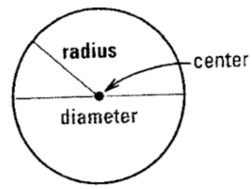
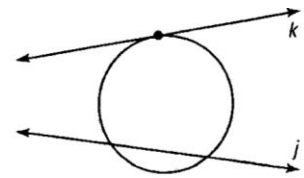
Name ____ Period _______

GEOMETRY – CHAPTER 10 Notes – CIRCLES

Section 12.1 Exploring Solids

Section 10.1 Tangents to Circles



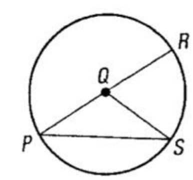


Line j is a secant.
Line k is a tangent.

Objectives: Identify segments and lines related to circles.
Use properties of a tangent to a circle.

Vocabulary:

A **Circle** is a set of points in a plane that are equidistant from a given point, called the **Center** of the circle.
The distance from the center to a point on the circle is the radius of the circle.
Two circles are **congruent** if they have the same radius.
The distance across the circle , though its center, is the **diameter** of the circle.
A **radius** is a segment whose endpoints are the center of the circle and a point In this circle.
A **cord** is a segment whose endpoints are points on the circle.
A **secant** is a line that intersects a circle in two points.
A **tangent** is a line in the plane of a circle that intersects the circle in exactly one place.



$\overline{PS}$ and $\overline{PR}$ are chords.

The diameter is equal to 2 times the radius: $d = 2r$

The radius is equal to half the diameter: $r = \frac{1}{2}d$

Identify Special Segments and Lines

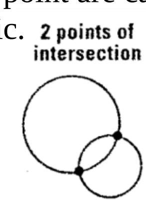
Example 1: The diameter of a circle is given. Find the radius.

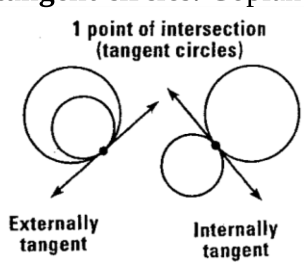
1. $d = 10$ in. 2. $d = 24$ ft 3. $d = 8.2$ cm 4. $d = 12.6$ in.

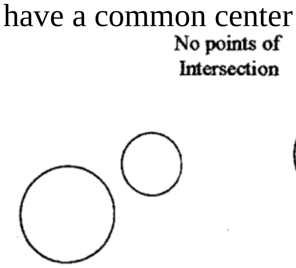
Example 2: The radius of a circle is given. Find the diameter.

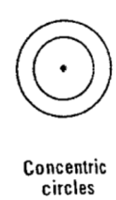
1. $r = 15$ cm 2. $r = 5.2$ ft 3. $r = 10$ in. 4. $r = 4.25$ cm

In a plane, two circles can intersect in two points, one point or no points. Coplanar circle that intersect in one point are called **tangent circles.** Coplaner circles that have a common center are called concentric.





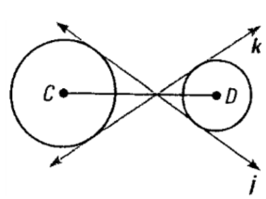




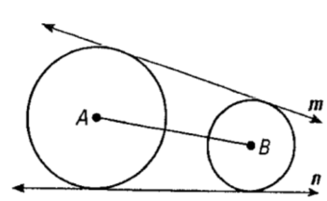
A line or segment that is tangent to two coplanar circles is called a **common tangent.** A *common internal tangent* intersects the segment that joins the centers of the two circles. A *common external tangent* does not intersect the segment that joins the centers of the two circles.

Example 3: Tell whether the **common tangents** are *internal* or *external*.

a.



b.

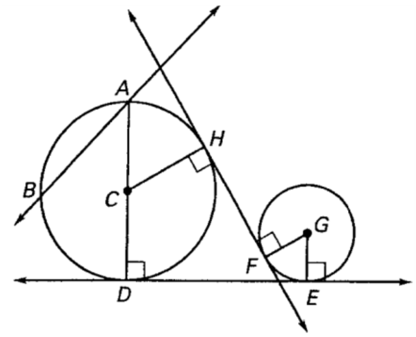


In a plane, the **Interior of a circle** consists of the points that are inside the circle. The **exterior of a circle** consists of the points that are outside the circle.

The point at witch a tangent line intersects the circle to witch it is tangent is the **point of tangency.**

Example 4: Match the notation with the term that best describes it.

