#419293

Topic: Coordinate System and Position Vectors

In which of the following examples of motion, can the body be considered approximately a point object:

(a) A railway carriage moving without jerks between two stations.

(b) A monkey sitting on top of a man cycling smoothly on a circular track.

(c) A spinning cricket ball that turns sharply on hitting the ground.

(d) A tumbling beaker that has slipped off the edge of a table.

Solution

(a) The size of a carriage is very small as compared to the distance between two stations. Therefore, the carriage can be treated as a point sized object.

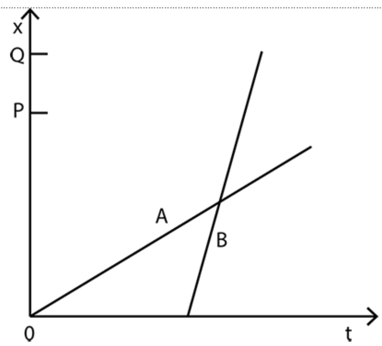
(b) The size of a monkey is very small as compared to the size of a circular track. Therefore, the monkey can be considered as a point sized object on the track.

(c) The size of a spinning cricket ball is comparable to the distance through which it turns sharply on hitting the ground. Hence, the cricket ball cannot be considered as a point object.

(d) The size of a beaker is comparable to the height of the table from which it slipped. Hence, the beaker cannot be considered as a point object.

#419294

Topic: Graphs in Kinematics



The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. above. Choose the correct entries in the brackets below :

(a) (A/B) lives closer to the school than (B/A)

(b) (A/B) starts from the school earlier than (B/A)

(c) (A/B) walks faster than (B/A)

(d) A and B reach home at the (same/different) time

(e) (A/B) overtakes (B/A) on the road (once/twice).

Solution

(a) As $OP < OQ$, A lives closer to the school than B.

(b) For $x = 0$, $t = 0$ for A; while t has some finite value for B. Therefore, A starts from the school earlier than B.

(c) Since the velocity is equal to slope of x-t graph in case of uniform motion and slope of x-t graph for$B$ is greater that that for A =, hence B walks faster than A.

(d) It is clear from the given graph that both A and B reach their respective homes at the same time.

(e) B moves later than A and his/her speed is greater than that of A. From the graph, it is clear that B overtakes A only once on the road.

#419298

Topic: Graphs in Kinematics

A woman starts from her home at 9.00 am, walks with a speed of $5\ km\,h^{-1}$ on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by an auto with a speed of $25\ km\,h^{-1}$. Choose suitable scales and plot the x-t graph of her motion.

Solution

Speed of the woman = $5\ km/h$

Distance between her office and home = 2.5 km

Time taken = Distance / Speed

$= 2.5/5 = 0.5\ h = 30\ min$

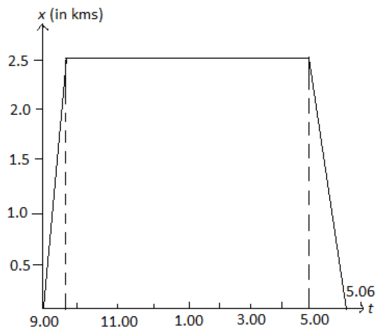
It is given that she covers the same distance in the evening by an auto.

Now, speed of the auto =$25\ km/h$

Time taken = Distance / Speed

$= 2.5/25 = 1/10 = 0.1\ h = 6\ min$

The suitable *x-t* graph of the motion of the woman is shown in the given figure.



#419299

Topic: Graphs in Kinematics

A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward,followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

Solution

time	step distance	
0	0	0
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	4
7	7	3
8	8	2
9	9	3
10	10	4
11	11	5
12	12	6
13	13	7
14	14	6
15	15	5
16	16	4
17	17	5
18	18	6
19	19	7
20	20	8
21	21	9
22	22	8

23	23	7
24	24	6
25	25	7
26	26	8
27	27	9
28	28	10
29	29	11
30	30	10
31	31	9
32	32	8
33	33	9
34	34	10
35	35	11
36	36	12
37	37	13

Lets call 1 cycle = going forward + coming back

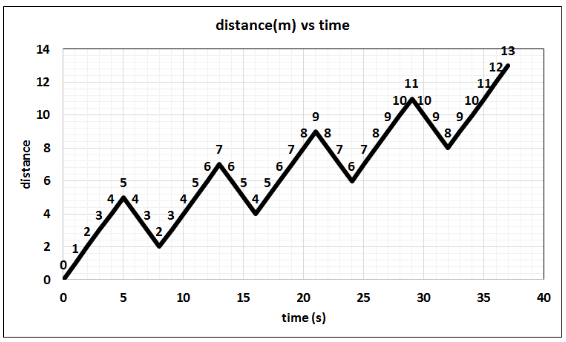
In 1 cycle, he goes 2 steps = $12\ m$ forward

In 1 cycle, he spends time of 8 units

So, in 4 cycles, displacement $= 8\ m$, time = 32 units

For 5th cycle, man reaches pit in 5 steps and falls in the pit

So, total time = $32 + 5 = 37$ units



#419302

Topic: Uniformly Accelerated Motion

A car moving along a straight highway with speed of $126\ km\ h^{-1}$ is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop ?

Solution

$u = 126\ km/h = 126 \times \frac{5}{18} m/s = 35 m/s$

$v = 0$

$s = 200\ m$

Newton's Equation of motion

$v^2 - u^2 = 2as$

$0^2 - 35^2 = 2a(200)$

$a = -3.0625 m/s^2$

Also

$v = u + at$

$0 = 35 - 3.06t$

$t = 11.4\ s$

#419305

Topic: Relative Motion

Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of $72\ km\,h^{-1}$ in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by $1\ m\,s^{-2}$. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them ?

