:
$-m'$ (= m/4) concentrated at O', *and m* (concentrated at O).
(The negative sign means that this portion has been removed from the original disc.)
Let X be the distance of the center of mass from O.
We know :
$X = (m_1 r_1 + m_2 r_2) / (m_1 + m_2)$
$X = [m \times 0 - m' \times (r/2)] / (M + (-M')) = -R/6$
(The negative sign indicates that the center of mass is R/6 towards the left of O.)

Q17. A meter rule stays still balanced at its center. If two objects, each having a mass of 2 g are stacked over each other at the 12 .0 cm mark, the stick balances at 40 cm. Find the mass of the rule.

Ans.

The above situation can be represented as :



Let W' and W be the weights of the coin and the meter rule respectively.

The center of mass of the meter rule acts from its center i.e., 50 cm mark

Mass of the meter stick = m'
Mass of each coin, m = 2 g
When the coins are placed 12 cm away from A, the centre of mass shifts by 10 cm from the midpoint R towards A.

The centre of mass is now at 40 cm from A.

For rotational equilibrium about R will
$10 \times g(40 - 12) = m'g(50 - 40)$
$\therefore m' = 38\ g$
Thus, the mass of the meter rule is 38 g.

Q18. A ball rolls down two inclined planes having equal heights but different angles of inclination. (a) Will the ball roll down the two planes at different velocities ? (b) Will the ball take a greater amount of time in rolling down one plane than the other ? (c) If yes, rolling down which plane will take a greater amount of time ? Assume the ball to be a solid sphere.

Ans.

(a)Let the mass of the ball = m
let the height of the ball = h
let the final velocity of the ball at the bottom of the plane = v

At the top of the plane, the ball possesses Potential energy = mgh

At the bottom of the plane, the ball possesses rotational and translational kinetic energies.
Thus, total kinetic energy = $(1/2)mv^2 + (1/2) I \omega^2$

Using the law of conservation of energy, we have:
$(1/2)mv^2 + (1/2)I\omega^2 = mgh$

For a solid sphere, the moment of inertia about its centre, $I = (2/5)mr^2$
Thus, equation (i) becomes:
$(1/2)mv^2 + (1/2)[(2/5)mr^2] \omega^2 = mgh$
$(1/2) v^2 + (1/5) r^2 \omega^2 = gh$
Also , we know $v = r\omega$
$\therefore$ We have : $(1/2)v^2 + (1/5)v^2 = gh$

v^2 (7/10) = gh

$v = \sqrt{\frac{10}{7}gh}$

Since the height of both the planes is the same, the final velocity of the ball will also be the same irrespective of which plane it is rolled down.

(b) Let the inclinations of the two planes be θ_1 and θ_2, , where:

$\theta_1 < \theta_2$

The acceleration of the ball as it rolls down the plane with an inclination of θ_1 is:

g $\sin\theta_1$

let R_1 be the normal reaction to the sphere.

Similarly, the acceleration in the ball as it rolls down the plane with an inclination of θ_2 is:

g $\sin \theta_2$

Let R_2 be the normal reaction to the ball.

Here, $\theta_2 > \theta_1$; $\sin \theta_2 > \sin \theta_1$ . . . . (*1*)

$\therefore a_2 > a_1$ . .. . (2)

Initial velocity, $u = 0$

Final velocity, v = Constant

Using the first equation of motion:

$v = u + at$

$\therefore t \propto$ (1/a)

For inclination θ_1 : $t_1 \propto (1 / a_1)$

For inclination θ_2 : $t_2 \propto (1 / a_2)$

As $a_2 > a_1$ we have :

$t_2 < t_1$

(c)Therefore, the ball will take a greater amount of time to reach the bottom of the inclined plane having the smaller inclination.

Q19. A ring of radius 1.5 m weighs 10 kg. If it rolls over a horizontal floor such that its center of mass has a speed of 20 cm/s. What amount of work needs to be done to bring the ring to a halt ?

Ans.

Given

Radius of the ring, r = 1.5 m

Mass of the ring, m = 10 kg

Velocity of the hoop, v = 20 cm/s = 0.2 m/s

Total energy of the loop = Rotational K.E + Translational K.E..

