PHYSICS

CLASS NOTES FOR CBSE

Chapter 27. Dual Nature of Radiation and Matter

01. Dual Nature of Radiation

The phenomena such as interference, diffraction and polarization were success-fully explained on the basis of were nature of light. On the other hand, photoelectric effect, Compton effect, etc can be explained on the basis of quantum nature of radiation.

02. Electron Emission

A metal has free electrons, but these electrons cannot come out of the metal surface. It is because, as an electron makes an attempt to come out of the metal surface, the metal surface acquires positive charge and the electron is pulled back into the metal. Thus, free electrons are held inside the metal surface by the positive ions, it contains. In order that an electron may come out of the metal surface, it must possess sufficient energy to overcome the attractive pull of the positive ions.

The minimum amount of energy required to eject an electron out of a metal surface is called **work function** of the metal. It is denoted by ω.

The work function of a metal depends on the nature of the metal. The following table gives the work functions of a few metals:

Work functions of some metals

Metal	Cs	K	Na	Ca	Mo	Al	Cu	Ni	Pt
Work function (eV)	2.14	2.30	2.75	3.20	4.17	4.28	4.65	5.15	5.65

By supplying energy atleast equal to work function, the electron emission can be caused from a metal surface by the following processes:

(i) **Thermionic emission.** It can be caused by supplying the minimum required energy by heating the metal surface to a suitable temperature.

(ii) **Field emission.** It can be caused by applying an electric field to the metal surface. If the electric field is sufficiently strong $\left(\approx 10^8\ \mathrm{V\,m^{-1}}\right)$, the electrons get pulled out of the metal surface.

(iii) **Photoelectric emission.** It can be caused by supplying the minimum required energy by illuminating the metal surface with light of suitable frequency.

(iv) **Secondary emission.** The process of emission of free electrons when highly energetic election beam is incident on a metal surface is called secondary emission. The electrons so emitted are called secondary electrons.

03. Photoelectric Effect

The phenomenon of emission of electrons from (preferably) metal surface exposed to light energy of suitable frequency is known as photoelectric effect.
The emitted electrons are called photo electrons and the current so produced is called photoelectric current.
Alkali metals (lithium, sodium, potassium, caesium etc.) show photoelectric effect with visible light, whereas the metals like zinc, cadmium magnesium etc. are sensitive only to ultraviolet light.

Hertz's Observations

The phenomenon of photoelectric emission was discovered in 1887 by Heinrich Hertz (1857–1894) while studying experimentally the production of electromagnetic waves by means of spark discharge. He found that when the emitter plate was illuminated by ultraviolet light, high-voltage sparks across the detector loop were enhanced. This observation led him to conclude that light facilitated the emission of some electrons.
From this it was concluded that when suitable radiation falls on a metal surface, some electrons near the surface absorb enough energy from the incident radiation to overcome the attraction of the positive ions in the material of the surface.

Hallwachs' and Lenard's Observations

Wilhelm Hallwachs and Philipp Lenard studied in detail the phenomenon of photoelectric effect during 1886–1902.
Lenard (1862–1947) observed flow of current when ultraviolet radiations is exposed on the emitter plate of an evacuated glass tube enclosing two electrodes (metal plate). The current flow stops as soon as the ultraviolet radiations were stopped. These observation indicated that electrons are ejected from emitter plate C, when ultraviolet radiations fall on it which are attracted towards the positive, collector plate A by the electric field.
Thus current in the external circuit is due to light falling on the surface of the emitter.
Hallwachs and Lenard studied variation of photocurrent with collector plate potential and with frequency and intensity of incident light.
Hallwachs, in 1888 for further study, connected a negatively charged zinc plate to an electroscope. He found that when zinc plate was illuminated by ultraviolet light it has lost its charge. When uncharged zinc plate was illuminated by ultraviolet light, it became positively charged. Further when positively charge zinc plate was illuminated by ultraviolet light it was found to be further enhanced. He concluded from these observations that under the action of ultraviolet light negatively charged particles were emitted from the zinc plate.
It became evident after the discovery of the electron in 1897 that the incident light causes electrons to be emitted from the emitter plate. The emitted electrons due to tis negative charge are pushed towards the collector plate by the electric field. Hallwachs and Lenard also observed that when the frequency of the incident light was smaller than a certain minimum value, no electrons were emitted at all from the emitter plate. This minimum frequency is called the threshold frequency and it depends on the nature of the material of the emitter plate.
It became evident after the discovery of the electron in 1897 that the incident light causes electrons to be emitted from the emitter plate. The emitted electrons due to tis negative charge are pushed towards the collector plate by the electric field.

Hallwachs and Lenard also observed that when the frequency of the incident light was smaller than a certain minimum value, no electrons were emitted at all from the emitter plate. This minimum frequency is called the threshold frequency and it depends on the nature of the material of the emitter plate.
It was found that some alkali metals such as lithium, sodium, potassium, caesium and rubidium were sensitive even to visible light whereas certain metals like zinc, cadmium, magnesium etc. responded only to ultraviolet light, having short wavelength for electron emission from the surface. Electrons are emitted, when photosensitive substances are illuminated by light. These electrons were termed as photoelectrons after the discovery of electrons. This phenomenon is known as photoelectric effect. The electric current constituted by photo-electrons is known as photoelectric current.

04. Experimental Study of Photoelectric Effect

The experimental set-up to study photoelectric effect is shown in figure. It consists of an evacuated glass or quartz tube having two electrodes. The electrode ‘C’ is a photosensitive plate, which emits photoelectrons when exposed to ultraviolet radiation. The electrode ‘A’ is a charge-collecting plate. The tube has a side window, which will allow the light of a particular wavelength to pass through it and falls on the photosensitive plate ‘C’.

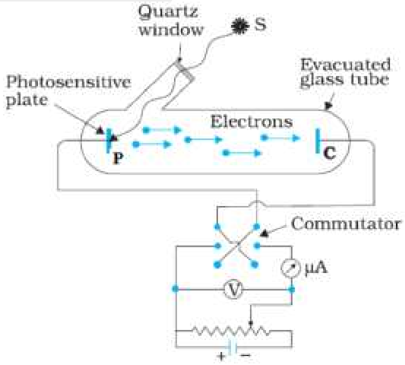


Fig.: Experimental arrangement for study of photoelectric effect.

