FLUID MECHANICS

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- The liquids and gases together are termed as fluids, in other words, we can say that the substances which can flow are termed as fluids.
- We assume fluid to be incompressible (i.e., the density of liquid is independent of variation in pressure and remains constant) and non-viscous (i.e. the two liquid surfaces in contact are not exerting any tangential force on each other).

1.1 Fluid Statics

1.1.1 Fluid Pressure

Pressure p at every point is defined as the normal force per unit area.

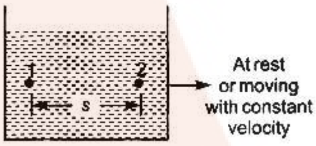
$$p = \frac{dF_{\perp}}{dA}$$

The SI unit of pressure is the Pascal and 1 Pascal = 1 N/m^2

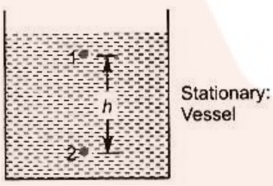
- Fluid force acts perpendicular to any surface in the fluid, no matter how that surface is oriented. Hence pressure, has no intrinsic direction of its own, it is a scalar.

Pressure

(a) Pressure at two points in a horizontal plane or at same level when the fluid is at rest or moving with constant velocity is same.

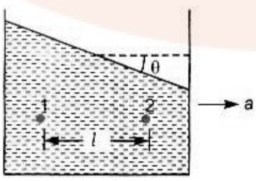


(b) Pressure at two points which are at a depth separation of h when fluid is at rest of moving with constant velocity is related by the expression



$p_2 - p_1 = \rho gh$, where ρ is the density of liquid.

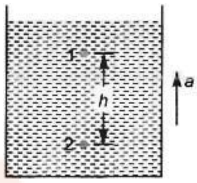
(c) Pressure at two points in a horizontal plane when fluid container is having some constant horizontal acceleration are related by the expression



$p_1 - p_2 = l\rho a$

and tan θ = a/g, where θ is the angle which the liquid's free surface is making with horizontal.

(d) Pressure at two points within a liquid at vertical separation of *h* when the liquid container is accelerating up are related by expression



$p_2 - p_1 = \rho\ (g + a)\ h$

If container is accelerating down, then $p_2 - p_1 = \rho(g - a)\ h$.

1.1.2 Atmospheric Pressure

- It is the pressure of the earth's atmosphere. Normal atmospheric pressure at sea level (an average value) is 1 atmosphere (atm) that is equal to 1.013×10^5 Pa.
- The excess pressure above atmospheric pressure is called gauge pressure, and total pressure is called absolute pressure.
- Barometer is a device used to measure atmospheric pressure while U–tube manometer or simply manometer is a device used to measure the gauge pressure.

1.1.3 Pascal's Law

- A change in the pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.
- There are a lot of practical applications of Pascal's law one such application is hydraulic lift.

1.1.4 Archimedes Principle

- When a body is partially or fully dipped into a fluid, the fluid exerts contact force on the body. The resulatant of all these contact forces is called buoyant force (upthrust).
- F = weight of fluid displaced by the body.
- This force is called buoyant force and acts vertically upwards (opposite to the weight of the body) through the centre of gravity of the displaced fluid.

$F = V\sigma g$

where, v = volume of liquid displaced

σ = density of liquid.

- Apparent decrease in weight of body = upthrust = weight of liquid displaced by the body.
- Floation :

(a) A body floats in a liquid if the average density of the body is less than that of the liquid.

(b) The weight of the liquid displaced by the immersed part of body must be equal to the weight of the body.

(c) The centre of gravity of the body and centre of buoyancy must be along the same vertical line.

1.2 Fluid Dynamics

- **Steady Flow (Stream Line Flow)**

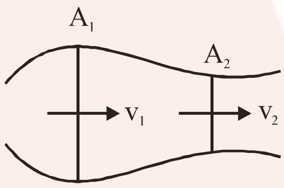
  The flow in which the velocity of fluid particles crossing a particular point is the same at all the times. Thus, each particle takes the same path as taken by a previous particle through that point.

- **Line of flow**

  It is the path taken by a particle in flowing liquid. In case of a steady flow, it is called streamline. Two steamlines can never intersect.

1.2.1 Equation of Continuity

In a time Δt, the volume of liquid entering the tube of flow in a steady flow is $A_1 V_1 \Delta t$. The same volume must flow out as the liquid is incompressible. The volume flowing out in Δt is $A_2 V_2 \Delta t$.



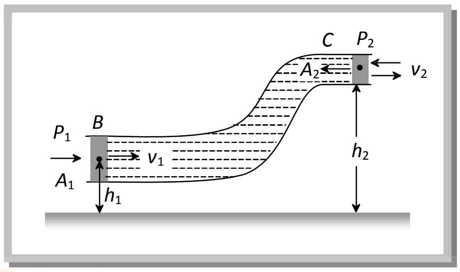
- $A_1 V_1 = A_2 V_2$
- mass flows rate = ρAV

  (where ρ is the density of the liquid.)

