# NCERT SOLUTIONS CLASS-XI PHYSICS <br> CHAPTER-8 <br> GRAVITATION 

Q.1: (a). By putting an electric charge inside a hollow conductor you can shield it from the influences of electrical forces. Can a body be shielded from the gravitational influences of nearby matter in a similar fashion?
(b). An astronaut in a shuttle orbiting Mars cannot feel gravity. If he was traveling in a very large space ship would he have detected gravity?
(c). The Sun's gravitational force on Earth is greater than the moon's. However, the Moon affects the tidal waves much more than the Sun. Why?

Sol: (a). No, as of now, no method has been devised to shield a body from gravity because gravity is independent of medium and it is the virtue of each and every matter. So the shield would exert the gravitational forces
(b). Yes, if the spaceship is large enough then the astronaut will definitely detect the Mars gravity
(c). Gravitational force is inversely proportional to the square of the distance whereas, Tidal effects are inversely proportional to the cube of the distance. So as the distance between the earth and moon is smaller than the distance between earth and sun, the moon will have a greater influence on the earth's tidal waves

Q2. Pick the right choice:
(a). Acceleration due to gravity does not depend upon the mass of the-[body/earth]

$$
\begin{aligned}
r_{2} & r_{2}
\end{aligned}
$$

(b). The formula, $\mathrm{mg}\left(\mathrm{r}_{2}-\mathrm{r}_{1}\right)$ is [more/less] accurate than the formula $\frac{G M n}{}$ for the difference of potential energy between two points at a distance of $r_{1}$ and $r_{2}$ from the earth's center. (c) Acceleration due to gravity [decreases/increases] with increasing depth. (Consider earth has a uniform density).
(d) Acceleration due to gravity [decreases/increases] with decreasing altitude Sol:
(a). body.
(b). more.
(c). decreases.
(d). increases.
Q.3: Imagine a planet that revolves around the sun at a double speed of the earth. Find the size of its orbit as compared to the orbit of the earth.

Sol:
Time taken by the earth for one complete revolution, $\mathrm{T}_{\mathrm{E}}=1$ Year
Radius of Earth's orbit, $\mathrm{R}_{\mathrm{E}}=1 \mathrm{AU}$
Thus, the time taken by the planet to complete one complete revolution:
$T_{P}=\frac{1}{2} T_{E}=\frac{1}{2}$ year
Let, the orbital radius of this planet $=R_{P}$
Now, according to the Kepler's third law of planetary motion:
$\left(\frac{R_{P}}{R_{E}}\right)^{3}=\left(\frac{T_{P}}{T_{E}}\right)^{2}$
$\frac{R_{P}}{R_{E}}=\left(\frac{T_{P}}{T_{E}}\right)^{\frac{2}{3}}$
$\frac{R_{P}}{R_{E}}=\left(\frac{T_{P}}{T_{E}}\right)^{\frac{2}{3}}=\left(\frac{\frac{1}{2} T_{E}}{T_{E}}\right)^{\frac{2}{3}}=\left(\frac{1}{2}\right)^{\frac{2}{3}}=0.63$
Therefore, radius of orbit of this planet is 0.63 times smaller than the radius of orbit of the Earth.

Given,
Orbital period of lo, $\mathrm{T}_{10}=1.769$ days $=1.769 \times 24 \times 60 \times 60 \mathrm{~s}$
Orbital radius of lo, $\mathrm{R}_{10}=4.22 \times 10^{8} \mathrm{~m}$
We know mass of Jupiter:
$\mathrm{M}_{\mathrm{J}}=4 \pi^{2} \mathrm{R}_{10}{ }^{3} / \mathrm{GT}_{10}{ }^{2}$
Where;
$M_{J}=$ Mass of Jupiter
$G=$ Universal gravitational constant
Also,
Orbital period of the earth,
$\mathrm{T}_{\mathrm{E}}=365.25$ days $=365.25 \times 24 \times 60 \times 60 \mathrm{~s}$
Orbital radius of the Earth, $R_{E}=1 \mathrm{AU}=1.496 \times 10^{11} \mathrm{~m}$
We know that the mass of sun is:
$\mathrm{M}_{\mathrm{S}}=\frac{4 \pi^{2} R_{E}^{3}}{G T_{E}^{2}}$. $\qquad$
Therefore, $\frac{M_{S}}{M_{J}}=\frac{4 \pi^{2} R_{E}^{3}}{G T_{E}^{2}} \times \frac{G T_{10}^{2}}{4 \pi^{2} R_{10}^{3}}=\frac{R_{E}^{3}}{T_{E}^{2}} \times \frac{T_{10}^{2}}{R_{10}^{3}}$
Now, on substituting the values, we will get:
$=\left[\frac{1.769 \times 24 \times 60 \times 60}{365.25 \times 24 \times 60 \times 60}\right]^{2} \times\left[\frac{1.496 \times 10^{11}}{4.22 \times 10^{8}}\right]^{3}=1045.04$
Therefore, $\frac{M_{S}}{M_{J}} \sim 1000$
$M_{S} \sim 1000 \times M_{J}$

## [Which proves that, the Sun's mass is 1000 times that of Jupiter's)

Q.5: Consider that the Sombrero galaxy has $3 \times 10^{11}$ stars, each of one solar mass. If the diameter of the galaxy is $10^{5}$ light years, calculate the time of a star, at a distance of 5000 light years from the galactic center, takes to complete one revolution.