$E_T = (1/2)mv^2 + (1/2)\, I\, \omega^2$

We know, the moment of inertia of a ring about its center, $I = mr^2$

$E_T = (1/2)mv^2 + (1/2)\, (mr^2)\omega^2$

Also, we know v = rω

$\therefore E_T = (1/2)mv^2 + (1/2)mr^2\omega^2$

=> $(1/2)mv^2 + (1/2)mv^2 = mv^2$

Thus the amount of energy required to stop the ring = total energy of the loop.

$\therefore$ The amount of work required, $W = mv^2 = 10 \times (0.2)^2 = 0.4$ J.

Q.20. An oxygen molecule has 5.30 × 10^{-26} kg of mass and a moment of inertia of 1.94×10^{-46} kg m^2 about an axis through its center. If this molecule has a mean speed of 450 m/s and its kinetic energy of rotation is 2 / 3 of its kinetic energy of translation. What is the molecule's average angular velocity ?

Ans.

Given,

Mass of one oxygen molecule, $m = 5.30 \times 10^{-26}$ kg

Thus, mass of each oxygen atom = $m/2$

Moment of inertia of jt, $I = 1.94 \times 10^{-46}$ kg m^2

Velocity of the molecule, v = 450 m/s

The distance between the two atoms in the molecule = 2r

Thus, moment of inertia I, is calculated as:

$I = (m/2)r^2 + (m/2)r^2 = mr^2$

$r = (I / m)^{1/2}$

=> $(1.94 \times 10^{-46} / 5.36 \times 10^{-26})^{1/2} = 0.60 \times 10^{-10}$ m

Given,

$K.E_{rot} = (2/3)K.E_{trans}$

$(1/2)\, I\, \omega^2 = (2/3) \times (1/2) \times mv^2$

$mr^2\omega^2 = (2/3)mv^2$

Therefore, $\omega = (2/3)^{1/2}\, (v/r)$

$= (2/3)^{1/2} (450 / 0.6 \times 10^{-10})\ 750 = 4.99 \times 10^{12}$ rad/s.

Q21.A solid cylinder rolls up plane inclined at 25°. The speed of the cylinder's center of mass at the bottom of the plane was 4 m/s.

(a) How much distance will the cylinder cover over the plane.
(b) What amount of time will the cylinder take to return back

Ans.

Given,

initial velocity of the solid cylinder, v = 4 m/s
Angle of inclination, θ = 25°
Assuming that the cylinder goes upto a height of h.We get :
$(½)\ mv^2 + (½)\ I\ \omega^2 = mgh$

$(½)\ mv^2 + (½)\ (½\ mr^2)\ \omega^2 = mgh$
$3/4\ mv^2 = mgh$
$h = 3v^2 / 4g = (3 \times 52) / 4 \times 9.8 = 1.224$ m

let d be the distance the cylinder covers up the plane, this means :
sin θ = h/d
d = h/sin θ = 1.224/sin 25° = 2.900m
Now, the time required to return back:

$$t = \sqrt{\frac{2d(1+\frac{K^2}{r^2})}{g sin\Theta}}$$

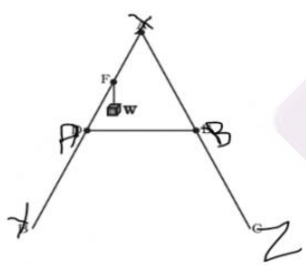
$$= \sqrt{\frac{2*2.9(1+\frac{1}{2})}{9.8 \sin 25^\circ}}$$

= 1.466 s

Thus, the cylinder takes 1.466s to return to the bottom.

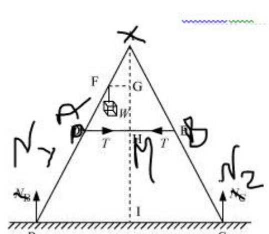
Q22. As depicted in the diagram below, the two sides of a ladder XY and XZ ,leaning on each other ,have a length of 1.6 m each. A string AB of length 0.5 m is tied half way up. A 30 kg body is hung from a point F, 1.2 m from Y along the ladder XY. Neglecting the ladder's weight and taking the floor to be friction less, calculate the tension in the string and forces exerted on the ladder by the floor. (Consider g = 9.8 m/s²)

(Hint: Consider the equilibrium of each side of the ladder separately.)



Ans.