The window is made of quartz covered with a filter. The electrons collected by the plate A (collector), are emitted by the plate C. Battery creates the electrical field between collector and emitter. The potential difference between the plates C and A is maintained by the battery, which can be varied. From a commutator the polarity of the plates C and A can be reversed. Thus with respect to emitter C, the plate A can be maintained at a desired positive or negative potential. The electrons are attracted when the collector plate A is positive with respect to the emitter plate C. Electron emission causes flow of electric current in the circuit. Voltameter (V) measures the potential difference between the emitter and collector plates. Microammeter (μA) measures the resulting photocurrent flowing in the circuit. The current flowing in the circuit can be increased or decreased by varying the potential between collector plate A and emitter plate C. We can also vary the intensity and frequency of the incident light.

To study the variation of photocurrent with (a) intensity of radiation (b) frequency of incident radiation (c) the potential difference between the plates A and C, and (d) the nature of the material of plate C, the experimental arrangement of above figure is used.
To get different frequency of light falling on the emitter C, suitable-coloured filter or coloured glass is used. The change is distance of light source from the emitter varies the intensity of light.

Effect of Intensity of Light on Photocurrent
To attract ejected electron from C towards collector A, the collector A is maintained at a positive potential with respect or emitter C. The intensity of light is varied, keeping the frequency of the incident radiation and the accelerating potential fixed and the resulting photoelectric current is measured each time. It is observed that the photocurrent increases linearly with intensity of incident light as shown in the figure.

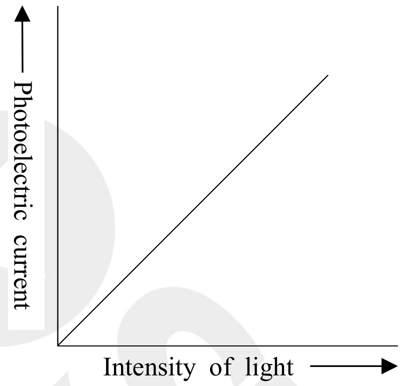


Fig.: Variation of Photoelectric current with intensity of light.

As we know the photocurrent is directly proportional to the number of photo electrons emitted per second, so the number of photo electrons emitted per second is directly proportional to the intensity of the incident radiations.

Effect of Potential on Photoelectric Current
Illuminate the plate C with radiation of fixed frequency (greater than threshold frequency) and fixed intensity, keeping the plate A at some accelerating positive with respect to plate C. Now we gradually vary the positive potential of plate A and measure the resulting photocurrent each time. For fixed frequency and fixed intensity of incident light, this photoelectric current increases with the increases in applied positive potential of plate 'A'. The photoelectric current has the maximum value, when all the photoelectrons emitted by electrode 'C' reach the plate 'A'. This maximum current is known as saturation current. Further increase in accelerating potential of plate A does not increase the current.
When the polarity is reversed (meaning applying a negative (retarding) potential to the plate A with respect to the plate C) and increase retarding potential gradually, then electrons are repelled. The photocurrent is found to decrease rapidly and at a certain, sharply defined critical value of the negative potential V_0 on the plate A, it drops to zero. The minimum negative (retarding) potential V_0 given to the plate A, for a particular frequency of incident radiation, for which the photocurrent stops or becomes zero is called the cut-off or stopping potential. At this stage photo electrons of maximum kinetic energy (the fastest photoelectron) cannot reach the plate A, therefore

$K_{max} = eV_0$ (K_{max} is maximum kinetic energy of photoelectron)

When we repeat this experiment with different intensity I_1, I_2 and $I_3 \left(I_3 > I_2 > I_1\right)$ of incident radiation and of the same frequency, we observe that the saturation currents have reached to higher values which implies that more electrons are emitted in a unit time, proportional to the intensity of incident radiation but there is no change in stopping potential for a given frequency of the incident radiation, graphically shown in figure.

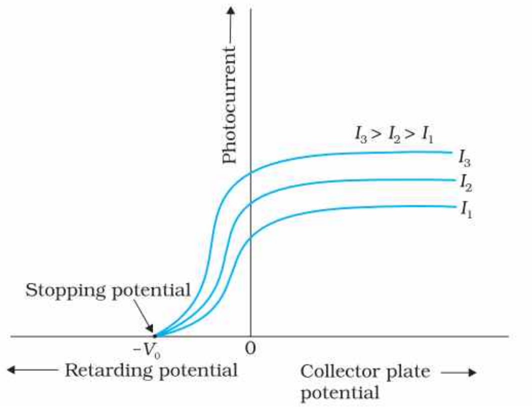


Fig.: Variation of photocurrent with collector plate potential for different intensity of incident radiation.

Thus stopping potential is independent of its intensity for a given frequency of the incident radiation or the maximum kinetic energy of photo electrons is independent of intensity of incident radiation but depends on frequency (color) of the light source and the emitter plate material.

Effect of Frequency of Incident Radiation on Stopping Potential

We now study the relation between stopping potential V_0 and the frequency v of the incident radiation. The resulting variation of photocurrent with collector plate potential for same intensity of light radiation at various frequencies is shown in figure.

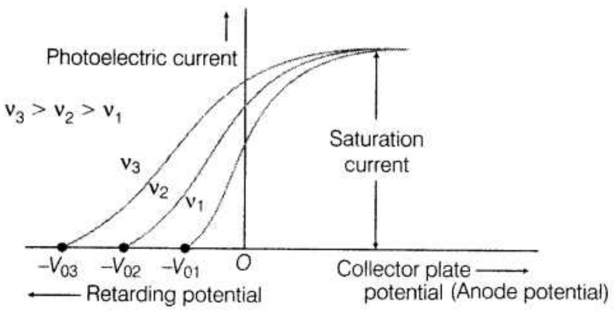


Fig.: Variation of photoelectric current with collector plate potential for different frequencies of incident radiation.

We get same value of the saturation current but different values of stopping potential for incident radiation of different frequencies. The emitted election energy depends on the incident radiations frequency. From figure, we note that stopping potentials are in the order $V_3 > V_2 > V_1$, for the frequencies in the order $v_3 > v_2 > v_1$. The stopping potential is more negative for higher frequencies of incident radiation.

This means, greater the frequency of incident light, greater is the maximum kinetic energy of the photo electrons. Therefore we require more retarding potential to stop them completely. We get a straight line if we plot a graph between the frequency of incident radiation and stopping potential for different metals as shown in figure.

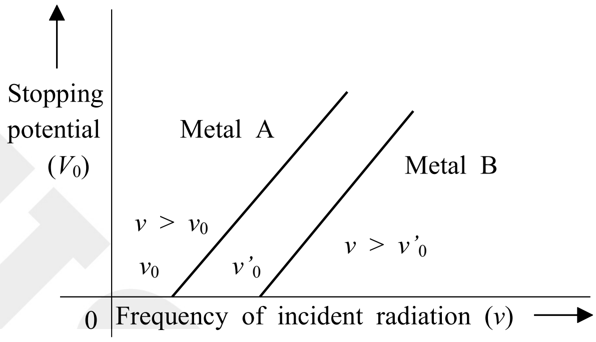


Fig.: Variation of stopping potential V_0 with frequency v of incident radiation for a given photosensitive material.