## Sol:

Mass of Sombrero Galaxy, M $=3 \times 10^{11}$ solar mass
Solar mass $=$ Mass of Sun $=2.0 \times 10^{36} \mathbf{~ k g}$
Mass of the galaxy, $M=3 \times 10^{11} \times 2 \times 10^{36}=6 \times 10^{41} \mathrm{~kg}$
Diameter of Sombrero Galaxy, $d=5 \times 10^{4} \mathrm{ly}$
Radius of Sombrero Galaxy, $\mathrm{r}=2.5 \times 10^{4} \mathrm{ly}$
We know that:
1 light year $=9.46 \times 10^{15} \mathrm{~m}$
Therefore, $\mathrm{r}=2.5 \times 10^{4} \times 9.46 \times 10^{15}=2.365 \times 10^{20} \mathrm{~m}$
As this star revolves around the massive black hole in center of the Sombrero galaxy, its time period can be found with the relation:
$\mathrm{T}=\left[\frac{4 \pi^{2} r^{3}}{G M}\right]^{\frac{1}{2}}$
$=\left[\frac{4 \times 3.14^{2} \times\left(2.365 \times 10^{20}\right)^{3}}{\left(6.67 \times 10^{-11}\right) \times\left(6 \times 10^{41}\right)}\right]^{\frac{1}{2}}=4.246 \times 10^{15} \mathrm{~s}$
Now we know, 1 year $=365 \times 24 \times 60 \times 60$ s
$1 \mathrm{~s}=\frac{1}{365 \times 24 \times 60 \times 60}$ years
Therefore, $4.246 \times 10^{15} \mathrm{~S}=\frac{4.246 \times 10^{15}}{365 \times 24 \times 60 \times 60}=1.34 \times 10^{8}$ years.

## Q.6: Pick the right alternative:

(a). The energy spent on sending a rocket vertically upwards out of the earth's gravitational influence is more I less than the energy spent on sending an orbiting satellite to the same height out of the earth's influence.
(b). If potential energy is zero at infinity, the total energy of an orbiting comet is negative of its potential / kinetic energy

Sol:
(a). more
(b). kinetic.

## Q7. Choose the factors upon, which the escape velocity of an object from earth depends upon:

(a) body's mass.
(b) location of projection.
(c) direction of projection.

Sol: (b)
Explanation:
Escape velocity is independent of the direction of projection and the mass of the body. It depends upon the gravitational potential at the place from where the body is projected. Gravitational potential depends slightly on the altitude and the latitude of the place, thus escape velocity depends slightly upon the location from where it is projected.
Q.8: An asteroid orbits a star in an elliptical orbit. Does the asteroid have a constant (a) angular speed, (b) kinetic energy, (c) total energy (d), linear speed, (e) potential energy and (f) angular momentum throughout its revolution?

## Sol:

An asteroid orbiting a star will have constant angular momentum and the constant value of total energy throughout its orbit.

Q9. Which of the following problems is likely to affect an astronaut in space? (a) Swollen feet, (b) bone loss, (c) orientational problem, (d) headache.

Sol:
(a). In zero gravity the blood flow to the feet isn't increased so the astronaut does not get swollen feet.
(b). Due to zero gravity, the weight the bones have to bear is greatly reduced, this causes bone loss in astronauts spending greater amounts of time in space.
(c). Space has different orientations, so orientational problems can affect an astronaut.
(d). Due to increased blood supply to their faces, astronauts can be affected by headaches.

## Q.10: Pick up the right answers from the given ones:

The direction of gravitational intensity inside a hemispherical sphere of a uniform mass is indicated by the arrow:
(i) c


Sol: (i) c

## Reason:

Inside a hollow sphere, gravitational forces on any particle at any point is symmetrically placed. However, in this case, the upper half of the sphere is removed. Since gravitational intensity is gravitational force per unit mass it will act in direction point downwards along ' $c$ '.

## Q.11: For the above problem the gravitational intensity at $P$ will be directed towards:

(i) h
(ii) f
(iii) m
(iv) v

Sol: (iii) m
Reason: Making use of the logic/explanation from the above answer we can conclude that the gravitational intensity at $P$ is directed downwards along $m$.

Q12. A rocket is launched off from the earth's surface for a massive comet nearing the earth. At what distance from the earth's center will the shuttle of the rocket be free from the earth's gravitational force. Mass of the comet $=7.4 \times 10^{22}$, mass of Earth $=6 \times 10^{24}$ and distance between earth and the comet $=3.84$ $\times 10^{10} \mathrm{~m}$.

Sol:

Given:
Mass of the comet, $\mathrm{M}_{\mathrm{C}}=7.4 \times 10^{24} \mathrm{~kg}$
Mass of the Earth, $M_{E}=6 \times 10^{24} \mathrm{~kg}$
Orbital radius, $\mathrm{r}=3.84 \times 10^{10} \mathrm{~m}$

## Mass of the shuttle $=\mathrm{mkg}$

Let ' $x$ ' be the distance from the center of the Earth where the gravitational force acting on the Shuttle ' $\mathbf{S}$ ' becomes zero.