The above situation can drawn as :



Here,

N_Z = Force being applied by floor point Z on the ladder
N_Y = Force being applied by floor point Y on the ladder
T = Tension in string.
YX = XZ = 1.6 m
AB = 0.5 m
YF = 1.2 m
Mass of the weight, m = 30 kg

Now,

Make a perpendicular from X on the floor YZ. This will intersect AB at mid-point M.
ΔXYI and ΔXIZ are similar
∴ZI = IY
This makes I the mid-point of ZY.
AB || ZY
ZY = 2 × AB = 1 m
XF = YX – YF = 0.4 m . . . . (1)
A is the mid-point of XY.
Thus, we can write:
XA = (1/2) × XY = 0.8 m . . . . (2)
Using equations (1) and (2), we get:
FB = 0.4 m
Thus, F is the mid-point of AX.
FG || AM and F is the mid-point of AX. This will make G the mid-point of XM.
ΔFXG and ΔXAM are similar
∴ FG / AM = XF / XA
FG / AM = 0.4 / 0.8 = 1 / 2
FG = (1/2) AM
= (1/2) × 0.25 = 0.125 m

In ΔXAM :
$XM = (XA^2 – AM^2)^{1/2}$
$= (0.8^2 – 0.25^2)^{1/2} = 0.76$ m

For translational equilibrium of the ladder, the downward force should be equal to the upward force.
$N_Y + N_Z = mg = 294$ N . . .. (3) [mg = 9.8 x 30]
Rotational equilibrium of the ladder about X is:
$-N_Y \times YI + FG \times mg + N_Z \times ZI – T \times XG + XG \times T = 0$
$-N_Y \times 0.5 + 294 \times 0.125 + N_Z \times 0.5 = 0$
$(N_Z – N_Y) \times 0.5 = 36.75$
$N_Z – N_Y = 73.5$ . . . . (4)
Adding equation (3) and equation (4), we get:
$N_Z = 183.75$ N
$N_Y = 110.25$ N
Rotational equilibrium about X for the ladder side XY
$-N_Y \times YI + FG \times mg + T \times XG = 0$
$-110.25 \times 0.5 + 294 \times 0.125 + 0.76 \times T = 0$
∴ T = 24.177 N.

Q23. A man is standing on a rotating disc with his arms stretched out and in each hand he is carrying a weight of 4 kg.The disc has an angular speed of 30 rev / min. He then closes his arms inward bring each weight from a distance of 80cm to 15cm from the axis of rotation. If the moment of the disc and the man together is constant and equal to 6.7 kg m2. Find
(a) His new angular speed? (Assume the system is frictionless.)
(b) Is there a change of kinetic energy in this process ?

Ans.

(a) Given,

Mass of each weight = 4 kg

Moment of inertia of the man-disc system = 6.7 kg m^2

Moment of inertia when his arms are fully stretched to 80 cm:
$2 \times m r^2$
$= 2 \times 4 \times (0.8)^2$
$= 5.12$ kg m^2

Initial moment of inertia of the system, $I_i = 6.7 + 5.12 = 11.82$ kg m^2

Angular speed, $\omega_i = 30$ rev / min
=> Angular momentum, $L_i = I_i\omega_i = 11.82 \times 30$

= 354.6 . . . . (i)
Moment of inertia when he folds his hands inward to 15 cm :
$2 \times mr^2$
$= 2 \times 4 (0.15)^2 = 0.18$ kg m^2

Final moment of inertia, $I_f = 6.7 + 0.18 = 6.88$ kg m^2
let final angular speed = ω_f
=> Final angular momentum, $L_f = I_f\omega_f = 6.88\ \omega_f$ . . . . . . (ii)
According to the principle of conservation of angular momentum:
$I_i\omega_i = I_f\omega_f$

$\therefore \omega_f = 354.6 / 6.88 = 51.54$ rev/min

(b) There is a change in kinetic energy , with the decrease in the moment of inertia kinetic energy increases. The extra kinetic energy is supplied to the system by the work done by the man in folding his arms inside.

Q24. A bullet of mass 8 g is shot at door at a muzzle velocity of 450 m/s. The bullet buries itself right at the center of the door. The door weighs 10 kg and is 1 m wide. It has a hinge in one end and it rotates about a vertical axis with no friction. Calculate the angular speed of the door right after the bullet buries into it.
(Hint: The moment of inertia of the door about the vertical axis at one end is $ML^2/3$.)

