CHAPTER 6: SYSTEMS OF TWO LINEAR EQUATIONS IN TWO VARIABLES

Contents

SECTION 6.1: SYSTEM OF EQUATIONS: GRAPHING

A. VERIFYING SOLUTIONS

In chapter 2 we solved single variable linear equations. For example, to solve for x given the linear equation

$$x + 3 = 4,$$

we must isolate the variable x. This is done by moving any term with an x to the left of the equal sign and the rest of the terms to the right of the equal sign. Doing this gives us $x = 1$.

What if we wanted to solve for two variables? For example, if we wanted to solve for x and y in the equation

$$x + y = 3$$

To solve for x all we would need to do is isolate it. Doing so gives us

$$x = -y + 3$$

The problem here is that even when x is by itself, we still don't know what x is equal to. Having y on the other side of the equal sign puts us in a tough spot since we have no idea what y is. It turns out that in order to solve a linear equation of two variables, we will need another equation. In other words, to solve for **two variables** x and y, we will need **two equations**. When solving for more than one equation and one variable, we call the set of equations a **system of equations.** When dealing with a system of equations, we are looking for the values that make both equations true. If only one equation is true, then we have the wrong answer and must try again.

A system of two linear equations in two variables is of the form

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$$

Where $a, b, c, d, e,$ and f are coefficients and x and y are variables.

Given an ordered pair (x, y), we can check to see if this is a solution to a system by plugging the ordered pair into both equations and verifying that both are true.

For example, to verify that the point $(4, 1)$ is a solution to the system

$$\begin{cases} \frac{1}{2}x + y = 3 \\ -\frac{3}{4}x + y = -2 \end{cases}$$

we will plug the point $(4, 1)$ into the first equation.

$$\frac{1}{2}(4) + (1) = 3$$
$$2 + 1 = 3$$
$$3 = 3$$

Seeing that this equation is true, let's verify the next one.

$$-\frac{3}{4}(4) + (1) = -2$$
$$-3 + 1 = -2$$
$$-2 = -2$$

Since both equations are true, we say the point $(4, 1)$ is a solution to the system.

	MEDIA LESSON Verifying solutions (Duration 2:18)

View the video lesson, take notes and complete the problems below

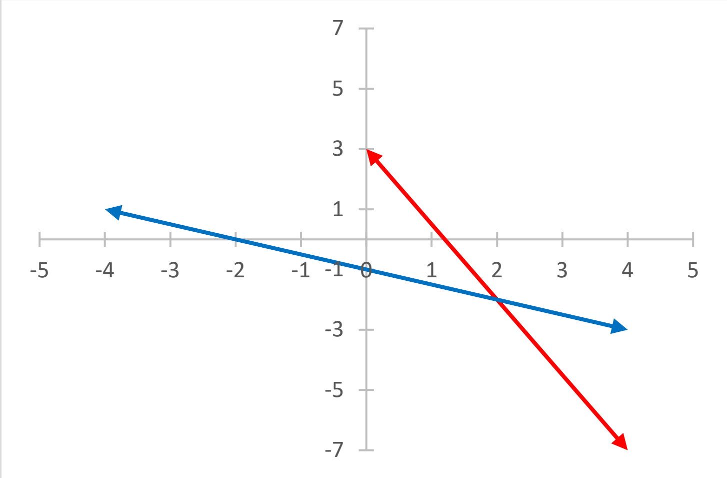
To solve a system of equations we want to find the value of __________ and the value of __________

that satisfies __________ equations.

The point of intersection is the point that lies on __________ lines

Example:

a) Given the graph, identify the solution to the system of equations. Verify the solution.



YOU TRY

a) Is the ordered-pair $(2, 1)$ the solution to the system

$$\begin{cases} 3x - y = 5 \\ x + y = 3 \end{cases}$$

B. SOLVE A SYSTEM BY GRAPHING

<u>One way</u> to solve a system of linear equations is by graphing each linear equation on the <u>same</u> xy-plane. When this is done, one of three cases will arise:

Case 1: Two Intersecting Lines

If the two lines intersect at a single point, then there is **one solution** for the system: the point of intersection.