9. D	A. Center
10. $\overleftrightarrow{FH}$	B. Chord
11. $\overline{CD}$	C. Diameter
12. $\overline{AB}$	D. Radius
13. C	E. Point of tangency
14. $\overline{AD}$	F. Common external tangent
15. $\overleftrightarrow{AB}$	G. Common internal tangent
16. $\overleftrightarrow{DE}$	H. Secant

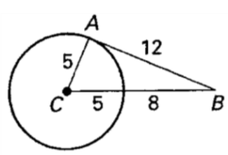


Theorem 10.1 If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

Theorem 10.2 In a lane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

Example 5: Tell whether $\overleftrightarrow{AB}$ is tangent to ©. Explain your reasoning.

a.



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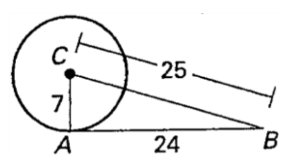
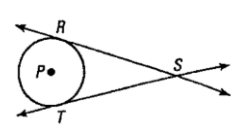
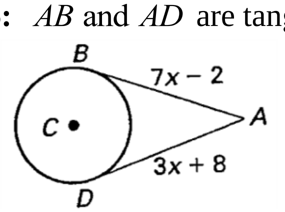


Theorem 10.3 If two segments from the same exterior point are tangent to a circle then they are congruent $\overline{RS} \cong \overline{TS}$

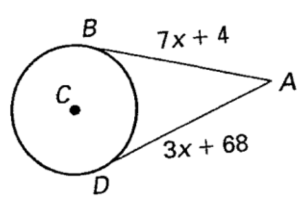


Example 6: $\overline{AB}$ and $\overline{AD}$ are tangent to ©. Find the value of x.

a.



b.

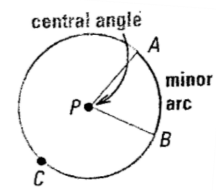


Section 10.2 Arcs and Chords

Objectives: Use properties of arcs of circles.
Use properties of chords of circles.

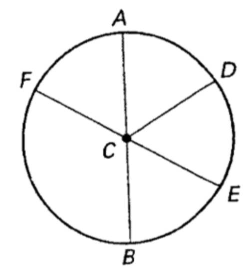
Vocabulary

- In a plane, an angle whose vertex is the center of a circle is a **central angle** of the circle.
- If the measure of a central angle, $\angle APB$, is less than $180°$, then A and B and the points of $\odot P$ in the interior of $\angle APB$ form a **minor arc of the circl.**
- The **measure of a minor arc** is defined to be the measure of its central angle.
- The **measure of a major arc** is defined as the difference between $360°$ and the measure of its associated minor arc.
- If the endpoints of an arc are the endpoints of a diameter, then the arc is a **semicircle**.



Example 1: Determine whther the arc is a minor arc, a major arc, or a semicircle of $\odot C$.

1. $\overset{\frown}{AE}$
2. $\overset{\frown}{AEB}$
3. $\overset{\frown}{FDE}$
4. $\overset{\frown}{DFB}$
5. $\overset{\frown}{FA}$
6. $\overset{\frown}{BE}$
7. $\overset{\frown}{BDA}$
8. $\overset{\frown}{FB}$

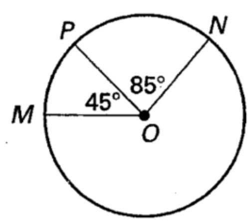


Postulate 26 The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

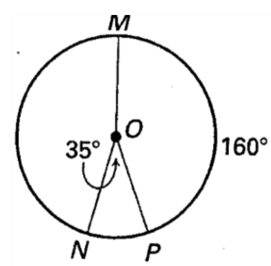
$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$

Example 3: Find the measure of $\overset{\frown}{MN}$.

19.

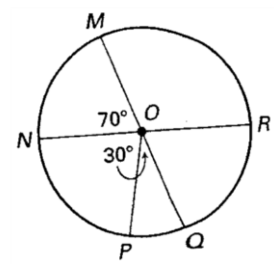


20.



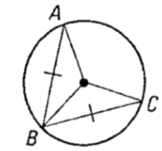
Example 2: $\overline{MQ}$ and $\overline{NR}$ are diameters. Find the indicated measures.

