MEASUREMENT AND UNITS & DIMENSIONS

Synopsis :

1. Every measurement has two parts. The first is a number (n) and the next is a unit (u). Q = nu.

   Eg : Length of an object = 40 cm.
2. The number expressing the magnitude of a physical quantity is inversely proportional to the unit selected.
3. If n_1 and n_2 are the numerical values of a physical quantity corresponding to the units u_1 and u_2, then $n_1u_1 = n_2u_2$. Eg : 2.8 m = 280 cm; 6.2 kg = 6200 g
4. The quantities that are independent of other quantities are called ***fundamental quantities***. The units that are used to measure these fundamental quantities are called ***fundamental units***.
5. There are four systems of units namely C.G.S, M.K.S, F.P.S and SI
6. The quantities that are derived using the fundamental quantities are called ***derived quantities***. The units that are used to measure these derived quantities are called ***derived units***.
7. The early systems of units :

Fundamental Quantity	**System of units**		
	C.G.S.	**M.K.S.**	**F.P.S.**
Length	centimetre	Metre	foot
Mass	Gram	Kilogram	pound
Time	second	Second	second

8. Fundamental and supplementary physical quantities in SI system (Systeme Internationale d'units) :

Physical quantity	**Unit**	**Symbol**
Length	Metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Intensity of light	candela	cd
Quantity of substance	mole	mol

Supplementary quantities:

Plane angle	radian	rad
Solid angle	steradian	sr

SI units are used in scientific research. SI is a coherent system of units.

13. A ***coherent system*** of units is one in which the units of derived quantities are obtained as multiples or submultiples of certain basic units.

    SI system is a comprehensive, coherent and rationalised M.K.S. Ampere system (RMKSA system) and was devised by Prof. Giorgi.

14. **Metre** : A metre is equal to 1650763.73 times the wavelength of the light emitted in vacuum due to electronic transition from $2p^{10}$ state to $5d^{5}$ state in Krypton–86. But in 1983, 17^{th} General Assembly of weights and measures, adopted a new definition for the metre in terms of velocity of light. According to this definition, metre is defined as the distance travelled by light in vacuum during a time interval of 1/299, 792, 458 of a second.
15. **Kilogram** : The mass of a cylinder of platinum–iridium alloy kept in the International Bureau of weights and measures preserved at Serves near Paris is called one kilogram.
16. **Second** : The duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of caesium–133 atom is called one second.
17. **Ampere** : The current which when flowing in each of two parallel conductors of infinite length and negligible cross–section and placed one metre apart in vacuum, causes each conductor to experience a force of 2×10^{-7} newton per metre of length is known as one ampere.
18. **Kelvin** : The fraction of 1/273.16 of the thermodynamic temperature of the triple point of water is called kelvin.
19. **Candela** : The luminous intensity in the perpendicular direction of a surface of a black body of area 1/600000 m^2 at the temperature of solidifying platinum under a pressure of 101325 Nm^{-2} is known as one candela.
20. **Mole** : The amount of a substance of a system which contains as many elementary entities as there are atoms in 12×10^{3} kg of carbon–12 is known as one mole.
21. **Radian** : The angle made by an arc of the circle equivalent to its radius at the centre is known as radian. 1 radian = $57^{\circ}17'45''$.
22. **Steradian** : The angle subtended at the centre by one square metre area of the surface of a sphere of radius one metre is known as steradian.
23. The quantity having the same unit in all the systems of units is **time**.
24. **Angstrom** is the unit of length used to measure the wavelength of light. 1 Å = 10^{-10} m.
25. **Fermi** is the unit of length used to measure nuclear distances. 1 fermi = 10^{-15} metre.
26. **Light year** is the unit of length for measuring astronomical distances.
27. **Light year** = distance travelled by light in 1 year = 9.4605×10^{15} m.
28. **Astronomical unit** = Mean distance between the sun and earth = 1.5×10^{11} m.
29. **Parsec** = 3.26 light years = 3.084×10^{16} m
30. **Barn** is the unit of area for measuring scattering cross–section of collisions. 1 barn = 10^{-28} m^2.
31. **Chronometer** and **metronome** are time measuring instruments.
32. **PREFIXES :** (or) Abbreviations for multiples and sub–multiples of 10.

