UNITS AND MEASUREMENT

CONCEPTS INVOLVED

- The International system of units
- Measurement of length
- Measurement of mass
- Measurement of Time
- Accuracy, Precision of instruments and errors in measurement
- Significant figures
- Dimensions of physical quantities
- Dimensional formulae and dimensional equations
- Dimensional analysis and its applications

Main points

- Physics is a quantitative science, based on measurement of physical quantities.Certain physical quantities have been chosen as fundamental or base quantities. The fundamental quantities that are chosen are Length, Mass, Time, electric current, thermodynamic temperature, amount of substance, and luminous intensity.
- Each base quantity is defined in terms of a certain basic arbitrarily chosenbut properly standardised reference standard called unit (such as metre,kilogram,second,ampere,kelvin,mole,and candela.The units for the fundamental base quantities are called fundamental or base units and two supplementary units in relation to quantities plane angle and solid angle radian, Ste radian..
- Other physical quantities derived from the base quantities can be expressed as a combination of the base units and are called derived units.A complete set of units both fundamental and derived units are called a system of units.
- The International System of units based on seven base unitsis at present internationally acceptedunit system and is widely used throughout the world
- The SI units are used in all physical measurements, for both the base quantitiesand the derived quantities obtained from them. Certain derived units are expressed by means of SI units of special names such as joule, newton, watt etc.
- In computing any physical quantity the units for derived quantities involved in the relationships are treated as though they were algebraic quantities till the desired units are obtained
- In SI system that is System Internationale d' Units there are 7 base units' andtwo supplementary units.
- Direct and indirect methods can be used for the measurement of physical quantities. In measured quantities while expressing theresult, the accuracy and precision of measuring instrumentsalong with errors in measurement should be taken into account.

- In measured and computed quantitiesproper significant figures only should be retained.

  <u>Rules for determining the number of significant figures,carrying out arithmetic operations with them and rounding off the uncertain digits must be followed.</u>
- The dimensions of base quantities and combination of these dimensions describe the nature of physical quantities .Dimensional analysis can be used to check the dimensional consistency of equations, deducing relations among physical quantities etc. A dimensionally consistent equation need not be actually an exact equation, but a dimensionally wrong or inconsistent equation must be wrong.
- The uncertainty in the measurement of a physical quantity is called an error.
- The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.
- Precision tells us to what limit the quantity is measured.
- The errors in measurement can be classified as
  (i) Systematic errors and (ii) Random errors
- SYSTEMATIC ERRORS: These are the errors that tend to be either positive or negative.Sources of systematic errors are
  (i) Instrumental errors
  (ii) Imperfection in experimental technique or procedure
  (iii) Personal errors
- RANDOM ERRORS :Those errors which occur irregularly .These errors arise due to unpredictable fluctuations in experimental conditions
- Least count error is the error associated with the resolution of the instrument.
- The magnitude of the difference between the individual measurement and the true value of the quantity is called the absolute error of the measurement.
  Ex: $\Delta a = | a - a_{mean} |$
- The relative error or the percentage error is the ratio of the mean absolute error to the mean value of the quantity measured. When the relative error is expressed in per cent it is called the percentage error.
  Ex: (i) **Relative error** = $\Delta a_{mean}/a_{mean}$ (ii) **% error** = $(\Delta a_{mean}/a_{mean}) \times 100$

<u>COMBINATION OF ERRORS</u>

- ✓ **<u>ERROR OF A SUM OR A DIFFERENCE</u>**
  When two quantities are added or subtracted, the absolute error in the final result is thesums of the absolute errors in the individual quantities.
  IF $Z=A+B$ **then the max possible error in Z,** $\Delta Z = \Delta A + \Delta B$
  IF $Z=A-B$ **then the max possible error in Z,** $\Delta Z = \Delta A + \Delta B$

- ✓ **<u>ERROR OF A PRODUCT OR A QUOTIENT</u>**
  When two quantities are multiplied or divided the relative error is the sum of the relative errors in the multipliers
  Suppose $Z = A*B$ **or** $Z=A/B$ **then the max relative error in 'Z'** = $\Delta Z/Z = (\Delta A/A) + (\Delta B/B)$
- ✓ **<u>ERROR IN CASE OF A QUANTITY RAISED TO A POWER</u>**

The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

Suppose $Z = A^k$ **then** $\Delta Z/Z = K (\Delta A/A)$

SIGNIFICANT FIGURES

The reliable digits plus the first uncertain digit in a measurement arecalled Significant figures.

