

6 Direct and inverse proportion

This unit will help you to solve problems where y is directly or inversely proportional to the square of x and to use and recognise graphs showing direct and inverse proportion.

AO1 Fluency check

- 1 a $y = 2x^2$ b $p = \frac{10}{q^2}$
- i Work out y when $x = 5$ ii Work out p when $q = 2$
- ii Work out x when $y = 128$ iii Work out q when $p = 62.5$
- 2 Make x the subject of each equation.
- a $4y = x^2$ b $y = 3x^2$ c $y = \frac{5}{x^2}$ d $ay = \frac{b}{x^2}$

3 Number sense

Circle the odd one out.

1 4 8 16 49 64

Key points

When y is directly proportional to the square of x , write $y \propto x^2$

When y is inversely proportional to the square of x , write $y \propto \frac{1}{x^2}$

These **skills boosts** will help you to solve problems involving direct and inverse proportion and to use and recognise graphs showing proportion.

1 Direct proportion involving squares

2 Inverse proportion involving squares

3 Direct and inverse proportion graphs

You might have already done some work on proportion. Before starting the first skills boost, rate your confidence using these questions.

- 1 y is directly proportional to the square of x .
When $y = 32$, $x = 8$.
Work out x when $y = 200$.

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.....

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How confident are you?

- 2 y is inversely proportional to the square of x .
When $y = 0.16$, $x = 5$.
Work out y when $x = 100$.

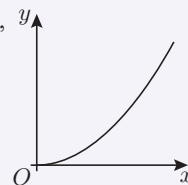
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- 3 In this graph, y is directly proportional to x ? Explain your answer.



.....

.....

.....



1

Direct proportion involving squares

$y \propto x^2$ means 'y is directly proportional to the square of x'.
So $y = kx^2$, where k is a number/constant.

Guided practice

y is proportional to x^2 . When $x = 5$, $y = 75$.

a Find a formula for y in terms of x.

b Work out the value of y when $x = 8$.

a $y \propto x^2$

So $y = kx^2$

$75 = k \times \dots\dots\dots^2$

$k = \dots\dots\dots$

The formula is $y = \dots\dots\dots x^2$

b When $x = 8$, $y = \dots\dots\dots$

Substitute $x = 5$ and $y = 75$ into the equation
 $y = kx^2$

Simplify and rearrange to work out the value of k.

Substitute your value of k into the equation $y = kx^2$

Substitute $x = 8$ into your formula to work out y.

1 y is proportional to the square of x. When $x = 3$, $y = 45$.

a Find a formula for y in terms of x.

b Work out the value of y when $x = 6$.

2 y is proportional to x^2 . When $x = 2$, $y = 16$.

a Find a formula for y in terms of x.

b Work out the value of y when $x = 7$.

c Work out the value of x when $y = 100$.

3 q is proportional to the square of p. When $p = 10$, $q = 20$.

a Find a formula for q in terms of p.

b Work out the value of q when $p = 4$.

c Work out the value of p when $q = 0.05$.

4 b is proportional to a^2 . When $a = 4$, $b = 8$.

a Find a formula for b in terms of a.

b Work out the value of b when $a = 12$.

c Work out the value of a when $b = 0.98$.

Hint To find x when $y = 100$, substitute $y = 100$ into your formula and rearrange it to find x.

Exam-style question

5 y is proportional to the square of x.

When $x = 10$, $y = 25$.

Work out the value of y when $x = 8.6$.

..... (3 marks)

Reflect

Explain what $y \propto x^2$ means and then use what you know to explain what $y \propto \sqrt{x}$ means.

2

Inverse proportion involving squares

$y \propto \frac{1}{x^2}$ means 'y is inversely proportional to the square of x'.

So $y = \frac{k}{x^2}$ where k is a number/constant.

Guided practice

y is inversely proportional to the square of x.