Solution

For train A:

Initial velocity, u = 72 km/h = 20 m/s

Time, t = 50 s

Acceleration, aI = 0 (Since it is moving with a uniform velocity)

From second equation of motion, distance (sI)covered by train A can be obtained as:

s = ut + (1/2)at^2

= 20 $\times$ 50 + 0 = 1000 m

For train B:

Initial velocity, u = 72 km/h = 20 m/s

Acceleration, a = 1 m/s2

Time, t = 50 s

From second equation of motion, distance (sII) covered by train A can be obtained as:

s_{II} = ut + (1/2)at^2

= 20 X 50 + (1/2) 1 (50)^2 = 2250 m

Length of both trains = 2 $\times$ 400 m = 800 m

Hence, the original distance between the driver of train A and the guard of train B is 2250 - 1000 - 800 = 450m.

#419309

Topic: Relative Motion

On a two-lane road, car A is travelling with a speed of $36\ km\,h^{-1}$. Two cars B and C approach car A in opposite directions with a speed of $54\ km\,h^{-1}$ each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

Solution

Suppose car A, B, and C are moving with velocity

$V_a, \quad V_b, and \quad V_c,$

let the posotive direction of X axis is taken as motion of car A Car B. now

$V_a = 36km/hr \quad V_b = 54km/hr \quad and \quad V_c = -54km/hr$

relative velocity of car B with car A

$V_{ba} = V_b - V_a = 54 - 36 = 18km/hr$

relative velocity of car with car A

$V_{ca} = V_c - V_a = -54 - 36 = -90km/hr$

negative sign shows Vca is in -ve x direction

now suppose car C takes time t to cover distance BA of 1 km in slightly less then (1/90)h at a speed of $V_{ba} = 18 \quad km/hr$

Substituting u=18km/hr t= 1/90h s = 1 km

$s = ut + \frac{1}{2}at^2$

$1000 = 5 \times 40 + \frac{1}{2}a(40)^2$

800a=1000 - 200

a = 800/800

$a = 1m/s^2$ (Ans.)

#419310

Topic: Relative Motion

Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of $20\ km\ h^{-1}$ in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?

Solution

Let V be the speed of the bus running between towns A and B.

Speed of the cyclist, v = 20 km/h

Relative speed of the bus moving in the direction of the cyclist

= V - v = (V-20) km/h

The bus went past the cyclist every 18 min i.e., 18 / 60 h (when he moves in the direction of the bus).

Distance covered by the bus = (V - 20) 18 / 60 km .... (i)

Since one bus leaves after every T minutes, the distance travelled by the bus will be equal to V T / 60 ....(ii)

Both equations (i) and (ii) are equal.

(V - 20) 18 / 60 = VT / 60 ......(iii)

Relative speed of the bus moving in the opposite direction of the cyclist

= (V + 20) km/h

Time taken by the bus to go past the cyclist = 6 min = 6 / 60 h

(V + 20) 6 / 60 = VT / 60 ....(iv)

From equations (iii) and (iv), we get

(V + 20) 6 / 60 = (V - 20) 18 / 60

V + 20 = 3V - 60

2V = 80

V = 40 km/h

Substituting the value of V in equation (iv), we get

(40 + 20) 6 / 60 = 40T / 60

T = 360 / 40 = 9 min

#419314

Topic: Free Fall

A player throws a ball upwards with an initial speed of $29.4\ m s^{-1}$.

(a) What is the direction of acceleration during the upward motion of the ball ?

(b) What are the velocity and acceleration of the ball at the highest point of its motion ?

(c) Choose x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.

(d) To what height does the ball rise and after how long does the ball return to the players hands ? (Take $g = 9.8 m/s^2$ and neglect air resistance).

Solution

(a) Irrespective of the direction of the motion of the ball, acceleration (which is actually acceleration due to gravity) always acts in the downward direction towards the centre of the Earth.

(b) At maximum height, velocity of the ball becomes zero. Acceleration due to gravity at a given place is constant and acts on the ball at all points (including the highest point) with a constant value i.e., $9.8 m/s^2$.

(c) During upward motion, the sign of position is positive, sign of velocity is negative, and sign of acceleration is positive. During downward motion, the signs of position, velocity, and acceleration are all positive.

(d) Initial velocity of the ball, $u = 29.4$ m/s

Final velocity of the ball, $v = 0$ (At maximum height, the velocity of the ball becomes zero)

Acceleration, $a = g = 9.8 m/s^2$

From third equation of motion, height (s) can be calculated as:

$v^2 - u^2 = 2gs$

$s = (v^2 - u^2)/2g = ((0)^2 - (29.4)^2)/2(-9.8) = 3s$

Time of ascent = Time of descent

Hence, the total time taken by the ball to return to the players hands = $3 + 3 = 6$ s.

#419315

Topic: Acceleration

Read each statement below carefully and state with reasons and examples, if it is true or false:

A particle in one-dimensional motion

(a) with zero speed at an instant may have non-zero acceleration at that instant.

(b) with zero speed may have non-zero velocity.

(c) with constant speed must have zero acceleration.

(d) with positive value of acceleration must be speeding up.

Solution

(a) True, when an object is thrown vertically up in the air, its speed becomes zero at maximum height. However, it has acceleration equal to the acceleration due to gravity (g) that acts in the downward direction at that point.

(b) Speed is the magnitude of velocity. When speed is zero, the magnitude of velocity along with the velocity is zero.

(c) Speed is the magnitude of velocity. Constant speed does not imply constant velocity as direction change may happen. Such a scenario is uniform circular motion of a particle about a center. In such a case, acceleration is not zero.

(d) This statement is false in the situation when acceleration is positive and velocity is negative at the instant time taken as origin. Then, for all the time before velocity becomes zero, there is slowing down of the particle. Such a case happens when a particle is projected upwards.

This statement is true when both velocity and acceleration are positive, at the instant time taken as origin. Such a case happens when a particle is moving with positive acceleration or falling vertically downwards from a height.

#419316

Topic: Graphs in Kinematics

A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.

