

Chapter 13

NUCLEI

- **Composition of Nucleus**

The Nucleus of an atom contains Protons and Neutrons. Protons are positively charged and Neutrons are chargeless. To bind a nucleus together there must be a strong attractive force, enough to overcome the coulomb's force of repulsion between the protons. This strong short range force which binds the nucleons (Protons & Neutrons) together is called Nuclear force.

The following terms and symbols are used to describe a nucleus.

Z = Atomic Number (Number of Protons)

N = Number of neutrons

$A = Z + N$, mass number (Total no. of Nucleons - protons and Neutrons)

- **Atomic Mass**

Major part of an atomic mass is concentrated in the nucleus. Atomic mass is expressed in atomic mass unit (amu or u)

$$1u = \frac{1^{th}}{12} \text{ mass of } C^{12} \text{ atom}$$

$$1u = 1.6605 \times 10^{-27} \text{ kg}$$

- **Size of Nucleus**

Assuming nucleus to be a sphere of radius R , its volume $\frac{4}{3}\pi R^3$ is found to be proportional to mass number A

$$\text{ie, } \frac{4}{3}\pi R^3 \propto A \quad \text{OR} \quad R \propto A^{1/3} \quad \text{OR} \quad R = R_0 A^{1/3}$$

$$\text{Where } R_0 = 1.2 \times 10^{-15} \text{ m}$$

Density of nucleus is very large and independent mass number A

$$\text{Nuclear density } \rho = \frac{\text{mass}}{\text{Volume}} = \frac{A \times 1.66 \times 10^{-27}}{\frac{4}{3}\pi R_0^3 A}$$

$$= \frac{1.66 \times 10^{-27}}{\frac{4}{3}\pi (1.2 \times 10^{-15})^3}$$

$$= 2.4 \times 10^{17} \text{ kg / m}^3$$

Nuclear radius of ${}^{27}_{13}\text{Al}$ is $3.9 \times 10^{-15} \text{ m}$. Find nuclear radius of ${}^{216}_{84}\text{Po}$

$$R_{Al} = R_0 (27)^{1/3}$$

$$R_x = R_0 (216)^{1/3}$$

$$\frac{R_x}{R_{Al}} = \left(\frac{216}{27} \right)^{1/3} = 3.9 \times 10^{-15} \left(\frac{216}{27} \right)^{1/3} = 7.8 \times 10^{-15} \text{ m}$$

- What was the mass defect and Binding Energy

The difference between total mass of nucleons (ie, protons and Neutrons) and the mass of nucleus is called mass defect.

$$\text{ie, mass defect } \Delta m = Zm_p + (A - Z)m_n - M$$

Where Z - Atomic number

A - Mass number

M_p - Mass of proton (1.0073u)

M_n - Mass of neutron (1.0087 u)

M - Mass of Nucleus

The energy equivalent of mass defect is called binding energy

$$BE = \Delta mc^2$$

$$= (Zm_p + (A - Z)m_n - M) C^2$$

$$\text{Note : For mass defect of 1u, } BE = \frac{1 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2}{1.6 \times 10^{-13}} \text{ meV} = 931 \text{ meV}$$

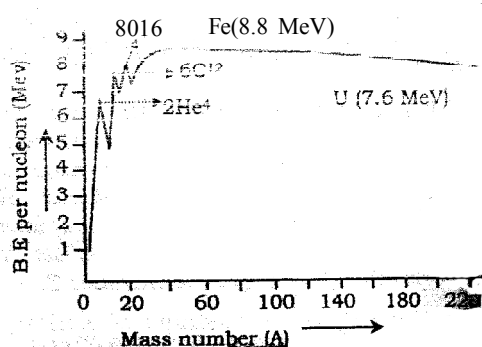
Note : BE is the energy needed to separate the nucleons apart

- Binding energy per nucleon $= \frac{BE}{A} = \frac{\Delta mc^2}{A}$

$$= \frac{(Zm_p + (A - Z)m_n - m) C^2}{A}$$

- Discuss the variation of Binding energy per nucleon with mass number.

- The Binding energy per nucleon is practically independent for nuclei of middle mass number ($30 < A < 170$). The curve has maximum about 8.75 meV for $A = 56$ (${}_{26}\text{Fe}^{56}$) and has a value 7.6 meV for $A = 238$ (${}_{92}\text{U}^{238}$)
- Binding energy per nucleon is lower for both light nuclei ($A < 30$) and heavy nuclei ($A > 170$)



Radioactivity

It is the phenomenon by which an unstable nucleus ($A > 206$) decays by emitting particles such as α , β , γ in order to achieve stability. This was discovered by Hency Becquerd in 1896.

