Direct and Inverse Proportion

12.1 Introduction to Proportions

We come across many such situations in our day-to-day life, where we need to see variation in one quantity bringing in variation in the other quantity.

For example:

i) If the number of articles purchased increases, the total cost also increases.

ii) More the money deposited in a bank, more is the interest earned.

iii) As the speed of a vehicle increases, the time taken to cover the same distance decreases.

iv) For a given job, more the number of workers, less will be the time taken to complete the work.

Observe that change in one quantity leads to change in the other quantity. This means that both the quantities are in proportion

12.2 Direct Proportion

If the cost of 1 kg of sugar is Rs 18, then what would be the cost of 3 kg sugar? It is Rs 54. Similarly, we can find the cost of 5 kg or 8 kg of sugar. Study the following table.

$\times 3$, $\times 5$, $\times 6$, $\times 8$, $\times 10$

Weight of sugar (in kg)	1	3	5	6	8	10
Cost (in Rs)	18	54	90	...	...	...

$\times 3$, $\times 5$, $\times 6$, $\times 8$, $\times 10$

Observe that as weight of sugar increases, cost also increases in such a manner that their ratio remains constant.

Take one more example. Suppose a car uses 4 litres of petrol to travel a distance of 60 km. How far will it travel using 12 litres? The answer is 180 km. How did we calculate it? Since petrol consumed in the second instance is 12 litres, i.e., three times of 4 litres, the distance travelled will also be three times of 60 km. In other words, when the petrol consumption becomes three-fold, the distance travelled is also three fold the previous one. Let the consumption of petrol be x litres and the corresponding distance travelled be y km. Now, complete the following table:

Petrol in litres (x)	4	8	12	15	20	25
Distance in km (y)	60	...	180	...	...	...

We find that as the value of x increases, value of y also increases in such a way that the

Ratio $\frac{x}{y}$ does not change; it remains constant (say k). In this case, it is $\frac{1}{15}$ We say that x and y

are in direct proportion, if $\frac{x}{y} = k$ or $x = ky$.

Example 1: The cost of 5 metres of a particular quality of cloth is Rs.210. Tabulate the cost of 2, 4, 10 and 13 metres of cloth of the same type.

Solution: Suppose the length of cloth is x metres and its cost, in Rs. is y.

x	2	4	5	10	13
y	y_2	y_3	210	y_4	y_5

As the length of cloth increases, cost of the cloth also increases in the same ratio. It is a case of direct proportion.

We make use of the relation of type $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

i) Here $x_1 = 5$, $y_1 = 210$ and $x_2 = 2$

Therefore, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ gives $\frac{5}{210} = \frac{2}{y_2}$ or $5y_2 = 2 \times 210$ or $y_2 = \frac{2 \times 210}{5} = 84$

ii) If $x_4 = 4$ then $\frac{5}{210} = \frac{4}{y_3}$ or $5y_3 = 4 \times 210$ or $y_3 = \frac{4 \times 210}{5} = 168$

iii) If $x_4 = 13$, then $\frac{5}{210} = \frac{13}{y_4}$ or $y_4 = \frac{10 \times 210}{5} = 420$

iv) If $x_5 = 13$, then $\frac{5}{210} = \frac{13}{y_5}$ or $y_5 = \frac{13 \times 210}{5} = 546$

Example 2: An electric pole, 14 metres high, casts a shadow of 10 metres. Find the height of a tree that casts a shadow of 15 metres under similar conditions.

Solution: Let the height of the tree be x metres. We form a table as shown below:

height of the object (in metres)	14	x
length of the shadow (in metres)	10	15

Note that more the hiehg tof an object, the more would be the length of its shadow.

Hence, this is a case of direct proportion. That is $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

We have $\frac{14}{10} = \frac{x}{15}$

or $\frac{14}{10} \times 15 = x$

or $\frac{14 \times 3}{2} = x$

So $21 = x$

Thus, height of the tree is 21 metres.

Example 3: If the weight of 12 sheets of thick paper is 40 grams, how many sheets of the same paper would weigh $2\frac{1}{2}$ kilograms?

Solution: Let the number of sheets which weigh $2\frac{1}{2}$ kg be x. We put the above information in the form of a table as shown below:

Number of sheets	12	x
Weight of sheets (in grams)	40	2500

More the number of sheets, the more would their weight be. So, the number of sheets and their weights are directly proportional to each other.

So, $\frac{12}{40} = \frac{x}{2500}$

or $\frac{12 \times 2500}{40} = x$

or $750 = x$

Thus, the required number of sheets of paper = 750

12.3 Inverse Proportion

Two quantities may change in such a manner that if one quantity increases, the other quantity decreases and vice versa. For example, as the number of workers increases, time taken to finish the job decreases. Similarly, if we increase the speed, the time taken to cover a given distance decreases. To understand this, let us look into the following situation.

Zaheeda can go to her school in four different ways. She can walk, run, cycle or go by car. Study the following table.

	Walking	Running	Cycling	By Car
Speed in km/hour	3	6	9	45
Time taken (in minutes)	30	15	10	2

Observe that as the speed increases, time taken to cover the same distance decreases. As Zaheeda doubles her speed by running, time reduces to half. As she increases her speed to three times by cycling, time decreases to one third. Similarly, as she increases her speed to 15 times, time decreases to one fifteenth. (Or, in other words the ratio by which time decreases is inverse of the ratio by which the corresponding speed increases). We can say that speed and time change inversely in proportion

Thus two quantities x and y are said to vary in inverse proportion, if there exists a relation of the type x y = k between them, k being a constant. If y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $x_1y_1 = x_2y_2$ (= k), or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$. We say that x and y are in inverse proportion.

Example 4: 6 pipes are required to fill a tank in 1 hour 20 minutes. How long will it take if only 5 pipes of the same type are used?

Solution: Let the desired time to fill the tank be x minutes. Thus, we have the following table.

Number of pipes	6	5
Time (in minutes)	80	x

Lesser the number of pipes, more will be the time required by it to fill the tank. So, this is a case of inverse proportion. Hence, 80 × 6 = x × 5 [x1 y1 = x2 y2]

or $\frac{80 \times 6}{5} = x$ or $x = 96$

Thus, time taken to fill the tank by 5 pipes is 96 minutes or 1 hour 36 minutes.

Example 8: There are 100 students in a hostel. Food provision for them is for 20 days. How long will these provisions last, if 25 more students join the group?

Solution: Suppose the provisions last for y days when the number of students is 125. We have the following table.

Number of students	100	125
Number of days	20	y

Note that more the number of students, the sooner would the provisions exhaust. Therefore, this is a case of inverse proportion.

So, 100 × 20 = 125 × y or $\frac{100 \times 20}{125} = y$ or 16 = y

Thus, the provisions will last for 16 days, if 25 more students join the hostel.

Example 9: If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?

Solution: Let the number of workers employed to build the wall in 30 hours be y. We have the following table.

Number of hours	48	30
Number of workers	15	y

Obviously more the number of workers, faster will they build the wall. So, the number of hours and number of workers vary in inverse proportion.
So 48 × 15 = 30 × y

Therefore, $\frac{48 \times 15}{30} = y$ or $y = 24$

i.e., to finish the work in 30 hours, 24 workers are required.