Solution

Ball is dropped from a height, s = 90 m

Initial velocity of the ball, u = 0

Acceleration, a = g = 9.8 m/s^2

Final velocity of the ball = v

From second equation of motion, time (t) taken by the ball to hit the ground can be obtained as:

$s = ut + (1/2)at^2$

$90 = 0 + (1/2) \times 9.8\ t^2$

$t = \sqrt{18.38} = 4.29$ s

From first equation of motion, final velocity is given as:

v = u + at

= 0 + 9.8 × 4.29 = 42.04 m/s

Rebound velocity of the ball, u_r = 9v / 10 = 9 × 42.04 / 10 = 37.84 m/s

Time (t) taken by the ball to reach maximum height is obtained with the help of first equation of motion as:

$v = u_r + at'$

0 = 37.84 + (– 9.8) t'

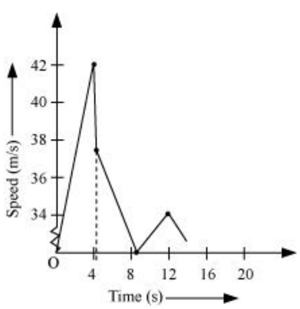
t' = -37.84 / -9.8 = 3.86 s

Total time taken by the ball = t + t' = 4.29 + 3.86 = 8.15 s

As the time of ascent is equal to the time of descent, the ball takes 3.86 s to strike back on the floor for the second time.

The velocity with which the ball rebounds from the floor = 9 × 37.84 / 10 = 34.05 m/s

Total time taken by the ball for second rebound = 8.15 + 3.86 = 12.01 s



#419321
Topic: Speed and Velocity

Explain clearly, with examples, the distinction between:

(a) Magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval.

(b) Magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval].

Show in both (a) and (b) that the second quantity is either greater than or equal to the first.

When is the equality sign true? [For simplicity, consider one-dimensional motion only].

Solution

(a) The magnitude of displacement over an interval of time is the shortest distance (which is a straight line) between the initial and final positions of the particle.

The total path length of a particle is the actual path length covered by the particle in a given interval of time.

For example, suppose a particle moves from point A to point B and then, comes back to a point, C taking a total time (t), as shown below. Then, the magnitude of displacement of the particle = AC.

(http://1.bp.blogspot.com/-E8N6SkT-Gzw/VPbuuzkPrxI/AAAAAAAAEQQ/rZg5iT7N7_c/s1600/fig-1-chapter-3-class-11th.PNG)Whereas, total path length = $AB + BC$.

It is also important to note that the magnitude of displacement can never be greater than the total path length. However, in some cases, both quantities are equal to each other.

(b) Magnitude of average velocity = Magnitude of displacement / Time interval

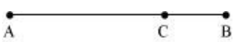
For the given particle,

Average velocity = AC / t

Average speed = Total path length / Time interval

= $(AB + BC) / t$

Since, $(AB + BC) > AC$, average speed is greater than the magnitude of average velocity. The two quantities will be equal if the particle continues to move along a straight line.



#419331

Topic: Speed and Velocity

A man walks on a straight road from his home to a market 2.5 km away with a speed of $5\ km\, h^{-1}$. Finding the market closed, he instantly turns and walks back home with a speed of $7.5\ km\, h^{-1}$. What is the magnitude of average velocity?

A 10 m/s

B 20 m/s

C 35 m/s

D 0 m/s

Solution

Average velocity = Total Displacement/ Total Time $= 0 m/s^2$

Because, at the end displacement = 0 m.

#419340

Topic: Speed and Velocity

(i) Explain clearly, with examples, the distinction between :

(a) magnitude of displacement over an interval of time and the total length of path covered by a particle over the same interval;

(b) magnitude of average velocity over an interval of time, and the average speed over the same interval.

Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true ?

(ii) A man walks on a straight road from his home to a market $2.5 km$ away with a speed of $5\ km\, h^{-1}$. Finding the market closed, he instantly turns and walks back home with a speed of $7.5\ km\, h^{-1}$. What is the:

(a) magnitude of average velocity, and

(b) average speed of the man over the interval of time (i) 0 to $30 min$, (ii) 0 to $50 min$, (iii) 0 to $40 min$?

Solution

i

a)-The magnitude of displacement over an interval of time is the shortest distance (which is a straight line) between the initial and final positions of the particle

The total path length of a particle is the actual path length covered by the particle in a given interval of time.

For example, suppose a particle moves from point A to point B and then, comes back to a point, C (between A and B) taking a total time t, . Then, the magnitude of displacement of the particle = AC.

Whereas, total path length = AB + BC

b)-Magnitude of velocity $=\dfrac{Magnitude\ of\ displacement}{time}$

For given figure average velocity $v = \dfrac{AC}{t}$

Average speed= $\dfrac{total\ path}{t}$

$S = \dfrac{AB + BC}{t}$

Since (AB + BC) > AC, average speed is greater than the magnitude of average velocity. The two quantities will be equal if the particle continues to move along a straight line.

(*ii*)

a) Time taken to reach market $t_1 = \dfrac{2.5}{5} = 0.5 hour = 30 min$

time taken to get back to home is $t_2 = dfrac2.57.5 = .33 hour = 20 min$

Average velocity for 0-30 in is $v = \dfrac{2.5}{.5} = 5 km/h$

Average speed for 0-30 in is $s = \dfrac{2.5}{.5} = 5 km/h$

total time he took for travelling $t = 30 + 20 = 50 min = \dfrac{5}{6} hour$

When he reached back then net displacement is zero

so for 0-50 min

Average velocity for 0-50 in is $v = \dfrac{0}{\frac{5}{6}} = 0 km/h$

Total distance he traveled when he arrive back is $2.5 + 2.5 = 5 km$

Average speed for 0-50 in is $v = \dfrac{5}{\frac{5}{6}} = 6 km/h$

Distance traveled in first 30 min is =2.5 km

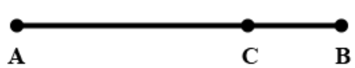
distance traveled (while returning) in 10 min is $d = 7.5 \times \dfrac{1}{6} = 1.25 km$

Total time is 40 min or $\dfrac{2}{3}$

So displacement in 0-40 min is $2.5 - 1.25 = 1.25 km$

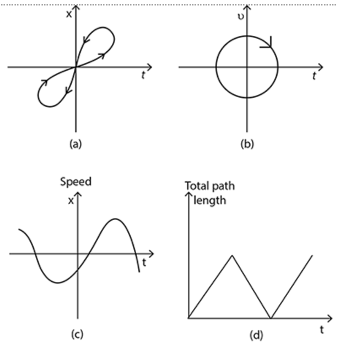
Average velocity for 0-40 in is $v = \dfrac{1.25}{\frac{2}{3}} = 1.875 km/h$

Average speed for 0-40 in is $v = \dfrac{3.75}{\frac{2}{3}} = 5.625 km/h$



#419341

Topic: Graphs in Kinematics



Look at the graphs (a) to (d) carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.