According to Newton's law of gravitation, we have:
$\Rightarrow \frac{G m M_{A}}{(r-x)^{2}}=\frac{G m M_{E}}{x^{2}}$
$\Rightarrow \frac{M_{A}}{M_{E}}=\left(\frac{r-x}{x}\right)^{2}$
$\Rightarrow \frac{r}{x}-1=\left(\frac{7.4 \times 10^{24}}{6 \times 10^{24}}\right)^{\frac{1}{2}}$
$\Rightarrow \frac{r}{x}-1=1.1106$
$\Rightarrow x=\frac{r}{2.1106}=\frac{3.84 \times 10^{10}}{2.1106}=1.819 \times 10^{10} \mathrm{~m}$
Q.13: If the earth's orbit around the sun is $1.5 \times 10^{8} \mathrm{~km}$ long, estimate the mass of the Sun.

Sol:
Given:
Earth's orbit, $\mathrm{r}=1.5 \times 10^{11} \mathrm{~m}$
Time taken by the Earth for one complete revolution,
$\mathrm{T}=1$ year $=365.25$ days
i.e. $T=(365.25 \times 24 \times 60 \times 60)$ seconds

Since, Universal gravitational constant, G $=6.67 \times 10^{-11} \mathrm{Nm}^{2} \mathrm{~kg}^{-2}$
Therefore, mass of the Sun, $\mathbf{M}=\frac{4 \pi^{2} r^{3}}{G T^{2}}$
$\Rightarrow M=\frac{4 \times(3.14)^{2} \times\left(1.5 \times 10^{11}\right)^{3}}{\left(6.67 \times 10^{-11}\right) \times(365.25 \times 24 \times 60 \times 60)^{2}}$
$\Rightarrow M=\frac{4 \times(3.14)^{2} \times\left(1.5 \times 10^{11}\right)^{3}}{\left(6.67 \times 10^{-11}\right) \times(365.25 \times 24 \times 60 \times 60)^{2}}$
$\Rightarrow M=\frac{1.331 \times 10^{35}}{66.425 \times 10^{3}}=2.004 \times 10^{30} \mathrm{~kg}$
Therefore, the estimated mass of the Sun is $2.004 \times 10^{30} \mathrm{Kg}$

Q14. Uranus is 84 times the earth year. If the Earth is $15 \times 10^{7} \mathrm{~km}$ away from the Sun, what is the distance between the Sun and Uranus?

Sol:
Given:
Distance between Earth and the Sun, $\mathrm{r}_{\mathrm{e}}=15 \times 10^{7} \mathrm{~km}=1.5 \times 10^{11} \mathrm{~m}$
Time period of the Earth $=T_{e}$
Time period of Uranus, $\mathrm{T}_{\mathrm{u}}=84 \mathrm{~T}_{\mathrm{e}}$
Let, the distance between the Sun and the Uranus be $\mathrm{r}_{\mathrm{u}}$
Now, according to the Kepler's third law of planetary motion:
$\mathrm{T}=\left(\frac{4 \pi^{2} r^{3}}{G M}\right)^{\frac{1}{2}}$
For Uranus and Sun, we can write:
$\Rightarrow \frac{r_{u}^{3}}{r_{e}^{3}}=\frac{T_{u}^{2}}{T_{e}^{2}}$
$\Rightarrow r_{u}=r_{e}\left[\frac{T_{u}}{T_{e}}\right]^{\frac{2}{3}}$
$\Rightarrow r_{u}=1.5 \times 10^{11}\left[\frac{84 T_{e}}{T_{e}}\right]^{\frac{2}{3}}=1.5 \times \mathbf{1 0}^{11} \times(84)^{\frac{2}{3}}$
$\Rightarrow r_{u}=28.77 \times 10^{12} \mathrm{~m}$
Therefore, Uranus is $28.77 \times 10^{12} \mathrm{~m}$ away from the Sun.
Q.15: A man weighing 60 N on the earth's surface is taken to the height that is equal to half of the earth's radius. What is the amount of gravitational force which the earth will exert on this man now?

Sol:
Given:
Weight of the man, $\mathrm{W}=60 \mathrm{~N}$
We know that acceleration due to gravity at height ' $h$ ' from the Earth's surface is:
$\mathbf{g}^{\mathbf{\prime}}=\frac{g}{\left[1+\left(\frac{h}{R_{e}}\right)\right]^{2}}$
Where, $\mathbf{g}=$ Acceleration due to gravity on the Earth's surface
And, $\mathbf{R}_{\mathrm{e}}=$ Radius of the Earth
For $\mathrm{h}=\frac{R_{\mathrm{e}}}{2}$
$\mathrm{g}^{\prime}=\frac{g}{\left[1+\left(\frac{R_{e}}{2 R_{e}}\right)\right]^{2}}$
$\Rightarrow \boldsymbol{g}^{\mathbf{\prime}}=\frac{g}{\left[1+\left(\frac{1}{2}\right)\right]^{2}}=\frac{4}{9} g$
Also, the weight of a body of mass ' $m$ ' $\mathbf{k g}$ at a height of ' $h$ ' meters can be represented as
$W^{\prime}=m g$
$=\mathrm{m} \times \frac{4}{9} \mathrm{~g}=\frac{4}{9} \mathrm{mg}$
$=\frac{4}{9} \mathrm{~W}$
$=\frac{4}{9} \times 60=26.66 \mathrm{~N}$.
Q.16: Considering the earth to be a sphere of uniform mass density. Find the weight of a body at onethird of the way down to the center of the earth if it weighed 200 N on the surface?