When $x = 10$, $y = 0.04$.

a Find a formula for y in terms of x.

b Work out the value of y when $x = 5$.

a $y \propto \frac{1}{x^2}$

So $y = \frac{k}{x^2}$

..... = $\frac{k}{10^2}$

k =

The formula is $y = \frac{\text{.....}}{x^2}$

b When $x = 5$, $y = \text{.....}$

Substitute, $x = 10$ and $y = 0.04$ into the equation

$y = \frac{k}{x^2}$

Simplify and rearrange to work out the value of k.

Substitute your value of k into the equation $y = \frac{k}{x^2}$

Substitute $x = 5$ into your formula to work out y.

Worked
exam
question



① y is inversely proportional to the square of x. When $x = 0.5$, $y = 20$.

a Find a formula for y in terms of x.

b Work out the value of y when $x = 2$

② y is inversely proportional to x^2 . When $x = 2$, $y = 0.125$.

a Find a formula for y in terms of x.

b Work out the value of y when $x = 0.4$

c Work out the value of x when $y = 1250$

③ q is inversely proportional to the square of p. When $p = 0.1$, $q = 80$.

a Find a formula for q in terms of p.

b Work out the value of q when $p = 5$

c Work out the value of p when $q = 5$

Hint To find x when $y = 1250$, substitute $y = 1250$ into your formula and rearrange it to find x.

Exam-style question

④ y is inversely proportional to the square of x.

When $x = 0.2$, $y = 6.25$.

Work out the value of y when $x = 0.1$.

..... (3 marks)

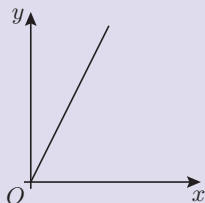
Reflect

Explain what $y \propto \frac{1}{x^2}$ means and then use what you know to explain what $y \propto \frac{1}{x^3}$ means.

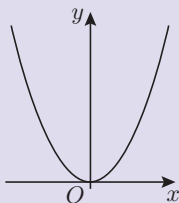
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Direct and inverse proportion graphs

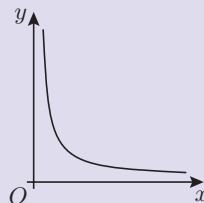
In this graph, y is directly proportional to x .



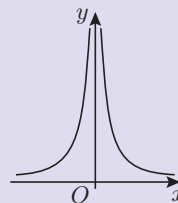
In this graph, y is directly proportional to the square of x .



In this graph, y is inversely proportional to x .

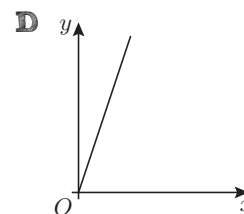
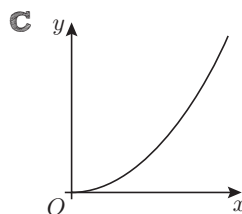
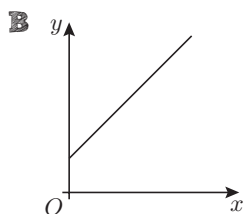
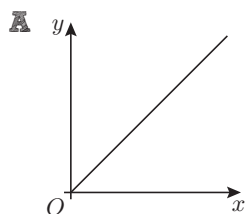


In this graph, y is inversely proportional to the square of x .



Guided practice

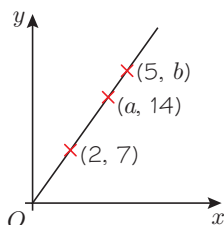
In which of these graphs is y directly proportional to x ?



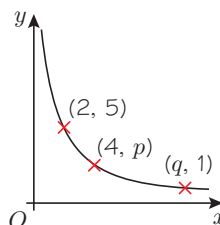
In graphs A and y is directly proportional to x .

If y is directly proportional to x , the graph will be a straight line that passes through the origin. Graph B does not pass through the origin.

- ① In this graph y is directly proportional to x . Find the values of a and b .

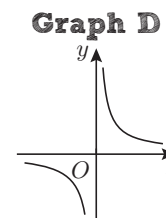
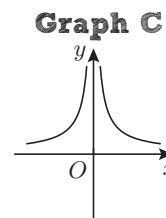
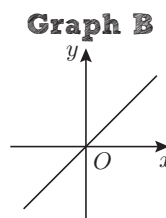
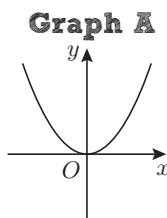


- ② In this graph y is inversely proportional to x . Find the values of p and q .



