#419639

**Topic:** Linear Momentum and its Conservation

A nucleus is at rest in the laboratory frame of reference. Show that if it disintegrates into two smaller nuclei the products must move in opposite directions.

**Solution**

Let $m$ $m_1$ and $m_2$ be the respective masses of the parent nucleus and the two daughter nuclei. The parent nucleus is at rest.Initial momentum of the system (parent nucleus) = 0

Let $v_1$ and $v_2$ be the respective velocities of the daughter nuclei having masses $m_1$ and $m_2$.

Total linear momentum of the system after disintegration =$m_1v_1 + m_2v_2$

According to the law of conservation of momentum:

Total initial momentum = Total final momentum

$0 = m_1v_1 + m_2v_2$

$v_1 = \frac{-m_2v_2}{m_1}$

Here, the negative sign indicates that the fragments of the parent nucleus move in directions opposite to each other.

#419642

**Topic:** Linear Momentum and its Conservation

A shell of mass $0.020kg$ is fired by a gun of mass $100kg$. If the muzzle speed of the shell is $80m/s$, what is the recoil speed of the gun?

**Solution**

Mass of the gun, *M* = 100 kg

Mass of the shell, *m* = 0.020 kg

Muzzle speed of the shell, *v* = 80 m/s

Recoil speed of the gun = *V*

Both the gun and the shell are at rest initially.

Initial momentum of the system = 0

Final momentum of the system = *mv* - *MV*

Here, the negative sign appears because the directions of the shell and the gun are opposite to each other.

According to the law of conservation of momentum:

Final momentum = Initial momentum

*mv* -*MV* = 0

*V* = mv / *M*

= 0.02$\times$ 80 / (100$\times$ 1000) = 0.016 m/s

#419718

**Topic:** Kinematics of Circular Motion

An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is $30^o$, what is the speed of the aircraft ?

**Solution**

In right angled triangle OAB, $\tan 30^o = \dfrac{h}{x}$

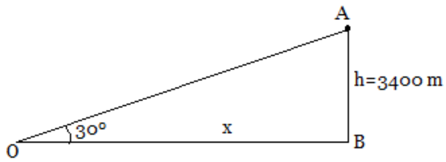
We get $x = \dfrac{h}{\tan 30^o}$

$\therefore\ x = 3400\sqrt{3} = 5889$m

Time taken $t = 10.0$ s

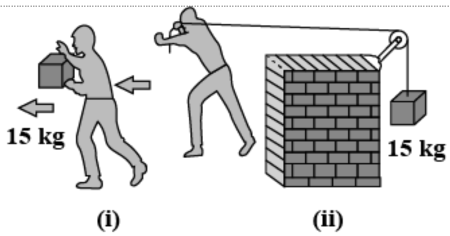
Thus speed of the aircraft $v = \dfrac{x}{t}$

$\therefore\ v = \dfrac{5889}{10.0} = 588.9\ m/s$




**#419815**

**Topic:** Linear Momentum and its Conservation




Answer the following :

(a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained. The rocket or the atmosphere?

(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comets velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?

(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?

(d) In Figure (i) the man walks $2\ m$ carrying a mass of $15\ kg$ on his hands. In Figure (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of $15\ kg$ hangs at its other end. In which case is the work done greater?

**Solution**

(a) Rocket: The burning of the casing of a rocket in flight (due to friction) results in the reduction of the mass of the rocket.

According to the conservation of energy:

Total Energy = Potential Energy + Kinetic Energy

$= mgh + (1/2)mv^2$

The reduction in the mass of the rocket causes a drop in the total energy. Therefore, the heat energy required for the burning is obtained from the rocket.

(b) Gravitational force is a conservative force. Since the work done by a conservative force over a closed path is zero, the work done by the gravitational force over every complete orbit of a comet is zero.

(c) When an artificial satellite, orbiting around the earth, moves closer to earth, its potential energy decreases because of the reduction in the height. Since the total energy of the system remains constant, the reduction in P.E. results in an increase in K.E. Hence, the velocity of the satellite increases. However, due to atmospheric friction, the total energy of the satellite decreases by a small amount.

(d)

Case(i)

Mass, $m$ = 15 kg

Displacement, $s$ = 2 m

Work done, $W = Fscos\theta$

Where, $\theta = Angle\, between\, force\, and\, displacement$

$W = mgscos\theta = 15 \times 2 \times 9.8 \times cos90^0 = 0$

Case(ii)

Mass, $m$ = 15 kg

Displacement, $s$ = 2 m

Here, the direction of the force applied to the rope and the direction of the displacement of the rope are same.

Therefore, the angle between them, $= 0$

Since $cos\ 0^0 = 1$

Work done, $W = Fscos\theta = mgs$

$= 15 \times 9.8 \times 2 = 294\ J$

Hence, more work is done in the figure (ii).

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**#419851**

**Topic:** Linear Momentum and its Conservation

A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the floor of the trolley at the rate of 0.05 kg $s^{-1}$. What is the speed of the trolley after the entire sand bag is empty ?

**Solution**

Since the hole is on the floor , that means sand is falling vertically with respect to trolley. Therefore there is no force in horizontal direction hence in horizontal direction momentum is conserved.

Let $M=$ mass of trolley

$m=$ mass of sandbag

$v_1=$ initial velocity

$v_2=$ final velocity ( to be found)

Then $P_1=(M+m)v_1$ when the sand bag is empty the momentum is

$P_2=(M+0)v_2$

Momentum is conserved in horizontal direction so

$P_1=P_2$

$\Rightarrow = v_2=\dfrac{(M+m)}{M}v_1=\dfrac{300+25}{300}\times 27\times\dfrac{5}{18}=8.215m/s$

**#419863**

**Topic:** Linear Momentum and its Conservation

A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of $4\,m\,s^{-1}$ relative to the trolley in a direction opposite to the its motion, and jumps out of the trolley. What is the final speed of the trolley ? How much has the trolley moved from the time the child begins to run ?

**Solution**

Mass of the trolley,$M=200kg$

Speed of the trolley,$v=36km/h=10m/s$

Mass of the boy, $m=20kg$

Initial momentum of the system of the boy and the trolley

= $(M+m)v$

= $(200+20)\times 10$

= $2200kg\ m/s$

Let $v'$ be the final velocity of the trolley with respect to the ground.