05. Laws of Photoelectric Emission

(i) The photoelectric current is directly proportional to the intensity of incident radiation (above the threshold frequency) for a given photosensitive material and frequency of incident radiation.

(ii) Saturation current is found to be proportional to the intensity of incident radiation whereas the stopping potential is independent of its intensity for a given photosensitive material and incident radiation.

(iii) There exists a certain minimum cut-of frequency, called threshold frequency of the incident radiation below which no emission takes place for a given photosensitive material irrespective of intensity of the incident radiation. The maximum kinetic energy or equivalently stopping potential above the threshold frequency of the emitted photo electrons increases linearly with the frequency of the incident radiation but is not a function of intensity.

(iv) The photoelectric emission is an instantaneous process. The time lag is very small between the incidence of radiation and emission of photo electrons ($\sim 10^{-9}$ or less), even when the incident radiation is extremely dim.

06. Photoelectric Effect and Wave Theory of Light

Attempts were made to example the laws of photoelectric effect on the basis of wave nature of radiation. However, it failed to explain the various features of the photoelectric effect as discussed below :

(a) According to wave model, when light is incident on a metal surface, it spreads evenly all over the metal surface. The energy of the incident light is shared by all the free electrons present at the surface of the metal.

As a result, an electron receives energy, which is too small to eject it out of the metal surface. However, with the passage of time, energy goes on accumulating with an electron, till it becomes just sufficient to eject it out. The calculations* show that it may take days or even months for an electron to gather the required energy. However, photoelectric effect is found to be instantaneous. Thus, wave nature of radiation can not explain the instantaneous emission of photoelectrons.

(b) According to wave model, no matter what the frequency of the incident light is, even less energetic light should be able to cause photoelectric emission in any metal, provided the metal surface is kept exposed to the light for a sufficient time. In other words, no threshold frequency exists for a metal surface. However, it is contrary to the experimentally observed fact. Thus, wave nature of radiation can not explain the existence of threshold frequency for a metal surface.

(c) According to wave model, the greater the intensity of incident light, greater should be the energy absorbed by each electron present at the surface of the metal. Therefore, kinetic energy of the emitted electrons should increase with the increase in the intensity of the incident light. However, kinetic energy of the emitted electrons is found to increase with the increase of frequency of the incident light and independent of the intensity light. Thus, *wave nature of radiation can not explain the fact that kinetic energy of the emitted electrons is independent of the intensity light and depends on its frequency.*

07. Particle Nature of Light : The Photon

Photoelectric effect gave strong evidence of particle-like behaviour of light, and light in interaction with matter behaved as if it was made of quanta or packets of energy hv. Einstein arrived at the important result that light quantum having a definite value of energy as well as momentum $\left(\frac{hv}{c}\right)$ is a strong indicator that the light quantum can be associated with a particle. Einstein was awarded the light quantum can be associated with a particle. Einstein was awarded the Nobel prize in physics for his contribution to theoretical physics and the photoelectric effect in 1921. This particle later on given the name photon. In 1924 by the experiment of A.H. Compton (1892-1962) on scattering of X-rays from electrons, the particle-like behaviour of high was further confirmed. Millikan for his work on the elementary charge of electricity and on the photoelectric effect was awarded the Nobel prize in physics in 1923.

Photon picture of electromagnetic radiation can be summarised as under:

(i) Radiation behaves as if it is made up of particles called photons in interaction with matter.

(ii) Each photon has energy E (= hv) and momentum $p\left(=\frac{hv}{c}\right)$ and speed c, the speed to light.

(iii) Photon energy is independent of intensity of radiation. All photons of light of a particular frequency v, or wavelength λ have the same energy $E\left(=hv=\frac{hc}{\lambda}\right)$ and momentum $p\left(=\frac{hv}{c}=\frac{h}{\lambda}\right)$. More intensity of light of given wavelength means number of photons per second crossing a given area, with each photon having the same energy.

(iv) Photons are electrically neutral and are not deflected by electric and magnetic fields.

(v) The total energy and total momentum are conserved in photon-particle collision (such as photon-electron collision). However, the photon may be absorbed or a new photon may be created in a collision. This implies photons may not be conserved in a collision.

Example Work function of sodium is 2.3 eV. Does sodium show photo-electric emission of light of wavelength 6800 A?

Solution The threshold wavelength of sodium

$$\lambda_0 = \frac{hc}{E} = \frac{6.6\times10^{-34}\times3\times10^{8}}{2.3\times1.6\times10^{-19}} = 5380\ \text{Å}$$

As given wavelength $\lambda = 6800$ Å > threshold wavelength $\lambda_0 = 5380$ Å, no photoelectric emission will take place.

08. Einstein's Photoelectric Equation

To explain photoelectric effect, Einstein postulated that the energy carried by a photon of radiation of frequency *v* is *h v*. According to him, the emission of a photoelectron was the result of the interaction of a single photon with an electron, in which the photon is completely absorbed by the electron.

The minimum amount of energy required to eject an electon out of the metal surface is called the ***work function*** *of the metal.* It is denoted by ω.

Thus, when a photon of energy *h v* is absorbed by an electron, an amount of energy at least equal to ω (provided $hv > \omega$) is used up in liberating the electron free and the difference $hv - \omega$ becomes available to the electron as its maximum kinetic energy. Thus,

$$\frac{1}{2}mv_{\max}^{2} = hv - \omega$$

or

$$hv = \omega + \frac{1}{2}m\,v_{\max}^{2} \qquad (1)$$

Here, *m* is the mass of electron and $v_{\max}$ is the maximum velocity of the photoelectrons. In fact, most of the electrons possess kinetic energy less than the maximum value as they lose a part of their kinetic energy due to collisions in escaping from the metal. Further, the work function of the metal is a characteristic of the metal and does not depend upon the nature of the incident radiation. It is sometimes also called the ***threshold energy*** of the metal. If v_0 is the frequency which corresponds to threshold energy of the metal, then

$$\omega = h\,v_0$$

Here, v_0 is called the ***threshold frequency***. In the equation (1), substituting $\omega = hv_0$, we obtain

$$hv = hv_0 + \frac{1}{2}m\,v_{\max}^{2} \qquad (2)$$

The above relation is called the ***Einstein's photoelectric equation***. Let us now deduce laws of photoelectric emission from the Einstein's photoelectric equation.