MACRO Prefixes	MICRO Prefixes
Kilo → K → 10^{3}	Milli → m → 10^{-3}
Mega → M → 10^{6}	micro → μ → 10^{-6}
Giga → G → 10^{9}	nano → n → 10^{-9}
Tera → T → 10^{12}	pico → p → 10^{-12}
Peta → P → 10^{15}	femto → f → 10^{-15}
Exa → E → 10^{18}	atto → a → 10^{-18}
Zetta → Z → 10^{21}	zepto → z → 10^{-21}
Yotta → y → 10^{24}	yocto → y → 10^{-24}

Note : The following are not used in SI system.

deca → 10^{1} deci → 10^{-1}

hecta → 10^{2} centi → 10^{-2}

33. Full names of the units, even when they are named after a scientist should not be written with a capital letter. Eg : newton, watt, ampere, metre.
34. Unit should be written either in full or in agreed symbols only.
35. Units do not take plural form. Eg : 10 kg but not
    10 kgs, 20 w but not 20 ws 2 A but not 2 As
36. No full stop or punctuation mark should be used within or at the end of symbols for units. Eg : 10 W but not 10 W.
37. ***Dimensions*** of a physical quantity are the powers to which the fundamental units are raised to obtain one unit of that quantity.
38. The expression showing the powers to which the fundamental units are to be raised to obtain one unit of a derived quantity is called the ***dimensional formula*** of that quantity.
39. If Q is the unit of a derived quantity represented by $Q = M^aL^bT^c$, then $M^aL^bT^c$ is called dimensional formula and the exponents a,b and c are called the dimensions.
40. Dimensional Constants : The physical quantities which have dimensions and have a fixed value are called dimensional constants. Eg : Gravitational constant (G), Planck's constant (h), Universal gas constant (R), Velocity of light in vacuum (C) etc.
41. Dimensionless quantities are those which do not have dimensions but have a fixed value.
    a) Dimensionless quantities without units.
    Eg : Pure numbers, π e, sinθ cosθ tanθ …. etc.,
    b) Dimensionless quantities with units.
    Eg : Angular displacement – radian, Joule's constant – joule/calorie, etc.,
42. Dimensional variables are those physical quantities which have dimensions and do not have fixed value. Eg : velocity, acceleration, force, work, power… etc.
43. Dimensionless variables are those physical quantities which do not have dimensions and do not have fixed value. Eg : Specific gravity, refractive index, coefficient of friction, Poisson's ratio etc.
44. Dimensional formulae are used to a) verify the correctness of a physical equation, b) derive relationship between physical quantities and c) to convert the units of a physical quantity from one system to another system.
45. **Law of homogeneity of dimensions** : In any correct equation representing the relation between physical quantities, the dimensions of all the terms must be the same on both sides. Terms separated by '+' or '–' must have the same dimensions.
46. A physical quantity Q has dimensions a, b and c in length (L), mass (M) and time (T) respectively, and n_1 is its numerical value in a system in which the fundamental units are L_1, M_1 and T_1 and n_2 is the numerical value in another system in which the fundamental units are L_2, M_2 and T_2 respectively, then
    $$n_2 = n_1\left[\frac{L_1}{L_2}\right]^a\left[\frac{M_1}{M_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$
47. Fourier laid down the foundations of dimensional analysis.
48. **Limitations of dimensional analysis :**

1. Dimensionless quantities cannot be determined by this method. Constant of proportionality cannot be determined by this method. They can be found either by experiment (or) by theory.
2. This method is not applicable to trigonometric, logarithmic and exponential functions.
3. In the case of physical quantities which are dependent upon more than three physical quantities, this method will be difficult.
4. In some cases, the constant of proportionality also possesses dimensions. In such cases we cannot use this system.
5. If one side of equation contains addition or subtraction of physical quantities, we can not use this method to derive the expression.