Case 2: Parallel Lines

If the two lines are parallel to each other (not touching), then there is **no solution** for the system.

Case 3: Same Lines

If one line is on top of the other line (equivalent lines), then there are **infinitely many solutions** for the system.

	MEDIA LESSON Solve by graphing (Duration 3:44)

View the video lesson, take notes and complete the problems below

When we talk about solving a system of equations what we're looking for is the combination of x and y that simultaneously make both equations__________. Another way of saying that is it's the point __________ that lies on __________ lines at the same time.

Example:

a) Solve the following system by graphing.

$$\begin{gathered} 2x + y = 5 \\ x - 3y = -8 \end{gathered}$$

YOU TRY

a) Solve the system by graphing. If possible, write the solution as an ordered pair.

$$\begin{cases} 6x - 3y = -9 \\ 2x + 2y = -6 \end{cases}$$

b) Solve the system by graphing. If possible, write the solution as an ordered pair.

$$\begin{cases} -\dfrac{3}{2}x + y = -4 \\ -\dfrac{3}{2}x + y = 1 \end{cases}$$

EXERCISES

Solve each system by graphing. When possible, write the solution as an ordered pair.

1)

$$y = -3$$
$$y = -x - 4$$

2)

$$y = \frac{1}{3}x + 2$$
$$y = -\frac{5}{3}x - 4$$

3)

$$x + 3y = -9$$
$$5x + 3y = 3$$

4)

$$2x + 3y = -6$$
$$2x + y = 2$$

5)

$$2x + y = -2$$
$$x + 3y = 9$$

6)

$$2x - y = -1$$
$$0 = -2x - y - 3$$

7)

$$-y + 7x = 4$$
$$-y - 3 + 7x = 0$$

8)

$$y = -\frac{5}{4}x - 2$$
$$y = -\frac{1}{4}x + 2$$

9)

$$y = 2x + 2$$
$$y = -x - 4$$

10)

$$y = \frac{1}{2}x + 4$$
$$y = \frac{1}{2}x + 1$$

11)

$$6x + y = -3$$
$$x + y = 2$$

12)

$$x + 2y = 6$$
$$5x - 4y = 16$$

13)

$$-2y + x = 4$$
$$2 = -x + \frac{1}{2}y$$

14)

$$16 = -x - 4y$$
$$-2x = -4 - 4y$$

15)

$$-5x + 1 = -y$$
$$-y + x = -3$$

SECTION 6.2: SYSTEMS OF EQUATIONS: THE SUBSTITUTION METHOD

A. THE SUBSTITUTION METHOD

In the previous section we saw that one way to solve a system of linear equations is to graph each equation on the same xy-plane. If the graph is not accurate, then it can be difficult to see the solution. In this section we will look at another method to solving a system of linear equations: the **substitution method.**

Let us look at an example.

$$\begin{cases} 6x - 3y = -9 & (a) \\ 2x + 2y = -6 & (b) \end{cases}$$

To the right of each equation is a label. This will make keeping track of our work much easier. The idea behind the substitution method is to solve one equation for one variable, then substitute this value into the second equation. Once this substitution is made, the second equation becomes a one variable equation. Let us walk through the example above.

Step 1: Solving for a variable (any variable you want)

Looking at the first equation (a), let us solve for y.

$$\begin{aligned} 6x - 3y &= -9 \\ -3y &= -6x - 9 \\ y &= 2x + 3 \end{aligned}$$

So $y = 2x + 3$. We don't know what y is since there is an x in it, but that's okay.