9. $m\overset{\frown}{MN}$
10. $m\overset{\frown}{NQ}$
11. $m\overset{\frown}{NQR}$
12. $m\overset{\frown}{MPR}$
13. $m\overset{\frown}{QR}$
14. $m\overset{\frown}{MR}$
15. $m\overset{\frown}{QMR}$
16. $m\overset{\frown}{PQ}$
17. $m\overset{\frown}{PRN}$
18. $m\overset{\frown}{MQN}$



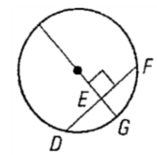
Theorem 10.4 In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

if and only if $\overline{AB} \cong \overline{BC}$



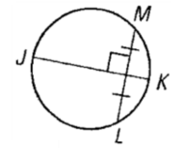
Theorem 10.5 If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

$$\overline{DE} \cong \overline{EF},\ \overset{\frown}{DG} \cong \overset{\frown}{GF}$$



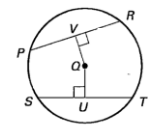
Theorem 10.6 If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

$\overline{JK}$ is a diameter of the circle.



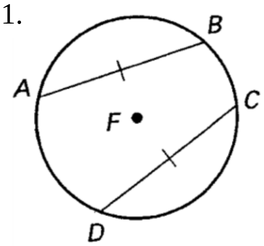
Theorem 10.7 In the same circle or congruent circles, two chords are congruent *if and only if* they are equidistant from the center.

if $\overline{QV} \cong \overline{QU}$ *then* $\overset{\frown}{PR} \cong \overset{\frown}{ST}$



Ex. 4 What can you conclude about the diagram? State a postulate or theorem that justifies your answer.

21.



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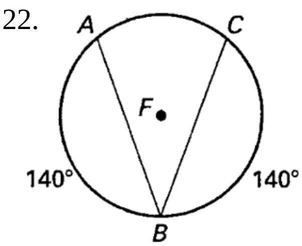

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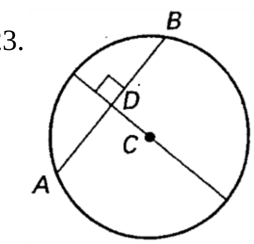
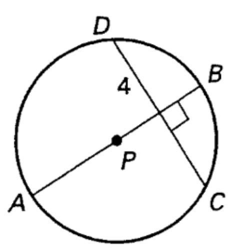
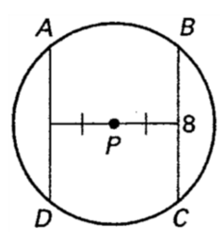


Ex. 5 Find the indicated measure of $\odot P$.

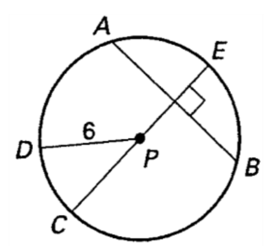
24. $DC =$ ___



25. $AD =$ ___



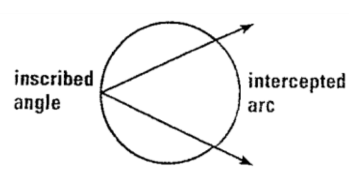
26. $EC =$ ___



Section 10.3 Inscribed Angles

An **inscribed angle** is an ____

____

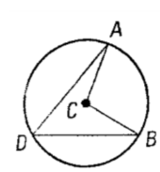


The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the _________________________ of the angle.

Theorem 10.8

If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.

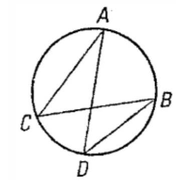
$$m\angle ADB = \frac{1}{2} m\overset{\frown}{AB}$$



Theorem 10.9

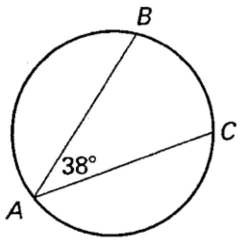
If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

$\angle C \cong \angle D$

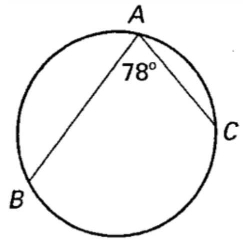


Example 1: Find the measure of the indicated arc or angle.