Ans.
Given, Velocity, v = 450 m/s

Mass of bullet, $m = 8\text{ g} = 8 \times 10^{-3}$ kg

Width of the door, L = 1 m
Radius of the door, $r = 1/2$
Mass of the door, M = 10 kg

Angular momentum imparted by the bullet on the door:
$L = mvr$
$= (8 \times 10^{-3}) \times (450) \times (1/2) = 1.8 \text{ kg m}^2 \text{ s}^{-1}$ ...(i)

Now, Moment of inertia of the door :
$I = ML^2 / 3$
$= (1/3) \times 10 \times 1^2 = 3.33 \text{ kgm}^2$
We know, $L = I\omega$
$\therefore \omega = L / I$
$= 1.8 / 3.33 = 0.54$ rad /s

Q25. Two turntables rotating at angular speeds of ω_1 and ω_2 and possessing moments of inertia I_1 and I_2 are brought into contact head on with their axes of rotation coincident.

(a) Calculate the angular speed of the two-disc system.

(b) Prove that the combined system has a lower kinetic energy than the sum of kinetic energies of the two turntables. Explain the loss of energy. Consider $\omega_1 \neq \omega_2$.

Ans.

(a)Given,

Let the moment of inertia of the two turntables be I_1 and I_2 respectively.

Let the angular speed of the two turntables be ω_1 and ω_2 respectively.

Thus we have;

Angular momentum of turntable 1, $L_1 = I_1\omega_1$
Angular momentum of turntable 2, $L_2 = I_2\omega_2$
=> total initial angular momentum $L_i = I_1\omega_1 + I_2\omega_2$

When the two turntables are combined together:
Moment of inertia of the two turntable system, $I = I_1 + I_2$
Let ω be the angular speed of the system.
=> final angular momentum, $L_T = (I_1 + I_2)\,\omega$
According to the principle of conservation of angular momentum, we have:
$L_i = L_T$
$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$
Therefore , $\omega = (I_1\omega_1 + I_2\omega_2) / (I_1 + I_2)$ . . . . (1)
(b) Kinetic energy of turntable 1, $K.E_1 = (1/2)\, I_1\omega_1^2$
Kinetic energy of turntable 2, $K.E_1 = (1/2)\, I_2\omega_2^2$
Total initial kinetic energy, $KE_I = (1/2)\,(I_1\omega_1^2 + I_2\omega_2^2)$
When the turntables are combined together, their moments of inertia add up.
Moment of inertia of the system, $I = I_1 + I_2$
Angular speed of the system = ω
Final kinetic energy KE_F : $= (1/2)\,(I_1 + I_2)\,\omega^2$

Using the value of ω from (1)
$= (1/2)\,(I_1 + I_2)\,[\,(I_1\omega_1 + I_2\omega_2) / (I_1 + I_2)\,]^2$
$= (1/2)\,(I_1\omega_1 + I_2\omega_2)^2 / (I_1 + I_2)$

Now, $E_I - E_F$
$= (I_1\omega_1^2 + I_2\omega_2^2)\,(1/2) - [\,(1/2)\,(I_1\omega_1 + I_2\omega_2)^2 / (I_1 + I_2)\,]$

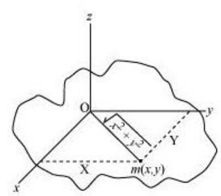
Solving the above equation, we get :
$= I_1 I_2 (\omega_1 - \omega_2)^2 / 2(I_1 + I_2)$
As $(\omega_1 - \omega_2)^2$ will only yield a positive quantity and I_1 and I_2 are both positive, the RHS will be positive.
Which means $KE_I - KE_F > 0$
Or, $KE_I > KE_F$

Some of the kinetic energy was lost overcoming the forces of friction when the two turntables were brought in contact.

Q26. (a) Verify the theorem of perpendicular axes.
(Clue: $x^2 + y^2$ is the square of the distance of a point (x, y) in the x–y plane from an axis through the origin normal to the plane.).
(b) Verify the theorem of parallel axes.
(Hint: $\sum m_i r_i = 0$ if the origin is chosen as the center of mass).

Ans.

(a) According to the theorem of perpendicular axes the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point where the perpendicular axis passes through it.
let us consider a physical body with center O and a point mass m,in the x–yplane at (x, y) is shown in the following figure.

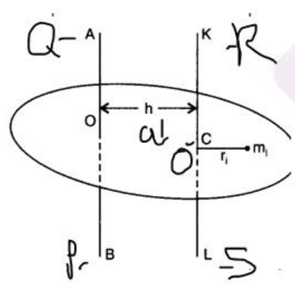


Moment of inertia about x-axis, $I_x = mx^2$

Moment of inertia about y-axis, $I_y = my^2$
Moment of inertia about z-axis, $I_z = m(x^2 + y^2)^{1/2}$

$I_x + I_y = mx^2 + my^2$
$= m(x^2 + y^2)$
$= m\,[(x^2 + y^2)^{1/2}]^{1/2}$
$I_x + I_y = I_z$
Thus, the theorem is verified.