(i) From the equation (2), the value of maximum kinetic energy of the emitted photoelectrons is given by

$$\frac{1}{2}m v_{\max}^{2} = h\nu - h\nu_0 \qquad (3)$$

For photoelectric emission to take place, the kinetic energy of the emitted electrons must be positive. From the equation (3), it follows that the photoelectrons will possess positive kinetic energy only if $h\nu > h\nu_0$ or if $\nu > \nu_0$. It proves that for photoelectric emission to take place, *the frequency of the incident radiation must be greater than the threshold frequency for the metal.*

(ii) From the equation (2), the value of maximum kinetic energy of the emitted photoelectrons is given by

$$\frac{1}{2}m v_{\max}^{2} = h\nu - h\nu_0 \qquad (3)$$

For photoelectric emission to take place, the kinetic energy of the emitted electrons must be positive. From the equation (3), it follows that the photoelectrons will possess positive kinetic energy only if $h\nu > h\nu_0$ or if $\nu > \nu_0$. It proves that for photoelectric emission to take place, *the frequency of the incident radiation must be greater than the threshold frequency for the metal.*

(iii) From the equation (3), it follows that the value of maximum kinetic energy of the emitted photoelectrons depends *linearly on the frequency.* It proves that *the maximum kinetic energy of the emitted photoelectrons increases, as the frequency of the incident radiation is increased.* Since the Einstein's equation does not involve a factor representing intensity of the incident radiation, it proves that *the maximum kinetic energy of the emitted photoelectrons is independent of the intensity of the incident radiation.*

(iv) According to Einstein, the photoelectric effect ariese, when a single photon is absorbed by a single electron i.e. it is a ***one photon-one electron phenomenon.*** Therefore, number of photoelectrons ejected will be large, when intense radiation is incident.

(v) According to Einstein, the basic process involved in photoelectric emission is absorption of a photon of light by an electron. Therefore as the photon is absorbed, *the emission of electron takes place instantaneously*

09. Verification of Einstein's Photoelectric Equation

In an attempt to disprove the Einstein's photoelectric equation, R.A. Millikan performed a series of experiments. However, in 1912, instead of disproving, his experiments rather proved the correctness of Einstein's photoelectric equation.

From the Einstein's photoelectric equation, we have

$$\frac{1}{2}m v_{\max}^{2} = h\nu - h\nu_0$$

If e is charge on electron and V_0 is the stopping potential, then

$$\frac{1}{2}m v_{\max}^{2} = eV_0$$

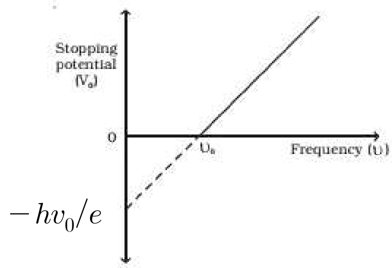
From the above two equations, we get

$$e V_0 = hv - hv_0$$

or

$$V_0 = \frac{hv}{e} - \frac{hv_0}{e}$$

Millikan plotted a graph between the frequency *v* (along X-axis) and the stopping potential V_0 (along Y-axis) over a wide range of frequencies. The graph was a straight line. It has a slope *h/e* and makes an intercept $-hv_0/e$ on Y-axis as shown in Figure. By measuring the slope of the graph and using the known value of *e,* Millikan determined the value of *h.*



Millikan also determined the value of the work function of the metal used in the experiment. By definition, when light of threshold frequency *(v_0)* is incident, photoelectrons just come out and no stopping potential is required. Therefore,

when $v = v_0,$ $V_0 = 0$

i.e. the intercept of the straight line graph on *v*-axis gives the threshold frequency $v_{0,}$ which when multiplied with *h,* gives the work function of the alkali metal.

The values of Planck's constant and work function of the metal determined by Millikan were in close agreement with values obtained from other experiments. It verified the correctness of Einstein's photoelectric equation.

Example **The energy of photoelectrons emitted from a photo-sensitive plate is 1.56 eV. If threshold wavelength is 2,500 Å, calculate the wavelength of incident light, Given,** $1\ eV = 1.6 \times 10^{-12}$ **erg and** $h = 6.62 \times 10^{-27}$ **erg s.**

Solution Now, $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + \frac{1}{2} m v_{\max}^2$

$$= \frac{6.62 \times 10^{-27} \times 3 \times 10^{10}}{2,500 \times 10^{-8}}$$

$$+ 1.56 \times 1.6 \times 10^{-12}$$

$$= 7.944 \times 10^{-12} + 2.496 \times 10^{-12}$$

$$= 10.44 \times 10^{-12} erg$$

$$\therefore \quad \lambda = \frac{hc}{10.44 \times 10^{-12}} \frac{6.62 \times 10^{-27} \times 3 \times 10^{10}}{10.44 \times 10^{-12}}$$

$$= 1.902 \times 10^{-5} cm = \mathbf{1{,}902\ Å}$$

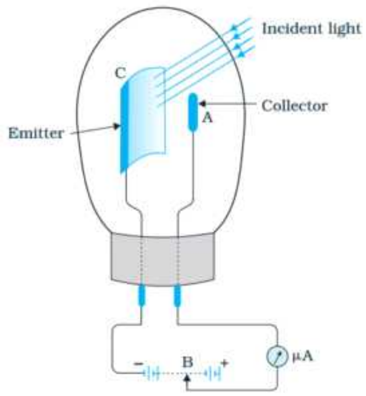
10. Photoelectric cells

Photoelectric cell is an arrangement which converts light energy into electrical energy.
A photocell is also sometimes called as an electric eye. Photoelectric cells are of the following three types:

(i) Photoemissive cells
(ii) Photovoltaic cells
(iii) Photoconductive cells.

A photoemissive cell may be of ***vacuum type*** or ***gas filled type.*** Let us discuss the working of a photoemissive cell.

Photoemissive cell. It consists of two electrodes, a cathode C and an anode A enclosed in a highly evacuated glass or quartz bulb [show in Figure]. The cathode C is a semi-cylindrical plate coated with a photosensitive material, such as a layer of cesium deposited on silver oxide. Sometimes a thin layer of photosensitive material is coated on the inner wall of the bulb. The anode A is in the form of a wire, so that it does not obstruct the pat of the light falling on the cathode.



Working

- A photocell converts a change in the intensity of incident light into a change in photocurrent.
- When light of frequency above the threshold frequency or the cathode surface is incident on the cathode, photoelectrons are emitted.
- If a potential difference of about 10 V is applied between the anode and cathode, the photoelectrons are attracted towards the anode and the microammeter connected in the circuit will record the current.