50. **Some important conversions :**

51. 1 bar = 0^6 dyne/cm^2 = 10^5 Nm^{-2} = 10^5 pascal

76 cm of Hg	=	1.013×10^6 dyne/cm^2
	=	1.013×10^5 pascal = 1.013 bar.
1 toricelli or torr	=	1 mm of Hg
	=	1.333×10^3 dyne/cm^2
	=	1.333 millibar.
1 kmph	=	5/18 ms^{-1}
1 dyne	=	10^{-5} N,
1 H.P	=	746 watt
1 kilowatt hour	=	36×10^5 J
1 kgwt	=	g newton
1 calorie	=	4.2 joule
1 electron volt	=	1.602×10^{-19} joule
1 erg	=	10^{-7} joule

52. **Some important physical constants :**

Velocity of light in vacuum (c)	=	3×10^8 ms^{-1}
Velocity of sound in air at STP	=	331 ms^{-1}
Acceleration due to gravity (g)	=	9.81 ms^{-2}
Avogadro number (N)	=	6.023×10^{23} /mol
Density of water at 4^oC	=	1000 kgm^{-3} or 1 g/cc.
Absolute zero	=	-273.15^oC or 0 K
Atomic mass unit	=	1.66×10^{-27} kg
Quantum of charge (e)	=	1.602×10^{-19} C
Stefan's constant(σ)	=	5.67×10^{-8} $W/m^2/K^4$
Boltzmann's constant (K)	=	1.381×10^{-23} JK^{-1}
One atmosphere	=	76 cm Hg
	=	1.013×10^5 Pa
Mechanical equivalent of heat (J)	=	4.186 J/cal
Planck's constant (h)	=	6.626×10^{-34} Js
Universal gas constant (R)	=	8.314 J/mol–K
Permeability of free space (μ_o)	=	$4\pi \times 10^{-7}$ Hm^{-1}

Permittivity of free space (ε_o) = 8.854×10^{-12} Fm^{-1}

Density of air at S.T.P. = 1.293 kgm^{-3}

Universal gravitational constant = 6.67×10^{-11} Nm^2kg^{-2}

53. **Derived SI units with special names :**

Physical quantity	**SI unit**	**Symbol**
Frequency	hertz	Hz
Energy	joule	J
Force	newton	N
Power	watt	W
Pressure	pascal	Pa
Electric charge or quantity of electricity	coulomb	C
Electric potential difference and emf	volt	V
Electric resistance	ohm	Ω
Electric conductance	siemen	S
Electric capacitance	farad	F
Magnetic flux	weber	Wb
Inductance	henry	H
Magnetic flux density	tesla	T
Illumination	lux	Lx
Luminous flux	lumen	Lm

Dimensional formulae for some physical quantities :

Physical quantity	**Unit**	**Dimensional formula**
Acceleration or acceleration due to gravity	ms^{-2}	LT^{-2}
Angle (arc/radius)	rad	$M^oL^oT^o$
Angular displacement	rad	$M^ol^oT^o$
Angular frequency (angular displacement / time)	$rads^{-1}$	T^{-1}
Angular impulse (torque x time)	Nms	ML^2T^{-1}
Angular momentum ($I\omega$)	kgm^2s^{-1}	ML^2T^{-1}
Angular velocity (angle / time)	$rads^{-1}$	T^{-1}
Area (length x breadth)	m^2	L^2
Boltzmann's constant	JK^{-1}	$ML^2T^{-2}\theta^{-1}$
Bulk modulus ($\Delta P.\frac{V}{\Delta V}$)	Nm^{-2}, Pa	$M^1L^{-1}T^{-2}$
Calorific value	Jkg^{-1}	L^2T^{-2}