1.2.2 Bernoulli's Theorem

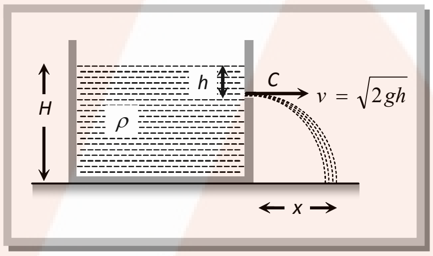
In a stream line flow of an ideal fluid, the sum of pressure energy per unit volume, potential energy per unit volume and kinetic energy per unit volume is always constant at all cross section of the liquid.

$$P+\rho gh+\frac{\rho V^2}{2}=\text{Constant}$$



- Bernoulli's equation is valid only for incompressible steady flow of a fluid with no viscosity.
- Application of Bernoulli's Theorem.

(a) Velocity of Efflux



Let us find the velocity with which liquid comes out of a hole at a depth h below the liquid surface.

Using Bernoulli's theorem,

$$P_A + \frac{1}{2}\rho V_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho V_B^2 + \rho g h_B$$

$$\Rightarrow \quad P_{atm} + \frac{1}{2}\rho V_A^2 + \rho g h = P_{atm} + \frac{1}{2}\rho V^2 + 0$$

(Note: $P_B = P_{atm}$, because we have opened the liquid to atmosphere)

$$\Rightarrow \quad V^2 = V_A{}^2 + 2gh$$

Using equation of continuity

$AV_A = aV$

A: area of cross-section of vessel

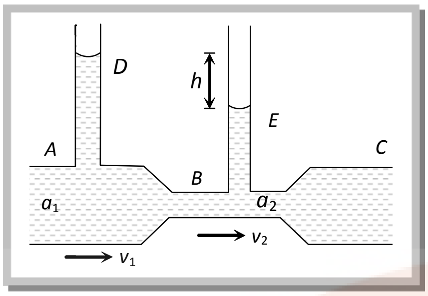
a: area of hole

$$\Rightarrow \quad V^2 = \frac{a^2}{A^2}V^2 + 2gh$$

$$\Rightarrow \quad V = \frac{\sqrt{2gh}}{\sqrt{1-a^2/A^2}} \approx \sqrt{2gh} \qquad \text{(if the hole is very small)}$$

(b) Venturi Meter

This is an instrument for measuring the rate of flow of fluids.



If P_A is pressure at A and P_B is pressure at B,

$P_A - P_B = h\rho g$ [h : difference of heights of liquids of density ρ in vertical tubes]

If V_1 is velocity at A and V_2 is velocity at B

$Q = A_1V_1 = A_2{}_2$ [equation of continuity]

$$P_A + \rho\frac{V_1^2}{2} = P_B + \rho\frac{V_2^2}{2} \quad \text{[Bernoulli's Theorem]}$$

$$\Rightarrow \quad V_2^2 - V_1^2 = \frac{2}{\rho}(P_A - P_B) = \frac{2}{\rho}h\rho g$$

$$\Rightarrow \quad \frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = 2hg \quad (Q = AV)$$

$$\Rightarrow \quad Q = A_1A_2\sqrt{\frac{2hg}{A_1^2 - A_2^2}}$$

1.3 Viscosity

The property of a fluid by virtue of which it opposes the relative motion between its different layers is known as viscosity and the force that is into play is called the viscous force.

Viscous force is given by :

$$F = -\eta A\frac{dv}{dx}$$

where η is a constant depending upon the nature of the liquid and is called the coefficient of viscosity and velocity gradient = dv/dx

S.I. unit of coefficient of viscosity is Pa.s or Nsm^{-2}.

CGS unit of viscocity is poise. (1 Pa.s = 10 Poise)

1.3.1 Stoke's Law

♦ When a solid moves through a viscous medium, its motion is opposed by a viscous force depending on the velocity and shape and size of the body.

♦ The viscous drag on a spherical body of radius r, moving with velocity v, in a viscous medium of viscosity η is given by

$F_{viscous} = 6\pi\eta rv$

This relation is called Stoke's law.

♦ **Importance of Stoke's law**

(a) This law is used in the determination of electronic charge with the help of milikan's experiment.

(b) This law accounts the formation of clouds.

(c) This law accounts why the speed of rain drops is less then that of a body falling freely with a constant velocity from the height of clouds.

(d) This law helps a man coming down with the help of a parachute.

1.3.2 Terminal Velocity

It is maximum constant velocity acquired by the body while falling freely in a viscous medium.

$$v_r = \frac{2r^2(\rho - \rho_0)g}{9\eta}$$

1.3.3 Poiseuille's Formula

Poiseuille studied the stream-line flow of liquid in capillary tubes.

Volume of liquid coming out of tube per second in given by

$$= \frac{\pi Pr^4}{8\eta\ell}$$

1.3.4 Reynold Number

♦ The stability of laminar flow is maintained by viscous forces. It is obverved, however that laminar or steady flow is disrupted it the rate of flow is large. Irregular, unsteady motion, turbulence, sets in at high flow rates.