Advantage :

(i) The chief advantages of this type of photocell are that there is no time-lag between the incident light and the emission of photoelectrons and that the photo-electric current is proportional to the intensity of light. The cell is extremely accurate in response. Hence, it is used in television and photometry.

(ii) The current may be increased by a factor of about 5 by filling the tube with an inert gas at a pressure of a few mm of mercury. When the potential difference between the electrodes exceeds the ionisation potential of the gas, the emitted photoelectrons ionise the gas atoms and now a much larger current flows.

Applications of Photoelectric cells

Photoelectric cells have a variety of applications in industries and daily life. A few important applications of photoelectric cells are as given below:

(i) It is used for the reproduction of sound from the sound track recorded on one edge of the cinema films.

(ii) It is used in a television studio to convert the light and shade of the object into electric currents for transmission of picture.

(iii) It is used in a photographic camera as a light meter for the automatic adjustment of aperture.

(iv) It is used to compare the illuminating powers of two sources of light and to measure the illumination of a surface.

(v) It is used for automatic counting of the number of persons entering a hall, a stadium, etc.

(vi) It is used for automatic switching of street lights and traffic-signals.

(vii) It is used for raising a fire alarm in the event of accidental fire in buildings, factories, etc.

(viii) It is used in burglar-alarms for houses, banks and treasuries.

(ix) A photo cell is also used in industries to locate flaws in metal sheets.

(x) It is also used to control the temperature during a chemical reaction and that of a furnace.

(xi) A photocell, whose cathode is coated with lead sulphide, is sensitive to infrared light. Such a photocell is used in electronic ignition circuits.

(xii) A photocell can be used for determining the opacity of solids and liquids.

11. Wave Nature of Matter

The wave nature of light shows the phenomena of interference, diffraction and polarisation. On the other hand, in Photoelectric effect and Compton effect which involve energy and momentum transfer, radiation behaves as if it is made up of a bunch of particles – the photons.

If radiation has a dual (wave-particle) nature, so should have matter. In 1924, the French physicist Louis Victor de Broglie (1892-1987) reasoned that nature was symmetrical and that basic physical entities – matter and energy, must have symmetrical character. De Broglie proposed that wavelength λ associated with a particle momentum p is given as $\lambda = \frac{h}{p} = \frac{h}{mv}$, where m is the mass of the particle and v is its speed.

The dual aspect of matter is evident in the above de Broglie relation, where λ is the attribute of wave while momentum p is a typical attribute of a particle.

λ is small for a heavier particle (large m) or more energetic particle (large v). This is th reason why macroscopic objects in our daily life do not show wave-like properties. The wave character of particles is significant and measurable in the sub-atomic domain.

Matter waves associated with an electron could be verified by crystal diffraction experiments parallel to X-ray diffraction because the wavelength associated with accelerated electron is of the same order as the spacing between the atomic planes in cystal.
It is interesting to see that this relation is also satisfied by a photon. That is, the de Broglie wavelength of a photon equals the wavelength of electromagnetic radiation of which the photon is a quantum of energy and momentum.

For particle having kinetic energy $E, p = \sqrt{2mE,}$ then $\lambda = \frac{h}{\sqrt{2mE}}$, for charged particle accelerated from rest by potential V, E = qV then $\lambda = \frac{h}{\sqrt{2mqV}}$.

Macroscopic objects in our daily life do not show wave-like character. Why?

From de Broglie relation $\lambda = \frac{h}{mv}$, clearly λ is smaller for a heavier particle or more energetic particle.

For example, let us consider a base-ball of 150 g traveling with a speed of 35.0 ms^{-1}

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34} J.s}{(150 \times 10^{-3} kg)(35.0 ms^{-1})}$$

$$= 1.26 \times 10^{-34} m$$

This wavelength is too small to measure. Hence macroscopic objects, do not show wave-like character.

Wave character of sub-atomic particles is significant and measurable

(i) **Electron** (mass m, charge e) accelerated from rest through a **potential V**

$$\therefore \quad \lambda = \frac{h}{\sqrt{2meV}} = \frac{6.63 \times 10 - 34 Js}{\sqrt{2 \times 9.1 \times 10^{-31} kg \times 1.6 \times 10^{-19} C \times V}}$$

or

$$\boxed{\lambda = \sqrt{\frac{150}{V}} \text{ Å} = \frac{12.27}{\sqrt{V}} \text{ Å}}$$

If we take $V = 120\, volt, \lambda = 0.112 nm = 1.12$ Å

This wavelength is about the size of a typical atom. This is also about the wavelength of X-rays. Hence the matter waves associated with an electron could be verified by crystal diffraction experiments analogous to X-rays diffraction.

(ii) **Proton :** $\lambda = \frac{0.286 \text{ Å}}{\sqrt{V}}$

(iii) **Deutron :** $\lambda = \frac{0.202 \text{ Å}}{\sqrt{V}}$

(iv) **α-particle :** $\lambda = \frac{0.101 \text{ Å}}{\sqrt{V}}$

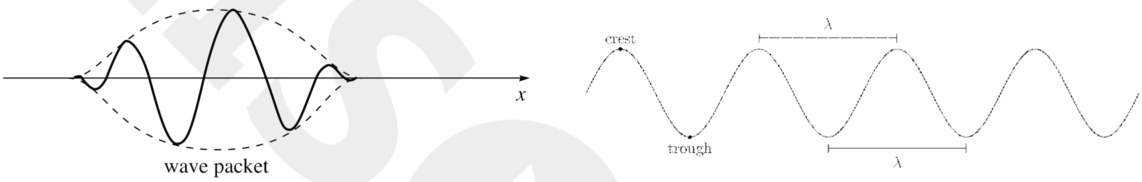
(v) **Neutron :** $\lambda = \frac{0.286 \text{ Å}}{\sqrt{E \in eV}}$

(vi) **Gas molecules :** $\lambda = \frac{h}{\sqrt{3mKT}}$ (m = mass of molecule, k = Boltzman Constant; T =Absolute temperature of gas)

According to the Heisenberg's uncertainty principle, it is not possible to measure both the position and momentum of an electron (or any other particle) at the same time exactly. There is always some uncertainty in the specification of both position ($\triangle$x) and momentum $(\triangle p)$.

The product of $\triangle$x and $\triangle$p is of the order of $h = \frac{h}{2\pi}$ $i.e., \triangle x \triangle p \simeq h.$

If any one from above is zero ($\triangle$x or $\triangle p$) then other must be infinite. If an electron has a definite momentum p, (i.e., $\triangle p = 0$), from the de Broglie relation, it has a definite wavelength λ. A wave of definite (single) wavelength extends allover the space. In general, the matter wave associated with the electron is not extended all over space. It is a wave packet extending over some finite region of space. It is made up of wavelengths spared around some central wavelength then momentum of the electron will also have a spread from the uncertainty principle.



