Answer keys

1	C	**2**	B	**3**	B	**4**	A	**5**	C	**6**	D	**7**	D
8	D	**9**	B	**10**	D	**11**	B	**12**	B	**13**	B	**14**	C
15	A	**16**	B	**17**		**18**	B	**19**		**20**	D	**21**	D
22	B	**23**	B	**24**	C	**25**		**26**	A	**27**	D	**28**	C
29	A	**30**		**31**	D	**32**	B	**33**	B	**34**	B	**35**	B
36		**37**	D	**38**		**39**	D	**40**	A	**41**	B	**42**	
43	A	**44**	C	**45**	C	**46**	A	**47**	A	**48**	C	**49**	C
50	D	**51**	A	**52**		**53**		**54**	C	**55**		**56**	D
57	C	**58**	D	**59**		**60**	C	**61**	C	**62**		**63**	B
64	D	**65**		**66**		**67**	B	**68**	B	**69**	D	**70**	
71		**72**		**73**		**74**	A	**75**	A	**76**		**77**	
78		**79**		**80**		**81**		**82**	D	**83**	B	**84**	
85													

Explanation:-

3. $\underset{x\to 8}{\mathrm{Lt}} \dfrac{x^{1/3}-2}{x-8}$, Applying L-Hospital's Rule , we get, $\underset{x\to 8}{\mathrm{Lt}} \dfrac{\frac{1}{3}x^{\frac{-2}{3}}}{1} = \dfrac{1}{12}$

4. Required probability $= 4_{C_3}\left(\dfrac{1}{2}\right)^3\left(\dfrac{1}{2}\right)^1 = \dfrac{1}{4}$

5. Given matrix is $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & P \end{bmatrix}$

Let λ_1, λ_2 and λ_3 be the Eigen values of the above matrix Let $\lambda_1 = 3(\text{Given})$

Now, $\lambda_1 + \lambda_2 + \lambda_3 = \text{sum of diagonal elements} = 1 + P$

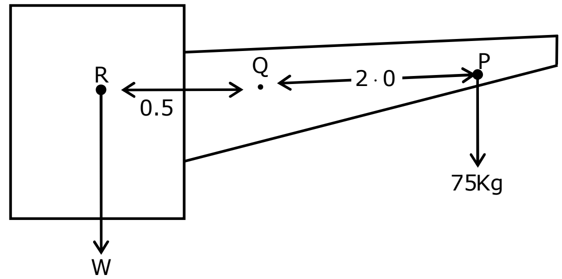
$\therefore\ \lambda_2 + \lambda_3 = P + 1 - 3 = P - 2$

6. Given vector is $F = (x-y)\,\hat{i} + (y-x)\,\hat{j} + (x+y+z)\,\hat{k}$

Divergence $= \dfrac{\partial F}{\partial x} + \dfrac{\partial F}{\partial y} + \dfrac{\partial F}{\partial z} = 1 + 1 + 1 = 3$

8. Both young's Modulus and shear Modulus are required as linear strain will be calculated by young modulus. Change in diameter can be calculated from Poisson's ratio which depends on young's modulus and shear modulus.

10. Let W be the weight of counterweight.
Taking moment about Q
$75 \times 2 = W \times 0.5$, $W = 300Kg$



11. Grubler's criterion is applied to mechanism with only single degree of freedom.
Given conditions satisfy Grubler's criterion i.e. $3\ell - 2j - 4 = 0$ where,
ℓ = link, j = No. of joints

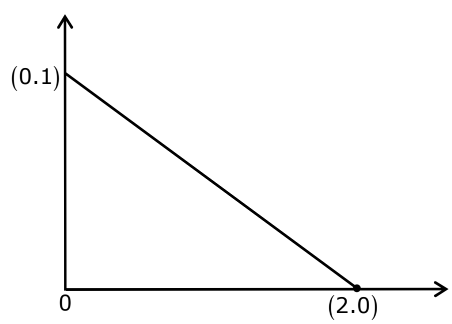
13. Since the final temperature is same as that of initial temperature

14. Prandtl Number, $Pr = \frac{\mu C_P}{K} = \frac{0.001 \times 1 \times 10^3}{1}$
Given δ = Hydroxynamic Boundary layer = 1
δ_t = Thermal boundary layer = ?
$\frac{\delta}{\delta_t} = Pr^{1/3} \Rightarrow \delta_t = 1$

18. Job with higher Processing time will be taken first since it will minimize the total holding cost.

21. $I = \int_0^2 \int_0^1 xydxdx$

$= \int_0^2 \left[\frac{y^2}{2}\right]_0^1 xdx = \int_0^2 \frac{x}{2} dx = \left[\frac{x^2}{4}\right]_0^2 = 1$



22. Gradient will $\nabla f = \hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z}$

$\nabla f = 2x\hat{i} + 4y\hat{j} + \hat{k}$
Now ∇f at the point $(1,1,2)$
$\nabla f = 2\hat{i} + 4\hat{j} + \hat{k}$
Directonal derivative of f in the direction $3\hat{i} + 4\hat{k}$ is
$= (2i + 4j + k).\frac{(3\hat{i} - 4\hat{j})}{\sqrt{3^2 + (-4)^2}} = \frac{6 - 16}{5} = \frac{-10}{5} = -2$

