Electrostatic Potential and Capacitance

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1. Two charges $5*10^{-8}C$ and $-3*10^{-8}C$ are located $16cm$ apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Solutions :

There are two charges,
$q_1 = 5*10^{-8}C$
$q_2 = -3*10^{-8}C$
Distance between the two charges, $d = 16cm = 0.16m$
Consider a point P on the line joining the two charges, as shown in the given figure.
r =Distance of point P from charge q_1
Let the electric potential (V) at point P be zero.
Potential at point P is the sum of potentials caused by charges q_1 and q_2 respectively.
$\therefore V = \frac{q_1}{4\pi \in_0 r} + \frac{q_2}{4\pi \in_0 (d-r)} ....(i)$
Where,
$\in_0$=Permittivity of free space
For $V = 0$, equation (i) reduces to
$0 = \frac{q_1}{4\pi \in_0 r} + \frac{q_2}{4\pi \in_0 (d-r)}$
$\frac{q_1}{4\pi \in_0 r} = -\frac{q_2}{4\pi \in_0 (d-r)}$
$\frac{q_1}{r} = \frac{-q_2}{d-r}$
$\frac{5*10^{-8}}{r} = -\frac{(-3*10^{-8})}{(0.16-r)}$
$\frac{0.16}{r} = \frac{8}{5}$
$\therefore r = 0.1m = 10cm$
Therefore, the potential is zero at a distance of $10cm$ from the positive charge between the charges.
Suppose point P is outside the system of two charges at a distance s from the negative charge, where potential is zero, as shown in the following figure. For this arrangement, potential is given by,
$V = \frac{q_1}{4\pi \in_0 s} + \frac{q_2}{4\pi \in_0 (s-d)} ....(ii)$
Where
$\in_0$=Permittivity of free space
For$V = 0$, equation (ii) reduces to
$\frac{q_1}{4\pi \in_0 s} = \frac{q_2}{4\pi \in_0 (s-d)}$
$\frac{q_1}{s} = \frac{-q_2}{s-d}$
$\frac{5*10^{-8}}{s} = -\frac{(-3*10^{-8})}{(s-0.16)}$
$1 - \frac{0.16}{s} = \frac{3}{5}$
$\frac{0.16}{s} = \frac{2}{5}$
$\therefore s = 0.4m = 40cm$
Therefore, the potential is zero at a distance of $40cm$ from the positive charge outside the system of charges.

2. A regular hexagon of side $10cm$ has a charge $5\mu C$ at each of its vertices. Calculate the potential at the centre of the hexagon.

Solutions :

The given figure shows six equal amount of charges, q , at the vertices of a regular hexagon.
Where,
Charge, $q = 5\mu C = 5*10^{-6}C$
Side of the hexagon,$I = AB = BC = CD = DE = EF = FA = 10cm$
Distance of each vertex from centre $O, d = 10cm$
Electric potential at point O,
$V = \frac{6*q}{4\pi \in_0 d}$
Where,$\in_0$=Permittivity of free space
$\frac{1}{4\pi \in_0} = 9*10^9 NC^{-2}m^{-2}$
$\therefore V = \frac{6*9*10^9*5*10^{-6}}{0.1}$
$= 2.7*10^6V$
Therefore, the potential at the centre of the hexagon is $2.7*10^6V$.

3. Two charges $2\mu C$ and $-2\mu C$ are placed at points A and $B, 6cm$ apart.
(a) Identify an equipotential surface of the system.
(b) What is the direction of the electric field at every point on this surface?

Solutions :

(a) The situation is represented in the given figure. An equipotential surface is the plane on which electric potential is same at every point. One of such plane is normal to line AB. The plane is located at the mid-point of line AB because the magnitude of charges is the same.
(b) The direction of the electric field at every point on this surface is normal to the plane in the direction of AB.

4. A spherical conductor of radius $12cm$ has a charge of $1.6 * 10^{-7}C$ distributed uniformly on its surface. What is the electric field
   (a) inside the sphere
   (b) just outside the sphere
   (c) at a point 18 cm from the centre of the sphere?

   ## Solutions :

   (a) Radius of the spherical conductor, $r = 12cm = 0.12m$
   Charge is uniformly distributed over the conductor, $q = 1.6 * 10^{-7}C$
   Electric field inside a spherical conductor is zero. This is because the net charge inside a conductor is zero.
   (b) Electric field E just outside the conductor is given by the relation,
   $E = \frac{q}{4\pi \in_0 r^2}$
   Where,
   $\in_0$=Permittivity of free space
   $\frac{1}{4\pi \in_0} = 9 * 10^9 Nm^2C^{-2}$
   $\therefore E = \frac{1.6 * 10^{-7} * 9 * 10^{-9}}{(0.12)^2}$
   $= 10^5 NC^{-1}$
   Therefore, the electric field just outside the sphere is $10^5 NC^{-1}$.
   (c) Electric field at a point $18m$ from the centre of the sphere $= E_1$
   Distance of the point from the centre, $d = 18cm = 0.18m$
   $E_1 = \frac{q}{4\pi \in_0 d^2}$
   $= \frac{9 * 10^9 * 1.6 * 10^{-7}}{(18 * 10^{-2})^2}$
   $= 4.4 * 10^4 N/C$
   Therefore, the electric field at a point $18cm$ from the centre of the sphere is $4.4 * 10^4 N/C$.