Final velocity of the boy with respect to the ground =$v'-4$

Final momentum = $Mv'+m(v'-4)$

= $200v'+20v'-80$

= $20v'-80$

As per the law of conservation of momentum:

Initial momentum = Final momentum

$2200=220v'-80$

$\therefore v'=2280/220=10.36m/s$

Length of the trolley, $l=10m$

Speed of the boy, $v''=4m/s$

Time taken by the boy to run, $t=10/4=2.5s$

$\therefore$ Distance moved by the trolley = $v''\times t$= $10.36\times 2.5=25.9m$

**#419880**

**Topic:** Linear Momentum and its Conservation

A bullet of mass 0.012 kg and horizontal speed 70 m/s strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

**Solution**

Mass of the bullet, m = 0.012 kg

Initial speed of the bullet, $u_b = 70 m/s$

Mass of the wooden block, $M = 0.4\ kg$

Initial speed of the wooden block, $u_B$ =0

Final speed of the system of the bullet and the block = $v\ \ m/s$

Applying the law of conservation of momentum:

$mu_b + Mu_B = (m + M)v$

$0.012 \times 70 + 0.4 \times 0 = (0.012 + 0.4)v$

$v = 0.84/0.412$

=$2.04$ m/s

For the system of the bullet and the wooden block:

Mass of the system, $m' = 0.412\ kg$

Velocity of the system = $2.04 m/s$

Height up to which the system rises = $h$

Applying the law of conservation of energy to this system:

Potential energy at the highest point $=$ Kinetic energy at the lowest point

$m'gh = (1/2)m'v^2$

$h = (1/2)(v^2/g)$

$= (1/2) \times (2.04)^2/9.8$

$= 0.2123 m$

The wooden block will rise to a height of $0.2123m$.

Heat produced = Kinetic energy of the bullet - Kinetic energy of the system

$= (1/2)mu^2 - (1/2)m'v^2$

$= (1/2) \times 0.012 \times (70)^2 - (1/2) \times 0.412 \times (2.04)^2$

$= 29.4 - 0.857 = 28.54J$

**#421118**

**Topic:** Moment of Inertia of Common Bodies

A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillation is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant $\alpha$ is defined by the relation $J = -\alpha\, \theta$, where J is the restoring couple and $\theta$ the angle of twist).

**Solution**

Mass of the circular disc, m $=$ 10 kg

Radius of the disc, r $=$ 15 cm $=$ 0.15 m

The torsional oscillations of the disc has a time period, T $=$ 1.5 s

The moment of inertia of the disc is:

$I = 1/2mr^2$

$= 1/2 \times (10) \times (0.15)^2$

$= 0.1125\ kg/m^2$

Time Period, $T = 2\pi\sqrt{I/\alpha}$

$\alpha$ is the torisonal constant.

$\alpha = 4\pi^2 I/T^2$

$= 4 \times (\pi)^2 \times 0.1125/(1.5)^2$

$= 1.972\ Nm/rad$

Hence, the torsional spring constant of the wire is $1.972\ Nm\, rad^{-1}$.

#455967

**Topic:** Centre of mass

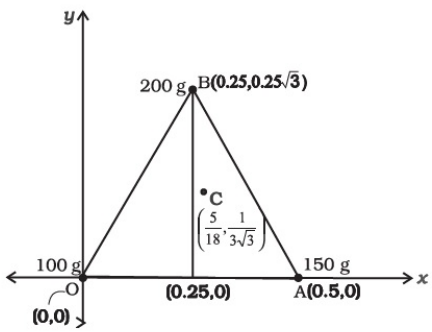
Find the centre of mass of three particles at the verticles of an equilateral triangle. The masses of the particles are $100g$, $150g$, and $200g$ respectively. Each side of the equilateral triangle is $0.5m$ long.

**Solution**

With the $x$- and $y$- axes chosen as shown in Fig., the coordinates of points O, A and B forming the equilateral triangle are respectively $(0,0)$, $(0.5,0)$, $(0.25, 0.25\sqrt{3})$ Let the masses $100g$, $150g$ and $200g$ be located at O, A and B be respectively. Then,

$$X = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$
$$= \frac{100(0) + 150(0.5) + 200(0.25)gm}{(100 + 150 + 200)g}$$
$$= \frac{75 + 50}{450}m = \frac{125}{450}m = \frac{5}{18}m$$
$$Y = \frac{100(0) + 150(0) + 200(0.25\sqrt{3})gm}{450g}$$
$$= \frac{50\sqrt{3}}{450}m = \frac{\sqrt{3}}{9}m = \frac{1}{3\sqrt{3}}m$$

The centre of mass C is shown in the figure. Note that it is not the geometric centre of the triangle OAB.




#455968

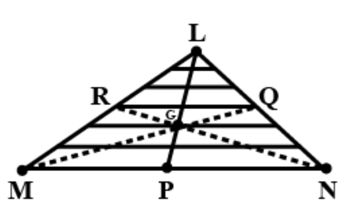
**Topic:** Centre of mass

Find the centre of mass of a triangular lamina.

**Solution**

The lamina $(\Delta LMN)$ may be subdivided into narrow strips each parallel to the base(MN) as shown in Fig.,

By symmetry each strip has its centre of mass at its midpoint. If we join the midpoint of all the strips we get the median LP. The centre of mass of the triangle as a whole therefore, has to lie on the median LP. Similarly, we can argue that it lies on the median MQ and NR. This means the centre of mass lies on the point of concurrence of the medians, i.e. on the centroid G of the triangle.



**#455969**

**Topic:** Centre of mass

Find the centre of mass of a uniform L-shaped lamina (a thin flat plate) with dimensions as shown. The mass of the lamina is $3$kg.

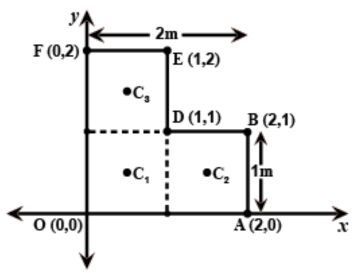
**Solution**

Choosing the X and Y axes as shown in Fig. we have the coordinates of the vertices of the L-shaped lamina as given in the figure. We can think of the L-shape to consist of $3$ squares each of length $1$m. The mass of each square is $1$kg, since the lamina is uniform. The centres of mass $C_1, C_2$ and $C_3$ of the squares are, by symmetry, their geometric centres and have coordinates $(1/2, 1/2), (3/2.1/2), (1/2, 3/2)$respectively. We take the masses of the squares to be concentrated at these points. The centre of mass of the whole L shape $(X, Y)$ is the centre of mass of these mass points.

Hence $X = \dfrac{[1(1/2) + 1(3/2) + 1(1/2)]kgm}{(1+1+1)kg} = \dfrac{5}{6}m$

$Y = \dfrac{[1(1/2) + 1(1/2) + 1(3/2)]kgm}{(1+1+1)kg} = \dfrac{5}{6}m$

The centre of mass of the L-shape lies on the line OD. We could have guessed this without calculations. Can you tell why? Suppose, the three squares that make up the L-shaped lamina of Fig. had different masses. How will you then determine the centre of mass of the lamina?




**#455971**

**Topic:** Torque

Find the torque of a force $7\hat{i} + 3\hat{j} - 5\hat{k}$ about the origin. The force acts on a particle whose position vector is $\hat{i} - \hat{j} + \hat{k}$

**Solution**

Here $r = \hat{i} - \hat{j} + \hat{k}$

and $F = 7\hat{i} + 3\hat{j} - 5k$

We shall use the determinant rule to find the torque $\tau = r \times F$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix} = (5-3)\hat{i} - (-5-7)\hat{j} + (3-(-7))\hat{k}$$

or $\tau = 2\hat{i} + 12\hat{j} + 10\hat{k}$

**#455972**

**Topic:** Rotational Equilibrium

A metal bar $70cm$ long and $4.00kg$ in mass supported on two knife-edges placed $10cm$ from each end. A $6.00$kg load is suspended at $30$cm from one end. Find the reactions at the knife-edges. (Assume the bar to be of uniform cross section and homogeneous.)

