

10. WAVE OPTICS

ONE MARK QUESTIONS

1. Define wavefront.
2. What is the shape of wavefront obtained from a point source at a (i) small distance (ii) large distance?
3. Under what conditions a cylindrical wavefront is obtained?
4. What type of wavefront is obtained when a plane wave is reflected by a concave mirror?
5. Who proposed the wave theory of light?
6. Name the physicist who experimentally studied the interference of light for the first time.
7. What is interference of light?
8. What is the maximum intensity of light in Young's double slit experiment if the intensity of light emerging from each slit is I_0 ?
OR What is the intensity of light due to constructive interference in Young's double slit experiment if the intensity of light emerging from each slit is I_0 ?
9. Define fringe width.
10. Instead of using two slits as in Young's experiment, if two separate but identical sodium lamps are used, what is the result on interference pattern?
11. What is the effect on interference fringes when yellow light is replaced by blue light in Young's double slit experiment?
12. How does the fringe width in interference pattern vary with the wavelength of incident light?
13. What is the effect on the interference fringes in a Young's double-slit experiment when the monochromatic source is replaced by a source of white light?
14. How does the fringe width in interference vary with the intensity of incident light?
15. Which colour of light undergoes diffraction to maximum extent?
16. Name a factor which affects the resolving power of a microscope.
17. How will the diffraction pattern due single slit change when violet light replaces green light?
18. Do all waves exhibit diffraction or only light?
19. We do not encounter diffraction effects of light in everyday observations. Why?
20. Why are diffraction effects due to sound waves more noticeable than those due to light waves?
21. Is the width of all secondary maxima in diffraction at slit same? If not how does it vary?
22. What is resolving power of microscope?
23. What about the consistency of the principle of conservation of energy in interference and in diffraction? **OR** Does the law of conservation of energy holds good in interference and in diffraction?
24. How can the resolving power of a telescope be increased?
25. Which phenomenon confirms the transverse nature of light?
26. What is meant by plane polarised light?
27. What is pass axis?
28. By what percentage the intensity of light decreases when an ordinary unpolarised (like from sodium lamp) light is passed through a polaroid sheet?
29. Let the intensity of unpolarised light incident on P_1 be I . What is the intensity of light crossing polaroid P_2 , when the pass-axis of P_2 makes an angle 90° with the pass-axis of P_1 ?
30. What should be the angle between the pass axes of two polaroids so that the intensity of transmitted light form the second polaroid will be maximum?
31. State Brewster's Law.
32. Write the relation between refractive index of a reflector and polarising angle.
33. Define Brewster's angle (**OR** Polarising angle).

TWO MARKS QUESTIONS

1. State Huygens' principle.
2. Name the wavefront obtained when a plane wave passed through (i) a thin convex lens (ii) thin prism.
3. What is the shape of the wavefront in each of the following cases:
(a) Light emerging out of a convex lens when a point source is placed at its focus.
(b) The portion of the wavefront of light from a distant star intercepted by the Earth.
4. What are coherent sources? Give an example.
5. Can two sodium vapour lamps be considered as coherent sources? Why?
6. Write the expression for fringe width in Young's double slit experiment.
7. What are the factors which affect the fringe width in Young's double slit experiment?
8. Let the fringe width in Young's double slit experiment be β . What is the fringe width if the distance between the slits and the screen is doubled and slit separation is halved?
9. What is diffraction of light? Give an example.
10. Mention the conditions for diffraction minima and maxima.
11. Give the graphical representation to show the variation of intensity of light in single slit diffraction.
12. Mention the expression for limit of resolution of microscope.
13. Write the expression for limit of resolution of telescope.
14. Give the two methods of increasing the resolving power of microscope.
15. Write the mathematical expression for Malus law. Explain the terms.
16. Represent polarised light and unpolarised light.
17. Unpolarised light is incident on a plane glass surface. What should be the angle of incidence so that the reflected and refracted rays are perpendicular to each other? (For glass refractive index = 1.5).
OR What is the Brewster angle for air to glass transition? (For glass refractive index = 1.5).
18. In a Young's double slit experiment, the angular width of a fringe formed on distant screen is 0.1° . The wavelength of light used is 6000 \AA . What is the spacing between the slits?
19. A beam of unpolarised is incident on an arrangement of two polaroids successively. If the angle between the pass axes of the two polaroids is 60° , then what percentage of light intensity emerges out of the second polaroid sheet?
20. Assume that light of wavelength 5000 \AA is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 5.08 m ?

THREE MARKS QUESTIONS

1. Using Huygen's wave theory of light, show that the angle of incidence is equal to angle of reflection in case of reflection of a plane wave by a plane surface.
2. Illustrate with the help of suitable diagram, action of the following when a plane wavefront incidents.
(i) a prism (ii) a convex lens and (iii) a concave mirror. (each three marks)
3. Briefly describe Young's experiment with the help of a schematic diagram.
4. Distinguish between interference of light and diffraction of light.
5. Briefly explain Polarisation by reflection with the help of a diagram.
6. Show that the refractive index of a reflector is equal to tangent of the polarising angle.
OR Arrive at Brewster's law.
7. What are Polaroids? Mention any two uses of polaroids.