Step 2: Substitution

Now let us take a look at the second equation (b). Since we solved for y in the first equation, we know what y is equal to. We will use this value and plug it into the second equation (the variable we are plugging into the equation is in red)

$$\begin{aligned} 2x + 2y &= -6 \\ 2x + 2(2x + 3) &= -6 \end{aligned}$$

Step 3: Simplify and solve

Notice how the second equation changes with the substitution we performed above! Our <u>two variable</u> equation $2x + 2y = -6$ just became the <u>one variable</u> equation $2x + 2(2x + 3) = -6$. And we know how to solve one variable equations.

$$\begin{aligned} 2x + 2(2x + 3) &= -6 \\ 2x + 4x + 6 &= -6 \\ 6x + 6 &= -6 \\ 6x &= -12 \\ x &= -2 \end{aligned}$$

Hence $x = -2$

Step 4: Finding the second value

We are almost done. We have what x is equal to and now we just need what y is equal to. Looking back at Step 1 we actually have an idea of what y is. Now that we have a value for x, let us plug this value (shown in blue) into the Step 1 equation and simplify

$$\begin{aligned} y &= 2x + 3 \\ y &= 2(-2) + 3 \\ y &= -4 + 3 \\ y &= -1 \end{aligned}$$

Therefore, our solution to this system is $x = -2$ and $y = -1$. Since the solution represents the <u>point of intersection</u>, let us write our solution as a point: $(-2, -1)$.

If there exists at least one solution to the system, then the system is called a **consistent system.** A consistent system with **one unique solution** (as shown above) is said to have an **independent solution**. A consistent system with **more than one solution** is said to have **a dependent solution.**
If no solution exists to the system, then the system is called an **inconsistent system**.

	MEDIA LESSON Substitute Expression (Duration 4:45)

View the video lesson, take notes and complete the problems below
Just as we can replace a variable with a number, we can also replace it with an ________________.

Whenever we substitute it is important to remember ________________.

Example:

a) Solve the system by substitution.

$$y = 5x - 3$$
$$-x - 5y = -11$$

b) Solve the system by substitution.

$$2x - 6y = -24$$
$$x = 5y - 22$$

	MEDIA LESSON Solve for a variable (Duration 5:00)

View the video lesson, take notes and complete the problems below
To use substitution we may have to ________________ a lone variable. If there are several lone

variables_______________.

Example:

a) Solve the system by substitution.

$$6x + 4y = -14$$
$$x - 2y = -13$$

b) Solve the system by substitution.

$$-5x + y = -17$$
$$7x + 8y = 5$$

YOU TRY

a) Solve the system by substitution.

$$2x - 3y = 7$$
$$y = 3x - 7$$

b) Solve the system by substitution.

$$3x + 2y = 1$$
$$x - 5y = 6$$

B. SUBSTITUTION: SPECIAL CASES

There are two special cases that arise in a system of linear equations that we should look at: when we have parallel lines and when we have duplicate lines. Once a system is graphed it can be readily seen what case we have, however, when solving a system algebraically it can be difficult to tell what case we are dealing with.

❖ *Infinitely many solutions (equivalent lines)*

Let us look at an example. We are still using the substitution method, so the steps we will use to arrive at our solution will mimic the ones used in part A of this section.

Solve the system by substitution.

$$y + 4 = 3x \quad (a)$$
$$2y - 6x = -8 \quad (b)$$

Step 1: Solving for a variable (any variable you want)

Looking at the first equation (a), let us solve for y.

$$y + 4 = 3x$$
$$y = 3x - 4$$

So $y = 3x - 4$.

Step 2: Substitution

$$2y - 6x = -8$$
$$2(3x - 4) - 6x = -8$$

Step 3: Simplify and solve

$$6x - 8 - 6x = -8$$
$$-8 = -8$$

What just happened? The variable x just got cancelled out and now we are left with no variable. However, we are left with a true statement: $-8 = -8$. This means that we have an infinitely many solutions. Graphically we have a line on top of another line (equivalent lines).

❖ *No solution (parallel lines)*

Let us look at another example. Solve the system by substitution.

$$6x - 3y = -9 \quad (a)$$
$$-2x + y = 5 \quad (b)$$

Step 1: Solving for a variable (any variable you want)

Looking at the first equation (a), let us solve for y.

$$6x - 3y = -9$$
$$-3y = -6x - 9$$
$$y = 2x + 3$$

So $y = 2x + 3$.