**Solution**

Figure shows the rod AB, the positions of the knife edges $K_1$ and $K_2$, the centre of gravity of the rod at G and the suspended load at P.

Note the weight of the rod W acts as its centre of gravity G. The rod is uniform in cross section and homogenous; hence G is at the centre of the rod; $AB = 70$cm. $AG = 35$cm, $AP = 30$cm, $PG = 5$cm, $AK_1 = BK_2 = 10$cm and $K_1G = K_2G = 25$cm. Also, $W =$ weight of the rod $= 4.00$kg and $W_1 =$ suspended load$= 6.00$kg ; $R_1$ and $R_2$ are the normal reactions of the support at the knife edges.

For translational equilibrium of the rod, $R_1 + R_2 - W_1 - W = 0$ ....$(i)$

Note $W_1$ and $W$ act vertically down and $R_1$ and $R_2$ act vertically up.

For considering rotational equilibrium, we take moments of the forces. A convenient point to take moments of the forces is G. The moments of $R_2$ and $W_1$ are anticlockwise $(+ve)$, whereas the moment of $R_1$ is clockwise $(-ve)$.

For rotational equilibrium,

$-R_1(K_1G) + W_1(PG) + R_2(K_2G) = 0 \qquad (ii)$

It is given that $W = 4.00$g N and $W_1 = 6.00$g N, where $g =$ acceleration due to gravity. We take $g = 9.8m/s^2$.

With numerical values inserted, from $(i)$

$R_1 + R_2 - 4.00g - 6.00g = 0$

or $R_1 + R_2 = 10.00gN \qquad (iii)$

$= 98.00N$

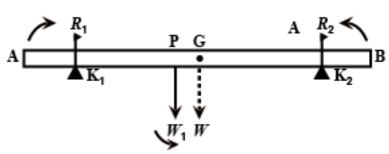
From $(ii)$, $-0.25R_1 + 0.05W_1 + 0.25R_2 = 0$

or $R_1 - R_2 = 1.2gN = 11.76N \qquad (iv)$

From $(iii)$ and $(iv)$, $R_1 = 54.88N$,

$R_2 = 43.12N$

Thus the reactions of the support are about $55N$ at $K_1$ and $43N$ at $K_2$.




#455973

**Topic:** Rotational Equilibrium

A $3m$ long ladder weighing $20$kg leans on a frictionless wall. Its feet rest on the floor $1$m from the wall as shown in Fig. Find the reaction forces of the wall and the floor.

**Solution**

The ladder AB is $3$m long, its foot A is at distance AC$=1$m from the wall. From Pythagons theorem, $BC=2\sqrt{2}$m. The forces on the ladder are its weight $W$ acting at its centre of gravity $D$, reaction forces $F_1$ and $F_2$ of the wall and the floor respectively. Force $F_1$ is perpendicular to the wall, since the wall is frictionless. Force $F_2$ is resolved into two components, the normal reaction N and the force of friction F. Note that F prevents the ladder from sliding away from the wall and is therefore directed toward the wall.

For translational equilibrium, taking the forces in the vertical direction,

$N-W=0 \qquad (i)$

Taking the forces in the horizontal direction,

$F-F_1=0 \qquad (ii)$

For rotational equilibrium, taking the moments of the forces about A,

$2\sqrt{2}F_1-(1/2)W=0 \qquad (iii)$

Now $W=20g=20\times 9.8N=196.0N$

From $(i) N=196.0$

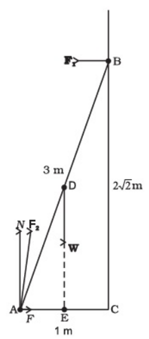
From $(iii) F_1=W/4\sqrt{2}=196.0/4\sqrt{2}=34.6N$

From $(ii) F=F_1=34.6N$

$F_2=\sqrt{F^2+N^2}=199.0N$

The force $F_2$ makes an angle $\alpha$ with the horizontal,

$\tan\alpha=N/F=4\sqrt{2}, \alpha=\tan^{-1}(4\sqrt{2})=80^o$

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**#455974**

**Topic:** Moment of Inertia of Common Bodies

What is the moment of intertia of a disc about one of its diameters?

**Solution**

We assume the moment of inertia of the disc about an axis perpendicular to it and through its centre to be known; it is $MR^2/2$, where M is the mass of the disc and R is its radius.

The disc can be considered to be a planar body. Hence the theorem of perpedicular axes is applicable to it. As shown in Fig., we take three concurrent axes through the centre of the disc, O as the $x, y, z$ axes ; $x$ and $y$-axes lie in the plane of the disc and $z$ is perpendicular to it. By the theorem of perpendicular axes,

$I_z=I_x+I_y$

Now, $x$ and $y$ axes are along two diameters of the disc, and by symmetry the moment of inertia of the disc is the same about any diameter. Hence

$I_x=I_y$

and $I_z=2I_x$

But $I_z=MR^2/2$

So finally, $I_x=I_z/2=MR^2/4$

Thus the moment of inertia of a disc about any of its diameter is $MR^2/4$.

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**#455975**

**Topic:** Moment of Inertia of Common Bodies

What is the moment of inertia of a rod of mass M, length l about an axis perpendicular to it through one end?

**Solution**

For the rod of mass M and length l,

$I = Ml^2/12$. Using the parallel axes theorem, $I' = I + Ma^2$ with $a = l/2$ we get,

$I' = M\frac{l^2}{12} + M\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{3}$

We can check this independently since I is half the moment of inertia of a rod of mass $2M$ and length $2l$ about its midpoint,

$I' = 2M.\, \frac{4l^2}{12} \times \frac{1}{2} = \frac{Ml^2}{3}$

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**#455976**

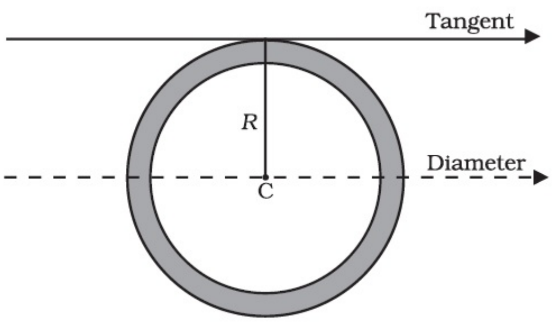
**Topic:** Moment of Inertia of Common Bodies

What is the moment of inertia of a ring about a tangent to the circle of the ring?

**Solution**

The tangent to the ring in the plane of the ring is parallel to one of the diameters of the ring. The distance between these two parallel axes is $R$, the radius of the ring. Using the parallel axes theorem,

$I_{tangent} = I_{dia} + MR^2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

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**#455977**

**Topic:** Kinematics of Circular Motion

Obtain equation of angular velocity as a function of time for rotating bodies with constant angular acceleration from the first principles.

**Solution**

The angular acceleration is uniform, hence

$\frac{d\omega}{dt} = \alpha =$ constant $\quad (i)$

Integrating this equation,

$\omega = \alpha dt + c$

$= \alpha t + c$(as $\alpha$ is constant)

At $t = 0, \omega = \omega_0$(give)

From $(i)$ we get at $t = 0, \omega = c = \omega_0$

Thus, $\omega = \alpha t + \omega_0$ as required.