FIVE MARKS QUESTIONS

1. Using Huygen's wave theory of light, derive Snell's law of refraction.
2. Obtain the expressions for resultant displacement and amplitude when two waves having same amplitude and a phase difference ϕ superpose. Hence give the conditions for constructive and destructive interference. OR Give the theory of interference. Hence arrive at the conditions for constructive and destructive interferences.

3. Derive an expression for the width of interference fringes in a double slit experiment.
4. Explain the phenomenon of diffraction of light due to a single slit and mention of the conditions for diffraction minima and maxima.

FIVE MARKS NUMERICAL PROBLEMS

1. A monochromatic yellow light of wavelength 589nm is incident from air on a water surface. What are the wavelength, frequency and speed of a refracted light? Refractive index of water is 1.33 .
2. In a double slit experiment angular width of a fringe is found to be 0.2° on a screen placed 80 cm away. The wave length of light used is 600 nm. Find the fringe width.
What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be $4/3$.
3. A beam of light consisting of two wavelengths 650 nm and 520 nm, is used to obtain interference fringes in Young's double slit experiment with $D = 60$ cm and $d = 1$ mm.
a) Find the distance of third bright fringe on the screen from central maximum for wavelength 650nm.
b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?
4. In Young's double-slit experiment using monochromatic light of wavelength λ , the intensity of light at a point on the screen where path difference is λ , is K units. What is the intensity of light at a point where path difference is (i) $\lambda/3$ (ii) $\lambda/2$?
5. A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1.25 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen.
Find (i) the width of the slit and (ii) angular position of the first secondary maximum.
6. In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.
Also find the distance of fifth dark fringe from the central bright fringe.
7. In Young's double slit experiment with monochromatic light and slit separation of 1mm, the fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5cm towards the slits, the change in fringe width is 30 μm . Calculate the wavelength of the light used.

ANSWERS FOR ONE MARK QUESTIONS

1. A locus of points, which oscillate in phase is called a wavefront.
OR A surface of constant phase is called wavefront.
2. (i) Spherical wavefront (ii) Plane wavefront.
3. A cylindrical wavefront is obtained at a small distance from a linear source of light.
4. Spherical wavefront (converging).
5. Christiaan Huygens.
6. Thomas Young.
7. The modification in the distribution of light energy due to the superposition of two or more waves of light is called interference of light.
8. Maximum intensity of light in Young's double slit experiment is $4I_0$.
9. The distance between two consecutive bright (or two consecutive dark) fringes is called fringe width.

10. Interference pattern disappears.
11. The fringe width decreases. Since $\beta \propto \lambda$ and λ is smaller for blue light than yellow light.
12. The fringe width is directly proportional to the wavelength of incident light.
13. The central fringe is white. The fringe closest on either side of the central white fringe is red and the farthest will appear blue.
14. The fringe width is not affected by the intensity of incident light.
15. Red.
16. The wave length of light or refractive index of medium between objective lens and the object.
17. The diffraction bands become narrower.
18. All the waves exhibit the phenomenon of diffraction.
19. Since the wavelength of light is much smaller than the dimensions of most of the obstacles.
20. The wavelength of sound waves is comparable with the size of the obstacles whereas for light, the wavelength is much smaller than the dimensions of most of the obstacles.
21. No. As the order of the secondary maximum increases its width decreases.
22. The resolving power of the microscope is defined as the reciprocal of the minimum separation of two points which are seen as distinct.
23. Interference and diffraction are consistent with the principle of conservation of energy.
24. Using objective of larger diameter.
25. Polarisation.
26. Plane polarised light is one which contains transverse linear vibrations in only one direction perpendicular to the direction of propagation.
27. When an unpolarised light wave is incident on a polaroid, the light wave will get linearly polarised with the electric vector oscillating along a direction perpendicular to the aligned molecules. This direction is known as the pass-axis of the polaroid.
28. 50%.
29. Zero. Since no light passes through the polaroids when they are crossed.
30. 0° .
31. Brewster's Law: The refractive index of a reflector is equal to tangent of the polarising angle.
32. $n = \tan i_B$, where n - refractive index of the reflector and i_B - polarising angle/Brewster's angle.
33. Brewster's angle/Polarising angle(i_B): The angle of incidence for which the reflected light is completely plane polarised is called Brewster's angle.