Coefficient of linear or areal or volume expansion	$^oC^{-1}$ or K^{-1}	θ^{-1}
Coefficient of surface tension (force/length)	Nm^{-1} or Jm^{-2}	MT^{-2}
Coefficient of thermal conductivity	$Wm^{-1}K^{-1}$	$MLT^{-3}\theta^{-1}$
Coefficient of viscosity ($F = \eta A \frac{dv}{dx}$)	poise	$ML^{-1}T^{-1}$
Compressibility (1/bulk modulus)	Pa^{-1}, m^2N^{-2}	$M^{-1}LT^2$
Density (mass / volume)	kgm^{-3}	ML^{-3}
Displacement, wavelength, focal length	m	L
Electric capacitance (charge / potential)	CV^{-1}, farad	$M^{-1}L^{-2}T^4I^2$
Electric conductance (1 / resistance)	Ohm^{-1} or mho or siemen	$M^{-1}L^{-2}T^3I^2$
Electric conductivity (1 / resistivity)	siemen/metre or Sm^{-1}	$M^{-1}L^{-3}T^3I^2$
Electric charge or quantity of electric charge (current x time)	coulomb	IT
Electric current	ampere	I
Electric dipole moment (charge x distance)	Cm	LTI
Electric field strength or Intensity of electric field (force / charge)	NC^{-1}, Vm^{-1}	$MLT^{-3}I^{-1}$
Electric resistance ($\frac{\text{potential difference}}{\text{current}}$)	ohm	$ML^2T^{-3}I^{-2}$
Emf (or) electric potential (work / charge)	volt	$ML^2T^{-3}I^{-1}$
Energy (capacity to do work)	joule	ML^2T^{-2}
Energy density ($\frac{\text{energy}}{\text{volume}}$)	Jm^{-3}	$ML^{-1}T^{-2}$
Entropy ($\Delta S = \Delta Q / T$)	$J\theta^{-1}$	$ML^2T^{-2}\theta^{-1}$
Force (mass x acceleration)	newton (N)	MLT^{-2}
Force constant or spring constant (force / extension)	Nm^{-1}	MT^{-2}
Frequency (1 / period)	Hz	T^{-1}
Gravitational potential (work / mass)	Jkg^{-1}	L^2T^{-2}
Heat (energy)	J or calorie	ML^2T^{-2}
Illumination (Illuminance)	lux (lumen/metre^2)	MT^{-3}
Impulse (force x time)	Ns or $kgms^{-1}$	MLT^{-1}
Inductance (L) (energy $= \frac{1}{2}LI^2$) or coefficient of self induction	henry (H)	$ML^2T^{-2}I^{-2}$

Intensity of gravitational field (F / m)	Nkg^{-1}	L^1T^{-2}
Intensity of magnetisation (I)	Am^{-1}	$L^{-1}I$
Joule's constant or mechanical equivalent of heat	$Jcal^{-1}$	$M^oL^oT^o$
Latent heat (Q = mL)	Jkg^{-1}	$M^oL^2T^{-2}$
Linear density (mass per unit length)	kgm^{-1}	ML^{-1}
Luminous flux	lumen or (Js^{-1})	ML^2T^{-3}
Magnetic dipole moment	Am^2	L^2I
Magnetic flux (magnetic induction x area)	weber (Wb)	$ML^2T^{-2}I^{-1}$
Magnetic induction (F = Bil)	$NI^{-1}m^{-1}$ or T	$MT^{-2}I^{-1}$
Magnetic pole strength (unit: ampere–metre)	Am	LI
Modulus of elasticity (stress / strain)	Nm^{-2}, Pa	$ML^{-1}T^{-2}$
Moment of inertia (mass x radius^2)	kgm^2	ML^2
Momentum (mass x velocity)	$kgms^{-1}$	MLT^{-1}
Permeability of free space ($\mu_o = \frac{4\pi Fd^2}{m_1m_2}$)	Hm^{-1} or NA^{-2}	$MLT^{-2}I^{-2}$
Permittivity of free space ($\varepsilon_o = \frac{Q_1Q_2}{4\pi Fd^2}$)	Fm^{-1} or $C^2N^{-1}m^{-2}$	$M^{-1}L^{-3}T^4I^2$
Planck's constant (energy / frequency)	Js	ML^2T^{-1}
Poisson's ratio (lateral strain / longitudinal strain)	—	$M^oL^oT^o$
Power (work / time)	Js^{-1} or watt (W)	ML^2T^{-3}
Pressure (force / area)	Nm^{-2} or Pa	$ML^{-1}T^{-2}$
Pressure coefficient or volume coefficient	$^oC^{-1}$ or θ^{-1}	θ^{-1}
Pressure head	m	M^oLT^o
Radioactivity	disintegrations per second	$M^oL^oT^{-1}$
Ratio of specific heats	—	$M^oL^oT^o$
Refractive index	—	$M^oL^oT^o$
Resistivity or specific resistance	Ω–m	$ML^3T^{-3}I^{-2}$
Specific conductance or conductivity (1 / specific resistance)	siemen/metre or Sm^{-1}	$M^{-1}L^{-3}T^3I^2$
Specific entropy (1/entropy)	KJ^{-1}	$M^{-1}L^{-2}T^2\theta$
Specific gravity (density of the substance / density of water)	—	$M^oL^oT^o$
Specific heat (Q = mst)	$Jkg^{-1}\theta^{-1}$	$M^oL^2T^{-2}\theta^{-1}$
Specific volume (1 / density)	m^3kg^{-1}	$M^{-1}L^3$
Speed (distance / time)	ms^{-1}	LT^{-1}