RULES FOR FINDING THE SIGNIFICANT FIGURES IN A MEASUREMENT

- ✓ All the non-zero digits are significant
- ✓ All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all
- ✓ If the number is less than 1, the zero(s) on the right side of decimal point but to the left of the first non-zero digit are not significant.
  Ex: In 0.000 35 the underlined zeros are not significant.

- ✓ The terminal or trailing zeros in a number without a decimal point are not significant
  Ex: 1795 m = 179500cm =1795000mm has four significant figures.
- ✓ The trailing zeros in a number with a decimal point are significant
  For ex: The numbers 75.00 or 0.06700 have four significant figures each.

RULES FOR ARITHMETIC OPERATIONS WITH SIGNIFICANT FIGURES.

- ✓ In multiplication or division, the final result should retain as many significant figures as there are in the original number with the least significant figures.

  Suppose $F = 0.04\ Kg \times 0.452\ m/sec^2 = 0.0108\ kg\text{-}m/sec^2$

The final result is $F = 0.01 Kg\text{-}m/Sec^2$

- ✓ In addition or subtraction, the final result should retain as many decimal places as there are in the number with the least decimal places.
  For ex: A= 334.5 kg; B= 23.45Kg then A + B =334.5 kg + 23.43 kg = 357.93 kg
  The result with significant figures is 357.9 kg

ROUNDING OFF:

While rounding off measurements the following rules are applied

- ✓ **Rule I:** If the digit to be dropped is smaller than 5,then the preceding digit should be left unchanged.
  For ex: 9.32 is rounded off to 9.3
- ✓ **Rule II:** If the digit to be dropped is greater than 5, then the preceding digit should be raised by 1
  For ex: 8.27 is rounded off to 8.3
- ✓ **Rule III:** If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit should be raised by 1

For ex: 9.351 on being rounded off to first decimal, becomes 9.4

- ✓ **Rule IV**: If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is not changed if it is even, is raised by 1 if it is odd.
  For ex: 5.45, on being rounded off,become 5.4
  5.450 on being rounded off, becomes 5.4

  (ii) 7.35 , on being rounded off, becomes 7.4

DIMENSIONS, DIMENSIONAL FORMULA AND DIMENSIONAL EQUATION

- Dimensions of a derived unit are the powers to which the fundamental units of mass, length and time etc. must be raised to represent that unit.
  For ex: Density = Mass / Volume= $M/L^3 = M^1L^{-3}$

- Dimensional formula is an expression which shows how and which of the fundamental units are required to represent the unit of a physical quantity.
  For Ex: $M^1 L^1 T^{-2}$ is the dimensional formula of Force.

CATEGORIES of PHYSICAL QUANTITIES

- Dimensional Cost ants: These are the quantities which possess dimensions and have a fixed value.
  Ex: Gravitational Constant
- Dimensional Variables These are the quantities which possess dimensions and do not have a fixed value
  For ex: velocity, acceleration etc.
- Dimensionless Constants: these are the quantities which do not possess dimensions and have a fixed value.
  For ex: πetc.
- Dimensionless Variables: These are the quantities which are dimensionless and do not have a fixed value.
  For ex: Strain, Specific Gravity etc.

PRINCIPLE OF HOMOGENEITY OF DIMENSIONS

- A given physical relation is dimensionally correct if the dimensions of the variousterms on either side of the relation are the same.

USES OF DIMENSIONAL EQUATIONS

- **TO CONVERT PHYSICAL QUANTITY FROM ONE SYSTEM OF UNITS TO ANOTHER**

  - ✓ Consider a physical quantity whose dimensions are $M^aL^bT^c$ Let n_1 be its numerical value in a system of fundamental units M_1, L_1, T_1. Then the magnitude of the physical quantity in this system is $n_1 [M_1^a L_1^b T_1^{c]}$.

Let n_2 be the numerical value in another system of fundamental units M_2, L_2 and T_2. The magnitude of the quantity in this system is $n_2 [M_2^a L_2^b T_2^c]$.
Since the value of the quantity is the same in all systems

$n_2 [M_2^a L_2^b T_2^c] = n_1 [M_1^a L_1^b T_1^c]$.
$n_2 = n_1 [M_1^a L_1^b T_1^c] / [M_2^a L_2^b T_2^c]$.