ANSWERS FOR TWO MARKS QUESTIONS

1. Each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave. These wavelets emanating from the wavefront are called as secondary wavelets and if we draw a common tangent to all these spheres, we obtain the new position of the wavefront at a later time.
2. (i) Spherical converging wavefront; (ii) Plane wavefront.
3. (a) Plane wavefront; (b) Plane wavefront.
4. The two sources are said to be coherent if the phase difference between the waves emitted by them at any point will not change with time. OR Any two sources continuously emitting waves having zero or constant phase difference are called coherent sources.
Example: In Young's double slit experiment the two slits behave like coherent sources.
5. No, because the phase difference between light coming from two independent sources continuously change.
6. The fringe width: $\beta = \frac{\lambda D}{d}$; where λ – wavelength of light, d – distance between the slits and D – distance between the screen and the slits.
7. The wavelength of light, distance between the slits and the screen or slit separation. [any two]

8. Initial fringe width: $\beta = \frac{\lambda D}{d}$ and the new fringe width: $\beta' = \frac{\lambda(2D)}{(d/2)} = 4\left(\frac{\lambda D}{d}\right) = 4\beta$

Thus, the new fringe width becomes four times the initial.

9. The phenomenon of bending of light waves around the edges (or corners) of the obstacles and entering into the expected geometrical shadow of the obstacle is called diffraction of light.

Example: Colours observed when a CD (Compact Disc) is viewed is due to diffraction of light.

10. Condition for secondary maxima is

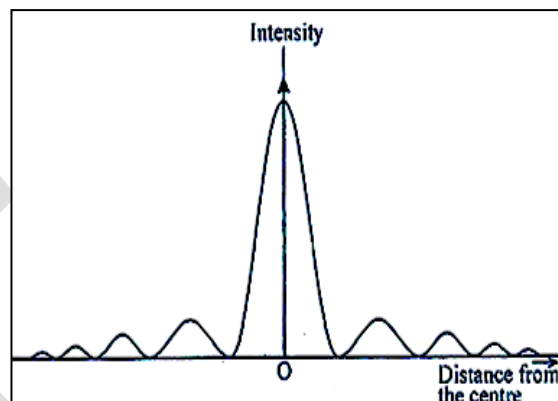
Angle of diffraction $\theta \approx \left(n + \frac{1}{2}\right) \frac{\lambda}{a}$, where $n = \pm 1, \pm 2, \pm 3, \dots$

Condition for diffraction minima:

Angle of diffraction $\theta \approx \frac{n\lambda}{a}$, Where $n = \pm 1, \pm 2, \pm 3, \dots$

where λ is wavelength of light used and a is slit width.

11. GRAPHICAL REPRESENTATION for the variation of intensity of light in single slit diffraction is as shown in the adjacent diagram.



12. Minimum separation OR Limit of resolution: $d_{\min} = \frac{1.22 \lambda}{2 n \sin \beta}$

where λ – wavelength of light, n – refractive index of the medium between the object and the objective lens and 2β – angle subtended by the object at the diameter of the objective lens at the focus of the microscope.

13. Expression for limit of resolution of telescope: Limit of resolution: $\Delta\theta = \frac{0.61 \lambda}{a} = \frac{1.22 \lambda}{2a}$

where λ – wavelength of light and $2a$ – diameter of aperture of the objective.

14. Resolving power of a microscope can be increased

(i) by choosing a medium of higher refractive index and (ii) by using light of shorter wavelength.

15. $I = I_0 \cos^2 \theta$, Where I is the intensity of the emergent light from second polaroid (analyser), I_0 is the intensity of plane polarised light incident on second polaroid after passing through first polaroid (polariser) and θ is the angle between the pass-axes of two polaroids (analyser and polariser).

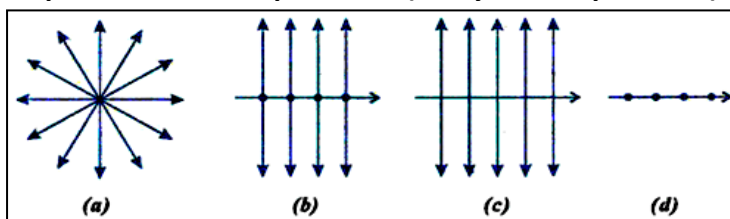
16. Unpolarized light is represented as shown in figure(a) and figure (b). [any one]

Plane polarized light with vibrations parallel to the plane of the paper is shown in figure(c).

Plane polarized light with vibrations

perpendicular to the plane of the paper is as shown in figure(d).

[any one]



17. $n = \tan i_B \Rightarrow$ Brewster's angle for glass: $i_B = \tan^{-1}(n) = \tan^{-1}(1.5) = 56^\circ 19'$.

18. Given $\theta = 0.1^\circ = 0.1 \left(\frac{\pi}{180} \right) = 1.745 \times 10^{-3} \text{ rad}$ and wavelength of light $= \lambda = 6000 \text{ \AA} = 6 \times 10^{-7} \text{ m}$

\therefore Spacing between the slits is $d = \frac{\lambda}{\theta} = \frac{6 \times 10^{-7}}{1.745 \times 10^{-3}} = 3.438 \times 10^{-4} \text{ m}$

19. Given $\theta = 60^\circ$, Intensity of light incident on the polaroid $= I_0$,

Intensity of light transmitted through the polaroid $I = ?$

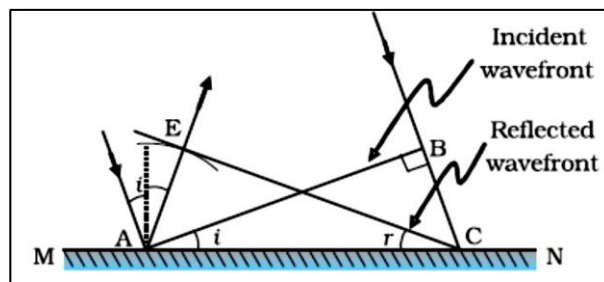
$I = I_0 \cos^2 \theta \Rightarrow I = I_0 \cos^2 60^\circ = I_0/4$. Thus 25% of the light intensity is transmitted through the polaroids.