Stefan's constant $\left(\frac{\text{heat energy}}{\text{area x time x temperature}^4}\right)$	$Wm^{-2}\theta^{-4}$	$ML^oT^{-3}\theta^{-4}$
Strain (change in dimension / original dimension)	—	$M^oL^oT^o$
Stress (restoring force / area)	Nm^{-2} or Pa	$ML^{-1}T^{-2}$
Surface energy density (energy / area)	Jm^{-2}	MT^{-2}
Temperature	^oC or θ	$M^oL^oT^o\theta$
Temperature gradient ($\frac{\text{change in temperature}}{\text{distance}}$)	$^oCm^{-1}$ or θm^{-1}	$M^oL^{-1}T^o\theta$
Thermal capacity (mass x specific heat)	$J\theta^{-1}$	$ML^2T^{-2}\theta^{-1}$
Time period	second	T
Torque or moment of force (force x distance)	Nm	ML^2T^{-2}
Universal gas constant (work / temperature)	$Jmol^{-1}\theta^{-1}$	$ML^2T^{-2}\theta^{-1}$
Universal gravitational constant ($F = G.\frac{m_1m_2}{d^2}$)	Nm^2kg^{-2}	$M^{-1}L^3T^{-2}$
Velocity (displacement/time)	ms^{-1}	LT^{-1}
Velocity gradient ($\frac{dv}{dx}$)	s^{-1}	T^{-1}
Volume (length x breadth x height)	m^3	L^3
Water equivalent	kg	ML^oT^o
Work (force x displacement)	J	ML^2T^{-2}

54. Quantities having the same dimensional formulae :
   a) impulse and momentum.
   b) work, energy, torque, moment of force, energy
   c) angular momentum, Planck's constant, rotational impulse
   d) stress, pressure, modulus of elasticity, energy density.
   e) force constant, surface tension, surface energy.
   f) angular velocity, frequency, velocity gradient
   g) gravitational potential, latent heat.
   h) thermal capacity, entropy, universal gas constant and Boltzmann's constant.
   i) force, thrust.
   j) power, luminous flux.

ERRORS AND SIGNIFICANT FIGURES

ACCURACY, PRECISION OF INSTRUMENTS AND ERRORS IN MEASUREMENT :

1. The measured value of a physical quantity is usually different from its true value. The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty. This uncertainty is called **error**.
2. Every calculated quantity which is based on measured values also has an error. We distinguish between two terms **accuracy** and **precision**.
3. The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.
4. In general, the errors in measurement can be broadly classified as (a) systematic errors and (b) random errors.
5. **Systematic errors** :

   The **systematic errors** are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are :

   a) **Instrumental errors** that arise from the errors due to imperfect design or calibration of the measuring instrument, etc. For example, in a Vernier calipers the zero mark of vernier scale may not coincide to the zero mark of the main scale, or simply an ordinary metre scale may be worn off at one end.

   b) **Imperfection for experimental technique or procedure.** For example, to determine the temperature of a human body, a thermometer placed under the armpit will always give a temperature lower than the actual value of the body temperature.

   c) **Personal errors** that arise due to an individual's bias, lack of proper setting of the apparatus of individuals, carelessness in taking observations without observing proper precautions, etc. For example, if you, by habit, always hold your head a bit too far to the right while reading the position of a needle on the scale, you will introduce an error due to **parallax**.
6. **Random errors** :

   The **random errors** are those errors, which occur irregularly and hence are random with respect to sign and size. These can arise due to random and unpredictable fluctuations in experimental conditions (e.g. unpredictable fluctuations in temperature, voltage supply).
7. **Least count error :**

   a) The **least count error** is the error associated with the resolution of the instrument. For example, a vernier calipers has a least count as 0.001 cm. It occurs with both systematic and random errors. The smallest division on the scale of the measuring instrument is called its **least count**.