With the definition of $\omega = d\theta/dt$ we may integrate Eq. to get Eq . This derivation and the derivation of Eq is eft as an excercise.

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**#455978**

**Topic:** Kinematics of Circular Motion

The angular speed of a motor wheel is increased from $1200$rpm to $3120$rpm in $16$ seconds. $(i)$ What is its angular acceleration to be uniform? $(ii)$ How many revolutions does the engine make during this time?

**Solution**

$(i)$ We shall use $\omega = \omega_0 + \alpha t$

$\omega_0 =$ initial anugular speed in rad/s

$= 2\pi \times \ angular\ speed\ in\ rev/s$

$= \dfrac{2\pi \times angular\ speed\ in\ rev/min}{60s/min}$

$= \dfrac{2\pi \times 1200}{60} rad/s$

$= 40\pi rad/s$

Similarly $\omega =$ final angular speed in rad/s

$= \dfrac{2\pi \times 3120}{60} rad/s$

$= 2\pi \times 52 rad/s$

$= 104\pi rad/s$

$\therefore$ Angular acceleration

$\alpha = \dfrac{\omega - \omega_0}{t} = 4\pi rad/s^2$

The angular acceleration of the engine $= 4\pi rad/s^2$

$(ii)$ The angular displacement in time $t$ is given by

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= (40\pi \times 16 + \frac{1}{2} \times 4\pi \times 16^2)$rad

$= (640\pi + 512\pi) rad$

$= 1152\pi rad$

Number of revolutions $= \dfrac{1152\pi}{2\pi} = 576$

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**#455979**

**Topic:** Conservation of Angular Momentum and Energy

A cord of negligible mass is wound round the rim of a fly wheel of mass $20$kg and radius $20$cm. A steady pull of $25N$ is applied on the cord as shown in Fig. The flywheel is mounted on a horizontal axle with frictionless bearings.

(a) Compute the angular acceleration of the wheel.

(b) Find the work done by the pull, when $2$m of the cord is unwound.

(c) Find also the kinetic energy of the wheel at this point. Assume that the wheel starts from rest.

(d) Compare answers to parts (b) and (c).

**Solution**

(a) We use $I\alpha = \tau$

the torque $\tau = FR$

$= 25 \times 0.20 Nm$(as $R = 0.20m$)

$= 5.0Nm$

$I = M.I.$ of flywheel about its axis $= \dfrac{MR^2}{2}$

$= \dfrac{20.0 \times (0.2)^2}{2} = 0.4kg\,m^2$

$\alpha =$ angular acceleration

$= 5.0Nm/0.4kg\,m^2 = 12.5s^{-2}$

(b) Work done by the pull unwinding $2m$ of the cord $= 25N \times 2m = 50J$

(c) Let $\omega$ be the final angular velocity. The kinetic energy gained $= \dfrac{1}{2}I\omega^2$,

since the wheel starts from rest. Now,

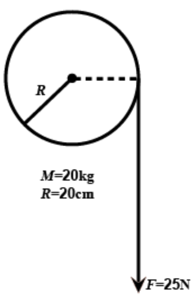
$\omega^2 = \omega_0^2 + 2a\theta, \omega_0 = 0$

The angular displacement $\theta =$ length of unwound string/ radius of wheel $= 2m/0.2m = 10$rad

$\omega^2 = 2 \times 12.5 \times 10.0 = 250(rad/s)^2$

$\therefore K.E.$ gained $= \dfrac{1}{2} \times 0.4 \times 250 = 50J$

(d) The answer are the same, i.e. the kinetic energy gained by the wheel$=$work done by the force. There is no loss of energy due to friction.




**#455980**

**Topic:** Rolling Motion

Three bodies, a ring, a solid cylinder and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of the bodies are identical. Which of the bodies reaches the ground with maximum velocity?

**Solution**

We assume conservation of energy of the rolloing body, i.e. there is no loss of energy due to friction etc. The potential energy lost by the body in rolling down the inclined plane $(= mgh)$ must, therefore, be equal to kinetic energy gained. Since the bodies start from rest the kinetic energy gained is equal to the final kinetic energy of the bodies. From Eq., $K = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$, where $v$ is the final velocity of (the centre of mass of) the body. Equating $K$ and $mgh,$

$mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$

or $v^2 = \left(\frac{2gh}{1+k^2/R^2}\right)$

Note is independent of the mass of the rolling body;

For a ring, $k^2 = R^2$

$v_{ring} = \sqrt{\frac{2gh}{1+1}},$

$= \sqrt{gh}$

For a solid cylinder $k^2 = R^2/2$

$v_{disc} = \sqrt{\frac{2gh}{1+1/2}}$

$= \sqrt{\frac{4gh}{3}}$

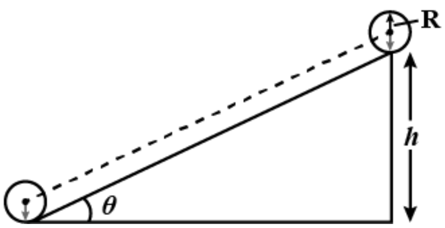
For a solid sphere $k^2 = 2R^2/5$

$v_{sphere} = \sqrt{\frac{2gh}{1+2/5}}$

$= \sqrt{\frac{10gh}{7}}$

From the results obtained it is clear that among the three bodies the sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

Suppose the bodies have the same mass. Which body has the greatest rotation kinetic energy while reaching the bottom of the inclined plane?




#458290

**Topic:** Centre of mass

Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body ?

**Solution**

In all the four cases, as the mass density is uniform, centre of mass is located at their respective geometrical centres.

i) Sphere - Centre of Sphere

ii) Cylinder - Middle Point on axis of cylinder

iii) Ring - At centre of ring (Outside the ring)

iv) Cube - At point of intersection of diagnols

No, it is not necessary that the centre of mass of a body should lie on the body. For example, in case of a circular ring, centre of mass is at the centre of the ring, where there is no mass.

#458291

**Topic:** Centre of mass

In the HCl molecule, the separation between the nuclei of the two atoms is about $1.27\ \mathring{A}(1\mathring{A} = 10^{-10}\ m)$. Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

**Solution**

Let Mass of H atom $= m$

Mass of Cl atom $= 35.5m$

Let the centre of mass of the system lie at a distance $x\ \mathring{A}$ from the Cl atom.

Distance of the centre of mass from the H atom =$(1.27 - x)\ \mathring{A}$

Let us assume that the centre of mass of the given molecule lies at the origin. Let Hydrogen lie to the left of the origin and chlorine to the right.

Position of Hydrogen atom $= -(1.27 - x)\ \mathring{A}$

Position of chlorine atom $= x\ \mathring{A}$

Therefore, we can have:

$$\frac{-m(1.27 - x) + 35.5mx}{m + 35.5m} = 0$$
$$\implies -m(1.27 - x) + 35.5mx = 0$$
$$\implies x = \frac{1.27}{35.5 + 1} = 0.035\ \mathring{A}$$

Hence, the centre of mass of the HCl molecule lies $0.035\ \mathring{A}$ from the Cl atom.