(b) According to the theorem of parallel axes the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its center of mass and the product of its mass and the square of the distance between the two parallel axes.



Suppose a rigid body is made up of n number of particles, having masses $m_1, m_2, m_3, \ldots, m_n$, at perpendicular distances $r_1, r_2, r_3, \ldots, r_n$ respectively from the center of mass O of the rigid body.
The moment of inertia about axis RS passing through the point O:

$$I_{RS} = \sum_{i=1}^{n} m_i r_i^2$$

The perpendicular distance of mass m_i from the axis QP = a+ r_i

$$I_{QP} = \sum_{i=1}^{n} m_i (a + r_i)^2$$

$$I_{QP} = \sum_{i=1}^{n} m_i (a^2 + r_i^2 + 2ar_i)$$

$$I_{QP} = \sum_{i=1}^{n} m_i a^2 + \sum_{i=1}^{n} m_i r_i^2 + \sum_{i=1}^{n} m_i 2ar_i$$

$$I_{QP} = I_{RS} = \sum_{i=1}^{n} m_i a^2 + 2\sum_{i=1}^{n} m_i a r_i^2$$

We know, the moment of inertia of all particles about the axis passing through the center of mass is zero.

$$2\sum_{i=1}^{n} m_i a r_i = 0$$

as a $\neq 0$

Therefore, $\sum m_i r_i = 0$

Also,

Therefore, $\sum m_i$

= M; M = Total mass of the rigid body

Therefore, $I_{QP} = I_{RS} + Ma^2$

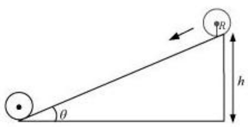
Therefore the theorem is verified.

Q27. $v^2 = 2gh / [1 + (k^2/R^2)]$
Using dynamical consideration (i.e. by taking forces and torques into account), show that the above equation gives the velocity v of translation of a rolling body (ring, sphere, disc etc) at the bottom of an inclined plane having a height h

$v^2 = 2gh / [1 + (k^2/R^2)]$
Note : R is the radius of the body and k is the radius of gyration of the body about its symmetry axis. The body starts rolling from rest from the top of the inclined plane

Ans.

The above situation can be represented as :



Here,

R = the body's radius

g = Acceleration due to gravity

K = the body's radius of gyration

v = the body's translational velocity

m = Mass of the body
h = Height of the inclined plane

Total energy at the top of the plane, E_T (potential energy) = mgh

Total energy at the bottom of the plane, $E_b = KE_{rot} + KE_{trans}$

= $(1/2)\, I\, \omega^2 + (1/2)\, mv^2$

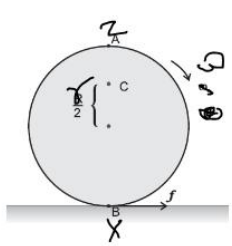
We know, $I = mk^2$ and $\omega = v / R$

Thus, we have $E_b = \frac{1}{2}mv^2 + \frac{1}{2}(mk^2)(\frac{v^2}{R^2})$
$= \frac{1}{2}mv^2(1 + \frac{k^2}{R^2})$

According to the law of conservation of energy:
$E_T = E_b$
mgh = $\frac{1}{2}mv^2(1 + \frac{k^2}{R^2})$

$\therefore$ v = 2gh / [1 + (k^2 / R^2)]
Thus, the given relation is proved.

Q28. A coin rotating about its axis at an angular speed of ω is placed gently (with no translational push) on a perfectly smooth plane mirror (zero friction on the surface). If the radius of the coin is r. Find the linear velocities of the points Z, X and C on the disc as depicted in the following figure. Can the coin roll in the indicated direction ?



Ans.

The respective linear velocities are :

For point A, $v_A = r\omega$ in the direction of the arrow

For point B, $v_B = r\omega$ in the direction opposite to the arrow

For point C, $v_c = (R/2)\omega_o$ in the same direction as that of v_A

Firstly there is no tangential push given to the coin in the initial state. Secondly, the force of friction was the only means of tangential force, but that too is absent as the surface is frictionless. Therefore, the coin cannot roll ahead.