   b) Systematic errors can be minimized by improving experimental techniques, selecting better instruments and removing personal bias as far as possible.

   c) Random errors are minimized by repeating the observations several times and taking the arithmetic mean of all the observations. The mean value would be very close to the true value of the measured quantity.
8. **Absolute Error, Relative Error and Percentage Error** :

   a) Suppose the values obtained in several measurements are a_1, a_2, a_3 … a_n. The arithmetic mean of these values is taken as the best possible value of the quantity under the given conditions of measurement as :

   $$a_{mean} = (a_1 + a_2 + a_3 + ... + a_n)/n \qquad \text{(or)}$$

   $$a_{mean} = \sum_{i=1}^{n} a_i / n$$

- The magnitude of the difference between the true value of the quantity and the individual measurement value is called the absolute error of the measurement. This is denoted by $|\Delta t|$ (As we do not know the true value of a quantity, let us accept the arithmetic mean of all measurements as the true value of the measured quantity). Then the absolute errors in the individual measurement values are $\Delta a_1 = a_{mean} - a_1$; $\Delta a_2 = a_{mean} - a_2$; … ; $\Delta a_n = a_{mean} - a_n$.
- The arithmetic errors may be positive in certain cases and negative in some other cases.

b) The arithmetic mean of all the absolute errors is taken as the final or mean absolute error of the value of the physical quantity 'a'. It is represented by Δa_{mean}.

Thus,

$$\Delta a_{mean} = (|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \dots + |\Delta a_n|) / n$$

$$= \sum_{i=1}^{n} |\Delta a_i| / n$$

- If we do a single measurement, the value we get may be in the range $a_{mean} \pm \Delta a_{mean}$.

i.e. $a = a_{mean} \pm \Delta a_{mean}$. or

$a_{mean} - \Delta a_{mean} \le a \le a_{mean} + \Delta a_{mean}$.

- This implies that any measurement of the physical quantity 'a' is likely to lie between $(a_{mean} + \Delta a_{mean})$ and $(a_{mean} - \Delta a_{mean})$

c) Instead of the absolute error, we often use the **relative error** or the **percentage error (δa).** The relative error is the ratio of the mean absolute error Δa_{mean} to the mean value a_{mean} of the quantity measured.

Relative error = $\Delta a_{mean} / a_{mean}$.

- When the relative error is expressed in percent, it is called the **percentage error** (δa)
- Thus, Percentage error

$\delta a = (\Delta a_{mean} / a_{mean}) \times 100\%$

9. **Combination of Errors :** If we do an experiment involving several measurements, we must know how the errors in all the measurements combine.

a) **Error of a sum or a difference :** Suppose two physical quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute errors. We wish to find the error ΔZ in the sum

$$Z = A + B$$

We have by addition, $Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$.

The maximum possible error in $Z = \Delta Z = \Delta A + \Delta B$

For the difference $Z = A - B$, we have

$$Z \pm \Delta Z = (A \pm \Delta A) - (B \pm \Delta B)$$
$$= (A - B) \pm \Delta A \pm \Delta B.$$

or

$$\pm \Delta Z = \pm \Delta A \pm \Delta B$$

The maximum value of the error ΔZ is again $\Delta A + \Delta B$.

- When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

b) **Error of a product or a quotient :** Suppose $Z = AB$ and the measured values of A and B are $A \pm \Delta A$ and $B \pm \Delta B$. Then

$$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B).$$

$= AB \pm B\,\Delta A \pm A\,\Delta B \pm \Delta A\,\Delta B.$

Dividing LHS by Z and RHS by AB we have, $1 \pm (\Delta Z / Z) = 1 \pm (\Delta A / A) \pm (\Delta B / B) \pm (\Delta A / A)(\Delta B / b)$.

$Z = \Delta Z / Z = (\Delta A / A) + (\Delta B / B)$

- When two quantities are multiplied or divided, the fractional error in the result is the sum of the fractional errors in the multipliers.

c) **Error due to the power of a measured quantity :**

$Z = A^2$, then

$\Delta Z / Z = (\Delta A / A) + (\Delta A / A) = 2\,(\Delta A / A)$

If $Z = A^p\, B^q / C^r$, then

$\Delta Z / Z = p\,(\Delta A / A) + q\,(\Delta B / B) + r\,(\Delta C / C)$

- The fractional error in a physical quantity raised to the power is the power times the fractional error in the individual quantity.