20. Given wavelength of light $= 5000 \text{ \AA} = 5 \times 10^{-7} \text{ m}$, Diameter of the objective $= 5.08 \text{ m}$

Limit of resolution: $\Delta\theta = \frac{1.22\lambda}{2a} = \frac{1.22 \times 5 \times 10^{-7}}{5.08} = 1.2 \times 10^{-7}$ radians

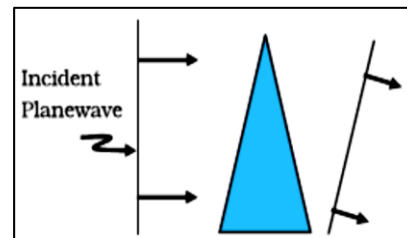
ANSWERS FOR THREE MARKS QUESTIONS:

1. Consider a plane wave AB incident at an angle i on a reflecting surface MN. If v represents the speed of the wave in the medium and if τ represents the time taken by the wavefront to advance from the point B to C then the distance $BC = v\tau$. In order to construct the reflected wavefront, a sphere of radius $= v\tau$, is drawn from the point A as shown in the adjacent figure. Let CE represent the tangent plane drawn from the point C to this sphere. $\therefore AE = BC = v\tau$, $\angle ABC = \angle CEA = 90^\circ$, AC is common. Triangles EAC and BAC are congruent. $\therefore i = r$.



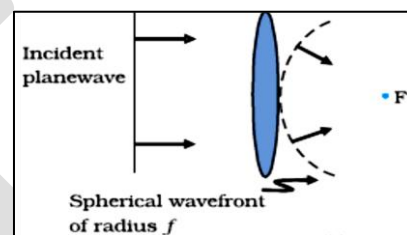
2. (i) Action of the prism when a plane wavefront incident on it:

In adjacent figure, consider a plane wave passing through a thin prism. Since the speed of light waves is less in glass, the lower portion of the incoming wavefront which travels through the greatest thickness of glass will get delayed resulting in a tilt in the emerging plane wavefront.



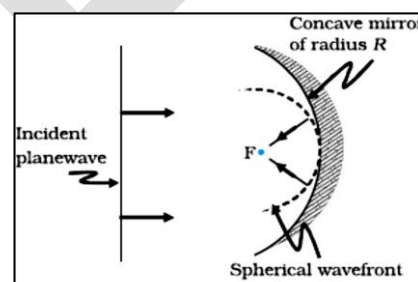
- (ii) Action of the convex lens when a plane wavefront incident on it:

In the adjacent figure, a plane wave incident on a thin convex lens; the central part of the incident plane wave traverses the thickest portion of the lens and is delayed the most. The emerging wavefront has a depression at the centre and therefore the wavefront becomes spherical (radius $= f$, focal length) and converges to the point focus F.



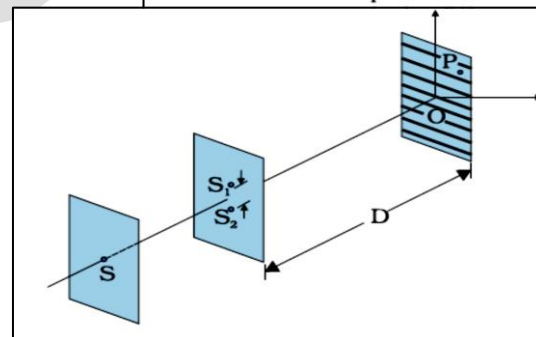
- (iii) Action of the concave mirror when a plane wavefront incident on it:

In adjacent figure, a plane wave is incident on a concave mirror and on reflection we have a spherical wave converging to the focus F.



3. Young's experiment : Description with a schematic diagram

S represents a pin hole illuminated by sunlight. The spherical wave front from S is incident on two pin holes S_1 and S_2 which are very close to each other and equidistant from S. Then the pin holes S_1 and S_2 act as two coherent sources of light of same intensity. The two sets of spherical wave fronts coming out of S_1 and S_2 interfere with each other in such a way as to produce a symmetrical pattern of varying intensity on the screen placed at a suitable distance D.



4. Differences between interference of light and diffraction of light:

	INTERFERENCE	DIFFRACTION
1	Interference fringes have equal width.	Diffraction bands have unequal width. (width of secondary maxima decreases with increase in order)
2	Interference is due to the superposition of two waves originating from two coherent sources	It is due to the superposition of secondary wavelets originating from different parts of single slit.