Solution

All the four graphs are impossible.

(a) A particle cannot have two different positions at the same time;

(b) A particle cannot have velocity in opposite directions at the same time;

(c) Speed is always non-negative;

(d) Total path length of a particle can never decrease with time

. (Note, the arrows on the graphs are meaningless).

#419342

Topic: Graphs in Kinematics

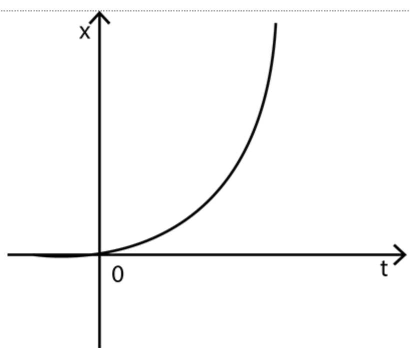


Figure shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for t < 0 and on a parabolic path for t > 0 ? If not, suggest a suitable physical context for this graph.

Solution

$t < 0$

x = const.

the particle is at rest

$t > 0$

$x = kt^2$ (parabola)

It moves in straight line, it is a 1d motion'=

No, wrong. x- t plot does not show the trajectory of a particle.

Context: A body is dropped from a tower (x = 0) at t = 0.

#419345

Topic: Relative Motion

A police van moving on a highway with a speed of $30\ km\,h^{-1}$ fires a bullet at a thiefs car speeding away in the same direction with a speed of $192\ km\,h^{-1}$. If the muzzle speed of the bullet is $150\ m\,s^{-1}$, with what speed does the bullet hit the thiefs car ? (Note:Obtain that speed which is relevant for damaging the thiefs car).

Solution

Speed of the police van, vp = 30 km/h = 8.33 m/s

Muzzle speed of the bullet, vb = 150 m/s

Speed of the thiefs car, vt = 192 km/h = 53.33 m/s

Since the bullet is fired from a moving van, its resultant speed can be obtained as:

= 150 + 8.33 = 158.33 m/s

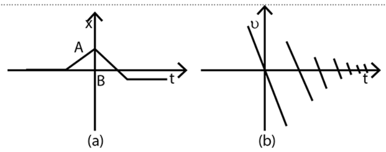
Since both the vehicles are moving in the same direction, the velocity with which the bullet hits the thiefs car can be obtained as:

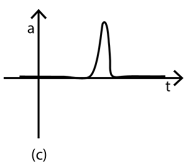
$v_{bt} = v_b - v_t$

= 158.33 - 53.33 = 105 m/s

#419346

Topic: Graphs in Kinematics





Suggest a suitable physical situation for each of the following graphs (Fig):

Solution

Keeping the following concepts

$velocity = \frac{dx}{dt}$

Slope on the graph(with sign, for direction)

Here are the examples of such situations.

(a) A ball at rest on a smooth floor is kicked, it rebounds from a wall with reduced speed and moves to the opposite wall which stops it;

(sudden change of sign of slope indicates sudden change of velocity)

Zero slope indicates that the object is at rest

(b) A ball thrown up with some initial velocity rebounding from the floor with reduced speed after each hit;

(c) A uniformly moving cricket ball turned back by hitting it with a bat for a very short time-interval.

#419351

Topic: Graphs in Kinematics

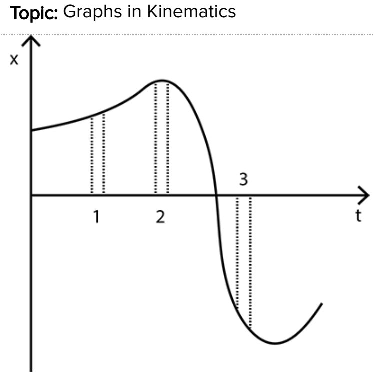


Figure gives the x-t plot of a particle in one-dimensional motion.Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least ? Give the sign of average velocity for each interval.

Solution

Interval 3 (Greatest), Interval 2 (Least)

Positive (Intervals 1 & 2), Negative (Interval 3)

The average speed of a particle shown in the *x-t* graph is obtained from the slope of the graph in a particular interval of time.

It is clear from the graph that the magnitude of slope is maximum and minimum restively in intervals 3 and 2 respectively. Therefore, the average speed of the particle is the greatest in interval 3 and is the least in interval 2. The sign of average velocity is positive in both intervals 1 and 2 as the slope is positive in these intervals. However, it is negative in interval 3 because the slope is negative in this interval.

#419352

Topic: Graphs in Kinematics

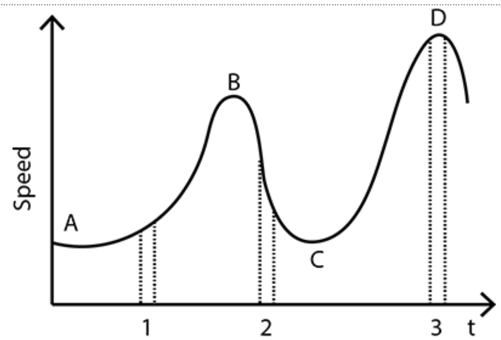


Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude ? In which interval is the average speed greatest ? Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. What are the accelerations at the points A, B, C and D ?

Solution

Average acceleration is greatest in interval 2

Average speed is greatest in interval 3

v is positive in intervals 1, 2, and 3

a is positive in intervals 1 and 3 and negative in interval 2

a = 0 at A, B, C, D

Acceleration is given by the slope of the speed-time graph. In the given case, it is given by the slope of the speed-time graph within the given interval of time.