(a) (b)

Fig.: Shows a schematic diagram of (a) a localised wave packet, and (b) an extended wave with fixed wavelength

12. Davisson and Germer Experiment for Wave Nature of Electron

The wave nature of electrons was first experimentally verified independently by C.J. Davisson and L.H. Germer in 1927 and By G.P. Thomos in 1928 while observing diffraction effects with beams of electrons scattered by crystals. Davisson and Thomson shared the Nobel prize in 1937 for same.
The experimental arrangement is schematically show in figure.

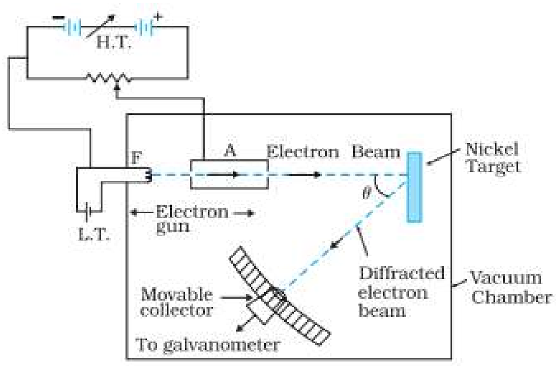
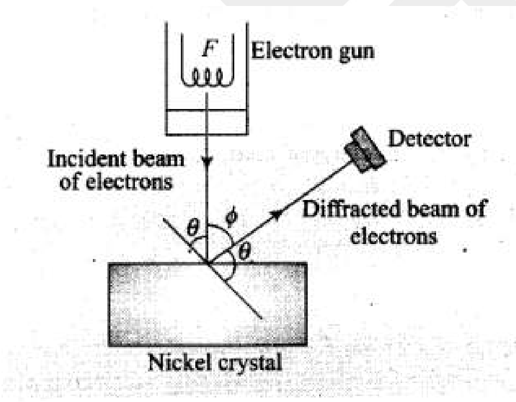
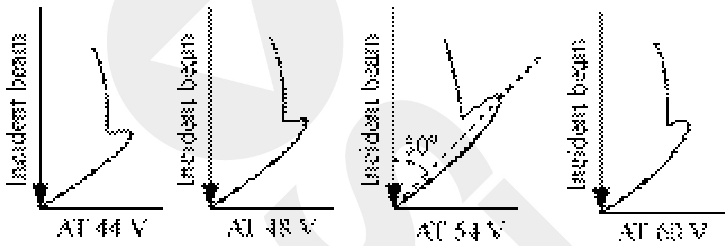


FIG Davisson-Germer electron diffraction arrangement.



It has an electron gun made up of a tungsten filament F, heated by a low voltage (L.T)battery and the filament is coated with barium oxide. Emitted electrons from filament are accelerated to a desired velocity by applying required potential/voltage from a high-voltage power supply (H.T. or battery). **C is a hollow metallic cylinder with a hole along the axis and is kept at negative potential to get a convergent beam of electrons emitted from filament. It acts as a cathode. A is a cylinder with fine hole along its axis acting as an anode.** The cathode and anode form an electron gun by which a fine beam of electrons can be obtained of different velocities by applying different accelerating potentials. N is a nickel crystal cut along cubical diagonal, D is an electron detector which can be rotated on a circular scale and is connected to a sensitive galvanometer which records the current.

Working : From electron gun a fine beam of accelerated electrons is made to fall normally on the surface of nickel crystal. The atoms of the crystal scatter the incident electrons in different directions. The detector detects the intensity of the electron beam scattered in particular direction by rotating the electron detector on circular scale at different positions. The accelerating voltage varied from 44 V to 68 V and found that at accelerating voltage 54 V the variation of intensity (I) and scattering angle (ϕ), (the angle between the incident and the scattered beam) is of the type as shown in figure.



Due to constructive interference of electrons scattered from different layers of regularly spaced atoms of the crystal, peak appears in a particular direction i.e., diffraction of electrons takes place. This establishes the wave nature of electron.

For the scattering angle $\phi = 50^o$, the angle of glancing θ

$$\theta + \phi + \theta = 180^o$$

or $$\theta = \frac{1}{2}(180^o - \phi) = 65^o$$

For the nickel crystal the interatomic separation is $d = 0.91$ Å

According to Bragg's law for first-order diffraction maxima $(n = 1)$

We have, $2d \sin\theta = 1 \times \lambda$

$$\lambda = 2 \times 0.91 \times \sin 65^o = 1.65 \text{ Å}$$

According to de Broglie hypothesis, the wavelength of the wave associated with electron is given by

$$\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}} = 1.66 \text{ Å}$$

Above value is in close agreement with the estimated value of de Broglie wavelength and the experimental value determined by Davisson and Germer.

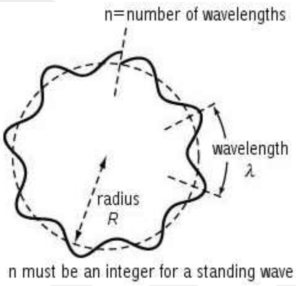
The development of modern quantum mechanics, whose basis is the de Broglie hypothesis, also led to the filed of electron optics. In the design of electron microscope, having higher resolution over the optical microscope, wave properties of electrons have been utilised.

de Broglie Wave Relation to Bohr Model

de Broglie relation $\left(\lambda = \frac{h}{mv}\right)$ can be used to obtain the Bohr quantum condition that the angular momentum $L = mvr = n.\frac{h}{2\pi}$, where $n = 1, 2, 3,$ ....

The method is analogous to determining th e normal-mode frequencies of standing waves. The electrons in Bohr-orbits radiate no energy, just as a standing wave on a string transmits no energy. So think of an electron as a standing wave fitted around a circle in one of the Bohr orbits.

For the wave to join onto itself smoothly, the circumference of this circle must include some whole number of wavelengths as suggested by figure.



For an orbit with radius r, we must have, $2\pi\lambda = n\lambda$, where $n = 1, 2, 3,$ ....

According to de Broglie, the wavelength λ of a particle with rest mass m, moving with non relativistic speed v, is

$$\lambda = \frac{h}{mv}$$

Combining, $2\pi r = n\lambda$

and $\lambda = \frac{h}{mv}$, we get

$$2\pi r = \frac{nh}{mv}$$

or, $mvr = n.\frac{h}{2\pi}$

Thus, the de Broglie relation is in agreement with the Bohr's quantum model of atoms.