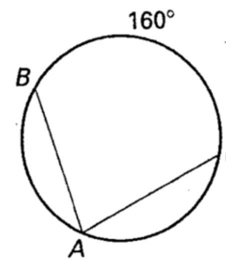
1. $m\overset{\frown}{BC} =$ ___



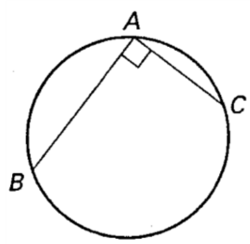
2. $m\overset{\frown}{BC} =$ ___



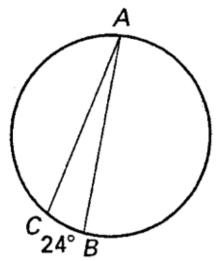
3. $m\angle BAC =$ ___



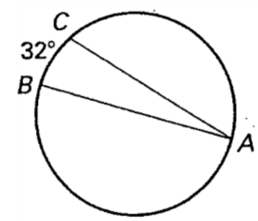
4. $m\overset{\frown}{BC} =$ ___



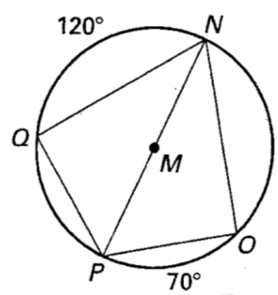
5. $m\angle BAC =$ ___



6. $m\angle BAC =$ ___

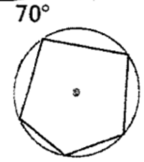


Ex. 2 Find the measure of the arc or angle in $\odot M$.



7. $m\angle QMP$
8. $m\angle NMO$
9. $m\angle PNO$
10. $m\angle QNP$
11. $m\overset{\frown}{QO}$
12. $m\overset{\frown}{NOP}$
13. $m\overset{\frown}{PQ}$
14. $m\overset{\frown}{OQN}$

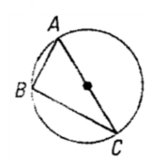
If all of the vertices of a polygon lie on a circle, the polygon is ______________ in the circle and the circle is __________________ about the polygon.



Theorem 10.10

If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of a circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.

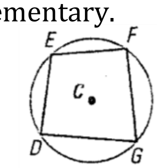
$\angle B$ is a right angle if and only if $\overline{AC}$ is a diameter of the circle.



Theorem 10.11

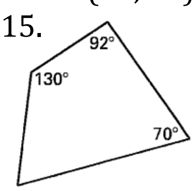
A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

D, E, F, and G lie on some circle, $\odot C$, if and only if $m\angle D + m\angle F = 180°$ and $m\angle E + m\angle G = 180°$



Ex. 3 (15, 16)Can a circle be circumscribed about the quad? (17, 18) Find x:

15.



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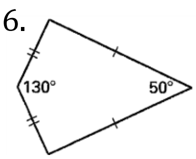
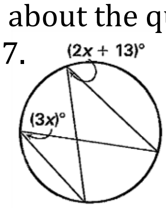


17.



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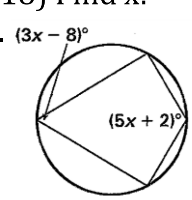
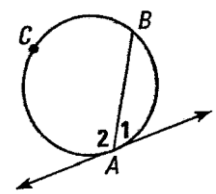


Section 10.4 Other Angle Relationships in Circles

From section 10.3, we found that the measure of an angle inscribed in a circle is half the measure of its intercepted arc. This is true even if one side of the angle is tangent to the circle.

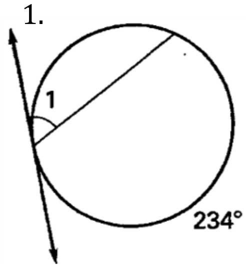
Theorem 10.12 If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its Intersected arc.



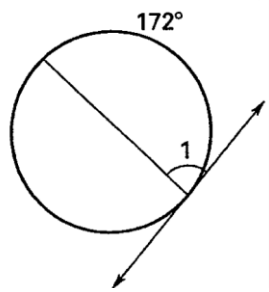
$$m\angle 1 = \frac{1}{2} m\widehat{AB} \qquad m\angle 2 = \frac{1}{2} m\widehat{BCA}$$

Ex. 1 Find the measure of $\angle 1$.

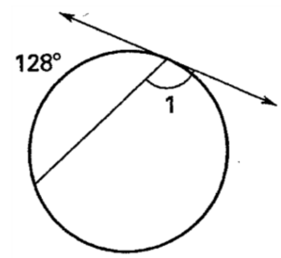
1.



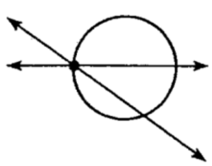
2.



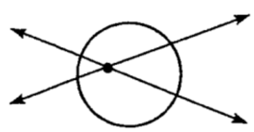
3.