Since the slope of the given speed-time graph is maximum in interval 2, average acceleration will be the greatest in this interval.

Height of the curve from the time-axis gives the average speed of the particle. It is clear that the height is the greatest in interval 3. Hence, average speed of the particle is the greatest in interval 3.

In interval 1:

The slope of the speed-time graph is positive. Hence, acceleration is positive. Similarly, the speed of the particle is positive in this interval.

In interval 2:

The slope of the speed-time graph is negative. Hence, acceleration is negative in this interval. However, speed is positive because it is a scalar quantity.

In interval 3:

The slope of the speed-time graph is zero. Hence, acceleration is zero in this interval. However, here the particle acquires some uniform speed. It is positive in this interval.

Points A, B, C, and D are all parallel to the time-axis. Hence, the slope is zero at these points. Therefore, at points A, B, C, and D, acceleration of the particle is zero.

#419508

Topic: Uniformly Accelerated Motion

A car, having acceleration $1 ms^{-2}$, is moving with an initial velocity $10 m/s$ and it stops after 10 seconds as the driver applied brakes. Find the distance travelled by the car.

A 30 m

B 40 m

C 50 m

D 60 m

Solution

As we know, $v^2 = u^2 - 2aS$

$S = \frac{u^2}{2a} = \frac{10^2}{2 \times 1} = 50\ m \quad \because$ final velocity(v) = 0

#419509

Topic: Relative Motion

Explain this common observation clearly : If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the trains motion, but the distant objects (hill tops, the Moon, the stars etc.)seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

Solution

Near objects make greater angle than distant (far off) objects at the eye of the observer.

When you are moving, the angular change is less for distant objects than nearer objects.

So, these distant objects seem to move along with you, but the nearer objects in opposite direction.

#419510

Topic: Relative Motion

A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to $49ms^{-1}$. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of $5ms^{-1}$ and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?

Solution

Initial velocity of the ball, u = 49 m/s

Acceleration, a = g = 9.8 m/s2

Case I:

When the lift was stationary, the boy throws the ball.

Taking upward motion of the ball,

Final velocity, v of the ball becomes zero at the highest point.

From first equation of motion, time of ascent (t) is given as:

$v = u + at$

t = (v - u) / a

= -49 / -9.8 = 5 s

But, the time of ascent is equal to the time of descent.

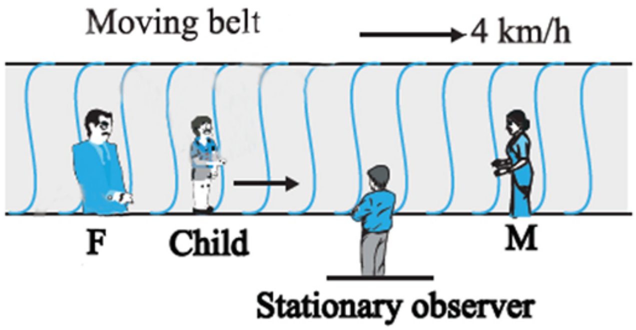
Hence, the total time taken by the ball to return to the boys hand = 5 + 5 = 10 s.

Case II:

The lift was moving up with a uniform velocity of 5 m/s. In this case, the relative velocity of the ball with respect to the boy remains the same i.e., 49 m/s. Therefore, in this case also, the ball will return back to the boys hand after 10 s.

#419512

Topic: Relative Motion



On a long horizontally moving belt (Fig.), a child runs to and fro with a speed $9 km h^{-1}$ (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of $4 km h^{-1}$. For an observer on a stationary platform outside, what is the

(a) speed of the child running in the direction of motion of the belt ?

(b) speed of the child running opposite to the direction of motion of the belt ?

(c) time taken by the child in (a) and (b)

Solution

Speed of the belt, $v_B = 4 km/h$

Speed of the boy, $v_b = 9 km/h$

(a)

Since the boy is running in the same direction of the motion of the belt, his speed (as observed by the stationary observer) can be obtained as:

$v_{bB} = v_b + v_B = 9 + 4 = 13 km/h$

(b)

Since the boy is running in the direction opposite to the direction of the motion of the belt, his speed (as observed by the stationary observer) can be obtained as:

$v_{bB} = v_b - v_B = 9 - 4 = 5 km/h$

(c)

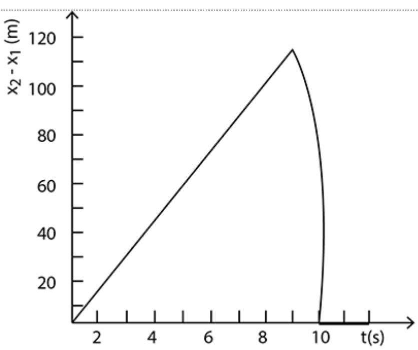
Distance between the child's parents = 50 m

As both parents are standing on the moving belt, the speed of the child in either direction as observed by the parents will remain the same i.e., 9 km/h = 2.5 m/s.

Hence, the time taken by the child to move towards one of his parents is 50 / 2.5 = 20 s

#419524

Topic: Graphs in Kinematics



Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of $15 ms^{-1}$ and $30 ms^{-1}$. Verify that the graph shown in Fig. 3.27 correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take $g = 10 ms^{-2}$. Give the equations for the linear and curved parts of the plot.

Solution

For first stone:

Initial velocity, $u_1 = 15 m/s$

Acceleration, $a = -g = -10 m/s^2$

Using the relation,

$x_1 = x_o + u_1 t + \frac{1}{2} a t^2$

Where, height of the cliff, $x_o = 200 m$

$x_1 = 200 + 15t - 5t^2$.....(i)

When this stone hits the ground, $x_1 = 0$

$\therefore -5t^2 + 15t + 200 = 0$

$\therefore - 5t^2 + 15t + 200 = 0$

t = 8 s or t = – 5 s

Since the stone was projected at time t = 0, the negative sign before time is meaningless.