SIGNIFICANT FIGURES :

10. Every measurement involves errors. Thus, the result of measurement should be reported in a way that indicates the precision of measurement.
11. Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus the first digit that is uncertain. The reliable digits plus the first uncertain digit are known as **significant digits** or **significant figures**.
12. If we say the period of oscillation of a simple pendulum is 1.62 s, the digits 1 and 6 are reliable and certain, while the digit 2 is uncertain. Thus, the measured value has three significant figures. The length of an object reported after measurement to be 287.5 cm has four significant figures, the digits 2, 8, 7 are certain while the digit 5 is uncertain.
13. Then a length of 16.2 cm means l = 16.20 ± 0.05 cm, i.e. it lies between 16.15 cm and 16.25 cm.
14. A choice of change of different units does not change the number of significant digits or figures in a measurement.

a) For example, the length 2.308 cm has four significant figures. But in different units, the same value can be written as 0.02308 m or 23.08 mm or 23080 μm.

The example gives the following rules :

i) All the non-zero digits are significant.

ii) All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.

iii) If the number is less than 1, the zeros on the right of decimal point but to the left of the first non-zero digit are not significant. (In $\underline{0}.\underline{00}2308$, the underlined zeros are not significant)

iv) The terminal or trailing zeros in a number without a decimal point are not significant.

v) (Thus 123 m = 12300 cm = 123000 mm has three significant figures, the trailing zeroes being not significant). However, you can also see the next observation.

vi) The trailing zeros in a number with a decimal point are significant. (The numbers 3.500 or 0.06900 have four significant figures each).

b) There can be some confusion regarding the trailing zeros. Suppose a length is reported to be 4.700 m. It is evident that the zeros here are meant to convey the precision of measurement and are, therefore, significant. (If these were not, it would be superfluous to write them explicitly, the reported measurement would have been simply 4.7 m).

4.700 m = 470.0 cm = 4700 mm = 0.004700 km. Since the last number has trailing zeroes in a number with no decimal, we would conclude erroneously from observation (1) above that the number has two significant figures, while infact it has four significant figures and a mere change of units cannot change the number of significant figures.

c) To remove such ambiguities in determining the number of significant figures, the best way is to report every measurement m scientific notation (in the power of 10). In this notation, every number is expressed as $a \times 10^b$, where 'a' is a number between 1 and 10, and b is any positive or negative exponent of 10.

$4.700 \text{ m} = 4.700 \times 10^2 \text{ cm}$

$\Rightarrow 4.700 \times 10^3 \text{ mm} = 4.700 \times 10^{-3} \text{ km}$

Thus, in the scientific notation, no confusion arises about the trailing zeros in the base number 'a'. They are always significant.

d) The scientific notation is ideal for reporting measurement. But if this is not adopted, we use the rules adopted in the preceding example :
   i) For a number greater than 1, without any decimal, the trailing zeros are not significant.
   ii) For a number with a decimal, the trailing zeros are significant.

15. **Rules for Arithmetic Operations with Significant Figures :**

a) The result of a calculation involving approximate measured values of quantities (i.e. values with limited number of significant figures) must reflect the uncertainties in the original measured values.

b) It cannot be more accurate than the original measured values themselves on which the result is based. In general, the final result should not have more significant figures than the original data from which it was obtained. Thus, if mass of an object is measured to be, say, 4.237 g (four significant figures) and its volume is measured to be 2.51 cm^3, then its density, by mere arithmetic division, is $1.68804780876 \text{ g/cm}^3$ upto 11 decimal places.

c) It would be clearly absurd and irrelevant to record the calculated value of density to such a precision when the measurements on which the value is based, have much less precision. The following rules for arithmetic operations with significant figures ensure that the final result of a calculation is shown with the precision that is consistent with the precision of the input measured values :
   i) In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

   Density = $\dfrac{4.237\text{g}}{2.51\,\text{cm}^3} = 1.69\text{g cm}^{-3}$

   Similarly, if the speed of light is given as $3.00 \times 10^8 \text{ ms}^{-1}$ (three significant figures).
   ii) In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.
   iii) For example, the sum of the numbers 436.32 g, 227.2 g and 0.301 g by mere arithmetic addition, is 663.821 g.
   iv) But the least precise measurement (227.2 g) is correct to only one decimal place. The final result should, therefore, be rounded off to 663.8 g.
   v) Similarly, the difference in length can be expressed as :
   $0.307 \text{ m} - 0.304 \text{ m} = 0.003 \text{ m} = 3 \times 10^{-3} \text{ m}.$
   vi) They do not convey the precision of measurement properly. For addition and subtraction, the rule is in terms of decimal places.