Sol:
Given,
The weight of a body having mass ' $m$ ' at the surface of earth, $W=m g=200 \mathrm{~N}$
Body of mass ' $m$ ' is located at depth, $d=\frac{1}{3} R_{e}$
Where, $\mathbf{R}_{\mathrm{e}}=$ Radius of the Earth
Now, acceleration due to gravity $g^{\prime}$ at depth (d) is given by the relation:
$\mathbf{g}^{\prime}=1-\left(\frac{d}{R_{e}}\right) g$
$\Rightarrow g^{\prime}=1-\left(\frac{R_{e}}{3 R_{e}}\right) g=\frac{2}{3} g$
Now, weight of the body at depth d,
$W^{\prime}=m g^{\prime}$
$W^{\prime}=m \times \frac{2}{3} g=\frac{2}{3} m g$
$W^{\prime}=\frac{2}{3} W$
$\Rightarrow W=\frac{2}{3} \times 200=133.33 \mathrm{~N}$
Therefore, the weight of a body at one-third of the way down to the center of the earth $=133.33 \mathrm{~N}$
Q.17: A missile is fired vertically upwards from the surface at $4 \mathrm{~km} \mathrm{~m} / \mathrm{s}$. What is the greatest height the missile can attain before falling back to the earth? Also find the final distance of the missile from the center of the earth. [Mass of earth $=6 \times 10^{24} \mathrm{~kg}$, mean radius of earth $=6.4 \times 10^{6} \mathrm{~m}$ and take $\mathrm{G}=6.67 \times 10^{-}$ ${ }^{11} \mathrm{~N} \mathrm{~m}^{2} \mathrm{~kg}^{-2}$ ]

Sol:
Given:
Velocity of the missile, $v=4 \mathrm{~km} / \mathrm{s}=4 \times 10^{3} \mathrm{~m} / \mathrm{s}$
Mass of the Earth, $M_{E}=6 \times 10^{24} \mathrm{~kg}$
Radius of the Earth, $R_{E}=6.4 \times 10^{6} \mathrm{~m}$
Let, the height reached by the missile be ' $h$ ' and the mass of the missile be ' $m$ '.
Now, at the surface of the Earth:
Total energy of the rocket at the surface of the Earth = Kinetic energy + Potential energy
$\mathrm{T}_{\mathrm{E} 1}=\frac{1}{2} m v^{2}+\frac{-G M_{E} m}{R_{E}}$
Now, at highest point ' $h$ ':
Kinetic Energy $=0 \quad$ [Since, $v=0$ ]
And, Potential energy $=\frac{-G M_{E} m}{R_{E}+h}$
Therefore, total energy of the missile at highest point ' $h$ ':
$\mathrm{T}_{\mathrm{E} 2}=0+\frac{-G M_{E} m}{R_{E}+h}$
$\Rightarrow \mathrm{T}_{\mathrm{E} 2}=\frac{-G M_{E} m}{R_{E}+h}$
According to the law of conservation of energy, we have :
Total energy of the rocket at the Earth's surface $T_{E 1}=$ Total energy at height ' $h$ ' $T_{E 2}$ :
$\Rightarrow \frac{1}{2} m v^{2}+\frac{-G M_{E} m}{R_{E}}=\frac{-G M_{E} m}{R_{E}+h}$
$\Rightarrow \frac{1}{2} v^{2}+\frac{-G M_{E}}{R_{E}}=\frac{-G M_{E}}{R_{E}+h}$
$\Rightarrow v^{2}=2 G M_{E} \times\left[\frac{1}{R_{E}}-\frac{1}{\left(R_{E}+h\right)}\right]=2 G M_{E}\left[\frac{h}{\left(R_{E}+h\right) R_{E}}\right]$
$\Rightarrow v^{2}=\frac{g R_{E} h}{R_{E}+h}$
Where, $g=\frac{G M}{R_{E}^{2}}=9.8 \mathrm{~ms}^{-2}$
Therefnre $v^{2}\left(R_{r}+h\right)=2 a R_{r} h$
$\Rightarrow v^{2} R_{E}=h\left(2 g R_{E}-v^{2}\right)$
$\Rightarrow h=\frac{R_{E} v^{2}}{2 g R_{E}-v^{2}}=\frac{\left(6.4 \times 10^{6}\right) \times\left(4 \times 10^{3}\right)^{2}}{2 \times 9.8 \times 6.4 \times 10^{6}-\left(4 \times 10^{3}\right)^{2}}$
$\Rightarrow \mathrm{h}=9.356 \times 10^{5} \mathrm{~m}$
Therefore, the missile reaches at height of $9.36 \times 10^{5} \mathrm{~m}$ from the surface.
Now, the distance from the center of the earth $=\mathbf{h}+\mathbf{R}_{\mathrm{E}}=9.36 \times 10^{5}+6.4 \times 10^{6}=\mathbf{7 . 3 3} \times 10^{6} \mathbf{m}$
Therefore, the greatest height the missile can attain before falling back to the earth is $9.356 \times 10^{5} \mathrm{~m}$ and the final distance of the missile from the center of the earth is $7.33 \times 10^{6} \mathrm{~m}$
Q.18: On the surface of the earth escape velocity is $11.2 \mathrm{~km} / \mathrm{s}$. If a body (rocket) is projected upward with twice this speed, find the speed of this rocket at a distance far away from the surface of the earth. Neglect the presence of other heavenly bodies like the sun and other planets.