Step 2: Substitution

$$-2x + y = 5$$
$$-2x + (2x + 3) = 5$$

Step 3: Simplify and solve

$$-2x + 2x + 3 = 5$$
$$3 = 5$$

The variable x got cancelled out again, however, we are now left with a false statement: $3 = 5$. This means that we have no solution. Graphically we have two parallel lines

	MEDIA LESSON Special cases (Duration 4:38)

View the video lesson, take notes and complete the problems below

If the variables subtract out to zero then it means either there is ___________________ or ________________.

Example:

a) Solve the system by substitution.

$$x + 4y = -7$$
$$21 + 3x = -12y$$

b) Solve the system by substitution.

$$5x + y = 3$$
$$8 - 3y = 15x$$

YOU TRY

Solve the system by substitution

$$\begin{aligned} 5x - 6y &= -14 \\ -2x + 4y &= 12 \end{aligned}$$

EXERCISES

Solve each system by substitution. Determine if each system is consistent, independent or dependent, or inconsistent.

1)

$$y = -2x - 9$$
$$y = 2x - 1$$

2)

$$y = 3x + 2$$
$$y = -3x + 8$$

3)

$$y = 6x - 6$$
$$-3x - 3y = -24$$

4)

$$y = -5$$
$$3x + 4y = -17$$

5)

$$y = -8x + 19$$
$$-x + 6y = 16$$

6)

$$x - 5y = 7$$
$$2x + 7y = -20$$

7)

$$-6x + y = 20$$
$$-3x - 3y = -18$$

8)

$$y = x + 5$$
$$y = -2x - 4$$

9)

$$y = 3x + 13$$
$$y = -2x - 22$$

10)

$$y = 7x - 24$$
$$y = -3x + 16$$

11)

$$6x - 4y = -8$$
$$y = -6x + 2$$

12)

$$y = x + 4$$
$$3x - 4y = -19$$

13)

$$x - 2y = -13$$
$$4x + 2y = 18$$

14)

$$6x + 4y = 16$$
$$-2x + y = -3$$

15)

$$-5x - 5y = -20$$
$$-2x + y = 7$$

16)

$$x + 5y = 15$$
$$-3x + 2y = 6$$

17)

$$-6x + 6y = -12$$
$$8x - 3y = 16$$

18)

$$2x + y = -7$$
$$5x + 3y = -21$$

SECTION 6.3: SYSTEM OF EQUATIONS: THE ADDITION METHOD

A. THE ADDITION METHOD

A third method to solving a system of linear equations is the addition method (also called the elimination method). Let us look at an example to see how the addition method is carried out. Given the system below, let us solve it by the addition method.

$$\begin{aligned} 3x - 4y &= 8 \\ 5x + 4y &= -24 \end{aligned}$$

Step 1: Lining up the variables

To begin the process of the addition method we want to start by lining up the variables. Notice in the system above how the first term in each linear equation have the same variable, x. Likewise how the second term in each linear equation have the same variable, y. Finally, on the right side of the equal sign, we have the numbers with no variables.

Step 2: Adding both equations

Now that the linear equations are lined up nicely, let us add them together

$$\begin{array}{r} 3x - 4y = 8 \\ +\ 5x + 4y = -24 \\ \hline 8x + \quad = -16 \\ x = -2 \end{array}$$

Notice that adding both equations together allows us to eliminate one variable and solve for the other simultaneously. So $x = -2$

Step 3: Finding the second value

Now that we have one value let us solve for the other. To get the second value all we need to do is plug the value we found into one of the initial linear equations. It doesn't matter which one we choose, so let us pick the first one.

$$\begin{aligned} 3x - 4y &= 8 \\ 3(-2) - 4y &= 8 \\ -6 - 4y &= 8 \\ -4y &= 14 \\ y &= -\frac{14}{4} \\ y &= -\frac{7}{2} \end{aligned}$$

Our solution is therefore $x = -2$ and $y = -\frac{7}{2}$. Since our solution represents a point of intersection, let us write our solution as an ordered pair: $\left(-2, -\frac{7}{2}\right)$.

	MEDIA LESSON Addition method (Duration 4:18)

View the video lesson, take notes and complete the problems below

If there is no lone variable, it may be better to use _______________. This method works by adding the _______________ and _______________ sides of the equations together.