16. **Rounding off the Uncertain Digits :**

a) The result of computation with approximate numbers, which contain more than one uncertain digit, should be rounded off.
b) Preceding digit is raised by 1 if the insignificant digit to be dropped (the underlined digit in this case) is more than 5, and is left unchanged if the latter is less than 5.
c) But what if the number is 2.745 in which the insignificant digit is 5. Here the convention is that if the preceding digit is even, the insignificant digit is simply dropped and, if it is odd, the preceding digit is raised by 1.

17. Rules for Determining the Uncertainty of Number in Arithmetic Operations :

a) The uncertainty or error in the measured value, as already mentioned, is normally taken to be half of the least count of the measuring instrument. The rules for determining the uncertainty of number in arithmetic operations can be understood from the following examples.

i) If the length and breadth of a thin rectangular sheet are measured as 16.2 cm and 10.1 cm respectively, there are three significant figures in each measurement. It means that the true length l may be written as

$$l = 16.20 \pm 0.05 \text{ cm} = 16.20 \text{ cm} \pm 0.3\%$$

Similarly, the breadth b may be written as

$$b = 10.10 \pm 0.05 \text{ cm} = 10.10 \text{ cm} \pm 0.5\%$$

To determine the uncertainty of the product of two (or more) experimental values, we often following a rule that is founded upon probability. If we assume that uncertainties combine randomly, we have the rule :

When two or more experimentally obtained numbers are multiplied, the percentage uncertainty of the final result is equal to the square root of the sum of the squares of the percentage uncertainties of the original numbers.

Following the square root of the sum of the squares rule, we may write for the product of length l and breadth b as

$$l\,b = 163.62 \text{ cm}^2 \pm \sqrt{(0.3\%)^2 + (0.5\%)^2}$$

$$= 163.62 \text{ cm}^2 \pm 0.6\%$$

$$= 163.62 \pm 1.0 \text{ cm}^2$$

The result leads us to quote the final result as

$$l\,b = 163.62 \pm 1.0 \text{ cm}^2$$

b) If a set of experimental data is specified to n significant figures, a result obtained by combining the data will also be valid to n significant figures.

i) However, if data are subtracted, the number of significant figures can be reduced.
ii) For example : 12.9 g – 7.06 g, both specified to three significant figures, cannot properly be evaluated as 5.84 g but only as 5.8 g, as uncertainties in subtraction or addition combine in a different fashion (smallest number of decimal places rather than the number of significant figures in any of the number added or subtracted).

c) The fractional error of a value of number specified to significant figures depends not only on n but also on the number itself.

i) For example, the accuracy in measurement of mass 1.02 g is ± 0.005 g whereas another measurement 9.89 g is also accurate to ± 0.005g.
ii) The fractional error in 1.02 g is
$= (\pm 0.005 / 1.02) \times 100\% \Rightarrow \pm 0.5\%$
iii) On the other hand, the fractional error in 9.89 g is $= (\pm 0.005 / 9.89) \times 100\% \Rightarrow \pm 0.05\%$
iv) Finally, remember that **intermediate results in a multi-step computation should be calculated to one more significant figure in every measurement than the number of digits in the least precise measurement.**

v) Theses should be justified by the data and then the arithmetic operations may be carried out; otherwise rounding errors can build up. For example, the reciprocal of 9.58, calculated (after rounding off) to the same number of significant figures (three) is 0.104, but the reciprocal of 0.104 calculated to three significant figures is 9.62. However, if we had written 1/9.58 = 0.1044 and then taken the reciprocal to three significant figures, we would have retrieved the original value of 9.58.

iv) This example justifies the idea to retain one more extra digit (than the number of digits in the least precise measurement) in intermediate steps of the complex multi-step calculations in order to avoid additional errors in the process of rounding off the numbers.

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