Sol:
Given,
Escape velocity of the Earth's surface, $\mathbf{v}_{\text {esc }}=11.2 \mathrm{~km} / \mathrm{s}$
Projection velocity of the rocket, $\mathrm{v}_{\mathrm{P}}=2 \mathrm{v}_{\text {esp }}$
Let, mass of the body $=\mathbf{m} \mathbf{~ k g}$
And, the velocity of the rocket at a distance very far away from the surface of earth $=\mathbf{v}_{\mathrm{F}}$
Total energy of the rocket on the surface $=\frac{1}{2} m v_{p}{ }^{2}-\frac{1}{2} m v^{2}{ }_{\text {esp }}$
Total energy of the rocket at a distance very far away from the Earth $=\frac{1}{2} \mathrm{mv}_{\mathrm{F}}{ }^{2}$
Now, according to the law of conservation of energy, we have:
$\frac{1}{2} m v_{P}{ }^{2}-\frac{1}{2} m v_{E S C}{ }^{2}=\frac{1}{2} m v_{F}{ }^{2}$
$m v_{E S C}{ }^{2}=\frac{1}{2} m v_{F}^{2}$
$\mathrm{v}_{\mathrm{F}}=\sqrt{\left(v_{p}\right)^{2}-\left(v_{e s c}\right)^{2}} \quad\left[\right.$ Since, $\mathrm{v}_{\mathrm{P}}=2 \mathrm{v}_{\text {esp }}$ ]
$\mathrm{v}_{\mathrm{F}}=\sqrt{\left(2 v_{\text {esp }}\right)^{2}-\left(v_{e s c}\right)^{2}}$
$\mathrm{v}_{\mathrm{F}}=\sqrt{3} \mathrm{v}_{\text {esc }}$
$\Rightarrow \mathrm{v}_{\mathrm{F}}=\sqrt{3} \times 11.2=19.39 \mathrm{~km} / \mathrm{s}$
Therefore, the speed of rocket at a distance far away from the surface of earth is $19.39 \mathrm{~km} / \mathrm{s}$.
Q.19: International Space Station (ISS) orbits the earth at a height of 380 km and has a mass of 420000 kg . Find the amount of energy required to take it out from the gravitational influence of earth. [Mass of earth $=6.0 \times 10^{24} \mathrm{~kg}$, radius of earth $=6.4 \times 10^{6} \mathrm{~m}$ and take $\mathrm{G}=6.67 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} \mathrm{~kg}^{-2}$ ]