5. A parallel plate capacitor with air between the plates has a capacitance of $8pF (1pF = 10^{-12}F)$. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

   ## Solutions :

   Capacitance between the parallel plates of the capacitor, $C = 8pF$
   Initially, distance between the parallel plates was d and it was filled with air. Dielectric constant of air, $k = 1$
   Capacitance, C, is given by the formula,
   $C = \frac{k \in_0 A}{d}$
   $= \frac{\in_0 A}{d} ....(i)$
   Where,
   A=Area of each plate
   $\in_0$= Permittivity of free space
   If distance between the plates is reduced to half, then new distance, $d' = \frac{d}{2}$
   Dielectric constant of the substance filled in between the plates,$k' = 6$
   Hence, capacitance of the capacitor becomes
   $C' = \frac{k' \in_0 A}{d} = \frac{6 \in_0 A}{d/2} .....(ii)$
   Taking ratios of equations (i) and (ii), we obtain
   $C' = 2 * 6C$
   $= 12C$
   $12 * 8 = 96pF$
   Therefore, the capacitance between the plates is $96pF$.

6. Three capacitors each of capacitance $9pF$ are connected in series.
   (a) What is the total capacitance of the combination?
   (b) What is the potential difference across each capacitor if the combination is connected to a $120V$ supply?

   ## Solutions :

   (a) Capacitance of each of the three capacitors, $C = 9pF$
   Equivalent capacitance (C') of the combination of the capacitors is given by the relation,
   $\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$
   $= \frac{1}{9} + \frac{1}{9} + = \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$
   $\therefore C' = 3\mu F$
   Therefore, total capacitance of the combination is $3\mu F$.
   (b) Supply voltage, $V = 100V$
   Potential difference (V') across each capacitor is equal to one-third of the supply voltate.
   $\therefore V' = \frac{V}{3} = \frac{120}{3} = 40V$

   Therefore, the potential difference across each capacitor is $40V$.

7. Three capacitors of capacitance $2pF, 3pF$ and $4pF$ are connected in parallel.
   (a) What is the total capacitance of the combination?
   (b) Determine the charge on each capacitor if the combination is connected to a $100V$ supply.

Solutions :

(a) Capacitances of the given capacitors are
$C_1 = 2pF; C_2 = 3pF; C_3 = 4pF$
For the parallel combination of the capacitors, equivalent capacitor C' is given by the algebraic sum,
$C' = C_1 + C_2 + C_3 = 2 + 3 + 4 = 9pF$
Therefore, total capacitance of the combination is $9pF$.
(b) Supply voltage, $V = 100V$
The voltage through all the three capacitors is same $= V = 100V$
Charge on a capacitor of capacitance C and potential difference V is given by the relation,
$q = VC \quad .....(i)$
For $C = 2pF$,
Charge $= VC = 100 * 2 = 200pC = 2 * 10^{-10}C$
For $C = 3pF$,
Charge $= VC = 100 * 3 = 300pC = 3 * 10^{-10}C$
For $C = 4pF$
Charge $= VC = 100 * 4 = 200pC = 4 * 10^{-10}C$

8. In a parallel plate capacitor with air between the plates, each plate has an area of $6 * 10^{-3}m^2$ and the distance between the plates is $3mm$. Calculate the capacitance of the capacitor. If this capacitor is connected to a $100V$ supply, what is the charge on each plate of the capacitor?

Solutions :

Area of each plate of the parallel plate capacitor, $A = 6 * 10^{-3}m^2$ Distance between the plates, $d = 3mm = 3 * 10^{-3}m$
Supply voltage, $V = 100V$
Capacitance C of a parallel plate capacitor is given by,
$C = \frac{\in_0 A}{d}$
Where,
$\in_0$= Permittivity of free space
$= 8.854 * 10^{-12}C^2N^{-1}m^{-2}$
$\therefore C = \frac{8.854 * 10^{-12} * 6 * 10^{-3}}{3 * 10^{-3}}$
$= 17.71 * 10^{-12}F$
$= 17.71pF$
Potential V is related with the charge q and capacitance C as
$V = \frac{q}{C}$
$\therefore q = VC$
$= 100 * 17.71 * 10^{-12}$
$= 17.71 * 10^{-9}C$
Therefore, capacitance of the capacitor is $17.71pF$ and charge on each plate is $1.771 * 10^{-9}C$

9. Explain what would happen if in the capacitor given in Exercise 8, a $3mm$ thick mica sheet (of dielectric constant =6) were inserted between the plates,
(a) while the voltage supply remained connected.
(b) after the supply was disconnected.

Solutions :

(a) Dielectric constant of the mica sheet, $k = 6$
Initial capacitance, $C = 1.771 * 10^{-11}F$
New capacitance,$C' = kC = 6 * 1.771 * 10^{-11} = 106pF$
Supply voltage, $V = 100V$
New charge, $q' = C'V = 106 * 100pC = 1.06 * 10^{-8}C$
Potential across the plates remains $100V$.
(b) Dielectric constant, $k = 6$
Initial capacitance, $C = 1.771 * 10^{-11}F$
New capacitance, $C' = kC = 1.771 * 10^{-11}F = 106pF$
If supply voltage is removed, then there will be no effect on the amount of charge on the plates.
Potential across the plates is given by,
$\therefore V' = \frac{q}{C'}$
$= \frac{1.771 * 10^{-9}}{106 * 10^{-12}} = 16.7V.$

10. A $12pF$ capacitor is connected to a $50V$ battery. How much electrostatic energy is stored in the capacitor?

Solutions :

Capacitor of the capacitance, $C = 12pF = 12 * 10^{-12}F$
Potential difference, $V = 50V$
Electrostatic energy stored in the capacitor is given by the relation,
$E = \frac{1}{2}CV^2$
$= \frac{1}{2} * 12 * 10^{-12} * (50)^2$
$= 1.5 * 10^{-8}J$
Therefore, the electrostatic energy stored in the capacitor is $1.5 * 10^{-8}J$.