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**#458292**

**Topic:** Motion of Centre of Mass

A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system ?

**Solution**

The child is running arbitrarily on a trolley moving with velocity V. However, the running of the child will produce no effect on the velocity of the centre of mass of the trolley. This is because the force due to the boy's motion is purely internal. Since no external force is involved in the boy–trolley system, the boy's motion will produce no change in the velocity of the centre of mass of the trolley.

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**#458305**

**Topic:** Angular Momentum

Find the components along the x, y, z axes of the angular momentum $l$ of a particle, whose position vector is $r$ with components x, y, z and momentum is $p$ with components $p_x, p_y$ and $p_z$. Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

**Solution**

Linear momentum of particle, $\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$

Position vector of the particle, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Angular momentum, $\vec{l} = \vec{r} \times \vec{p}$

$$\implies l_x\hat{i} + l_y\hat{j} + l_z\hat{k} = \hat{i}(yp_z - zp_y) - \hat{j}(xp_z - zp_x) + \hat{k}(xp_y - yp_x)$$

Therefore on comparison of coefficients,

$l_x = yp_z - zp_y$

$l_y = zp_x - xp_z$

$l_z = xp_y - yp_x$

The particle moves in the x-y plane. Hence the z component of the position vector and linear momentum vector becomes zero.

$z = p_z = 0$

Thus $l_x = 0$

$l_y = 0$

$l_z = xp_y - yp_x$

Thus when particle is confined to move in the x-y plane, the angular momentum of particle is along the z-direction.

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**#458307**

**Topic:** Angular Momentum

Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken.

**Solution**

Let at a certain instant two particles be at points P and Q, as shown in the following figure.

Consider a point R, which is at a distance $y$ from point Q, i.e.,

$QR = y$

$\therefore PR = d - y$

Angular momentum of the system about point P:

$L_P = mv \times 0 + mv \times d = mvd \ldots\ldots (i)$

Angular momentum of the system about point Q:

$L_Q = mv \times d + mv \times 0 = mvd \ldots. (ii)$

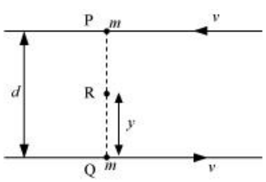
Angular momentum of the system about point R:

$L_R = mv \times (d - y) + mv \times y = mvd \ldots. (iii)$

Comparing equations **(i)**, **(ii)**, and **(iii)**, we get:

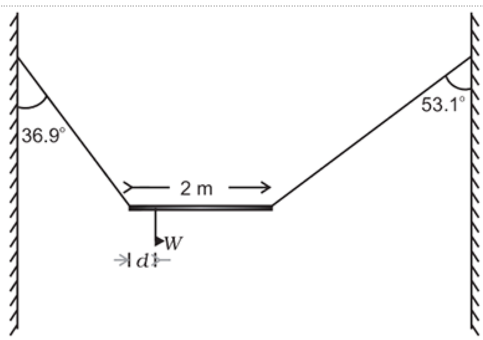
$L_P = L_Q = L_R \ldots\ldots (iv)$

We infer from equation $(iv)$ that the angular momentum of a system does not depend on the point about which it is taken.




**#458308**

**Topic:** Rotational Equilibrium




A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig. The angles made by the strings with the vertical are $36.9^o$ and $53.1^o$ respectively. The bar is 2 m long. Calculate the distance d of the center of gravity of the bar from its left end.

**Solution**

The free body diagram of the bar is shown in the following figure.

Length of the bar is ,l=2m

$T_1, T_2$ be the tensions produced in the left and right strings respectively.

At translational equilibrium, we have,

$$T_1 sin36.9^\circ = T_2 sin53.1^\circ$$
$$\implies \frac{T_1}{T_2} = \frac{4}{3}$$

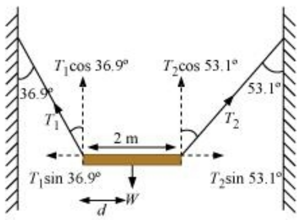
For rotational equilibrium, on taking the torque about the centre of gravity, we have:

$$T_1(cos36.9^\circ) \times d = T_2 cos53.1^\circ(2-d)$$

Using both equations,

$$d = 0.72m$$

Hence, the centre of gravity of the given bar lies 0.72 m from its left end.




#458309

**Topic:** Rotational Equilibrium

A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

**Solution**

Mass of car $m = 1800kg$

Distance between front and back axles $d = 1.8m$

Distance between center of gravity and front axle$= 1.05m$

Let $R_b$ and $R_f$ be the forces exerted by the level ground on the back and front wheels respectively.

At translational equilibrium:

$$R_f + R_b = mg = 17640\ N.....(i)$$

For rotational equilibrium, on taking the torque about the C.G., we have:

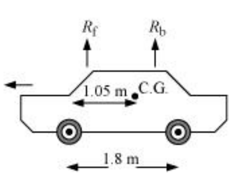
$$R_f(1.05) = R_b(1.8-1.05)$$
$$\implies R_b = 1.4R_f.......(ii)$$

Solving $(i)$ and $(ii)$ gives

$$R_f = 7350\ N$$
$$R_b = 10290\ N$$

The force exerted on each front wheel =$7350/2 = 3675\ N$

The force exerted on each back wheel =$10290/2 = 5145N$




#458310

**Topic:** Moment of Inertia of Common Bodies

(a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.

(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

**Solution**

(a)

The moment of inertia (M.I.) of a sphere about its diameter$= 2MR^2/5$

According to the theorem of parallel axes, the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

The M.I. about a tangent of the sphere =$2MR^2/5 + MR^2 = 7MR^2/5$

(b)

The moment of inertia of a disc about its diameter =$MR^2/4$

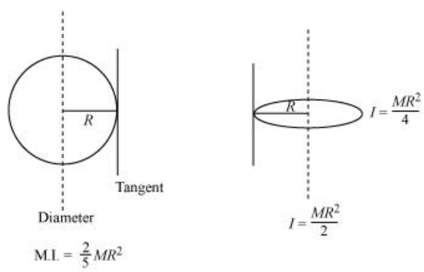
According to the theorem of perpendicular axis, the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

The M.I. of the disc about an axis normal to the disc through the center $= MR^2/4 + MR^2/4 = MR^2/2$

Now, Applying the theorem of parallel axes:

The moment of inertia about an axis normal to the disc and passing through a point on its edge

$= MR^2/2 + MR^2 = 3MR^2/2$




#458311

**Topic:** Moment of Inertia of Common Bodies

Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time.

**Solution**

The moment of inertia for the hollow cylinder=$I_1 = mr^2$

The moment of inertia for the solid sphere=$I_2 = \dfrac{2}{5}mr^2$

For hollow sphere we have $\tau = I_1\alpha_1$

For solid sphere we have $\tau = I_2\alpha_2$

$\implies \dfrac{\alpha_2}{\alpha_1} = \dfrac{I_1}{I_2} = \dfrac{5}{2} > 1$

Thus $\alpha_2 > \alpha_1$

$\omega(t) = \omega_0 + \alpha t$

The angular velocity($\omega$) at a certain time will be greater for solid sphere.