♦ Reyonlds defined a dimensionless number whose value gives one an approximate idea, whether the flow rate would be turbulent.

This number, called the Reynolds number R_e is defined as,

$$R_e = \frac{\rho vD}{\eta}$$

where, ρ = the density of the fluid flowing with a speed v.

D = the diameter of the tube.

η = the coefficient of viscosity of the fluid.

♦ It is found that flow is streamline or laminar for R_e less than 1000. The flow is turbulent for $R_e > 2000$. The flow becomes unsteady for R_e between 1000 and 2000.

1.4 Surface Tension

The surface tension of a liquid is defined as the force per unit length in the plane of the liquid surface at right angles to either side of an imaginary line drawn on that surface.

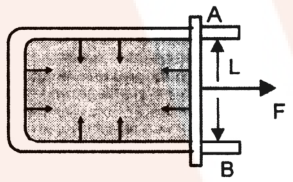
So, $S = \frac{F}{\ell}$ where S = surface tension of liquid.

Unit of surface tension in MKS system : N/m, J/m^2

CGS system : Dyne/cm, erg/cm^2

1.4.1 Surface Energy

In order to increase the surface area, the work has to be done over the surface of the liquid. This work done is stored in the liquid surface as its potential energy. Hence the surface energy of a liquid can be defined as the excess potential energy per unit area of the liquid surface.



W = SΔA, where ΔA = increase in surface area.

1.4.2 Excess Pressure

- Excess pressure in a liquid drop or bubble in a liquid is

$$P = \frac{2T}{R}$$

- Excess pressure in a soap bubble is $P = \frac{4T}{R}$

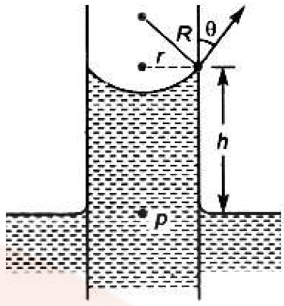
(because it has two free surfaces)

1.4.3 Angle of Contact

- The angle between the tangent to the liquid surface at the point of contact and the solid surface inside the liquid is called the angle of contact.
- If the glass plate is immersed in mercury, the surface is curved and the mercury is depressed below. Angle of contact is obtuse for mercury.
- If the plate is dipped in water with its side vertical, the water is drawn-up along the plane and assumes the curved shape as shown. Angle of contact is acute for water.

1.4.4 Capillary Tube and Capillarity Action

- A very narrow glass tube with fine bore and open at both ends is known as capillary tube. When a capillary tube in dipped in a liquid, then liquid will rise or fall in the tube, this action is termed as capillarity.



$$h = \frac{2S\cos\theta}{r\rho g} = \frac{2S}{R\rho g}$$

where, S = surface tension,

θ = angle of contact,

r = radius of capillary tube,

R = radius of meniscus, and

ρ = density of liquid.

- **Capillary rise in a tube of insufficient length :**

If the actual height to which a liquid will rise in a capillary tube is 'h' then a capillary tube of length less than 'h' can be called a tube of "insufficient length".

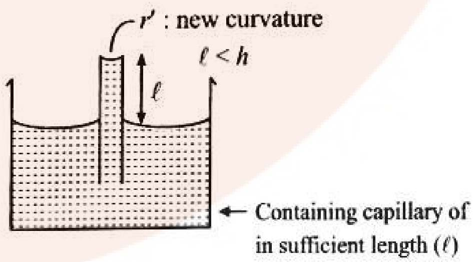
In such a case, liquid rises to the top of the capillary tube of length l ($l < h$) and adjusts the radius of curvature of its meniscus until the excess pressure is equalised by the pressure of liquid column of length l. (Note liquid does not overflow).

$\Rightarrow \quad \frac{2\sigma}{r'} = \ell \rho g \qquad$ ... (i)

If r were the actual radius of curvature,

$\Rightarrow \quad \frac{2\sigma}{r} = h \rho g \qquad$ ...(ii)

Comparing (i) and (ii)



$$\frac{2\sigma}{\rho g} = \ell r' = hr$$

$\Rightarrow \quad r' = \frac{hr}{\ell}$ i.e. radius of curvature r′ can be calculated.

Adhesion > Cohesion	Adhesion = Cohesion	Adhesion < Cohesion
1. Liquid will wet the solid.	Critical.	Liquid will not wet the solid.
2. Meniscus is concave.	Meniscus is plane.	Meniscus is convex.
3. Angle of contact is acute ($\theta = 90°$).	Angle of contact is 90°.	Angle of contact is obtuse ($\theta = 90°$).
4. Pressure below the meniscus is lesser than above it by (2T/r), i.e. $P = P_0 - \frac{2T}{r}$.	Pressure below the meniscus is same as above it, i.e. $P = P_{0.}$	Pressure below the meniscus more then above it by (2T/r), i.e., $P = P_0 + \frac{2T}{r}$.
5. In capillary there will be ascend.	No capillarity.	In capillary there will be descend.