Exam-style question

- ③ Each graph shows a proportionality relationship between y and x .



Match each graph with a statement in the table below.

Proportionality statement	Graph
y is directly proportional to x .	
y is inversely proportional to x .	
y is directly proportional to the square of x .	
y is inversely proportional to the square of x .	

(2 marks)

Reflect

Describe the features of the graphs for direct and inverse proportion.

Practise the methods

Answer this question to check where to start.

Check up

Tick the equation that represents the relationship 'y is inversely proportional to the square of x'.

A

$$y = kx$$



B

$$y = kx^2$$



C

$$y = \frac{k}{x^2}$$



D

$$y = \frac{x^2}{k}$$



If you ticked C, work out the value of k given that $x = 2$ when $y = 2.5$. Then go to Q4.

If you ticked A, B or D go to Q1 for more practice.

- 1 Match each proportionality statement with an equation.

A y is directly proportional to x .

B y is inversely proportional to x .

C y is directly proportional to the square of x .

D y is inversely proportional to the square of x .

i $y = \frac{k}{x}$

ii $y = kx^2$

iii $y = \frac{k}{x^2}$

iv $y = kx$

- 2 y is directly proportional to the square of x .

When $x = 1.5$, $y = 18$.

Find a formula for y in terms of x .

- 3 y is inversely proportional to the square of x .

When $x = 0.2$, $y = 250$.

Find a formula for y in terms of x .

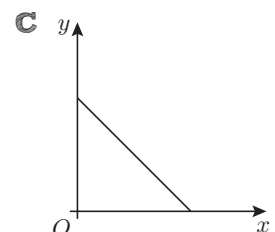
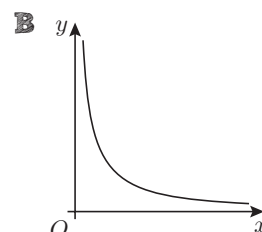
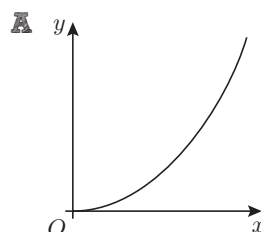
- 4 y is directly proportional to the square of x .

When $x = 4$, $y = 20$.

a Find a formula for y in terms of x .

b Work out the value of y when $x = 18$.

- 5 Circle the graph in which y is inversely proportional to x .



Exam-style question

- 6 T is inversely proportional to d^2 .

$T = 180$ when $d = 6$.

Find the value of T when $d = 0.5$.

..... (3 marks)

Problem-solve!

Exam-style question

- 1 D is directly proportional to the square of n .
Carly says that when n is doubled, the value of D is doubled.
Carly is wrong.
Explain why. (1 mark)

- 2 In an experiment, measurements of a and b were taken.

a	2	6	10
b	4	108	500

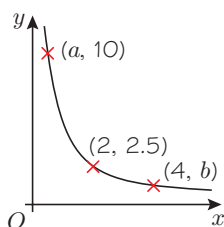
Which of these relationships fits the result?

$b \propto a$ $b \propto a^2$ $b \propto a^3$ $b \propto \sqrt{a}$

Exam-style questions

- 3 y is inversely proportional to the square of x .
When $x = 4$, $y = 1440$.
Work out the value of y when $x = 0.25$ (3 marks)
- 4 y is directly proportional to the square of x .
When $x = 20$, $y = 240$.
Work out the value of y when $x = 3$ (3 marks)
- 5 d is inversely proportional to the square of c .
When $c = 5$, $d = 300$.
Work out the value of d when $c = 2.5$ (3 marks)

- 6 In this graph y is inversely proportional to x .
Find the values of a and b .



Now that you have completed this unit, how confident do you feel?

1 Direct proportion
involving squares



2 Inverse proportion
involving squares



3 Direct and inverse
proportion graphs