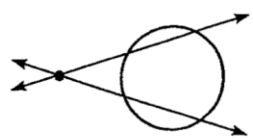
If two lines intersect a circle, there are three places where the lines can intersect.



on the circle



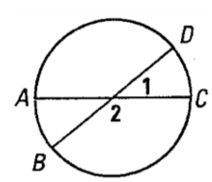
inside the circle



outside the circle

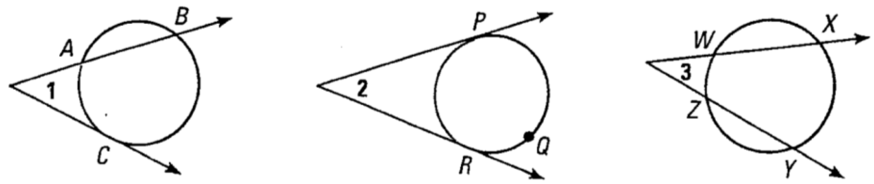
So far, we have learned how to find angle and arc measures when lines intersect *on* the circle. In Theorems 10.13 and 10.14, you will be able to find arcs and angles when the lines intersect *inside* or *outside* the circle.

Theorem 10.13
If two cords intersect in the interior of a circle, then the measure of each angle is one half the sum of the measures of the arcs intercepted by the angle and its vertical angle.



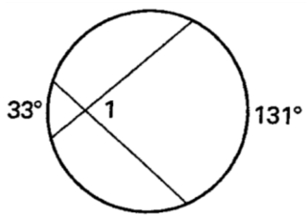
$$m\angle 1 = \frac{1}{2}(m\widehat{CD} + m\widehat{AB}),\ m\angle 2 = \frac{1}{2}(m\widehat{BC} + m\widehat{AD})$$

Theorem 10.14
If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is one half the difference of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$ $m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$ $m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

Ex. 2 Find the measure of $\angle 1$.

5.


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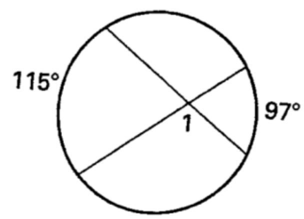

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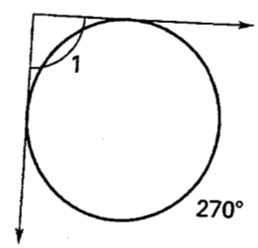

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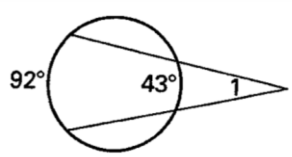

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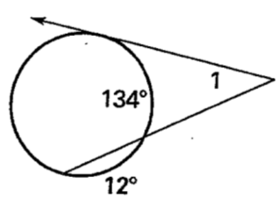

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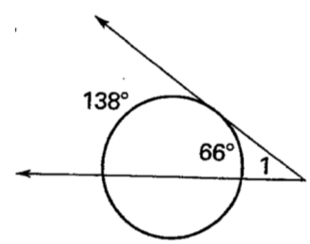
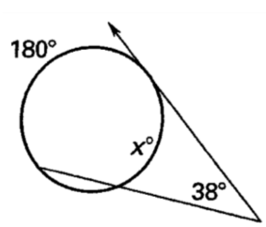


Ex. 3 Find the value of *x*.

11.


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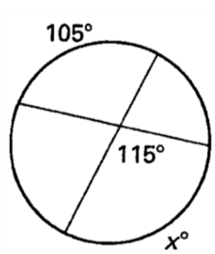

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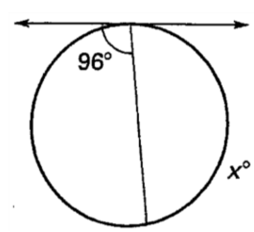
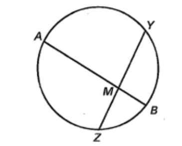


Section 10.5 Segment Lengths in Circles

When two cords intersect on the interior of a circle, each chord is divided into two segments which are called *segments of a chord*.

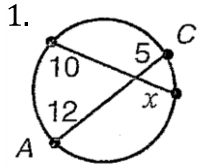
Theorem 10.15 If two cords intersect on the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



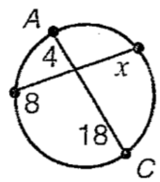
$ZM \cdot MY = AM \cdot MB$

Ex 1 Find *x*:

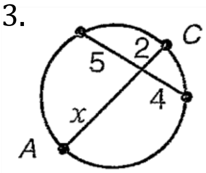
1.