3	Intensity of all bright fringes is equal and Intensity of dark fringes is zero.	Intensity of central maximum is highest, Intensity of secondary maxima decreases with increase in order.
4	At an angle of λ/a , maximum intensity for two narrow slits separated by a distance 'a' is found.	At an angle of λ/a , the first minimum of the diffraction pattern occurs for a single slit of width a .
5	In an interference pattern there is a good contrast between dark and bright fringes.	In a diffraction pattern the contrast between the bright band and dark band is comparatively lesser.

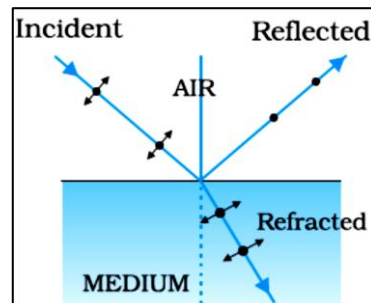
5. POLARIZATION BY REFLECTION:

It is found that when a beam of ordinary light is reflected by the surface of a transparent medium like glass or water, the reflected light is partially polarized.

The degree of polarization depends on the angle of incidence.

As the angle of incidence is gradually increased from a small value, the degree of polarization also increases. At a particular angle of incidence the reflected light is completely plane polarized. This angle of incidence is called Brewster's angle or polarizing angle (i_B).

If the angle of incidence is further increased, the degree of polarization decreases.

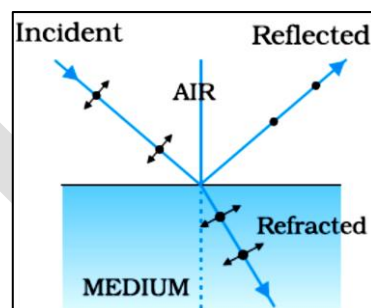


6. From Snell's law, $n = \frac{\sin i}{\sin r} = \frac{\sin i_B}{\sin r}$, since $i = i_B$.

For Brewster's angle of incidence, $r + i_B = 90^\circ \Rightarrow r = 90^\circ - i_B$

$$\Rightarrow n = \frac{\sin i_B}{\sin (90^\circ - i_B)} = \frac{\sin i_B}{\cos i_B}$$

\Rightarrow Refractive index of reflector: $n = \tan i_B$



7. Polaroids are the devices used to produce plane polarised light.

Uses of polaroids: 1) To control the intensity of light in sunglasses, windowpanes, etc..

2) In photographic cameras and 3D movie cameras.

ANSWERS FOR FIVE MARKS QUESTIONS:

1. Derivation of Snell's law of refraction.

Let PP' represent the surface separating medium-1 and medium-2.

Let v_1 and v_2 be the speed of light in medium-1 and medium-2 respectively.

Consider a plane wave front AB incident in medium-1 at angle 'i' on the surface PP'.

According to Huygens principle, every point on the wave front AB is a source of secondary wavelets.

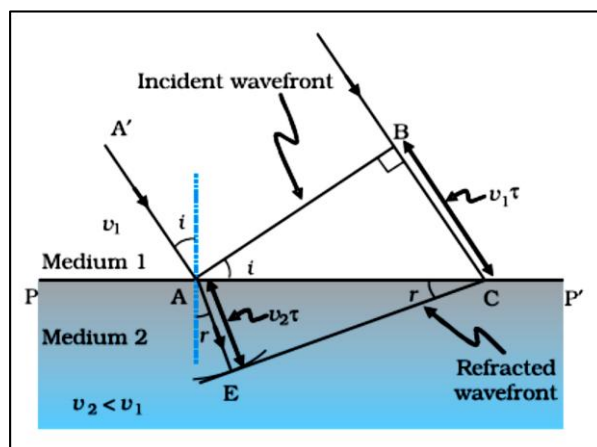
Let the secondary wavelet from B strike the surface PP' at C in a time τ . Then $BC = v_1\tau$.

The secondary wavelet from A will travel a distance $v_2\tau$ as radius; draw an arc in medium 2. The tangent from C touches the arc at E. Then $AE = v_2\tau$ and CE is the tangential surface touching all the spheres of refracted secondary wavelets. Hence, CE is the refracted wave front. Let r be the angle of refraction.

In the above figure, $\angle BAC = i =$ angle of incidence and $\angle ECA = r =$ angle of refraction

$$BC = v_1\tau \quad \text{and} \quad AE = v_2\tau$$

$$\text{From triangle BAC, } \sin i = \frac{BC}{AC} \quad \text{and} \quad \text{from triangle ECA, } \sin r = \frac{AE}{AC}$$



$$\therefore \frac{\sin i}{\sin r} = \frac{BC/AC}{AE/AC} = \frac{BC}{AE} = \frac{v_1 \tau}{v_2 \tau} = \frac{v_1}{v_2}$$

Since v_1 is a constant in medium-1 and v_2 is a constant in medium-2, $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \text{constant} \dots\dots (*)$

Now, refractive index (n) of a medium: $n = \frac{c}{v}$ or $v = \frac{c}{n}$, where c – speed of light in vacuum.