Sol:
Given:
Mass of the satellite, $\mathrm{m}=420000 \mathrm{~kg}$
Radius of the Earth, $\mathrm{R}_{\mathrm{E}}=6.4 \times 10^{6} \mathrm{~m}$
Mass of the Earth, $\mathrm{M}_{\mathrm{E}}=6.0 \times 10^{24} \mathrm{~kg}$
Universal gravitational constant, G $=6.67 \times 10^{-11} \mathrm{Nm}^{2} \mathrm{~kg}^{-2}$
Height of the satellite, $\mathrm{h}=380 \mathrm{~km}=0.38 \times 10^{6} \mathrm{~m}$
We know that the Total energy of the satellite at height ' h ' $=\frac{1}{2} m v^{2}+\left[\frac{-G M_{E} m}{R_{E}+h}\right]$
Also, orbital velocity of the satellite, $\mathbf{v}=\left[\frac{G M_{E}}{R_{E}+h}\right]^{\frac{1}{2}}$
Total energy at height, $\mathrm{h}=\frac{1}{2}\left[\frac{G M_{E}}{R_{E}+h}\right]-\left[\frac{G M_{E} m}{R_{E}+h}\right]$
Therefore, $\mathrm{T}_{\mathrm{E}}=\frac{-1}{2}\left[\frac{G M_{E} m}{R_{E}+h}\right]$
This negative sign means that the ISS is bounded to Earth. This is the bound energy of the satellite. Now, Total Energy required to send the satellite out of its orbit $=\boldsymbol{-}($ Bound energy $)$
$\mathrm{T}_{\mathrm{c}}=\frac{1}{n}\left\lceil\underline{G M_{E} m}\right\rceil$
$-\quad 2\left\lfloor K_{E}+h\right\rfloor$
$\mathrm{T}_{\mathrm{E}}=\frac{1}{2}\left[\frac{\left(6.67 \times 10^{-11}\right) \times\left(6 \times 10^{24}\right) \times\left(4.2 \times 10^{5}\right)}{\left(6.4 \times 10^{6}\right)+\left(0.380 \times 10^{6}\right)}\right]=\frac{1}{2} \times\left[\frac{1.681 \times 10^{20}}{6.78 \times 10^{6}}\right]$
$\Rightarrow \mathrm{T}_{\mathrm{E}}=1.2396 \times 10^{13} \mathrm{~J}$
Therefore, the amount of energy required to take ISS out from the gravitational influence of earth is $1.2396 \times 10^{13} \mathrm{~J}$
Q.20: Two planets each of mass $2 \times 10^{31} \mathrm{~kg}$ are on a path of collision with each other. If they have negligible speeds at a distance of $10^{10} \mathrm{~km}$. Find the speed at which they collide if each planet has a radius of $10^{3} \mathrm{~km}$. [Take $\mathrm{G}=6.67 \times 10^{-11}$ ]
Sol:
Given
Radius of each planet, $R=10^{3} \mathrm{~km}=10^{6} \mathrm{~m}$
Distance between the planet, $\mathbf{r}=10^{10} \mathrm{~km}=10^{13} \mathrm{~m}$
Mass of each planet, $\mathbf{M}=2 \times 10^{31} \mathrm{~kg}$
For negligible speeds, $\mathbf{v}=\mathbf{0}$
So the total energy of two planets separated by a distance ' $\mathbf{r}$ ':
$\mathrm{T}_{\mathrm{E}}=\frac{-G M M}{r}+\frac{1}{2} m v^{2}$
Since, $\mathrm{v}=0$;
Therefore, $\mathrm{T}_{\mathrm{E}}=\frac{-G M M}{r}$ $\qquad$
Now, when the planets are just about to collide:
Let, the velocity of the planets $=\mathbf{v}$
The centers of the two planets are at a distance of $=\mathbf{2 R}$
Total kinetic energy of the two planets $=\frac{1}{2} \mathrm{Mv}^{2}+\frac{1}{2} \mathrm{Mv}^{2}=\mathrm{Mv}^{2}$
Total potential energy of the two planets $=\frac{-G M M}{2 R}$
Therefore, Total energy of the two stars $=\mathbf{M v}^{2}-\frac{G M M}{2 R}$ $\qquad$
Now, according to the law of conservation of energy
$M v^{2}-\frac{G M M}{2 R}=\frac{-G M M}{r}$
$\mathrm{v}^{2}=\frac{-G M}{r}+\frac{G M}{2 R}$
$\mathrm{v}^{2}=\mathrm{GM} \times\left[\frac{-1}{r}+\frac{1}{2 R}\right]$
$\Rightarrow v^{2}=6.67 \times 10^{-11} \times 2 \times 10^{31} \times\left[\frac{-1}{\left(10^{13}\right)}+\frac{1}{\left(2 \times 10^{6}\right)}\right]$
$\Rightarrow v^{2}=6.67 \times 10^{14}$
Therefore, $v=\left(6.67 \times 10^{14}\right)^{1 / 2}=2.583 \times 10^{7} \mathrm{~m} / \mathrm{s}$.
Hence, the speed at which they will collide $=2.583 \times 10^{7} \mathrm{~m} / \mathrm{s}$.
Q.21: Two spheres each having a radius of 0.15 m and mass 1000 kg are placed 1.0 m apart from each other on a horizontal plane. Calculate the gravitational potential and force at the midpoint of the line connecting the centers of the two spheres. If an object is kept at that point will it be in equilibrium? If yes, is this equilibrium position is unstable or stable?

Sol:
Given:
Radius of spheres, $\mathrm{R}=0.15 \mathrm{~m}$
Distance between two spheres, $r=1.0 \mathrm{~m}$
Mass of each sphere, $\mathrm{M}=1000 \mathrm{~kg}$


From the above figure, ' $A$ ' is the mid-point and since each sphere will exert the gravitational force in opposite direction. Therefore, the gravitational force at this point will be zero.

Gravitational potential at the midpoint $(\mathrm{A})$ is;
$\mathrm{U}=\left[\frac{-G M}{\frac{\tau}{2}}+\frac{-G M}{\frac{\tau}{2}}\right]$
$\mathrm{U}=\left[\frac{-4 G M}{r}\right]$
$\mathrm{U}=\left[\frac{-4 \times\left(6.67 \times 10^{-11}\right) \times(1000)}{1.0}\right]$
$\Rightarrow \mathrm{U}=-2.668 \times 10^{-7} \mathrm{~J} / \mathrm{kg}$
Therefore, the gravitational potential and force at the mid-point of the line connecting the centers of the two spheres is $=-2.668 \times 10^{-7} \mathrm{~J} / \mathrm{kg}$

The net force on an object, placed at the mid-point is zero. However, if the object is displaced even a little towards any of the two bodies it will not return to its equilibrium position. Thus, the body is in unstable equilibrium.
Q.22: Find the gravitational potential due to Earth's gravity on a geo-stationary satellite orbiting earth at 36000 km above the surface. [Mass of earth $=6 \times 10^{24} \mathrm{~kg}$, radius $=6400 \mathrm{~km}$ and Assume that the potential energy is zero at infinity]