- Activity
Activity of radioactive sample is the rate of decay of the nucleus.

$$\text{Activity } A = \frac{dN}{dt}$$

Where N is the number radioactive nuclei in a sample.

- SI unit of Activity is Becquerd (Bq)
1 Bq = 1 disintegration/second

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

$$1 \text{ Rutherford} = 1 \text{ Rd} = 10^6 \text{ Bq}$$

- **Radioactive Decay law**

This law states that rate of disintegration is directly proportional to total number of nuclei in that sample.

If N is the number of nuclei present in a sample at a time ' t '

$$\frac{dN}{dt} \propto N$$

$$\text{or } \frac{dN}{dt} = -\lambda N \text{ Where } \lambda \text{ is called decay const. or disintegration const.}$$

$$\text{or } \frac{dN}{dt} = -\lambda dt$$

$$\text{Integrating } \int_{N_0}^N \frac{dN}{N} = - \int_0^t \lambda dt$$

$$\log N - \log N_0 = -\lambda t$$

Where N_0 is the initial number of nuclei present in the sample (at $t=0$)

$$\text{or } \log \left(\frac{N}{N_0} \right) = -\lambda t$$

$$N(t) = N_0 e^{-\lambda t}$$

This eqn. gives the number of radioactive nuclei present in the sample after a given time ' t '. If N_0 is the number of nuclei at $t=0$

- **Half life :** It is the time taken by the radioactive sample to decay to half the initial number.

$$(\text{ie, } N \text{ becomes } \frac{N_0}{2})$$

$$\therefore \text{ When } t = T_{1/2}$$

$$N(t) = \frac{N_0}{2}$$

$$N(t) = N_0 e^{-\lambda t}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$e^{-\lambda T_{1/2}} = \frac{1}{2}$$

$$\lambda T_{1/2} = \log 2$$

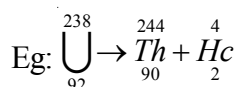
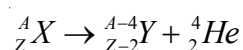
$$T_{1/2} = \frac{\log 2}{\lambda} = \frac{0.693}{\lambda}$$

$$\text{ie, } T_{1/2} = \frac{0.693}{\lambda} \text{ or } T_{1/2} = \tau 0.693$$

Where $\tau = \frac{1}{\lambda}$ is called mean life of the sample.

- **α - decay**

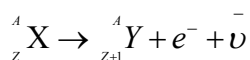
In α - decay the mass number of daughter nucleus is four less than parent nucleus, While atomic number decreases by 2.



Energy released during α decay

$$Q = [m_x - (m_y + m_\alpha)] c^2$$

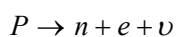
- **β^- decay**



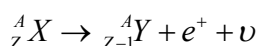
e^- - Beta minus, $\bar{\nu}$ - antineutrino

(Here a neutron in the nucleus decays to proton, electron and antineutrino. This electron is emitted as β^- and the proton increases the atomic number by one)

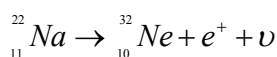
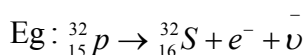
For β^+ plus decay, a proton transform into,



Here e^+ is emitted as β^+ plus and atomic number decreased by one.



ν - neutrino $e^+ \rightarrow \beta^+$ plus



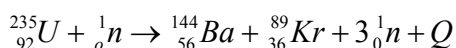
Note : -ve electron - negatron
+ve electron - positron
They are collectively called positronium

γ - Decay

When a nucleus in a excited state (Daughter nucleus formed after the α or β decay) spontaneously decays to ground state, a photon is emitted with energy equal to the difference in the two energy levels of the nucleus. This decay is called γ - decay.

Nuclear Fission

It is the process by which a heavy nucleus splits into two or more light nuclei.



The self sustained nuclear fission process is called chain reaction. In a nuclear Reactor chain reaction takes place in a controlled manner.

Only slow neutrons can induce fission in a fissionable material so the slowing down of fast neutrons liberated in the initial fission process will sustain the chain reaction.

In a nuclear reactor this process is achieved by neutron moderators (Eg: Heavy water, graphite) These moderators should not absorb neutrons, they should only reduce the speed of neutrons.

In a nuclear reactor to control the chain reaction by absorbing extra neutrons, control rods are used (Eg: Calcium, Boron, Cadmium)

Nuclear Fusion

It is the process by which two or more light nucleus combine to form a heavy nucleus with the release of energy. It is the source of energy in stars. The energy released during fusion is called Thermonuclear energy. Fusion process need extremely high temperature to initiate the process.