28 $f = y^x$

$\ell nf = x\ell ny$

differentiating with respect to x

$$\frac{1}{f}\frac{\partial f}{\partial x} = \ell ny \Rightarrow \frac{\partial f}{\partial x} = f\ell ny$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}(f\ell ny) = \frac{\partial}{\partial y}\left(y^x \ell ny\right)$$

$$= y^x \frac{1}{y} + \ell nyxy^{x-1} \Rightarrow y^{x-1}(x\ell ny + 1)$$

Now $x = 2,\ \ y = 1$

So $\dfrac{\partial^2 f}{\partial x \partial y} = 1$

29. $y'' + 2y' + y = 0 \Rightarrow D^2 + 2D + 1 = 0$

i.e. $(D+1)^2 = 0, \quad D = -1, -1$

So solution will be $y = (C_1 + C_2)e^{-x}$

Now given, y=0 at x=0 and y=0 at x=1

So we get $C_1 = C_2 = 0$

$y = \text{constant}$

$y(0.5) = 0$

32. Let F_S be the shear stress

$T = \dfrac{\pi}{16} \times f_s \times d^3 \Rightarrow f_s = 51\text{MPa}, \quad f_t = \text{Tensile stress} = 50\text{MPa}$

Maximum principal stress, $\sigma_{max} = \dfrac{f_t}{2} + \sqrt{\left(\dfrac{f_t}{2}\right)^2 + {f_s}^2} \approx 82\text{MPa}$

34. At node P

$T_{PQ}\cos 45^\circ + T_{PR}\cos 60^\circ + F = 0$........(1)

$T_{PQ}\sin 45^\circ = T_{PR}\sin 60^\circ$..................(2)

from these two equations

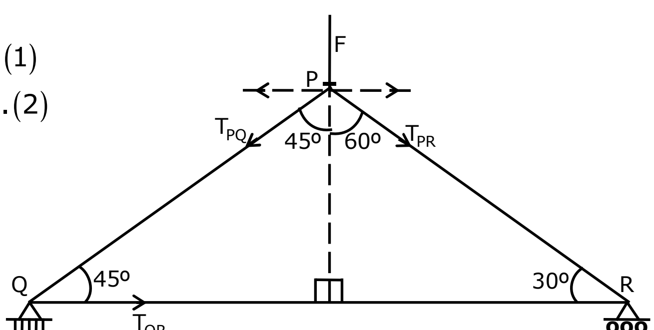
we can find out

T_{PQ} and T_{PR} in terms of F.

Now, At node Q.

$T_{QR} = T_{PQ}\cos 45^\circ$

On solving we get, $T_{QR} = 0.63F$



35. Given spring system forms a parallel combination

$K_{eq} = K_1 + K_2 = 4000 + 1600 = 5600 N/m$

Natural frequency $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = 10Hz$

39. $K_1 = \frac{G.d_1^4}{8D_1^3.n_1}$ & $K_2 = \frac{G.d_2^4}{8D_2^3.n_2}$

$d_1 = d_2 = 2mm$ [dia of spring wire]

$G = 80GPa$

$n_1 = n_2 = 10$

$D_1 = 20mm, D_2 = 10mm$

$\therefore \frac{K_1}{K_2} = \left(\frac{D_2}{D_1}\right)^3 = \frac{1}{8} \Rightarrow K_2 = 8K_1$

56. Direction of heat flow is always normal to surface of constant temperature.

So, for surface P , $\frac{dT}{dx} = 0$

From energy conservation, heat rate at P = Heat rate at Q

$0.1 \times 1 \times \left.\frac{dT}{dy}\right|_P = 0.1 \times 2 \times \left.\frac{dT}{dx}\right|_Q$

$\therefore \frac{dT}{dy} = 20 K/m$

63. Riser takes care of solidification/contraction in liquid state and phase transition. So volume of metal compensated from the riser = 3% + 4% = 7%

67. Heat supplied by power source = Heat required

melting efficiency × transfer efficiency × welding power

= cross sectional area × welding speed × 10

$.5 \times .7 \times 2 \times 10^3 = 5 \times 10 \times V \Rightarrow V = 14 mm/s$

41. Torque carrying capacity, $T = \frac{2}{3}\mu w \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$

$w = P \times \pi\left(R_0^2 - R_i^2\right)$

Given R_0 = 50mm, R_i = 20mm, P = 2MPa and μ =0.4

So, T=196NM

45. Given $m_c = 2m_h$ [Mass flow rate]

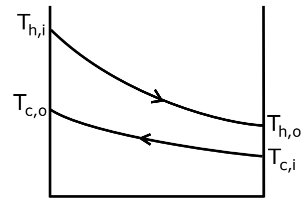
$c_h = 2c_c$ [specific heat]

So, we get

$[\text{Heat capacity}]_{\text{Hot fluid}} = [\text{Heat capacity}]_{\text{Cold fluid}}$

$\therefore$ LMTD $= \Delta T_1 = T_{h,i} - T_{c,o}$

$20 = 100 - T_{c,o} \Rightarrow T_{c,o} = 80^{o}C$



82. $\tau_s = 250\text{MPa}.\ V = 180\text{m/min},\ F = 0.20\text{mm/rev}$

$r = 0.5, \alpha = \text{rake angle} = 7^{o}$

$\phi = \text{shear angle}$

$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} \Rightarrow \phi = 28^{o}$

Now shear force

$F_s = \frac{wt_1\tau_s}{\sin\phi}$ $\qquad w = \text{depth of cut} = 3\text{mm}, t_1 = \text{feed} = 0.02\text{mm}$

$\therefore F_s = 320\text{KN}$

83. From Merchant's theory

$2\phi + \lambda - \alpha = 90^{o} \therefore\ \lambda = \text{Friction Angle} = 90^{o} + 7^{o} - 2 \times 28^{o} = 41^{o}$

$\mu = \tan\lambda = .87$

Form Merchant circle

$F_C = R\cos(\lambda - \alpha) ....(1)$ and $R = \frac{F_s}{\cos(\phi + \lambda - \alpha)} ....(2)$

$R = \text{Resultant force} \therefore F_C = \frac{F_S\cos(\lambda - \alpha)}{\cos(\phi + \lambda - \alpha)}, F_C = 565\text{N}$