∴t = 8 s

For second stone:

Initial velocity, $u_2 = 30 m/s$

Acceleration, $a = -g = -10 m/s^2$

Using the relation,

$x_2 = x_o + u_2 t + \frac{1}{2} a t^2$

$= 200 + 30t - 5t^2$ ....(ii)

At the moment when this stone hits the ground; $x_2 = 0$

$-5t^2 + 30t + 200 = 0$

t = 10 s or t = – 4 s

Here again, the negative sign is meaningless.

∴ t = 10 s

Subtracting equations (i) and (ii), we get

$x_2 - x_1 = (200 + 30t - 5t^2) - (200 + 15t - 5t^2)$

$x_2 - x_1 = 15t$.......(iii)

Equation (iii) represents the linear path of both stones. Due to this linear relation between $(x_2 - x_1)$ and t, the path remains a straight line till 8 s.

Maximum separation between the two stones is at t = 8 s.

$(x_2 - x_1)_{max} = 15 \times 8 = 120 m$

This is in accordance with the given graph.

After 8 s, only second stone is in motion whose variation with time is given by the quadratic equation:

$x_2 - x_1 = 200 + 30t - 5t^2$

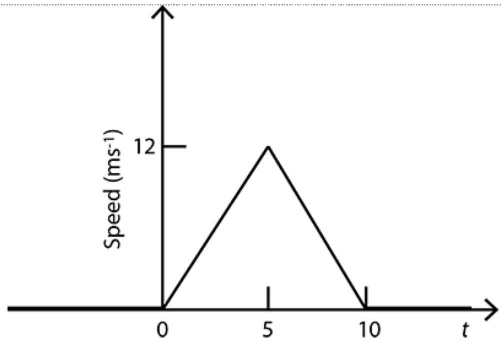
Hence, the equation of linear and curved path is given by

$x_2 - x_1 = 15t$ (Linear path)

$x_2 - x_1 = 200 + 30t - 5t^2$ (Curved path)

#419529

Topic: Graphs in Kinematics



The speed-time graph of a particle moving along a fixed direction is shown in Fig. Obtain the distance traversed by the particle between time t = 0 s to 10 s.

Solution

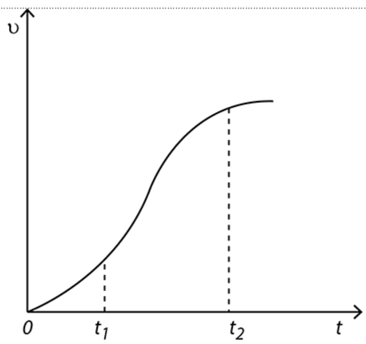
Distance travelled by the particle = Area under the given graph

$= (1/2) \times (10 - 0) \times (12 - 0) = 60\ m$

Average speed $= \frac{\text{Distance}}{\text{Time}} = \frac{60}{10} = 6 m/s$

#419542

Topic: Graphs in Kinematics



The velocity-time graph of a particle in one-dimensional motion is shown in FIg. 3.29:

Which of the following formula are correct for describing the motion of the particle over the time-interval $t_1 to t_2$.

(a) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + (\frac{1}{2})a(t_2 - t_1)^2$

(b) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(c) $v_{average} = (x(t_2) - x(t_1))/(t_2 - t_1)$

(d) $a_{average} = (v(t_2) - v(t_1))/(t_2 - t_1)$

(e) $x(t_2) = x(t_1) + v_{average}(t_2 - t_1) + (\frac{1}{2})a_{average}(t_2 - t_1)^2$

(f) $x(t_2) - x(t_1) =$ area under the v-t curve bounded by the t-axis and the dotted line shown.

Solution

The correct formulae describing the motion of the particle are (c), (d) and, (f)

The given graph has a non-uniform slope. Hence, the formulae given in (a), (b), and (e) cannot describe the motion of the particle. Only relations given in (c), (d), and (f) are correct equations of motion.

#419598

Topic: Speed and Velocity

A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is

(a) the average speed of the taxi.

(b) the magnitude of average velocity.

Are the two equal?

Solution

Total distance traveled = $23\ km$

Total time taken = $28\ min = 28/60\ hr$

(a) Average speed of the taxi = Total distance traveled / Total time taken

= $23/(28/60) = 49.29\ km/h$

(b) Distance between the hotel and the station = $10\ km$ = Displacement of the car

Average velocity = $10/(28/60) = 21.43\ km/h$

So, the two physical quantities (average speed and average velocity) are not equal.

#419609

Topic: Relative Motion

In a harbor, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbor flutters along the N-E direction. If the boat starts moving at as speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat ?

Solution

The flag is fluttering in the north-east direction. It shows that the wind is blowing toward the north-east direction. When the ship begins sailing toward the north, the flag will move along the direction of the relative velocity (v_{wb}) of the wind with respect to the boat.

(http://2.bp.blogspot.com/-QioirFc7j9Q/VPgjQOD_fEI/AAAAAAAAEU8/nY2PBBZeMyA/s1600/fig-5-chapter-4-class-11th.PNG)

The angle between v_w and $v_b = 90 + 45$

$tan\theta = 51sin(90 + 45)/(72 + 51cos(90 + 45)$

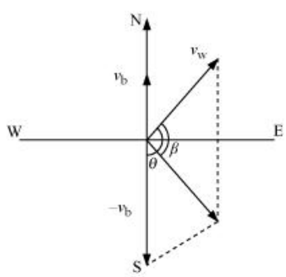
Substituting and solving we get,

$tan\theta = 51/50.80 = 1.0038$

$\theta = tan^{-1}(1.0038) = 45.11^0$

Angle with respect to the east direction = $45.11 - 45 = 0.11$

Hence, the flag will flutter almost due east.



#419646

Topic: Uniformly Accelerated Motion

A particle starts from the origin at t = 0 s with a velocity of 10.0 $\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $\left(8.0\hat{i} + 2.0\hat{j}\right)$ m/s^2.

(a) At what time is the x- coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?

(b)What is the speed of the particle at the time ?

