2.3 Work, Energy and Power

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C Kari Eloranta 2015 2.3 Work, Energy and Power

2.3 Work as Energy Transfer

Work as Energy Transfer

Work W is a quantity that gives the amount of energy transferred between a system and its surroundings by mechanical means.

- The SI unit of work is the unit of energy, 1 J (joule).
- When forces do work on an object, they tend to accelerate the object.
- When the work done by a force on an object is positive (W > 0), the force transfers energy to the object.
- When the work done by a force on an object is negative (W < 0), the force transfers energy away from the object.
- For example, the gravitational force accelerates a falling apple. The work done by the gravitational force on the apple is positive. The air resistance opposes the motion. The work done by the air resistance on the apple is negative.

2.3 Definition of Work

Definition of Work Work is the product of a force on an object, and the displacement of the object in the direction of the force.

Work

Work W done by a constant force \overrightarrow{F} acting on an object is

 $W = Fs\cos\theta$

(1)

where $s\cos\theta$ is the displacement of the object in the direction of the force.

• In Equation 1 the symbol θ (greek theta) is the angle between the displacement vector \vec{s} and force \vec{F} .

2.3 Kinetic Energy $E_{\rm K}$

Definition of Kinetic Energy

The kinetic energy of an object is the energy the object has due to its motion in a reference frame with respect to the speed of the object is measured.

Kinetic Energy $E_{\rm K}$ If an object moves by the speed of v, its kinetic energy is

$$E_{\rm K} = \frac{1}{2}mv^2$$

where m is the mass of the object.

- The unit of kinetic energy is $[E_K] = [m][v]^2 = kgm^2s^{-2} = 1J$ (joule).
- Kinetic energy is a scalar quantity.
- In the equation the mass is measured in kilograms and speed in ms^{-1} .

2.3 Work-Energy Theorem

• When a non-zero net force acts on an object which can be regarded as a point particle, the kinetic energy of the object increases. As a result, the net force force does work on the object.

Work-Energy Theorem (NOT IN DATABOOKLET)

The work done W by the net force on the object is equal to the change in the kinetic energy of the object

$$W = \Delta E_{\rm K} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \tag{3}$$

where v is the final speed of the object, u the initial speed of the object, and m the mass of the object.

Example

Your physics book falls from rest down to the floor from a 82 cm high table.

- a) Calculate the work done by the gravitational force on the book.
- b) State the change in the kinetic energy of the object.
- c) Where does the energy come from?

The mass of the book is 1450 g. You may ignore the effect of air resistance.

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The mass of the book is 1450 g. You may ignore the effect of air resistance.

a) $m = 1450 \text{ g} = 1.450 \text{ kg}, h = 82 \text{ cm} = 0.82 \text{ m}, g = 9.81 \text{ ms}^{-2}$

Because the weight of the book is G = mg, and the book moves in the direction of the weight, the angle between the displacement and gravitational force is zero. The work done by the gravitational force on the book is thus

$$W = Fs\cos 0^\circ = Gh \times 1$$

$$= mgh = 1.450 \,\mathrm{kg} \times 9.81 \,\mathrm{ms}^{-2} \times 0.82 \,\mathrm{m} \approx 12 \,\mathrm{J}$$

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b) 12 J. (By the work energy theorem the work done by the net force equals the change in kinetic energy. Neglecting the air resistance, the net force equals the gravitational force. Because the book starts from rest, the change in kinetic energy equals the final kinetic energy.)

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The mass of the book is 1450 g. You may ignore the effect of air resistance.

c) The energy comes from the gravitational field. When the book falls and gains kinetic energy, the work done by the gravitational force equals the amount by which the energy of the gravitational field decreases.

Topic 10 (AHL): Field of Force

- Mass creates a gravitational field around it, and electric charge an electric field.
- Fields carry energy in them. When a field changes, the energy of the field changes.
- Distant interaction can be understood in terms of fields. The types of distant interaction at macroscopic level are gravitational interaction, electric interaction and magnetic interaction.
- The interacting objects form a system, in which the distant forces contribute to the total potential energy of the system.

2.3 Introduction to Potential Energy

Definition of Potential Energy

Potential energy is energy stored in a system as a result of internal forces that depend on the position of interacting objects in the system.