Example **Find the de Broglie wavelength of revolving electron for the Bohr's first orbit of circumference** $2\pi r$.

Solution According to Bohr's theory $mvr = \frac{nh}{2\pi}$

$$2\pi r = \frac{nh}{mv} = n\left(\frac{h}{mv}\right) = n\lambda \text{ for } n = 1$$

$$\lambda = 2\pi r$$

CBSE
Exercise (1)

(Q 1 to 2) One Mark

1. The frequency (ν) of incident radiation is greater than threshold frequency (ν_0) in a photocell. How will the stopping potential vary, if frequency (ν) is increased, keeping other factors constant?

2. If the frequency of the incident radiation is equal to the threshold frequency, what will be the value of the stopping potential?

(Q 3 to 4) Two Marks

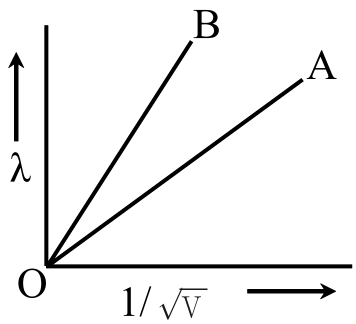
3. A proton and an α-particle are accelerated, using the same potential difference. How are de-Broglie wave-lengths λ_p and λ_α related to each other?

4. Two monochromatic beams A and B of equal intensity I, hit a screen. The number of photons hitting the screen by beam A is twice that by beam B. Then, what inference can you make about their frequencies?

(Q 5 to 6) Three Marks

5. Ultra-violet light of wavelength 2,271 Å from a 100 W mercury source irradiates a photocell made of molybdenum metal. If the stopping potential is –1.3 V, estimate the work function of the metal. How should the photocell respond to a high intensity ($\approx 10^5$ W m^{-2}) red light of wavelength 6,328 Å produced by a He-Ne laser?

6. The two lines marked A and B in Fig. show a plot of de-Broglie wavelength (λ) as a function of $1/\sqrt{V}$ (V is the accelerating potential) for two nuclei $_1H^2$ and $_1H^3$.
   (i) What does the slope of the lines represent?
   (ii) Identify, which lines correspond to these nuclei?



(Q 7) Four Marks

7. When the surface of a certain metal is illuminated with the light of wavelength λ, the emitted photoelectrons possess a maximum kinetic energy K_{max}. Show that when light of wavelength

$\frac{hc\lambda}{hc + K_{\max}\lambda}$ is incident on the metal, the emitted electrons will have a maximum kinetic energy of 2 K_{max}.

8. Illuminating the surface of a certain metal alternately with the light of wavelength λ_1 and λ_2 ($\lambda_1<\lambda_2$), it was found that the corresponding maximum velocities of photoelectrons differ by a factor of *n*. Show that the work function of the metal is given by

$$\omega = hc\frac{\left(n^2 - \lambda_2/\lambda_1\right)}{\lambda_2\left(n^2 - 1\right)}$$

9. Draw suitable graphs to show the variation of photoelectric current with collector plate potential for
   (i) a fixed frequency but different intensities $L_1 > L_2 > L_3$ of radiation.
   (ii) a fixed intensity but different frequencies $v_1 > v_2 > v_3$ of radiation.

(Q 10) Five Marks

10.

(a) Explain (*i*) what is meant by a photon and (*ii*) why most electrons are emitted with kinetic energy less than the maximum.

(b) The photoelectric effect provides evidence for the particulate nature of electromagnetic radiation. State three experimental observations that support this conclusion.

(c) Electromagnetic radiation of wavelength λ and intensity I, when incident on a metal surface, causes *n* electrons to be ejected per unit time. The maximum kinetic energy of the electrons is E_{max}. State and explain the effect, if any, on n and E_{max}. When (*i*) the intensity is reduced to I/2 but the wavelength λ is unchanged and (*ii*) the wavelength λ is reduced but the intensity I is not changed.

ANSWER

Q1

When the frequency is increased, the kinetic energy of the emitted photoelectrons will increase and hence the stopping potential will also increase.

Q2

From Einstein's photoelectric equation, we have

$$hv = h\,v_0 + e\,V_0$$

When $v = v_0$, we have

$$h\,v_0 = h\,v_0 + e\,V_0$$

or $V_0 = 0$

Q3

Let m_p and m_a be the masses of proton and a- particle respectively. Suppose that they are accelerated through the same potential difference V. In terms of accelerating potential V, the de-Broglie wavelength of a charged particle is given by

$$\lambda = \frac{h}{\sqrt{2meV}},$$

where e is charge and m, mass of the particle.

The de-Broglie wavelengths of proton and the alpha particle are given by

$$\lambda_p = \frac{1}{\sqrt{m_p\,e_p\,V}} \qquad ...(i)$$

and

$$\lambda_a = \frac{1}{\sqrt{m_a\,e_a\,V}} \qquad ...(ii)$$

Here, e_p and e_a are the charges on proton and a-particle respectively.

From the equations (i) and (ii), we have

$$\frac{\lambda_p}{\lambda_a} = \frac{\sqrt{m_a\,e_a}}{\sqrt{m_p\,e_p}}$$

Now, $m_a = 4\,m_p$ and $e_a = 2\,e_p$

$$\therefore \quad \frac{\lambda_p}{\lambda_a} = \frac{\sqrt{4m_p \times 2e_p}}{\sqrt{m_p\,e_p}} = \sqrt{8}$$

Q4

Let n_A and n_B be the number of photons of the beams A and B respectively, which hit the screen per second per unit area. Further, let v_A and v_B be their respective frequencies. Then intensities of the two beams are given by

$$I_A = n_A h v_A \quad \text{and} \quad I_B = n_B h v_B$$

Since $I_A = I_B$, we have

$$n_A h v_A = n_B h v_B$$

or

$$\frac{v_A}{v_B} = \frac{n_B}{n_A} = \frac{n_B}{n_A} = \frac{n_B}{2n_B} = \frac{1}{2} \qquad (\because\ n_A = 2\,n_B)$$

or $\boldsymbol{v_B = 2v_A}$

Q5

Here, $V_0 = 1.3\ V$;
$\lambda = 2{,}271\ \text{Å} = 2{,}271 \times 10^{-10}$ m

We know, $h\nu = h\ \nu_0 + \frac{1}{2} m\ v^2_{max}$

or $$\frac{hc}{\lambda} = \omega + eV_0$$

or $$\omega = \frac{hc}{\lambda} - eV_0$$

Taking $h = 6.62 \times 10^{-34}$ J s ; $e = 1.6 \times 10^{-19}$ C

$$\omega = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2{,}271 \times 10^{-10}} - 1.6 \times 10^{-19C} \times 1.3$$

$$= 8.745 \times 10^{-19} - 2.08 \times 10^{-19} = 6.665 \times 10^{-19}\ J$$

$$= \frac{6.665 \times 10^{-19}}{1.6 \times 10^{-10}} = 4.166\ eV$$

The threshold wavelength is given by

$$\lambda_0 = \frac{hc}{\omega} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{6.665 \times 10^{-19}}$$

$$= 2.98 \times 10^{-7}\ m = 2{,}980\ \text{Å}$$

Since wavelength 6,328 Å produced by He-Ne laser and incident on the photocell is greater than λ_0, the photocell ***will not respond***.