- **TO CHECK THE DIMENSIONAL CORRECTNESS OF A GIVEN PHYSICAL RELATION**

Ex: v = u + a t, Here v represents the velocity of the body after t secs, a, is the acceleration and u the initial velocity of the body.
Dimensional formula of u is $M^0 L^1 T^{-1}$
Dimensional formula of V is $M^0 L^1 T^{-1}$
Dimensional formula of at is $\{M^0L^1 T^{-2}\}\{T^1\} = \{M^0L^1 T^{-1}\}$
The dimensions of every term in the given physical relation is same, hence according to principle of homogeneity the given physical relation is dimensionally correct.

- **TO ESTABLISH RELATION BETWEEN DDIFFERENT PHYSICAL QUANTITIES**

To find an expression for the time period of a simple pendulum given that the time period(t) may depend upon (i) massof the bob (ii) length of the pendulum (iii) acceleration due to gravity,(iv) angle of swing Θ

Let (i) $t \propto m^a$ (ii) $t \propto l^b$ (iii) $t \propto g^c$ (iv) $t \propto \Theta^d$

Or $t = K m^a l^b g^c \Theta^d$ Where K is a Dimensionlessconstant of proportionality.

Writing down the dimensions on either side of the equation we get
$[T] = [M^a][L^b][L T^{-2}]^c = [M^a L^{b+C} T^{-2c}]$
Comparing the dimensions on either side

a=0; b+c=0; -2c=1 i.e., c= -1/2 , b= +1/2 ,a = 0

$t = K l^{1/2} g^{-1/2}$ or $t = K\sqrt{l/g}$

The value of K as found by experiment comes out to be 2π

And **hence $t = 2\pi\sqrt{l/g}$**

- **LIMITATIONS OF DIMENSIONAL ANALYSIS**
  - It supplies no information about dimensionless constants. They have to be determined either by experiment or by mathematical investigation.
  - This method applicable only in the case of power functions. It fails in case of exponential and trigonometric relations.
  - It fails to derive a relation which contains two or more than two quantities of like nature.

- It can only check whether a physical relation is dimensionally correct or not. It cannot tell whether the relation is absolutely correct or not
- It cannot identify all the factors on which the given physical quantity depends upon.

ANSWER THE FOLLOWING EACH QUESTION CARRIES 1 MARK.

1. Define physical quantities

2.Distinguish between fundamental and derived quantities

3. Define one metre

4.Define one kilogram

5.Define one second

6.Define the SI unit of the following physical quantities

(i) Temperature (ii) Luminous intensity

7.Define one radian

8. Define one Ste radian

9.Give the relation between light year and metre

10. Write two advantages in choosing the wavelength of light radiation as a standard

of length

11.What is the difference between 5.0 and 5.000?

12.Write the uses of the dimensional analysis

13.Write the dimensional equation for force

14.Write the dimensional representation for torque

15. Give the relationship between calorie and joule

16.Write two advantages in choosing the wavelength of light radiation as a standard

of length.

17.What is the difference between 4.0 and 4.0000?

18. Write the uses of Dimensional Analysis.

19. Define the term significant figures.

Answer the following. Each question carries 2 marks.

20. Write four limitations of dimensional analysis

21. If $(P + a/V^2)(V-b) = RT$, Where the difference symbols have their usual meaning then what are the dimensions of (a/ V^2) and b.

22.Write the dimensions of the following

(i) Electric intensity (ii) Electric Potential (iii) E.M.F. of a cell (iv)Electrical resistance

23 Write the dimensions of the following

(i)Specific Resistance (II) Magnetic flux (III) Electric flux (IV) Magnetic Induction

24Write the dimensions of the following

(i) Conductance (ii)Electric Permittivity (iii)Magnetic Permeability (iv)Coefficient of Self Inductance

23.Solve the following to correct significant figures

(i) 5.1 + 13.235 (ii)7.54 + 18.1295 (iii)14.632 – 5.52345 (iv)3.021 X 11

24.Define mean scalar second

25.Define second in terms of Cs-133 vibrations

Answer the following. Each question carries 3 marks.

26. Answer the following

(a) You aregiven a thread and a meter scale. How will you estimate the diameter of this thread?

(b)A screw gaugehas a pitch of 1.00mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the gaugearbitrarily by increasing the number of divisions on the circular scale?

27. Explain briefly how you will estimate the size of the molecule of oleic acid.

28.Explain how will you estimate the distance of a planet or star by using parallax method.

29.Find the area of a circle of radius 3.458 cm up to correct significant figures.

30. If the % errorin the measurement of the radius R of a sphere is 0.2%, then calculate the % error in its volume.

31. A laser light beamed towards the moon takes 2.56 sec to return to the earth after the reflection on themoon's surface. What is the distance of the moon from theearth?