For the first medium: $v_1 = \frac{c}{n_1}$ and for the second medium: $v_2 = \frac{c}{n_2} \Rightarrow \frac{v_1}{v_2} = \frac{n_2}{n_1}$

(*) becomes $\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$ or $n_1 \sin i = n_2 \sin r$. This is the Snell's law of refraction.

2. THEORY OF INTERFERENCE: Expression for the amplitude of resultant displacement and intensity

If the displacement produced by source S_1 is given by $y_1 = a \cos(\omega t)$

Then the displacement produced by S_2 would be $y_2 = a \cos(\omega t + \phi)$, where ϕ is the phase difference between the two waves.

The resultant displacement: $y = y_1 + y_2 = [a \cos(\omega t) + a \cos(\omega t + \phi)] = a [\cos(\omega t + \phi) + \cos(\omega t)]$

$$= 2a \cos\left(\frac{2\omega t + \phi}{2}\right) \cos\left(\frac{\phi}{2}\right); \quad \text{Using } \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$= 2a \cos\left(\frac{\phi}{2}\right) \cos\left(\omega t + \frac{\phi}{2}\right)$$

The amplitude of the resultant displacement is $2a \cos\left(\frac{\phi}{2}\right)$

Conditions for constructive Interference:

If the two coherent sources S_1 and S_2 vibrating in phase, then at an arbitrary point P,

The phase difference: $\phi = 0, \pm 2\pi, \pm 4\pi \dots$ [OR path difference: $\delta = n\lambda$, (Where $n = 0, 1, 2, 3, \dots$)] constructive interference takes place leading to maximum intensity $= 4I_0$ and Resultant amplitude $= 2a$

Conditions for destructive Interference:

If the point P is such that the phase difference: $\phi = \pm \pi, \pm 3\pi, \pm 5\pi \dots$ [OR path difference: $\delta = \left(n + \frac{1}{2}\right)\lambda$

(Where $n = 0, 1, 2, 3, \dots$)] , destructive interference takes place, leading to zero amplitude and zero intensity.

3. Derivation of expression for fringe width:

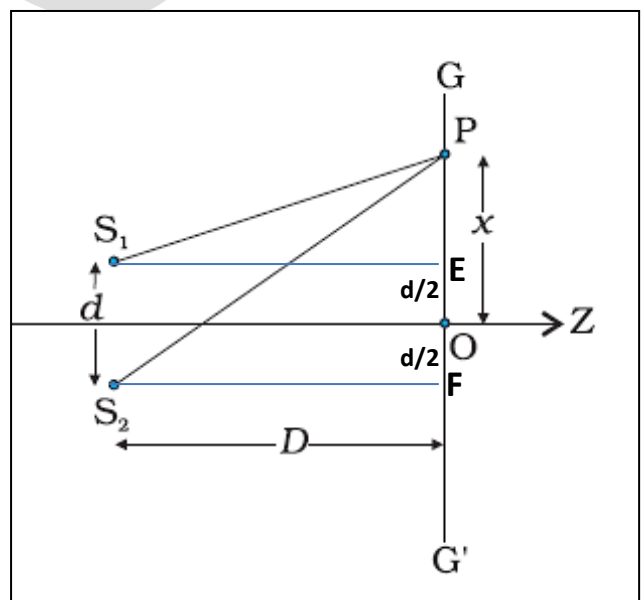
In the adjacent figure S_1 and S_2 represent two coherent source (slits in Young's double slit experiment) separated by a distance 'd'.

Let a screen be placed at a distance 'D' from the coherent sources.

The point O on the screen is equidistant from S_1 and S_2 so that the path difference between the two light waves from S_1 and S_2 reaching O is zero. Thus the point O has maximum intensity. Consider a point P at a distance x from O. The path difference between the light waves from S_1 and S_2 reaching the point P is $\delta = S_2P - S_1P$

From the figure, $(S_2P)^2 = (S_2F)^2 + (FP)^2 = D^2 + \left(x + \frac{d}{2}\right)^2$

Similarly, $(S_1P)^2 = (S_1E)^2 + (EP)^2 = D^2 + \left(x - \frac{d}{2}\right)^2$



$$\begin{aligned}\therefore (S_2P)^2 - (S_1P)^2 &= \left[D^2 + \left(x + \frac{d}{2} \right)^2 \right] - \left[D^2 + \left(x - \frac{d}{2} \right)^2 \right] \\ &= \left[D^2 + x^2 + \frac{d^2}{4} + 2(x)\left(\frac{d}{2}\right) \right] - \left[D^2 + x^2 + \frac{d^2}{4} - 2(x)\left(\frac{d}{2}\right) \right] = 2xd\end{aligned}$$

i.e., $(S_2P - S_1P)(S_2P + S_1P) = 2xd$ OR $(S_2P - S_1P) = \frac{2xd}{(S_2P + S_1P)}$