Q6

In terms of accelerating potential V, the de-Broglie wavelength of a charged particle is given by

$$\lambda = \frac{h}{\sqrt{2meV}}, \qquad ...(i)$$

where e is charge and m, mass of the particle.

The equation (i) represents a straight line, whose slope is $h/\sqrt{2me}$.

(i) The slope of the line is inversely proportional to $\sqrt{m}$.

(ii) Since the slope of line A is lesser, it represents the particle of heavier mass.

Q7

According to Einstein's photoelectric equation,

$$h\nu = \omega + \frac{1}{2} m\, v_{max}{}^2$$

or $$K_{max} = \frac{hc}{\lambda} - \omega \qquad ...(i)$$

Let λ' be the wavelength of incident light, which will cause the emission of photoelectrons with maximum kinetic energy of $2K_{max}$. Then, in accordance with the equation (i), we have

$$2K_{max} = \frac{hc}{\lambda'} - \omega \qquad ...(ii)$$

Subtracting the equation (i) from (ii), we have

$$2K_{max} = \frac{hc}{\lambda'} - \frac{hc}{\lambda}$$

or $$\frac{hc}{\lambda'} = \frac{hc}{\lambda} + K_{\max}$$

or $$\frac{hc}{\lambda'} = \frac{hc + K_{\max}\lambda}{\lambda}$$

or $$\lambda' = \frac{hc\lambda}{hc + K_{\max}\lambda}$$

Q8

According to Einstein's photoelectric equation,

$$hv = \omega + \frac{1}{2} m\, v_{\max^2}$$

or $$\frac{1}{2} m\, v_{\max^2} = \frac{hc}{\lambda} - \omega$$

Let v_1 and v_2 be the maximum velocities of the emitted photoelectrons, when light of wavelengths λ_1 and λ_2 is incident on the metal surface. Then,

$$\frac{1}{2} m\, v_1^2 = \frac{hc}{\lambda_1} - \omega \qquad ...(i)$$

and $$\frac{1}{2} m\, v_2^2 = \frac{hc}{\lambda_2} - \omega \qquad ...(ii)$$

Dividing the equation (i) by (ii), we have

$$\frac{\frac{1}{2} m\, v_1^2}{\frac{1}{2} m\, v_2^2} = \frac{hc/\lambda_1 - \omega}{hc/\lambda_2 - \omega}$$

or $$\frac{v_1^2}{v_2^2} = \frac{hc/\lambda_1 - \omega}{hc/\lambda_2 - \omega}$$

Now, $v_1/v_2 = n$

Therefore, the above equation becomes

$$n^2 = \frac{hc/\lambda_1 - \omega}{hc/\lambda_2 - \omega}$$

or $$hc/\lambda_1 - \omega = n^2\,(hc/\lambda_2 - \omega)$$

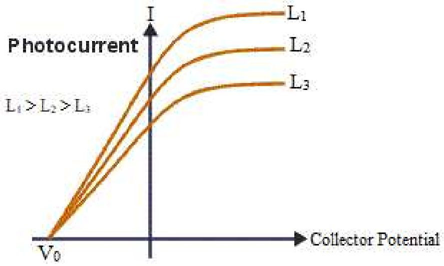
or $$n^2\omega - \omega = n^2 hc/\lambda_2 - hc/\lambda_1$$

or $$\omega(n^2 - 1) = \frac{hc}{\lambda_2}\left(n^2 - \frac{\lambda_2}{\lambda_1}\right)$$

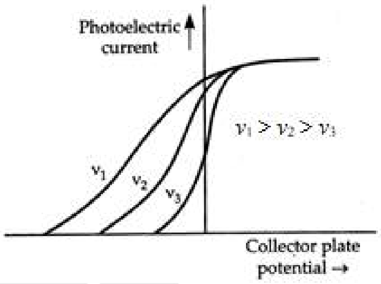
or $$\omega = hc\frac{(n^2 - \lambda_2/\lambda_1)}{\lambda_2\,(n^2 - 1)}$$

Q9

(i) The variation of photoelectric current with collector plate potential for a fixed frequency but different intensities $L_1 > L_2 > L_3$ of radiation is as shown in Fig. below.



(ii) The variation of photoelectric current with collector plate potential for a fixed intensity but different frequencies $\nu_1 > \nu_2 > \nu_3$ of radiation is as shown in Fig. above.



Q10

(a) (i) The photons are quantum of electromagnetic radiation. A photon of frequency ν has energy equal to $h\ \nu$ and momentum equal $h\ \nu/c$.

(ii) The maximum kinetic energy emitted corresponds to the electrons emitted from the surface of the metal. Those electrons, which are emitted from the inside of the metal, lose a part of this energy in coming to the surface of the metal. Therefore, most electrons are emitted with kinetic energies less than the maximum value of the kinetic energy.

(b) Following experimental observations support the conclusion that the photoelectric effect provides evidence for the particle nature of electromagnetic radiation :

(i) The photoelectric effect is instantaneous.

(ii) For every metal surface, there is a threhold frequency and if the frequency of the photon is below it, photoelectrons will not be emitted.

(iii) The number of photoelectrons emitted depends on the intensity of the radiation (independent of the frequency of the radiation) and maximum kinetic energy of the emitted electrons depends on the frequency of the radiation (independent of the intensity of the radiation).

(c) (i) When the intensity is reduced to I/2, the number of photons falling on the metal surface will become I/2. Since photoelectric emission is one-one phenomenon, the number of electrons emitted will also become ***one half***. Further, as the wavelength λ is unchanged, energy of the photon will remain the same and therefore, E_{max} will not be ***affected***.

(ii) When the wavelength λ is reduced, energy of photons will increase. As a result, E_{max} will ***increase***. On the other hand, if the intensity I is not changed, the number of photons falling on the metal surface will remain the same and therefore, the number of electrons emitted will also remain ***the same***.