Solution

$\vec{u} = 10\hat{j} m/s$

$\vec{a} = 8\hat{i} + 2\hat{j}$

(a)

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$x\hat{i} + y\hat{j} = 4t^2\hat{i} + (10t + t^2)\hat{j}$

$x = 4t^2$

$x = 16 \implies t = 2s$

$y = 10t + t^2$

At t=2, $y = 24m$

(b)

$\vec{v} = \vec{u} + \vec{a}t$

$\vec{v} = 10\hat{j} + (8\hat{i} + 2\hat{j})2$

$\vec{v} = 14\hat{j} + 16\hat{i}$

$|\vec{v}| = \sqrt{14^2 + 16^2} = 21.26 m/s$

#419664

Topic: Graphs in Kinematics

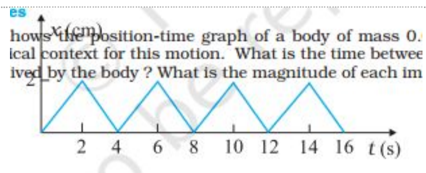


Figure above shows the position-time graph of a body of mass $0.04\ kg$. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the body? What is the magnitude of each impulse?

Solution

Physical Context:

Consider a system of two parallel walls, perpendicular to x-axis at x=0 cm and x=2 cm with frictionless ground surface between them. Ball is kept at x=0 and given an initial velocity of 1 cm/s in x direction by some external medium. Ball moves in x-direction, suffers an elastic collision with wall at x=2 cm, and reverses its direction. The above continues indefinitely long.

Instant of impulses are the sharp points on the graph.

From the graph, time between two consecutive impulses = 2s.

v=dx/dt

For positive slope region, v = 2/2 = 1 cm/s in positive x direction

For negative slope region, v = -(2/2) = 1 cm/s in negative x direction

Impulse = Change in momentum

Magnitude of impulse $= 0.04 \times (1 - (-1) \times 10^{-2}) = .8 \times 10^{-3} Ns$

#419698

Topic: Acceleration

For any arbitrary motion in space, which of the following relations are true:

(a) $v_{average} = (1/2)(v(t_1) + v(t_2))$

(b) $v_{average} = [r(t_2) - r(t_1)]/(t_2 - t_1)$

(c) $v(t) = v(0) + at$

(d) $r(t) = r(0) + v(0)t + (1/2)a\,t^2$

(e) $a_{average} = [v(t_2) - v(t_1)]/(t_2 - t_1)$

(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

Solution

(a) Invalid for non uniform acceleration

(b) This is the mathematical definition of average velocity.

(c) This is valid only for constant acceleration

(d) Valid only for constant accelerations

(e) Mathematical definition of average acceleration

(b) and (e) are valid.

#419867

Topic: Free Fall

The escape speed of a projectile on the earth's surface is $11.2\ kms^{-1}$. A body is projected out with thrice this speed. What is the speed of the body far away from the earth?

Ignore the presence of the sun and other planets.

Solution

Escape velocity of a projectile from the Earth, $v_{esc} = 11.2\ km/s$

Projection velocity of the projectile, $v_p = 3v_{esc}$

Mass of the projectile $= m$

Let velocity of the projectile far away from the Earth $= v_f$

Total energy of the projectile on the Earth $= \frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2$

Gravitational potential energy of the projectile far away from the Earth is zero.

Total energy of the projectile far away from the Earth $= \frac{1}{2}mv_f^2$

From the law of conservation of energy, we have

$\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2 = \frac{1}{2}mv_f^2$

$v_f = \sqrt{v_p^2 - v_{esc}^2}$

$= \sqrt{8}v_{esc}$

$= \sqrt{8} \times 11.2$

$= 31.68\ km/s$

#420277

Topic: Uniformly Accelerated Motion

A stone dropped from the top of a tower of height 300 m high splashes into

the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 m s^{-1}? (g = 9.8 ms^{-2})

Solution

The time at which stone hits the ground can be found by $s = ut + \frac{1}{2}gt^2$

$\therefore 300 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$

$\therefore t = 7.82s$

Now, the time for sound to travel 300m is $t = \frac{300}{340} = 0.88s$

Therefore, total time will be addition of the two times.

$t = 7.82 + 0.88 = 8.7s$

#464640

Topic: Speed and Velocity

Joseph jogs from one end A to the other end B of a straight $300\ m$ road in 2 min 30 seconds and then turns around and jogs $100\ m$ back to point C in another 1 minute. What are Joseph's average speeds and velocities in jogging (a) from A to B and (b) from A to C?

Solution

(a)

From A to B,

Average velocity = *Total displacement*/ *Total time*

$v_{average} = \frac{300}{150} = 2m/sec$

so velocity will be $2\ m/sec$ in rightward direction.

Average speed = *Total distance*/ *time*

$= 300/150 = 2\ m/s$

(b)

From A to C,

Average velocity = *Total displacement*/ *Total time*

$= 200/210 = 0.952\ m/s$

$Average\ Speed = \frac{total\ \ distance\ \ travelled}{total\ \ time\ \ taken}$

$= \frac{400}{210} = 1.905\ m/sec$

#464642

Topic: Speed and Velocity

Abdul, while driving to school, computes the average speed for his trip to be 20km h^{-1}. On his return trip along the same route, there is less traffic and the average speed is 30 km h^{-1}. What is the average speed for Abdul's trip?

Solution

We know that $Averagespeed = \frac{Totaldistance}{Totaltime}$

Let the distance be S

Let Time taken while going to school $= t_1 = \frac{S}{20}$

Let Time taken while coming from school $= t_2 = \frac{S}{30}$

To calculate the average speed to need to know the total time taken to go and come from school, $t_1 + t_2 = \frac{S}{20} + \frac{S}{30}$

$\Rightarrow \frac{3S+2S}{60} = \frac{S}{12}$

So, total time $= \frac{S}{12}$

Total Distance $= S + S = 2S$

So, $Averagespeed = \frac{Totaldistance}{Totaltime} = \frac{2S}{\frac{S}{12}} = 24$

$\Rightarrow 24 km/hr$

Let us assume that Abdul is travelling x kilometer distance while going to school.