Since P is very close to O and $d \ll D$, $(S_2P + S_1P) \approx 2D$

\therefore Path difference: $(S_2P - S_1P) = \frac{2xd}{2D} = \frac{xd}{D}$ (1)

Equation (1) represents the path difference between light waves from S_1 and S_2 superposing at the point P. For bright fringe or maximum intensity at P, the path difference must be multiple of λ , where λ is the wavelength of the light used. i.e., $S_2P - S_1P = n\lambda$; $n = 0, 1, 2 \dots$

From equn.(1), $\frac{xd}{D} = n\lambda$ or $x = n\left(\frac{\lambda D}{d}\right)$

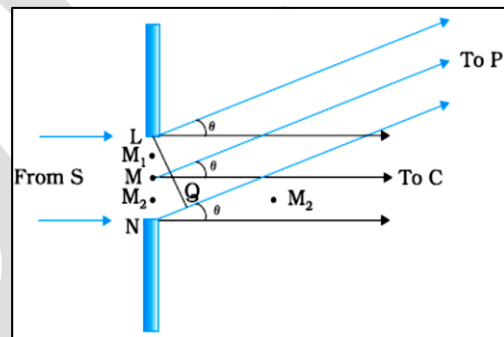
The distance of the n^{th} bright fringe from the centre O of the screen is $x_n = n\left(\frac{\lambda D}{d}\right)$

The distance of $(n+1)^{\text{th}}$ bright fringe from the centre of the screen is $x_{n+1} = (n+1)\left(\frac{\lambda D}{d}\right)$

The fringe width, $\beta = x_{n+1} - x_n = (n+1)\left(\frac{\lambda D}{d}\right) - n\left(\frac{\lambda D}{d}\right) = \frac{\lambda D}{d}$ $\therefore \beta = \frac{\lambda D}{d}$

4. DIFFRACTION OF LIGHT AT SINGLE SLIT:

- When single narrow slit illuminated by a monochromatic light source, a broad pattern with a central bright region is seen. On both sides, there are alternate dark and bright regions; the intensity becomes weaker away from the centre.
- A parallel beam of light falling normally on a single slit LN of width a . The diffracted light goes on to meet a screen. The midpoint of the slit is M. A straight line through M perpendicular to the slit plane meets the screen at C.
- The straight lines joining P to the different points L, M, N, etc., can be treated as parallel, making an angle θ with the normal MC. This is to divide the slit into smaller parts, and add their contributions at P with the proper phase differences.
- Different parts of the wavefront at the slit are treated as secondary sources. Because the incoming wavefront is parallel to the plane of the slit, these sources are in phase.
- The path difference between the two edges of the slit N and P is $NP - LP = NQ = a \sin \theta \approx a \theta$
- At the central point C on the screen, the angle θ is zero. The path difference is zero and hence all the parts of the slit contribute in phase. This gives maximum intensity at C, the central maximum.
- Secondary maxima is formed at $\theta \approx \left(n + \frac{1}{2}\right) \frac{\lambda}{a}$, where $n = \pm 1, \pm 2, \pm 3, \dots$
- Minima (zero intensity) is formed at $\theta \approx \frac{n\lambda}{a}$, Where $n = \pm 1, \pm 2, \pm 3, \dots$



ANSWERS FOR FIVE MARKS NUMERICAL PROBLEMS:

1. For refracted light, wavelength: $\lambda' = \lambda/n = 5.89 \times 10^{-7} / 1.33 = 4.43 \times 10^{-7} \text{ m}$

As frequency remain unaffected on entering another medium $= \nu' = \nu = c/\lambda = 5.09 \times 10^{14} \text{ Hz}$

Speed of light in water, $v = c/n = 2.25 \times 10^8 \text{ m/s}$

2. Angular fringe width: $\theta = \frac{\lambda}{d} = 0.2^\circ = 0.2 \left(\frac{\pi}{180} \right) = 3.49 \times 10^{-3} \text{ rad.}$

Fringe width $\beta = \frac{\lambda D}{d} = \left(\frac{\lambda}{d} \right) D = 3.49 \times 10^{-3} \times 0.8 = 2.79 \text{ mm.}$

When entire experimental apparatus is immersed in water, wavelength of light $\lambda' = \lambda / n$

Hence angular fringe width in water, $\theta' = \frac{\lambda'}{d} = \frac{\lambda}{n d} = \frac{0.2^\circ}{4/3} = 0.15^\circ.$

3. Given $\lambda_1 = 650 \text{ nm} = 650 \times 10^{-9} \text{ m}$ and $\lambda_2 = 520 \times 10^{-9} \text{ m.}$

a) The distance of n^{th} bright fringe on the screen from central maximum, $x_n = \frac{n D \lambda}{d}$

\therefore The distance of third bright fringe on the screen from central maximum

$$x_3 = \frac{3 \times 0.6 \times 650 \times 10^{-9}}{1 \times 10^{-3}} = 1.17 \times 10^{-3} \text{ m} = 1.17 \text{ mm}$$

b) For least distance x , the n^{th} bright fringe due to longer wavelength (λ_1) coincides with the $(n+1)^{\text{th}}$ bright fringe due to shorter wavelength (λ_2).

$$\frac{n D \lambda_1}{d} = \frac{(n+1) D \lambda_2}{d} \quad \text{or} \quad n \lambda_1 = (n+1) \lambda_2 \Rightarrow n = 4$$

\therefore Required least distance, $x_4 = \frac{4 D \lambda_1}{d} = 1.56 \text{ mm}$

4. If the phase difference between the two waves is ϕ , then the intensity at that point is $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right).$

Given path difference $= \lambda$, the corresponding phase difference is 2π .