Example:

a) Solve the system using the addition method.

$$\begin{aligned} -8x - 3y &= -12 \\ 2x + 3y &= -6 \end{aligned}$$

b) Solve the system using the addition method.

$$\begin{aligned} -5x + 9y &= 29 \\ 5x - 6y &= -11 \end{aligned}$$

YOU TRY

a) Solve the system by using the addition method.

$$\begin{aligned} 4x + 2y &= 0 \\ -4x - 9y &= -28 \end{aligned}$$

B. THE ADDITION METHOD WITH MULTIPLICATION

Sometimes the systems we are dealing with don't add together nicely, meaning that one of the variables is not readily able to cancel out. In cases like these we may multiply one of the equations by some constant, allowing us to create a scenario where we can cancel out one of the variables. For example,

$$\begin{aligned} -6x + 5y &= 22 \quad (a) \\ 2x + 3y &= 2 \qquad (b) \end{aligned}$$

If we were to add both of these equations together we would get $-4x + 8y = 24$.This result is pointless for us. All we did was end up with a third equation with both variables still in it.
To eliminate a variable, we must first decide what variable we want to get rid of first. It doesn't matter which variable we choose to eliminate nor does it matter which equation we choose to modify, we just need to pick one. Let us choose to eliminate the variable x. To start the elimination process we will multiply equation (b) by 3. Equation (b) turns into

$$\begin{aligned} 3 \cdot (2x + 3y) &= 2 \cdot 3 \\ 6x + 9y &= 6 \end{aligned}$$

Rewriting the system with the new (b) equation

$$\begin{aligned} -6x + 5y &= 22 \quad (a) \\ 6x + 9y &= 6 \qquad (b) \end{aligned}$$

Notice how the coefficients belonging to the x variable in each linear equation is now the exact opposite of each other. This gives us the scenario we had in the previous section. Now all we need to do is follow our previous steps.

Step 1: Lining up the variables

This is done for us already.

Step 2: Adding both equations

$$\begin{array}{r} -6x + 5y = 22 \\ +\ 6x + 9y = 6 \\ \hline 0x\ + 14y\ = 28 \\ y = 2 \end{array}$$

Step 3: Finding the second value

$$\begin{aligned} -6x + 5y &= 22 \\ -6x + 5(2) &= 22 \\ -6x + 10 &= 22 \\ -6x &= 12 \\ x &= -2 \end{aligned}$$

Our solution is therefore $(-2, 2)$.

	MEDIA LESSON Addition method with multiplication (Duration 4:58)

View the video lesson, take notes and complete the problems below

Addition only works if one of the variables have ________________________.

To get opposites we can multiply ________________ of an equation to get the values we want.

Be sure when multiplying to have a ______________ and ______________ in front of a variable.

Example:

a) Solve the system using the addition method.

$$\begin{aligned} 2x - 4y &= -4 \\ 4x + 5y &= -21 \end{aligned}$$

b) Solve the system using the addition method.

$$\begin{aligned} -5x - 3y &= -3 \\ -7x + 12y &= 12 \end{aligned}$$

YOU TRY

Solve the system by using the addition method.

$$-x-5y=28$$
$$-x+4y=-17$$

C. MULTIPLYING TWO EQUATIONS

In the previous section we multiplied one equation by some constant to change it to our liking. However, there are systems that require us to multiply both equations by some constant. Let us solve the system below by adding.

$$3x+6y=-9 \quad (a)$$
$$2x+9y=-26 \quad (b)$$

To eliminate x we will multiply both equations by a constant to obtain the LCM $(3,2)=6$ with opposite signs: We'll multiply equation (a) by 2 and multiply equation (b) by -3

$$2\cdot(3x+6y)=-9\cdot 2$$
$$-3\cdot(2x+9y)=-26\cdot -3$$

This gives us

$$6x+12y=-18$$
$$-6x-27y=78$$

Notice how the coefficients of x are the same but with opposite signs. This gives us a scenario we are familiar with. Let us solve the system.