Sol:
Given:
Radius of the Earth, $\mathbf{R}=6400 \mathrm{~km}=0.64 \times 10^{7} \mathrm{~m}$
Mass of Earth, M = $6 \times 10^{24} \mathrm{~kg}$
Height of the geo-stationary satellite from earth's surface, $\mathrm{h}=36000 \mathrm{~km}=3.6 \times 10^{7} \mathrm{~m}$
Therefore, gravitational potential at height ' $h$ ' on the geo-stationary satellite due to the earth's gravity
$\mathrm{G}_{\mathrm{P}}=\frac{-G M}{R+h}$
$\Rightarrow G_{P}=\frac{-\left(6.67 \times 10^{-11}\right) \times\left(6 \times 10^{24}\right)}{\left(0.64 \times 10^{7}\right)+\left(3.6 \times 10^{7}\right)}$
$\Rightarrow G_{P}=\frac{-40.02 \times 10^{13}}{4.24 \times 10^{7}}=-9.439 \times 10^{6} \mathrm{~J} / \mathrm{Kg}$
Therefore, the gravitational potential due to Earth's gravity on a geo-stationary satellite orbiting earth is $-9.439 \times 10^{6} \mathrm{~J} / \mathrm{Kg}$
Q.23: A star 10 times the size of the sun collapses on itself and gets reduced to a black hole of radius 10 km , rotating at a speed of 10 revolutions per second. Prove that an object placed on its equator will remain stuck on the equator due to its gravity. [Mass of the sun $=2 \times 10^{3} \mathrm{~kg}$ ]

Sol.
Any matter/ object will remain stuck to the surface if the outward centrifugal force is lesser than the inward gravitational pull.

Gravitational force, $\mathrm{f}_{\mathrm{G}}=\frac{G M m}{R^{2}}$ [Neglecting negative sign]
Here,
$\mathbf{M}=$ Mass of the star $=10 \times 2 \times 10^{30}=2 \times 10^{31} \mathrm{~kg}$
$\mathrm{m}=$ Mass of the object
$\mathrm{R}=$ Radius of the star $=10 \mathrm{~km}=1 \times 10^{4} \mathrm{~m}$
Therefore, $\mathrm{f}_{\mathrm{G}}=\frac{\left(6.67 \times 10^{-11}\right) \times\left(2 \times 10^{31}\right) \mathrm{m}}{\left(1 \times 10^{4}\right)^{2}}=1.334 \times 10^{13} \mathrm{~m} \mathrm{~N}$
Now, Centrifugal force, $\mathrm{f}_{\mathrm{C}}=\mathrm{mr} \omega^{2}$
Here, $\omega=$ Angular speed $=2 \pi v$
$v=$ Angular frequency $=10 \mathrm{rev} \mathrm{s}^{-1}$
$\mathrm{f}_{\mathrm{c}}=\mathrm{mR}(2 \pi \mathrm{v})^{2}$
$f_{c}=m \times\left(10^{4}\right) \times 4 \times(3.14)^{2} \times(10)^{2}=\left(3.9 \times 10^{7} \mathrm{~m}\right) \mathrm{N}$

Q.24: Find the amount of energy required to launch a spaceship stationed on Mars to out of the solar system.
(Mass of the Sun $=2 \times 10^{30} \mathrm{~kg}$; mass of the space ship $=2000 \mathrm{~kg}$, mass of mars $=6.4 \times 10^{23} \mathrm{~kg}$; radius of mars $=3395 \mathrm{~km}$; radius of the orbit of mars $=2.28 \times 10^{8} \mathrm{~km} ; \mathrm{G}=6.67 \times 10^{-11} \mathrm{~m}^{2} \mathrm{~kg}^{-2}$ )