#458312

**Topic:** Angular Momentum

A solid cylinder of mass 20 kg rotates about its axis with angular speed $100\ rad\ s^{-1}$. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

**Solution**

The moment of inertia of a solid cylinder=$mr^2/2$

$= \frac{1}{2} \times 20 \times (0.25)^2$

$= 0.625 kgm^2$

Therefore kinetic energy=$\frac{1}{2} I\omega^2$

$= 3125J$

Angular momentum ,$L = I\omega$

$= 0.625 \times 100$

$= 62.5Js$

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**#458314**

**Topic:** Conservation of Angular Momentum and Energy

(a) A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value ? Assume that the turntable rotates without friction.

(b) Show that the childs new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account for this increase in kinetic energy?

**Solution**

(a) Since no external force acts on the boy, the angular momentum is a constant.

Thus $I_1\omega_1 = I_2\omega_2$

$\implies \omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{5}{2} \times 40 rev/min = 100 rev \min$

(b) Initial kinetic energy=$E_1 = \frac{1}{2} I_1\omega_1^2$

Final kinetic energy=$E_2 = \frac{1}{2} I_2\omega_2^2$

Thus $\frac{E_2}{E_1} = \frac{I_2}{I_1} \times \frac{\omega_2^2}{\omega_1^2} = 2.5$

The increase in the rotational kinetic energy is attributed to the internal energy of the boy.

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**#458315**

**Topic:** Torque

A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N ? What is the linear acceleration of the rope ? Assume that there is no slipping.

**Solution**

The moment of inertia of the hollow cylinder about its geometric axis:

$I = mr^2 = 3 \times 0.4^2 kgm^2$

Torque $\tau = rF = 0.4 \times 30Nm = 12Nm$

We know that $\tau = I\alpha$

$\implies \alpha = 25rads^{-2}$

Thus linear acceleration=$r\alpha = 0.4 \times 25m/s^2 = 10m/s^2$

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**#458316**

**Topic:** Torque

To maintain a rotor at a uniform angular speed of $200\ rad\ s^{-1}$, an engine needs to transmit a torque of 180 N m. What is the power required by the engine ? (Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100% efficient.

**Solution**

The power of the rotor required to transmit energy to apply a torque $\tau$ to rotate a motor with angular speed $\omega$,

$P = \tau\omega$

$= 180 \times 200W = 36kW$

**#458317**

**Topic:** Centre of mass

From a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

**Solution**

Let mass per unit area of the original disc=$\sigma$

Thus mass of original disc=$M = \sigma\pi R^2$

Radius of smaller disc=$R/2$.

Thus mass of the smaller disc=$\sigma\pi(R/2)^2 = M/4$

After the smaller disc has been cut from the original, the remaining portion is considered to be a system of two masses. The two masses are:

M(concentrated at O), and -M(=M/4) concentrated at O'

(The negative sign indicates that this portion has been removed from the original disc.)

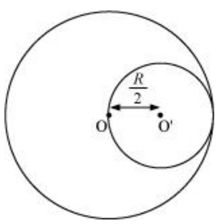
Let *x* be the distance through which the centre of mass of the remaining portion shifts from point O.

The relation between the centres of masses of two masses is given as:

$x = (m_1r_1 + m_2r_2)/(m_1 + m_2)$

$= (M \times 0 - (M/4) \times (R/2))/(M - M/4) = -R/6$

(The negative sign indicates that the centre of mass gets shifted toward the left of point O)




**#458319**

**Topic:** Rotational Equilibrium

A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?

**Solution**

Let $W$ and $W'$ be the respective weights of the metre stick and the coin.

The mass of the metre stick is concentrated at its mid-point, i.e., at the 50 cm mark.

Mass of the metre stick=$m'$

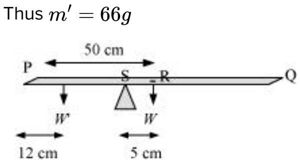
Mass of each coin=5g

When the coins are placed 12 cm away from the end P, the centre of mass gets shifted by 5 cm from point R toward the end P. The centre of mass is located at a distance of 45 cm from point P.

The net torque will be conserved for rotational equilibrium about point R.

$10g(45-12) - m'g(50-45) = 0$

Thus $m' = 66g$




**#458320**

**Topic:** Rolling Motion

A solid sphere rolls down on two different inclined planes of the same heights but different angles of inclination. (a) Will it reach the bottom with the same speed in each case? (b) Will it take longer to roll down one plane than the other? (c) If so, which one and why?

**Solution**

(a)

Total energy of sphere at the top= $mgh$

At the bottom, the sphere has both translational and rotational energy=$\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$

Using conservation of energy,

$mgh=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$......(i)

For solid sphere $I=\frac{2}{5}mr^2$......(ii)

Also we have the relation, $v=r\omega$......(iii)

Solving above equations gives $v=\sqrt{\frac{10gh}{7}}$

Clearly the speed of sphere at the bottom does not depend on the angle of inclination, both g and h are independent of the angle of inclination.

(b)

Assuming rolling without slipping,

$I\alpha=Fr$

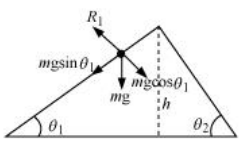
$(7/5)mr^2\times a/r=mgsin(\theta)r$

$a=(5/7)gsin(\theta)$

Since $\theta_1<\theta_2$,

(c )Acceleration for the less inclined plane is less.

Thus it takes longer time to reach the bottom along the plane with $\theta_1$ inclination.




**#458321**

**Topic:** Combined Rotational and Translational Motion

A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it?

**Solution**

Total energy of the hoop=Translational Kinetic energy+ Rotational Kinetic energy=$\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$

For hoop, $I=mr^2$

Also $v=r\omega$

Thus total energy=$\frac{1}{2}mv^2+\frac{1}{2}mr^2(\frac{v}{r})^2=mv^2$

Hence the work required to stop the hoop=Its total energy=$mv^2=100\times 0.2^2J=4J$

**#458323**

**Topic:** Conservation of Angular Momentum and Energy

The oxygen molecule has a mass of $5.30\times 10^{-26}kg$ and a moment of inertia of $1.94\times 10^{-46}kg\,m^2$ about an axis through its center perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

**Solution**

Let the separation between the two atoms be $2r$

and mass of each oxygen atom be $m/2$

Hence moment of inertia of the oxygen molecule=$(\frac{m}{2})r^2 + (\frac{m}{2})r^2 = mr^2$

It is given that $KE_{rot} = \frac{2}{3}KE_{trans}$

$\implies \frac{1}{2}I\omega^2 = \frac{2}{3} \times \frac{1}{2}mv^2$

$\implies r^2\omega^2 = \frac{2}{3}v^2$

$\implies \omega = \sqrt{\frac{2}{3}}\frac{v}{r} = 6.80 \times 10^{12} rad/s$

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**#458324**

**Topic:** Rolling Motion

A solid cylinder rolls up an inclined plane of angle of inclination $30^o$. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m/s.

(a) How far will the cylinder go up the plane?

(b) How long will it take to return to the bottom?

**Solution**

(a) Let the cylinder roll up to a height $h$.