Step 1: Lining up the variables
This is done for us already.

Step 2: Adding both equations

$$\begin{array}{r} 6x+12y=-18 \\ +-6x-27y=78 \\ \hline 0x-15y=60 \\ y=-4 \end{array}$$

Step 3: Finding the second value

$$6x+12y=-18$$
$$6x+12(-4)=-18$$
$$6x-48=-18$$
$$6x=30$$
$$x=5$$

Our solution is therefore $(5,-4)$.

	MEDIA LESSON Multiplying two equations (Duration 4:58)

View the video lesson, take notes and complete the problems below

Sometimes we may have to multiply _______________ by something to get opposites.

The opposites we look for is the _____________ of both coefficients.

Example:

a) Solve the system using the addition method.

$$-6x + 4y = 26$$
$$4x - 7y = -13$$

b) Solve the system using the addition method.

$$3x + 7y = 2$$
$$10x + 5y = -30$$

YOU TRY

Solve the system by using the addition method.

$$-8x - 8y = -8$$
$$10x + 9y = 1$$

D. ADDITION: SPECIAL CASES

Like the substitution method, the addition method will also bring about true or false statements. These are the special cases that we have talked about before, but it is important to go over them once more using the addition method.

❖ *Infinitely many solutions (equivalent lines)*

Solve the system by using the addition method

$$2x - 5y = 3$$
$$-6x + 15y = -9$$

Let's get rid of the x variable. We'll multiply the top equation by 3.

$$3 \cdot (2x - 5y) = 3 \cdot 3$$
$$6x - 15y = 9$$

Now that the x variables match and are opposite each other, let us add them together.

$$\begin{array}{r} 6x - 15y = 9 \\ + \ -6x + 15y = -9 \\ \hline 0x + 0y = 0 \end{array}$$

So we have $0 = 0$. Since this is a true statement this implies that we have infinitely many solutions.

❖ *No solution (parallel lines)*

Solve the system by using the addition method.

$$4x - 6y = 8$$
$$6x - 9y = 15$$

Let's get rid of the x variable. We'll multiply the top equation by -6 and the bottom by 4.

$$-6 \cdot (4x - 6y) = 8 \cdot -6$$
$$4 \cdot (6x - 9y) = 15 \cdot 4$$

This gives us

$$\begin{array}{r} -24x + 36y = -48 \\ + \ 24x - 36y = 60 \\ \hline 0x + 0y = 12 \end{array}$$

So we have $0 = 0$. Since this is a false statement this implies that we have no solution.

MEDIA LESSON
Special cases (Duration 4:21)

View the video lesson, take notes and complete the problems below

If the variables subtract out to zero then it means either there is ________________ or ____________________.

Example:

a) Solve the system using the addition method.

$$2x - 4y = 16$$
$$3x - 6y = 20$$

b) Solve the system using the addition method.

$$-10x + 4y = -6$$
$$25x - 10y = 15$$

YOU TRY

a) Solve the system by using the addition method.

$$-x - 2y = -7$$
$$x + 2y = 7$$

EXERCISES

Solve each system by addition. Determine if each system is consistent, independent or dependent, or inconsistent.

1)

$$-9x + 5y = -22$$
$$9x - 5y = 13$$

2)

$$4x - 6y = -10$$
$$4x - 6y = -14$$

3)

$$2x - y = 5$$
$$5x + 2y = -28$$

4)

$$2x + 4y = 24$$
$$4x - 12y = 8$$

5)

$$5x + 10y = 20$$
$$-6x - 5y = -3$$

6)

$$9x - 2y = -18$$
$$5x - 7y = -10$$

7)

$$-7x + 5y = -8$$
$$-3x - 3y = 12$$

8)

$$9y = 7 - x$$
$$-18y + 4x = -26$$

9)

$$-7x + y = -10$$
$$-9x - y = -22$$

10)

$$5x - 5y = -15$$
$$5x - 5y = -15$$

11)

$$-10x - 5y = 0$$
$$-10x - 10y = -30$$

12)

$$x + 3y = -1$$
$$10x + 6y = -10$$

13)