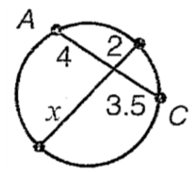
2.



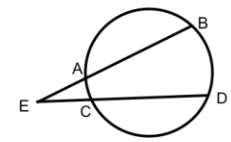
3.



4.

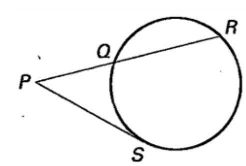


Theorem 10.16 If two secant segments share the same endpoint outside a circle, then the product of the length of one secant segment and the length of its external segment equals the product of the length of the other secant segment and the length of its external segment.



$EA \cdot EB = EC \cdot ED$

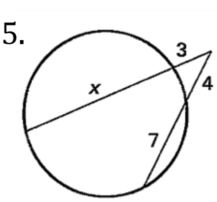
Theorem 10.17 If a secant segment and a tangent share an endpoint outside a circle, then the product of the length of the secant segment and the length of its external segment equals the square of the length of the tangent segment.



$PQ \cdot QR = (PS)^2$

Ex 2 Find *x*:

5.



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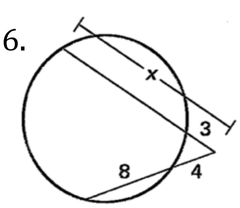

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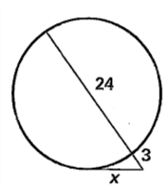

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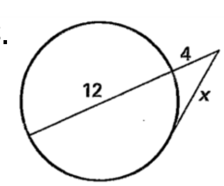


Section 10.6 Equations of Circles

Objective: Write the equation of a circle.

Vocabulary: The standard equation of a circle with radius r and center (h, k) is $(x-h)^2+(y-k)^2=r^2$. If the center is the origin, then $x^2+y^2=r^2$.

Example 1: Match the equation of a circle with its description.

1. $x^2+y^2=4$ — A. Center (-1, 4), radius 4
2. $x^2+y^2=9$ — B. Center (-2, -3), radius 3
3. $(x+1)^2+(y-4)^2=16$ — C. Center (0, 0), radius 2
4. $(x+2)^2+(y+3)^2=9$ — D. Center (2, 5), radius 3
5. $(x+3)^2+(y-5)^2=16$ — E. Center (-3, 5), radius 4
6. $(x-2)^2+(y-5)^2=9$ — F. Center (0, 0), radius 3

Example 2: Give the center and radius of the circle.

7. $x^2+y^2=25$
   C:
   R:

8. $x^2+(y-4)^2=9$
   C:
   R:

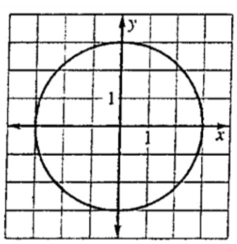
9. $(x-5)^2+y^2=16$
   C:
   R:

10. $(x+1)^2+(y-1)^2=4$
   C:
   R:

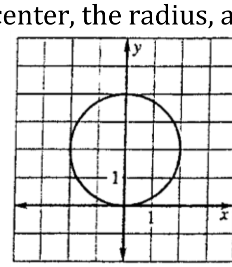
11. $(x-2)^2+(y-4)^2=81$
   C:
   R:

12. $(x+4)^2+(y-2)^2=25$
   C:
   R:

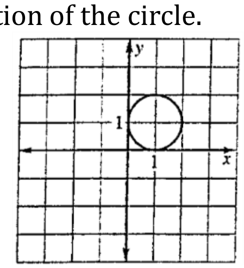
Example 3: Give the coordinates of the center, the radius, and the equation of the circle.

13. 

C:
R:

14. 

C:
R:

15. 

C:
R:

Example 4: Write the standard equation of the circle with the given center and radius.

16. Center (0, 0), radius 2

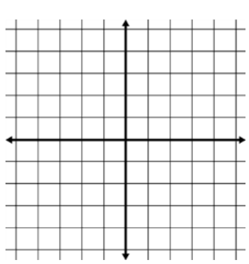
17. Center (-3, 5), radius 4

18. Center (2, 0), radius 3

19. Center (3, 3), radius 4

Example 5: Graph the equation

20. $(x-3)^2+(y+4)^2=4$



21. $(x+1)^2+(y-2)^2=9$