From conservation of energy between the initial and final states,

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$

$I = \frac{1}{2}mr^2$

$\implies \frac{1}{2}mv^2 + \frac{1}{2}(\frac{1}{2}mr^2)\omega^2 = mgh$

Also for rolling, $v = r\omega$

Thus $h = \frac{3v^2}{4g} = 1.913m$

If s is distance it goes up the plane, $sin\theta = \frac{h}{s}$

Thus $s = \frac{h}{sin\theta} = 3.826m$

(b) Time taken to return the bottom=$\sqrt{\frac{2s(1+\frac{K^2}{r^2})}{g\sin\theta}} = \sqrt{\frac{2 \times 3.826 \times (1+\frac{1}{2})}{9.8 \times \frac{1}{2}}} = 1.53s$

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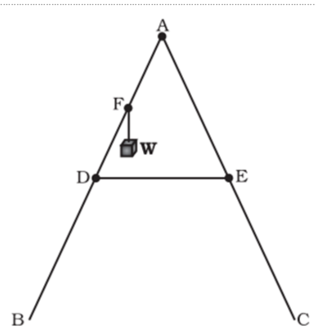
**#458327**

**Topic:** Rotational Equilibrium




As shown in Fig., the two sides of a step ladder BA and CA are 1.6 m long and hinged at A. A rope DE, 0.5 m is tied half way up. A weight 40 kg is suspended from a point F, 1.2 m from B along the ladder BA. Assuming the floor to be frictionless and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. (Take g = 9.8 m/s$^2$) (Hint: Consider the equilibrium of each side of the ladder separately.)

**Solution**

The normal forces from the floor on the ladder and the tension in the rope is as shown in the figure.

Since D is the mid point of AB, from geometry, $BI = 2DH$

Thus $BC = 2DE = 1m$

and $AD = \frac{1}{2}BA = 0.8m$

$FG = \frac{1}{2}DH = 0.125m$

In $\triangle ADH$

$AH = \sqrt{AD^2 - DH^2} = 0.76m$

From translational equilibrium of the ladder,

$N_B + N_C = mg$.....(i)

From rotational equilibrium of the ladder, balancing moment about A,

$N_B \times BI + T \times AH = N_C \times CI + T \times AH + mg \times FG$

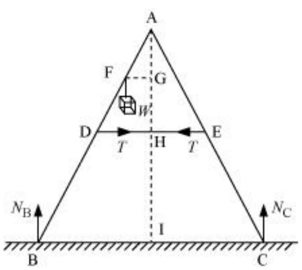
$\implies N_C - N_B = 98$......(ii)

Solving (i) and (ii) gives

$N_B = 147N$

$N_C = 245N$

Hence T=96.7N




#458328

**Topic:** Conservation of Angular Momentum and Energy

A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minute. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90cm to 20cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m$^2$.

(a) What is his new angular speed? (Neglect friction.)

(b) Is kinetic energy conserved in the process? If not, from where does the change come about?

**Solution**

Moment of inertia of man+platform system=$7.6kgm^2$

Moment of inertia of the weights=$2 \times mr^2 = 2 \times 5 \times 0.9^2 = 8.1kgm^2$

Thus initial moment of inertia=$7.6 + 8.1 = 15.7kgm^2$

Thus initial angular momentum=$I_i\omega_i = 15.7 \times 30$.....(i)

Moment of inertia of weights when man brings arms close=$2 \times mr'^2$

$= 2 \times 5 \times (0.2)^2 = 0.4kgm^2$

Thus final moment of inertia=$7.6 + 0.4 = 8.0kgm^2$

Let the final angular momentum be $\omega'$

Then from conservation of angular momentum,

$I_i\omega_i = I_f\omega'$

$\implies \omega' = \frac{15.7 \times 30}{8} = 58.88rev/min$

(b) Kinetic energy is not conserved in the given process. In fact, with the decrease in the moment of inertia, kinetic energy increases. The additional kinetic energy comes from the work done by the man to fold his hands toward himself.

#458329

**Topic:** Angular Momentum

A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the center of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it. (Hint: The moment of inertia of the door about the vertical axis at one end is $ML^2/3$.)

**Solution**

Angular momentum imparted by bullet on the door=$mvr$

$= (10 \times 10^{-3}) \times 500 \times 0.5 kgm^2/s$

Moment of inertia of the door, $I = ML^2/3 = \frac{1}{3} \times 12 \times 1^2 = 4kgm^2$

Angular momentum of the system after the bullet gets embedded$\approx I\omega$

From conservation of angular momentum about the rotation axis,

$mvr = I\omega$

$\implies \omega = 0.625 rad/s$

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**#458331**

**Topic:** Angular Momentum

Two discs of moments of inertia $I_1$ and $I_2$ about their respective axes (normal to the disc and passing through the centre), and rotating with angular speeds $\omega_1$ and $\omega_2$ are brought into contact face to face with their axes of rotation coincident. (a) What is the angular speed of the two-disc system? (b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy? Take $\omega_1 \neq \omega_2$.

**Solution**

(a) Total initial angular momentum=$I_1\omega_1 + I_2\omega_2$

When the discs are joined together, total moment of inertia about the axis becomes=$I_1 + I_2$

Let the angular speed of the two-disc system be $\omega$.

Then from conservation of angular momentum=$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

Thus $\omega = \dfrac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

(b) Total initial kinetic energy of the system=$E_i = \frac{1}{2} I_1\omega_1^2 + \frac{1}{2} I_2\omega_2^2$

Final kinetic energy of the system=$\frac{1}{2}(I_1 + I_2)\omega^2$

$= \frac{1}{2}(I_1\omega_1 + I_2\omega_2)^2/(I_1 + I_2)$

Thus $E_i - E_f = I_1 I_2(\omega_1 - \omega_2)^2/2(I_1 + I_2) > 0$

The loss of KE can be attributed to the frictional force that comes into play when the two discs come in contact with each other.

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**#458332**

**Topic:** Moment of Inertia of Common Bodies

(a) Prove the theorem of perpendicular axes.

(Hint : Square of the distance of a point (x, y) in the xy plane from an axis through the origin and perpendicular to the plane is $x^2 + y^2$).

(b) Prove the theorem of parallel axes.

(Hint : If the centre of mass is chosen to be the origin $\sum m_i r_i = 0$ ).

**Solution**

(a)

Let perpendicular axes x,y and z (which meet at origin O) so that the body lies in the x-y plane, and the z-axis is perpendicular to the plane of the body. Let $I_x, I_y, I_z$ be moments of inertia about axis x, y, z respectively, the perpendicular axis theorem states that

$I_z = I_x + I_y$

Proof:

$I_x = \int y^2 dm$ because distance of point (x,y) from x-axis is y

Similarly,

$I_y = \int x^2 dm$

$I_z = \int (x^2 + y^2) dm$

$\implies I_z = I_x + I_y$

(b)

Suppose a body of mass m is made to rotate about an axis z passing through the body's center of gravity. The body has a moment of inertia $I_{cm}$ with respect to this axis. The parallel axis theorem states that if the body is made to rotate instead about a new axis z' which is parallel to the first axis and displaced from it by a distance d, then the moment of inertia $I$ with respect to the new axis is related to $I_{cm}$ by:

$I = I_{cm} + md^2$

Proof:

$I_{cm} = \int r^2 dm$

$I = \int (r \pm d)^2 dm$

$= \int r^2 dm + \int d^2 dm \pm \int 2drdm$

$= I_{cm} + md^2$

Last term is zero since it gives center of mass.