Q29.Considering the above case answer the following questions :

(a) What is the direction of the force of friction at X, and the sense of frictional torque, before perfect rolling starts.
(b) Give the frictional force after perfect rolling starts?

Ans.

(a) Frictional force acts towards right as it opposes the direction of velocity of point X which is towards left. The sense of frictionless torque will be normal to the plane of the coin and outwards.

(b) As force of friction acts in the direction opposite to the velocity at point X, perfect rolling starts only when the force of friction at that point equals zero. This makes the force of friction acting on the coin equal to zero.

Q30. A ring and a solid disc, both having a radius of 5 cm are kept on a horizontal plane simultaneously, with initial angular speed of 8 π rad s^{-1}. Find which of the two will start rolling faster. (Take the co-efficient of kinetic friction , $\mu_K = 0.2$).

Ans.

Given,

Radii of the ring and the disc, r = 5 cm = 0.05 m
Initial angular speed, $\omega_0 = 8\pi$ rad s^{-1}
Coefficient of kinetic friction, $\mu_k = 0.2$
Initial velocity of both the objects, u = 0a

Motion of the two objects is caused by force of friction. According Newton's second, force of friction, f = ma
$\mu_k mg = ma$
Where,
a = Acceleration produced in the disc and the ring
m = Mass
$\therefore a = \mu_k g$ . . . . (1)

Using the first equation of motion :
$v = u + at$
$= 0 + \mu_k gt$
$= \mu_k gt$ . . . . (2)

The frictional force applies a torque in perpendicularly outward direction and reduces the initial angular speed.
Torque, $T = -I\alpha$
Where, α = Angular acceleration
$\mu_k mgr = -I\alpha$
$\therefore \alpha = -\mu_k mgr / I$ . . . . (3)

According to the first equation of rotational motion, we have :
$\omega = \omega_0 + \alpha t$
$= \omega_0 + (-\mu_k mgr / I)t$ . . . .(4)
Rolling starts when linear velocity, $v = r\omega$
$\therefore v = r(\omega_0 - \mu_k mgrt / I)$ …(v)

Using equation (2) and equation (5), we have:
$\mu_k gt = r(\omega_0 - \mu_k mgrt / I)$
$= r\omega_0 - \mu_k mgr^2 t / I$ . . . . (6)

For the ring:
$I = mr^2$
$\therefore \mu_k gt = r\omega_0 - \mu_k mgr^2 t / mr^2$
$= r\omega_0 - \mu_k gt$
$2\mu_k gt = r\omega_0$
$\therefore t = r\omega_0 / 2\mu_k g$
$= (0.05 \times 8 \times 3.14) / (2 \times 0.2 \times 9.8) = 0.32$ s . . . . (7)
For the disc: $I = (1/2)mr^2$
$\therefore \mu_k gt = r\omega_0 - \mu_k mgr^2 t / (1/2)mr^2$
$= r\omega_0 - 2\mu_k gt$

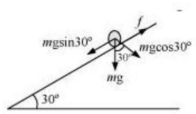
$3\mu_k gt = r\omega_0$
$\therefore t = r\omega_0 / 3\mu_k g$
$= (0.05 \times 8 \times 3.14) / (3 \times 0.2 \times 9.8) = 0.213$ s .....(8)

Since $t_D > t_R$, the disc will start rolling before the ring.

Q31. A cylinder of radius 10 cm and mass 8 kg is rolling perfectly over a surface inclined at 25°. Given that the coefficient of static friction, μ_s = 0.25.
(a) Find the magnitude of force of friction acting on the cylinder.
(b) Find the amount of work done against friction while rolling.
(c) At what value of the inclination will the cylinder begin to skid instead of rolling perfectly?

Ans.

The above situation can be depicted as:



Given,

mass, m = 8 kg
Radius, r = 10 cm = 0.1 m
Co-efficient of kinetic friction, μ_k = 0.25
Angle of inclination, θ = 25°

We know, moment of inertia of a solid cylinder about its geometric axis, $I = (1/2)mr^2$

The acceleration of the cylinder is given as:

$a = mg \text{ Sin}\theta / [m + (I/r^2)]$
$= mg \text{ Sin}\theta / [m + \{ \frac{1}{2} mr^2 / r^2 \}]$
$= (2/3)$ g Sin 25°
$= (2/3) \times 9.8 \times \;\; = 2.72 \text{ ms}^{-2}$

(a) Using Newton's second law of motion, we can write net force as:
$f_{NET} = ma$
mg Sin 25° – f = ma
f = mg Sin 25° – ma
= 8 × 9.8 × 0.422 – 10 × 2.72
= 5.88 N

(b) There is no work done against friction during rolling.