Distance = Speed x Time

1)When it's average speed is 20 km/hr

$x = v_1 \times t_1 = 20 \times t_1$

2)When it's average speed is 30 km/hr

$x = v_2 \times t_2 = 30 \times t_2$

$V_{average} = \frac{total \quad distance \quad travelled}{total \quad time \quad taken}$

$V_{average} = \frac{x + x}{\frac{x}{v_1} + \frac{x}{v_2}} = \frac{2x}{\frac{x}{20} + \frac{x}{30}} = 24 \ km\ h^{-1}$

#464644

Topic: Uniformly Accelerated Motion

A motorboat starting from rest on a lake accelerates in a straight line at a constant rate of $3.0 m/s^2$ for 8.0s. How far does the boat travel during this time?

Solution

We know that Diatance is given by second equation of motion , $S = ut + \frac{1}{2}at^2$

Here $S = ?$ = Distance

$u = 0$= initial velocity

$a = 3m/s^2$ = acceleratin

$t = 8s =$ time

So, using the equation we get

$S = ut + \frac{1}{2}at^2$

$\Rightarrow S = 0 + \frac{1}{2} \times 3 \times 8^2$

$\Rightarrow S = 96m$

Initial velocity of the motorboat $u = 0$

Acceleration of the boat $a = 3.0 \ ms^{-2}$

Time $t = 8$ s

Let the distance covered during this time be d

Using 2nd equation of motion, $d = ut + \frac{1}{2}at^2$

$d = (0) \times 8 + \frac{1}{2} \times (3) \times 8^2$

$d = 96 \ m$

#464645

Topic: Graphs in Kinematics

A driver of a car travelling at 52km h^{-1} applies the brakes and accelerates uniformly in the opposite direction. The car stops in 5s. Another driver going at 3km h^{-1} in another car applies his brakes slowly and stops in 10s. On the same graph paper, plot the speed versus time graphs for the two cars. Which of the two cars travelled farther after the brakes were applied?

Solution

(A) : Initial speed of the car A $u = 52 \ kmh^{-1} = 52 \times \frac{5}{18} = 14.44 \ ms^{-1}$

The car stops in 5 seconds i.e $v = 0$ at $t = 5$

(A) : Initial speed of the car A $u = 3 \ kmh^{-1} = 3 \times \frac{5}{18} = 0.83 \ ms^{-1}$

The car stops in 10 seconds i.e $v = 0$ at $t = 10$

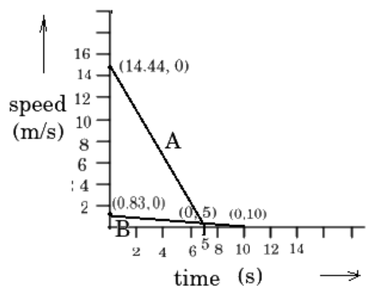
With the help of these initial and final points for both the cases, we can plot the graph of speed vs time.

Area under the speed-time graph gives the distance covered.

$\therefore$ Distance covered by car A $x_A = \frac{1}{2} \times 14.44 \times 5 = 36.1$ m

$\therefore$ Distance covered by car A $x_B = \frac{1}{2} \times 0.83 \times 10 = 4.15$ m

Thus car A travels more distance than B.



#464647

Topic: Uniformly Accelerated Motion

A ball is gently dropped from a height of 20 m. If its velocity increases uniformly at the rate of 10 $m\,s^{-2}$, with what velocity will it strike the ground? After what time will it strike the ground?

Solution

Height, $s = 20m$

Acceleartion , $a = 10m/s^2$

By 3rd equation of motion

$v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 10 \times 20 = 400$

$\Rightarrow v = 20m/s$

So, we conclude final velocity , $v = 20m/s$

again, by 1st equation of motion , $v = u + at$

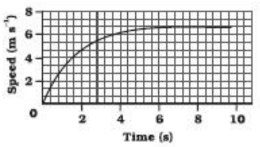
$\Rightarrow t = \frac{v - u}{a}$

$\Rightarrow t = \frac{20}{10} = 2s$

So, it will strike the ground after $2s$

#464648

Topic: Graphs in Kinematics



The speed-time graph for a car is shown in Fig.

(*a*) Find how far does the car travel in the first 4 seconds. Shade the area on the graph that represents the distance travelled by the car during the period.

(*b*) Which part of the graph represents uniform motion of the car?

Solution

(a)

Distance travelled by the car = Area under the graph

Hence, the area of the shaded region in the graph is equal to the distance travelled in first four seconds.

Area of shaded region = Area of 58 full squares + Area of four half squares

= Area of 60 full squares

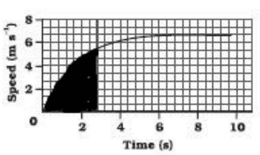
Area of 1 square $= (2/10) \times (2/3)$

$= 2/15\ m$

Total area shaded $= 60 \times 2/15 = 8\ m$

(b)

During uniform motion, velocity is constant and hence speed is constant. In the graph, this is valid for $t > 6\ s$



#464651

Topic: Speed and Velocity

An artificial satellite is moving in a circular orbit of radius 42250km. Calculate its speed if it takes 24 hours to revolve around the earth.

Solution

We know that for any body total distance covered is equals to the circumference of that circular track.

so,circumference of circular track=2πr=2 $\times$ π $\times$ 42250=265572km

So,Distance=Speed $\times$ time

$Speed = \frac{Distance}{time} = \frac{265572}{24} = 11065.4 km/hr = 3073.74 m/sec$

Time taken by the satellite to revolve around the earth once $t = 24$ hours

$\therefore\ t = 24 \times 3600$ s $= 86400$ s

Radius of the orbit $R = 42250$ km $= 42250000$ m

Let the velocity of the satellite be v

$$\therefore\ v = \frac{2\pi R}{t} = \frac{2 \times 3.14 \times 42250000}{86400}$$

$\Rightarrow\ v = 3070.95\ ms^{-1} = 3.1\ kms^{-1}$