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**#458333**

**Topic:** Combined Rotational and Translational Motion

Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by

$v^2 = \frac{2gh}{\frac{1+k^2}{R^2}}$

using dynamical consideration (i.e. by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

**Solution**

Total energy at the top of the plane=$mgh$

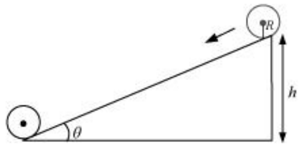
Total energy at the bottom of the plane=$KE_{rot} + KE_{trans}$

$= \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

$I = mk^2, v = R\omega$

Thus $mgh = \frac{1}{2} mk^2 (\frac{v}{R})^2 + \frac{1}{2} mv^2$

$\implies v^2 = \frac{2gh}{\frac{1+k^2}{R^2}}$

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**#458336**

**Topic:** Torque

A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to $10\pi$ rad $s^{-1}$. Which of the two will start to roll earlier ? The co-efficient of kinetic friction is $\mu_k = 0.2$

**Solution**

The motion of the two objects is caused by the frictional force acting on the objects=$\mu_k mg$

Thus acceleration due to frictional force=$\mu_k g$

Thus linear velocity attained in time t,$v = u + at = 0 + \mu_k gt = \mu_k gt$

Also a torque would act due to friction causing a rotation about the center.

Thus $\tau = I\alpha$

$\implies fr = I\alpha$

Thus angular acceleration, $\alpha = \dfrac{fr}{I} = \dfrac{\mu_k mgr}{I}$

Thus angular velocity attained after time t, $\omega = \omega_0 - \alpha t$

When rolling starts, $v = r\omega$

$\implies \mu_k gt = r(\omega_0 - \dfrac{\mu_k mgr}{I}t)$

$\implies t = \dfrac{r\omega_0}{\mu_k g + \dfrac{\mu_k mgr^2}{I}}$

Since $I$ for ring is greater than that for solid disc, ring takes longer time to achieve pure rolling. Thus the solid disc starts to roll earlier.

**#458338**

**Topic:** Instantaneous Axis of Rotation

Read each statement below carefully, and state, with reasons, if it is true or false;

(a) During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.

(b) The instantaneous speed of the point of contact during rolling is zero.

(c) The instantaneous acceleration of the point of contact during rolling is zero.

(d) For perfect rolling motion, work done against friction is zero.

(e) A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling) motion

**Solution**

(a) False

Frictional force acts opposite to the direction of motion of the centre of mass of a body. In the case of rolling, the direction of motion of the centre of mass is backward. Hence, frictional force acts in the forward direction.

(b) True

Rolling can be considered as the rotation of a body about an axis passing through the point of contact of the body with the ground. Hence, its instantaneous speed is zero.

(c) False

This is because when a body is rolling, its instantaneous acceleration is not equal to zero. It has some value.

(d) True

This is because once the perfect rolling begins, force of friction becomes zero. Hence work done against friction is zero.

(e) True

This is because rolling occurs only on account of friction which is a tangential force capable of providing torque. When the inclined plane is perfectly smooth, the wheel will simply slip under the effect of its own weight.

**#458339**

**Topic:** Angular Momentum

Separation of Motion of a system of particles into motion of the centre of mass and motion about the centre of mass :

(a) Show $p = p'_i + m_i V$

where $p_i$ is the momentum of the $i$th particle (of mass $m_i$) and $p'_i = m_i v'_i$. Note $v'_i$ is the velocity of the $i$th particle relative to the centre of mass.

Also, prove using the definition of the center of mass $\sum p'_i = 0$

(b) Show $K = K' + \frac{1}{2} MV^2$

where $K$ is the total kinetic energy of the system of particles, $K'$ is the total kinetic energy of the system when the particle velocities are taken with respect to the centre of mass and $MV^2/2$ is the kinetic energy of the translation of the system as a whole (i.e. of the centre of mass motion of the system). The result has been used in Sec. 7.14.

(c) Show $L = L' + R \times MV$

where $L' = r'_i \times p'_i$ is the angular momentum of the system about the centre of mass with velocities taken relative to the centre of mass. Remember $r'_i = r_i - R$ rest of the notation is the standard notation used in the chapter. Note $L'$ and $MR \times V$ can be said to be angular momenta, respectively, about and of the centre of mass of the system of particles.

(d) Show $\frac{dL'}{dt} = \sum r'_i \times \frac{dp'}{dt}$

Further, show that

$$\frac{dL'}{dt} = \tau'_{ext}$$

where $\tau'_{ext}$ is the sum of all external torques acting on the system about the centre of mass.

(Hint : Use the definition of centre of mass and Newtons Third Law. Assume the internal forces between any two particles act along the line joining the particles.)

**Solution**

(a) Let us consider a system of n particles of masses $m_1$ $m_2$ .......... $m_n$. Let their velocities wrt ground be $\overrightarrow{V_1}g$ $\overrightarrow{V_2}g, \; ......\overrightarrow{V_n}g$

The moment of the system of the particles is $\vec{p} = m_1\overrightarrow{V_1}g + m_2\overrightarrow{V_2}g + ....... + m_n\overrightarrow{V_n}g$

$\Rightarrow \vec{p} = m_1(\overrightarrow{V_{1c}} + \overrightarrow{V_g}) + m_2(\overrightarrow{V_{2c}} + \overrightarrow{V_g}) + ........$ where $\overrightarrow{V_{1c}}, \; \overrightarrow{V_{2c}}$ are velocities if individual masses with respect to the centre of mass and $V_g$ is the velocity of the centre of mass of the system wrt ground.

After adding up all of the term we get,

$$\vec{p} = p'_i + m_i\vec{V}$$

(b) The kinetic energy of n- particle of the systme

$$K = 1/2m_1V_{1g}^2 + 1/2m_2V_{2g}^2 + .....$$

On simplification we get

$K = K' + 1/2MV^2$ where $V = V - g$

(c) The angular momentum of the sphere about O is $L = I_0\omega$

But according to the perpendicular axes theorem

$$I_0 = I_C + MR^2$$

$$\therefore L = (I_C + MR^2)\omega$$

$$L = I_C\omega + MR^2\omega$$

$$L = I_C\omega + M(R\omega)R$$

$$L = I_C\omega + MVR$$

$$L = \vec{L}' + \vec{R} \times M\vec{V}$$

(d) The angular momentum of the system wrt centre of mass

$$\vec{L}' = \sum r'_i \times p'_i$$

Differentiating wrt time, we get

$$\frac{dL'}{dt} = \sum \overrightarrow{r_i}' \times \frac{dp'}{dt}$$

We know that $\frac{dL'}{dt} = \overrightarrow{\tau_{ext}}$

where $\overrightarrow{\tau_{ext}}$ is the sum of the external torques acting on the system about the centre of the mass.