Thus, $I = 4 I_0 \cos^2\left(\frac{2\pi}{2}\right) = 4 I_0 = K \text{ (given).}$

(i) Given path difference $= \lambda/3 \Rightarrow$ phase difference $= 2\pi/3$

The intensity at that point is $I = 4I_0 \cos^2\left(\frac{2\pi/3}{2}\right) = K(1/2)^2 = K/4.$

(ii) Given path difference $= \lambda/2 \Rightarrow$ phase difference $= \pi.$

The intensity at that point is $I = 4I_0 \cos^2\left(\frac{\pi}{2}\right) = 0.$

5. Given $\lambda = 500 \text{ nm} = 5 \times 10^{-7} \text{ m}$, $D = 1.25 \text{ m}$, $x = 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m.}$

Angular position of the first minimum, $\tan \theta \approx \theta = \frac{x}{D} = \frac{2.5 \times 10^{-3}}{1.25} = 2 \times 10^{-3} \text{ rad.}$ (Assuming θ to be small)

(i) The width of the slit, $a = \frac{\lambda}{\theta} = \frac{5 \times 10^{-7}}{2 \times 10^{-3}} = 2.5 \times 10^{-4} \text{ m.}$

(ii) Secondary maxima is formed at $\theta \approx \left(n + \frac{1}{2}\right) \frac{\lambda}{a}$

For the first secondary maximum $n = 1$,

Thus, the angular position of the first secondary maximum, $\theta \approx \frac{3}{2} \left(\frac{\lambda}{a} \right) = \frac{3}{2} \left(\frac{5 \times 10^{-7}}{2.5 \times 10^{-4}} \right) = 3 \times 10^{-3} \text{ rad.}$

6. Given $D = 1.4 \text{ m}$, $x_4 = 1.2 \text{ cm} = 1.2 \times 10^{-2} \text{ m}$, $d = 0.28 \text{ mm} = 0.28 \times 10^{-3} \text{ m}$, $\lambda = ?$ and for fifth dark fringe $x = ?$

Distance of n^{th} bright fringe from central bright fringe: $x_n = \frac{n \lambda D}{d}$

$$\therefore \text{Wavelength of light: } \lambda = \frac{x_n d}{n D} = \frac{(1.2 \times 10^{-2})(0.28 \times 10^{-3})}{4(1.4)} = 6 \times 10^{-7} \text{ m} = 600 \text{ nm}$$

Distance of dark fringe from central bright fringe: $x = \left(n + \frac{1}{2} \right) \left(\frac{\lambda D}{d} \right)$

$$n = 4 \text{ for fifth dark fringe, } x = \left(4 + \frac{1}{2} \right) \frac{(6 \times 10^{-7})(1.4)}{(0.28 \times 10^{-3})} = 1.35 \times 10^{-2} \text{ m} = 1.35 \text{ cm}$$

7. Given $d = 1 \text{ mm} = 10^{-3} \text{ m}$. Let the initial distance of the screen from the slits be D .

When the screen is moved 5cm towards the slits the fringe width decreases by $30 \mu\text{m}$.

$$D' = D - 5 \times 10^{-2} \text{ m and } \beta' = \beta - 30 \times 10^{-6} \text{ m.}$$

$$\text{Initial fringe width: } \beta = \frac{\lambda D}{d} \dots\dots\dots(1)$$

$$\text{The new fringe width: } \beta' = \frac{\lambda(D - 5 \times 10^{-2})}{d} \Rightarrow \beta - 30 \times 10^{-6} = \frac{\lambda D}{d} - \frac{\lambda(5 \times 10^{-2})}{d} \dots\dots\dots(2)$$

$$\text{Using equn.(1) in RHS of equn.(2), } \beta - 30 \times 10^{-6} = \beta - \frac{\lambda(5 \times 10^{-2})}{d} \Rightarrow 30 \times 10^{-6} = \frac{\lambda(5 \times 10^{-2})}{d}$$

$$\Rightarrow 30 \times 10^{-6} = \frac{\lambda(5 \times 10^{-2})}{10^{-3}} \Rightarrow \lambda = \frac{(30 \times 10^{-6})(10^{-3})}{(5 \times 10^{-2})} = 6 \times 10^{-7} \text{ m} = 600 \text{ nm}$$