$$-6x + 4y = 4$$
$$-3x - y = 26$$

14)

$$-5x + 4y = 4$$
$$-7x - 10y = -10$$

15)

$$-4x - 5y = 12$$
$$-10x + 6y = 30$$

16)

$$-7x + 10y = 13$$
$$4x + 9y = 22$$

17)

$$-6 - 42y = -12x$$
$$x - \frac{1}{2} - \frac{7}{2}y = 0$$

SECTION 6.4: APPLICATIONS WITH SYSTEMS OF EQUATIONS

A. VALUE & INTEREST PROBLEMS

Here we will look at applications that we've gone over before, but using a different approach to solve. Now that we know how to solve a system of linear equations, we will solve the applications in this sections using two variables (instead of one like in previous sections).

Example. There were 41 tickets sold for an event. Tickets for children cost $\$1.50$ and tickets for adults cost $\$2.00$. Total receipts for the event were $\$73.50$. How many of each type of ticket were sold?

To solve this problem we'll make a table to help organize our information.

	Amount	**Value (in $\$$)**	**Total Value**
Adult tickets	a	$\$2$	$2a$
Children tickets	c	$\$1.50$	$1.5c$
Total	41		$\$73.50$

We need to find out how many children tickets were sold and how many adult tickets were sold.
We will let $\boldsymbol{c}$ represent children tickets and let $\boldsymbol{a}$ represent adult the tickets.
It is important to understand that each equation we make represents something from the problem. Our first equation will represent the amount of tickets. The total amount of tickets sold is 41, which includes children $\boldsymbol{c}$, and adults $\boldsymbol{a}$. This leads us to conclude that our first equation is

$$c + a = 41$$

We will make the next equation represent the total value of the purchase. If one child ticket was sold, then the purchase will be $\$1.50$. If two were sold the purchase would be $\$1.50 \cdot 2$. If three were sold the purchase would be $\$1.50 \cdot 3$. Since we have no idea how many children tickets were sold, we will multiply by $\boldsymbol{c}$. Likewise for adults. Remember, that the total purchase for children and adult tickets is given to be $\$73.50$. Our second equation is then

$$1.5c + 2a = 73.50$$

Our system is

$$\begin{aligned} c + a &= 41 \\ 1.5c + 2a &= 73.50 \end{aligned}$$

To solve this system let's multiply the first equation by -2.

$$\begin{aligned} -2 \cdot (c + a) &= 41 \cdot -2 \\ -2c - 2a &= -82 \end{aligned}$$

Now adding this to the second equation

$$\begin{array}{r} -2c - 2a = -82 \\ + \quad 1.5c + 2a = 73.50 \\ \hline -0.5c = -8.50 \\ c = 17 \end{array}$$

So the number of children tickets sold is $c = 17.$
To find the number of adult tickets, let us plug in the value we just got into the first equation.

$$\begin{aligned} c + a &= 41 \\ 17 + a &= 41 \\ a &= 24 \end{aligned}$$

The number of adult tickets sold is $a = 24$.
In conclusion, the number of adult tickets sold is 24 and the number of children tickets sold is 17.

	MEDIA LESSON Word Problem 1 (Duration 4:28)

View the video lesson, take notes and complete the problems below

Example:

a) If 105 people attended a concert and tickets for adults costs $\$2.50$ while tickets for children cost $\$1.75$ and total receipts for the concert were $\$228$, how many children and how many adults went to the concert?

YOU TRY

Aaron invests $\$9,700$ in two different accounts. The first account paid 7%, the second account paid 11% in interest. At the end of the first year he had earned $\$863$ in interest. How much was in each account?

B. MIXTURE PROBLEMS

Example: A farmer has two types of milk, one that is 24% butterfat and another which is 18% butterfat. How much of each should he use to end up with 42 gallons of 20% butterfat?

