Mathematics Assignment – Squares and Square Roots.

Square Basic Concepts

When any number is multiplied by itself we get the product of two same numbers

If 'n' is any number multiplied by itself i.e. 'n' we get $n \times n$ OR n^2

So the square of a number is that number whose power is raised by 2.

i.e, square of $2 = 2^2 = 2 \times 2 = 4$

" " $3 = 3^2 = 3 \times 3 = 9$

" " $4 = 4^2 = 4 \times 4 = 16$

Perfect square – A natural number is called a perfect square or a square number, if it is the square of some natural number

e.g. $1 = 1^2$

$4 = 2^2$

$9 = 3^2$ etc.

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3, 5, 7, ---- are not perfect squares as it cannot be expressed as the product of two equal factors.

Some properties of Squares

(a) A number ending with 2, 3, 7, 8 is never a perfect square.

(b) A number ending in an odd number of zeros is never a perfect square

(c) Squares of even numbers are even.

(d) Squares of odd numbers are odd.

(e) For every natural number n, we have $(n+1)^2 - n^2 = (n+1) + n = 2n+1$

(f) A triplet (a, b, c) is called a pythagorean triplet if $a^2+b^2=c^2$

For every natural number m $(2m, m^2-1, m^2+1)$ is a pythagorean triplet e.g. if $m=3$ then $(6, 8, 10)$ is a pythagorean triplet

$\because 6^2+8^2=10^2$ is true

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(g) The square of a natural number 'n' is equal to the sum of first 'n' odd numbers.

So for any natural number 'n' we have n^2 = Sum of first n odd nos.

e.g, $1^2 = 1$ = Sum of first 1 odd no.

$2^2 = 4 = 1+3$ = " " 2 " "s.

$3^2 = 9 = 1+3+5$ = " " 3 " "

and so on.

(h) Between the squares of the numbers n and (n+1), there are '2n' non perfect square numbers

$\therefore (n+1)^2 - n^2 - 1 = 2n$

(i) If a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.

e.g,

17 can not be expressed as a sum of odd nos. starting with 1

$17 \neq 1+3+5+7+9$

Whereas $25 = 1+3+5+7+9$ so 25 is a perfect sq.

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(j) The square of any odd number can be expressed as a sum of two consecutive positive integers.

Let n be any odd no. Now its square (n^2) is expressed as sum of two consecutive positive integer as

$$\frac{n^2-1}{2} \text{ and } \frac{n^2+1}{2}$$

$$\left[\text{Sum } \frac{n^2-1}{2} + \frac{n^2+1}{2} = \frac{n^2-\cancel{1}+n^2+\cancel{1}}{2} = \frac{2n^3}{2} = n^2 \right]$$

Square-Root – It is the inverse operation of square.

Let the number be 'n'

The square root of 'n' is that number which when multiplied by itself gives n as the product.

It is denoted by $\sqrt{\ }$

e.g, $\sqrt{4} = 2$, $\sqrt{9} = 3$ etc.

Square root of a number can be obtained by different method

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Method No. 1 : By prime factorization method.

Following are the main steps

(i) Resolve the given number into prime factors.

(ii) Make pairs of similar factors.

(iii) Take the product of prime factors by choosing one out of every pair.

This gives you the square root of the required number.

Method No. 2 - (By division method) -

This method is used when the number is large because doing factorization of large number is difficult and lengthy also. To overcome this problem we use long division method.

Following are the most important steps to find the square root by long division method.

Step 1. Place a bar over every pair of digits starting from unit place. If

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by chance at last one digit left then put the bar over this single digit also.

e.g. $\overline{10}\;\overline{24}$ $\qquad$ $\overline{6}\;\overline{25}$

Step 2. Now find a number whose square is less than or equal to the number under the extreme left bar. Write this number at three places as shown below. and find the remainder

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{10}\;\overline{24} \\ \times 3 & 9 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & \overline{6}\;\overline{25} \\ \times 2 & 4 \\ \hline & 2 \end{array}$$

Step 3. Bring the next number under the bar along with remainder as shown below

$$\begin{array}{r|l} & 3 \\ \hline 3 & \overline{10}\;\overline{24} \\ \times 3 & 9 \\ \hline & 1\,24 \end{array} \qquad \begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\;\overline{25} \\ \times 2 & 4 \\ \hline & 2\,25 \end{array}$$

Step 4. Now to divide the remainder, we need the new divisor this will be the combination of double the quotient along with the new number (This is to be searched). Divide

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The remainder by the number (which is double the quotient) approximately and write this number at three places shown below

```
              3 2
     3  | ‾10 ‾24
    x3  |   9
Double  |  ------
    6 2 |   1 24
    x 2 |   1 24
        |  ------
        |     x
```

```
              2 5
     2  | ‾6 ‾25
    x 2 |  4
Double  | ------
    4 5 |  2 25
    x 5 |  2 25
        | ------
        |    x
```

Finally procede as shown above. If the number is large, proceede as explained in step No. 3, 4. and we will get the square root

$$\sqrt{1024} = 32 \quad ; \quad \sqrt{625} = 25$$

<u>Square Roots of Decimals</u> — It should be noted that a decimal number has two parts, integral part and decimal part. Now method of putting the bars will be different. In integral part bars to be placed from right to left and in decimal part, from left to right. In decimal part, we can take help of zero if the

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last number is not in pair. placing the bars over decimal numbers shown below.

$\overleftarrow{2}\,\overleftarrow{35}.\overrightarrow{56}\,\overrightarrow{75}$; $\overleftarrow{2}.\overrightarrow{35}\,\overrightarrow{70}$

Rest the method is same as explaned earlier. After finishing the steps of square root for integral part, put the decimal and follow the same steps as in integral part.

Estimating Square Root- It means, to find the square root of a number not exactly but near by. This, I will explain with the help of an example. Suppose we have to find the square root of 250. Its square root will have two digit (This you can find just by counting the bars (2) as $\overset{②}{\overline{2}}\overset{①}{\overline{50}}$)

Now we will find a two digit no. whose square is just less than 250

ie, $15^2 = 225 < 250$

Its next no. $16^2 = 256 > 250$

It means $225 < 250 < 256$

from above 256 is very near to 250 so approximate root of 250 is 16.