(c) We know for rolling without skidding :
μ = (1/3) tan θ
tan θ = 3μ = 3 × 0.25
$\therefore \theta = \tan^{-1}(0.75) = 36.87°$

Q32. State whether the following are true or false. Provide appropriate explanations :
(a) Work done against friction is always zero during perfect rolling.
(b) A wheel moving down a perfectly smooth plane will be slipping not rolling.
(c) The point of contact during rolling has an instantaneous speed equal to zero.
(d) The force of friction acting on a rolling body acts in the same direction as its center of mass is moving.
(e) The point of contact during rolling has an instantaneous acceleration equal to zero.

Ans.

(a) True. This is because during perfect rolling frictional force is zero so work done against it is zero.

(b) True. Rolling occurs only when there is frictional force to provide the torque so in the absence of friction the wheel simply slips down the plane under the influence of its weight.

(c) True. During rolling the point of the body in contact with the ground does not move ahead (this would be slipping) instead it only touches the ground for an instant and lifts off following a curve. Thus, only if the point of contact remains in touch with the ground and moves forward will the instantaneous speed not be equal to zero.

(d) False. Force of friction acts in the direction opposite to the direction of motion of the center of mass of the body.

(e) False. The point of contact during rolling as an acceleration in the form of centrifugal force directed towards the center.

Q33. Breaking down Motion of a system of particles into motion about the center of mass and motion of the center of mass:

(i) Show $\overrightarrow{p_i} = \overrightarrow{p_i'} + m_i\vec{V}$

Where p_i ***is the momentum of the*** i^{th} ***particle (of mass*** m_i***) and*** $\overrightarrow{p_i} + m_i\overrightarrow{v_i'}$***. Note*** $\overrightarrow{v_i'}$ ***is the velocity of the*** i^{th} ***particle with respect to the center of mass.***

Also, verify using the definition of the center of mass that $\sum \overrightarrow{p_i} = 0$

(ii) Prove that K = K' + ½MV²

Where K is the total kinetic energy of the system of particles, K' is the total kinetic energy of the system when the particle velocities are taken relative to the center of mass and MV² /2 is the kinetic energy of the translation of the system as a whole.

(iii) Show $\vec{L} = m_i\overrightarrow{L'} + M\vec{R} * \vec{V}$

Where $\vec{L} = \sum \overrightarrow{r_i'} * \overrightarrow{p_i'}$ ***is the angular momentum of the system about the centre of mass with velocities considered with respect to the centre of mass. Note*** $\overrightarrow{r_i'} = \overrightarrow{r_i} - \vec{R}$***; rest of the notation is the standard notation used in the lesson. Remember*** $\overrightarrow{L'} and M\overrightarrow{R} * \vec{v}$ ***can be said to be angular momenta, respectively, about and of the center of mass of the system of particles.***

(iv) Prove that : $\frac{\overrightarrow{dl'}}{dt} = \sum \overrightarrow{r_i'} * \frac{\overrightarrow{dp'}}{dt}$

Further prove that :

$\frac{\overrightarrow{dL'}}{dt} = \overrightarrow{T_{ext}}$

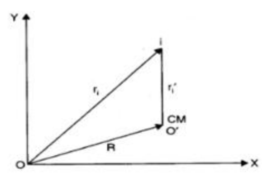
Where $\overrightarrow{T_{ext}}$ ***is the sum of all external torques acting on the system about the center of mass. (Clue : apply Newton's Third Law and the definition of center of mass . Consider that internal forces between any two particles act along the line connecting the particles.)***

Ans.

Here $\overrightarrow{r_i} = \overrightarrow{r_i'} + \vec{R} + R$ . . . . . . (1)

and, $\overrightarrow{V_i} = \overrightarrow{V_i'} + \vec{V}$ . . . .(2)

Where, $\overrightarrow{r_i'}$ and $\overrightarrow{v_i'}$ represent the radius vector and velocity of the i^{th} particle referred to center of mass O' as the new origin and $\vec{V}$ is the velocity of center of mass with respect to O.