Organizing the information in a table:

	Amount	Concentration	Total Butterfat
24% butterfat	x	0.24	$0.24x$
18% butterfat	y	0.18	$0.18y$
20%butterfat	42	0.20	$0.20(42)$

Using our chart to set up the equation as before, we get

$$x + y = 42$$

$$0.24x + 0.18y = 8.40$$

Multiplying the top equation by -0.18 we get

$$-0.18 \cdot (x + y) = 42 \cdot -0.18$$
$$-0.18x - 0.18y = -7.56$$

Adding the equation together gives us

$$-0.18x - 0.18y = -7.56$$

$$+ \quad 0.24x + 0.18 = 8.40$$

$$0.06x = 0.84$$

$$x = 14$$

The amount of 24% butterfat is therefore 14 gallons. This implies that the amount of 18% butterfat is 28 gallons.

	MEDIA LESSON Word problem 2 (Duration 3:23)

View the video lesson, take notes and complete the problems below

Example:

a) A chemist needs to create 100mL of 38% acid solution. On hand she has a 20% acid solution and a 50% acid solution. How many mL of each solution should she use?

YOU TRY

A solution of pure antifreeze is mixed with water to make a 65% antifreeze solution. How much of each should be used to make 70 liters?

C. UNIFORM MOTION WITH UNKNOWN RATES

Example: Turkey the Pigeon travels the same distance of 72 miles in 4 hours against the wind as it does traveling 3 hours with the wind in the local skies. What is the rate of Turkey the Pigeon in still air and the rate of the wind?

First we'll make a table to organize the given information and then create an equation.

Let r represent the rate of Turkey the Pigeon and w represent the rate of the wind.

	Rate	**Time**	**distance**
With the wind	$r + w$	3	$3(r + w)$
Against the wind	$r - w$	4	$4(r - w)$

Turkey travels a distance of 72 miles with the wind and against it. The equations are then

$$3(r + w) = 72$$

$$4(r - w) = 72$$

To make things easier for us let us divide the first equation by 3 and the second by 4. Doing so gives us

$$r + w = 24$$

$$r - w = 18$$

Adding both equations together we get $2r = 42$ which implies that $r = 21$. Hence $w = 3$.

MEDIA LESSON
Word problem 3 (Duration 5:43)

View the video lesson, take notes and complete the problems below

Example:

a) An airplane travels 1,200 miles in 4 hours with the wind. The same trip takes 5 hours against the wind. What is the speed of the plane in still air and what is the wind speed?

YOU TRY

A boat travels upstream for 156 miles in 3 hours and returns in 2 hours traveling downstream in a local stream of water. What is the rate of the boat in still water and the rate of the current?

EXERCISES

1) There were 429 people at a play. Admission was $\$1$ each for adults and 75 cents each for children. The receipts were $\$372.50$. How many adults and how many children attended?

2) There were 200 tickets sold for a women's basketball game. Tickets for students were 50 cents each and for adults 75 cents each. The total amount of money collected was $\$132.50$. How many of each type of ticket was sold.

3) A total of $\$27{,}000$ is invested, part of it at 12% and the rest at 13%. The total interest after one year is $\$3385$. How much was invested at each rate?

4) Millicent earned $\$435$ last year in interest. If $\$3{,}000$ was invested at a certain rate of return and $\$4{,}500$ was invested in a fund with a rate that was 2% lower, find the two rates of interest.

5) A syrup manufacturer has some pure maple syrup and some which is 85% maple syrup. How many liters of each should be mixed to make 150 liters which is 96% maple syrup.

6) A goldsmith combined an alloy that costs $\$4.30$ per ounce with an alloy that costs $\$1.80$ per ounce. How many ounces of each were used to make a mixture of 200 ounces costing $\$2.50$ per ounce?

7) A boat travels upstream for 336 miles in 4 hours and returns in 3 hours traveling downstream in a local stream of water. What is the rate of the boat in still water and the rate of the current.

CHAPTER REVIEW

KEY TERMS AND CONCEPTS	
Look for the following terms and concepts as you work through the workbook. In the space below, explain the meaning of each of these concepts and terms in your own words. Provide examples that are not identical to those in the text or in the media lesson.	
System of equations	
System of equations with one solution	
System of equations with no solution	
System of equations with infinitely many solutions	
Consistent system	
Inconsistent system	