Sol:
Given,
Mass of the Sun, $M=2 \times 10^{30} \mathrm{~kg}$
Mass of the spaceship, $\mathrm{m}_{\mathrm{S}}=2000 \mathrm{~kg}$
Radius of Mars, $r=3395 \mathrm{~km}=3.395 \times 10^{6} \mathrm{~m}$
Mass of Mars, $M_{m}=6.4 \times 10^{23} \mathrm{~kg}$
Orbital radius of Mars, $R=2.28 \times 10^{8} \mathrm{~km}=2.28 \times 10^{11} \mathrm{~m}$
Universal gravitational constant, G $=6.67 \times 10^{-11} \mathrm{~m}^{2} \mathrm{~kg}^{-2}$
Now,
Potential energy of the spaceship due to the gravity of Sun $=\frac{-G M m_{s}}{R}$
Potential energy of the spaceship due to gravity of Mars $=\frac{-G M_{M} m_{s}}{r}$
As the spaceship is stationed on Mars, its velocity is 'zero' and thus, its kinetic energy is also 'zero'.
Thus, Total energy of the spaceship $=\frac{-G M_{M} m_{s}}{r}-\frac{G M m_{s}}{R}$
$\mathrm{T}_{\mathrm{E}}=-G m_{s}\left[\frac{M}{R}+\frac{M_{M}}{r}\right]$
The negative sign means that the system is in bound state.
Therefore, energy required to launch the spaceship out of the solar system:
$\mathrm{T}_{\mathrm{E}}=-$ (bound energy)
$\mathrm{T}_{\mathrm{E}}=G m_{s}\left[\frac{M}{R}+\frac{M_{M}}{r}\right]$
Now, on substituting the values of $\mathbf{G}, \mathbf{M}, \mathbf{R}, \mathrm{m}_{\mathrm{s}}, \mathrm{M}_{\mathrm{M}}$ and r we will get:
$\mathrm{T}_{\mathrm{E}}=\left(6.67 \times 10^{-11}\right) \times\left(2 \times 10^{3}\right) \times\left[\frac{2 \times 10^{30}}{2.28 \times 10^{11}}+\frac{6.4 \times 10^{23}}{3.395 \times 10^{6}}\right]$
$\Rightarrow 13.34 \times 10^{-8} \times\left[87.7 \times 10^{17}+1.885 \times 10^{17}\right]=\mathbf{1 . 1 9 5} \times \mathbf{1 0}^{\mathbf{1 2}} J$
Therefore, the total amount of energy required to launch a spaceship stationed on Mars to out of the solar system $=1.195 \times 10^{12} \mathrm{~J}$
Q.25: A missile is shot vertically upward from the surface of Venus; if $25 \%$ of its initial energy is lost in overcoming Venus's atmospheric resistance, find the maximum height the missile will achieve before returning back to the surface. Take Initial velocity of the missile $=3 \mathrm{~km} \mathrm{~s}^{-1}$, Mass of Venus $=4.8 \times 10^{24}$ kg and the radius of Venus $=6052 \mathrm{~km}$. [G $\left.=6.67 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} \mathrm{~kg}^{-2}\right]$

## Sol.

Given:
Mass of Venus, $M=4.8 \times 10^{24} \mathrm{~kg}$
Initial velocity of the missile, $v=3 \mathrm{~km} / \mathrm{s}=3 \times 10^{3} \mathrm{~m} / \mathrm{s}$
Radius of Venus, $R=6052 \mathrm{~km}=6.05 \times 10^{6} \mathrm{~m}$
Universal gravitational constant, G $=6.67 \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} \mathrm{~kg}^{-2}$
Let the mass of the missile $=\mathbf{m} \mathbf{~ k g}$
Since, the Initial kinetic energy of the missile $=\frac{1}{2} m v^{2}$
and the Initial potential energy of the missile $=\frac{-G M m}{R}$
Thus, total initial energy $=\frac{1}{2} m v^{2}+\left(\frac{-G M m}{R}\right)$

Hence, the total available initial energy $=\left[\frac{75}{100} \times \frac{1}{2} m v^{2}\right]-\frac{G M m}{R}=0.375 m v^{2}-\frac{G M m}{R}$
Let ' $h$ ' be the maximum height attained by the missile.
Now, at height ' $h$ ' the final velocity $=0$ and hence, the kinetic energy $=0$
Therefore, the total energy of the missile at height ' h ' $=\frac{-G M m}{R+h}$
Now, according to the law of conservation of energy:
$\Rightarrow 0.375 m v^{2}-\frac{G M}{R}=\frac{-G M m}{R+h}$
$\Rightarrow \quad 0.375 v^{2}=G M m\left(\frac{1}{R}-\frac{1}{R+h}\right)$
$\Rightarrow 0.375 v^{2}=G M \times\left[\frac{h}{R(R+h)}\right]$
$\Rightarrow \frac{R+h}{h}=\frac{G M}{0.375 v^{2} R}$
$\Rightarrow \quad \frac{R}{h}=\frac{G M}{0.375 v^{2} R}-1$
$\Rightarrow \quad \frac{R}{h}=\frac{G M-0.375 v^{2} R}{0.375 v^{2} R}$
$\Rightarrow \quad \frac{1}{h}=\frac{G M-0.375 v^{2} R}{0.375 v^{2} R^{2}}$
$\Rightarrow h=\frac{0.375 v^{2} R^{2}}{G M-0.375 v^{2} R}$
Now, on substituting the values of $\mathbf{G}, \mathrm{M}, \mathrm{v}$ and R we will get:
$\Rightarrow h=\frac{0.375 \times\left(3 \times 10^{3}\right)^{2} \times\left(6.05 \times 10^{6}\right)^{2}}{\left[\left(6.67 \times 10^{-11}\right) \times\left(4.8 \times 10^{24}\right)\right]-\left[0.375 \times\left(3 \times 10^{3}\right)^{2} \times 6.05 \times 10^{6}\right]}$
$\Rightarrow h=\frac{0.375 \times\left(3 \times 10^{3}\right)^{2} \times\left(6.05 \times 10^{6}\right)^{2}}{\left[\left(6.67 \times 10^{-11}\right) \times\left(4.8 \times 10^{24}\right)\right]-\left[0.375 \times\left(3 \times 10^{3}\right)^{2} \times 6.05 \times 10^{6}\right]}$
$\Rightarrow \frac{1.235 \times 10^{20}}{3.2016 \times 10^{14}-2.042 \times 10^{13}}=412023.75 \mathrm{~m}=4.12 \mathrm{~km}$
Therefore, the maximum height that is achieved by the missile before returning back to the surface $=$ 4.12 km .