- The types of potential energy include gravitational potential energy, elastic potential energy, electrical potential energy, nuclear energy, and chemical energy.
- Each type of potential energy relates to a different type of force acting in a system.

2.3 Gravitational Potential Energy

- The Earth creates a gravitational field around it. When an object is placed in Earth's gravitational field, the object and the Earth form a system that is bound together by gravity.
- Gravitational potential energy is potential energy associated with the gravitational force acting inside a system.

Definition of Gravitational Potential Energy (NOT IBO)

Gravitational potential energy is energy an object has due to its position in gravitational field, and the gravitational force acting on the object.

- The gravitational potential energy is a property of the gravitational field formed by the object and the Earth.
- Instead of considering the actual value of gravitational potential energy, we consider the changes in it.

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2.3 Change in Gravitational Potential Energy

• When an object moves in a gravitational field, the work done by the field on the object depends only on the initial and final position of the object.

Change in Gravitational Potential Energy When the vertical position of an object placed in Earth's gravitational field changes by Δh , the change in the gravitational potential energy of the object is

$$\Delta E_{\rm P} = mg\Delta h,$$

where m is the mass of the object, and $g = 9.81 \,\mathrm{m\,s^{-2}}$ is the acceleration due gravity.

- The gravitational potential energy is a property of the gravitational field formed by the object and the Earth.
- Instead of considering the actual value of gravitational potential energy, we consider the changes in it.

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2.3 Conservation of Energy

Definition of an Isolated System

An isolated system is a system that cannot exchange energy or matter with its surroundings.

• The law of conservation of energy states that the energy of an isolated system remains constant over time:

Law of Conservation of Energy

The total energy of an isolated system is conserved.

Energy Transformations

In any process, energy is neither created or destroyed. It merely transforms from one from into another.

• For example, in the absence of resistive forces, the gravitational potential energy of the falling apple changes into kinetic energy. Or, chemical energy of the fuel is released as thermal energy in a composition engine. Or, thermal energy of hot water is transformed into rotational kinetic energy of the turbine.

Definition of Mechanical Energy

The mechanical energy of a system is the sum of the gravitational potential and kinetic energy in the system $(E = E_{\rm K} + E_{\rm P})$.

• When an object moves in a gravitational field, the work done by the gravitational force on the object depends only on the initial and final position of the object. It does not depend on the path the object travelled.

Conservative Force

A force is conservative, if the work done by the force on the object, as the object moves from point A to point B, depends only on the points A and B, not on the path along which the object travelled from A to B.

• Gravitational and electric force are examples of conservative forces, whereas kinetic friction and air resistance are examples of nonconservative forces.

Conservation Law of Mechanical Energy When only conservative forces are acting in a system, the mechanical energy of the system is conserved (does not change).

- When an object is in the gravitational field of the Earth, the Earth does not move with respect to the object. As a result, the kinetic energy of the system formed by the object and Earth equals the kinetic energy of the object in the reference frame attached to the Earth.
- Likewise, when the position of the object changes in the Earth's gravitational field, we may say say that the gravitational potential energy of the object changes, instead of saying that the potential energy of the system changes.

2.3 Zero Level of Gravitational Potential Energy

• Only the changes in gravitational potential energy are physically relevant. However, if we choose a reference level of potential energy, and attach a value of $E_{\rm P} = 0 \,\text{J}$ to that level, we may speak of the potential energy of an object at vertical distance h from the reference level.

As an example, consider a ball on the table. If the height of the table is denoted by h, the ball on the table has gravitational potential energy $E_p = mgh$.



Center of Mass

• The motion of an object under the influence of a net force can be described in terms of the centre of mass of the object.

Definition of Centre of Mass

The centre of mass of an object is the point where the mass of the object can be thought of residing such that the net force on the object acts on that point.

• For a homogeneous and symmetric object the centre of mass lies at the centre of the object.

Center of Mass Motion

The photograph illustrates the path of a bouncing basketball.



Even though the ball is rotating its centre of mass follows a parabolic path. The shape of the path can be understood in terms of net force acting on the centre of mass.

Center of Mass and Change in Gravitational Potential Energy

When an object falls a distance h, the vertical position of its center of mass changes by h as well. That explains why the magnitude of the change in gravitational potential energy is given by $E_p = mgh$.