(i) Momentum of i^{th} particle

$\overrightarrow{p'} = m_i\overrightarrow{V_i'}$

= $m_i(\overrightarrow{V_i'} + \vec{V})$ [From equation (1)]

Or, $\vec{P} = m_i\vec{V} + \overrightarrow{P_i}$

(ii) Kinetic energy of system of particles

K = $\frac{1}{2}\sum m_i V_i^2$

= $\frac{1}{2}\sum m_i \overrightarrow{V_i}.\overrightarrow{V_i}$

= $\frac{1}{2}\sum m_i(\overrightarrow{V_i'} + \vec{V})(\overrightarrow{V_i'} + \vec{V})$

= $\frac{1}{2}\sum m_i(\overrightarrow{V_i'^2} + \overrightarrow{V^2} + 2\overrightarrow{V_i'}\vec{V})$

= $\frac{1}{2}\sum m_i V_i'^2 + \frac{1}{2}\sum m_i V^2 + \sum m_i \overrightarrow{V_i'}\vec{V}$

= ½ MV² + K'

Where M = $\sum m_i$ = total mass of the system.

K' = $\frac{1}{2}\sum m_i V_i'^2$

= kinetic energy of motion about the center of mass.

Or, ½ Mv² = kinetic energy of motion of center of mass.(Proved)

Since, $\sum_i m_i \vec{V_i'}\vec{V} = \sum m_i \frac{d\vec{r_i}}{dt}\vec{V}$

=0

(iii) Total angular momentum of the system of particles.

$\vec{L} = \vec{r_i} * \vec{p}$

= $(\vec{r_i} + \vec{R}) * \sum_i m_i(\vec{V_i'} + \vec{V})$

= $\sum_i(\vec{R} * m_i\vec{V}) + \sum_i(\vec{r_i'} * m_i\vec{V_i'}) + (\sum_i m_i\vec{r_i'}) * \vec{V} + \vec{R}* \sum_i m_i\vec{V_i}$

= $\sum_i(\vec{R} * m_i\vec{V}) + \sum_i(\vec{r_i'} * m_i\vec{V_i'}) + (\sum_i m_i\vec{r_i'}) * \vec{V} + \vec{R} * \frac{d}{dt}(\sum_i m_i\vec{r_i'})$

However, we know $\sum_i m_i\vec{r_i'}$ = 0

Since, $\sum_i m_i\vec{r_i'}$ = $\sum_i m_i(\vec{r_i'} - \vec{R}) = M\vec{R} - M\vec{R}$ = 0

According to the definition of center of mass,

$\sum_i(\vec{R} * m_i\vec{V}) = \vec{R} * M\vec{V}$

Such that, $\vec{L} = \vec{R} * M\vec{V} + \sum_i \vec{r_i'} * \vec{P_i}$

Given, $\vec{L} = \sum \vec{r_i'} * \vec{p_i'}$

Thus, we have ; $\vec{L} = \vec{R} * M\vec{V} + \vec{L'}$

(iv) From previous solution

$\vec{L'} = \sum \vec{r_i} * \vec{P_i}\ \frac{d\vec{L'}}{dt} = \sum \vec{r_i} * \frac{d\vec{P_i}}{dt} + \sum \frac{d\vec{r_i'}}{dt} * \vec{P_i}$

= $\sum \vec{r_i'} * \frac{d\vec{P_i'}}{dt}$

= $\sum \vec{r_i'} * \vec{F_i^{ext}}$ = $\vec{T'_{ext}}$

Since, $\sum \frac{d\vec{r_i'}}{dt} * \vec{P_i}$ = $\sum \frac{d\vec{r_i'}}{dt} * m\vec{v_i}$ = 0

Total torque = $\vec{T'_{ext}} = \sum \vec{r_i'} * \vec{F_i^{ext}}$

= $\sum(\vec{r_i'} + \vec{R}) * \vec{F_i^{ext}}$

= $\vec{T'_{ext}} + \vec{T_0^{(ext)}}$

Where, $\vec{T'_{ext}}$ is the net torque about the center of mass as origin and $\vec{T_0^{ext}}$ is about the origin O.

$\vec{T'_{ext}}$ = $\sum \vec{r_i'} * \vec{F_i^{ext}}$

=$\sum \vec{r_i'} * \frac{d\vec{P_i'}}{dt}$

= $\frac{d}{dt}\sum(\vec{r_i'} * \vec{P_i'})$ = $\frac{d\vec{L'}}{dt}$

Thus we have , $\frac{d\overrightarrow{L'}}{dt} = \overrightarrow{T'_{ext}}$