When a ball is at rest on a table, its mechanical energy with respect to the floor is

$$E = E_{\rm p} + E_{\rm k} = mgh + \frac{1}{2}mu^2 = mgh, \tag{5}$$

because the initial speed is u = 0, and the kinetic energy is thus $E_k = \frac{1}{2}mu^2 = \frac{1}{2}m \times (0 \text{ m s}^{-1})^2 = 0 \text{ J}$. The ball has only potential energy and no kinetic energy.



Because the ball is relatively heavy and the distance fallen is small, we may neglect the effect of air resistance. As a result, the only force acting on the falling ball is the gravitational force.

By the work energy theorem, the work done by the gravity on the ball equals the change in the kinetic energy of the ball.

$$E_{\rm p} = mgh, E_{\rm k} = 0 \, {\rm J}$$

$$\Delta E_{\rm p} = -\Delta E_{\rm k}$$
height h

As the ball reaches the ground, the potential energy has transformed entirely into kinetic energy. The mechanical energy of the ball is just kinetic energy:

$$E = E_{\rm p} + E_{\rm k} = 0 \,\mathrm{J} + \frac{1}{2} m v^2 = \frac{1}{2} m v^2. \tag{6}$$

$$E = E_{p} + E_{k} = mgh + 0J = mgh$$
$$E = E_{p} + E_{k} = 0J + \frac{1}{2}mv^{2} = \frac{1}{2}mv^{2}$$

Now we are finally ready to introduce the law of conservation of mechanical energy in symbolic form.

At point 1, the object has gravitational potential energy $E_{\rm P,1} = mgh_1$, and kinetic energy $E_{\rm K,1} = \frac{1}{2}mu^2$. At point 2, the potential energy is $E_{\rm P,2} = mgh_2$, and kinetic energy $E_{\rm K,2} = \frac{1}{2}mv^2$.

Conservation Law of Mechanical Energy

When an object moves from point 1 to point 2 in Earth's gravitational field, in the absence of resistive forces the mechanical energy of the object is conserved so that

$$E_{\rm P,1} + E_{\rm K,1} = E_{\rm P,2} + E_{\rm K,2} \tag{7}$$

That is,

$$mgh_1 + \frac{1}{2}mu^2 = mgh_2 + \frac{1}{2}mv^2.$$
 (8)

Energy Transformation in Free Fall

In the absence of resistive forces, the mechanical energy of an object in Earth's gravitational field is conserved. When the ball falls from rest to the floor, its gravitational potential energy transforms entirely into kinetic energy.



Energy Transformation in Free Fall

As an equation the conversion of gravitational potential energy into kinetic energy is expressed as

$$mgh = \frac{1}{2}mv^2. \tag{9}$$

Solving for the final speed gives

$$mgh = \frac{1}{2}mv^{2} \text{ (mass } m \text{ is cancelled)}$$
$$v^{2} = 2gh$$
$$v = \sqrt{2gh}$$

In free fall from rest, the final speed depends only on the distance fallen h and acceleration due gravity $g = 9.81 \text{ ms}^{-2}$, not on mass m.

2.3 Definition of Power

- When a force does work on an object, energy is exchanged between the object and its surroundings.
- The rate at which energy is exchanged is called power.
- Because work relates to energy transfer, the power can be defined in terms of work done.

Definition of Power in Terms of Work Power is the rate at which work is done on an object.

• Alternatively, the power can be defined in terms of energy.

Definition of Power in Terms of Energy Power is the rate at which energy is transferred.

2.3 Power as the Average Rate of Work Done

- For example, if the power of an incandescent lamp is 15 W, it transforms electrical energy into thermal energy and light at the rate of 15 J in one second.
- If the power of a car engine is 120 kW, the engine can do work at the rate of 120 kJ in one second. Of that power, only a fraction is used to actually accelerate the car.

Average Power

Average power is

$$P = \frac{W}{t}$$

(10)

where W is the work done on the object in time t.

• The unit of power is $[P] = \frac{[W]}{[t]} = Js^{-1} = 1W$ (watt = joule per second).

2.3 Work, Energy and Power Work and Kinetic Energy

2.3 Instantaneous Power

Instantaneous Power

Instantaneous power is

$$P = Fv$$

where F is the magnitude of a force acting in the direction of motion of an object moving at speed v.