NCERT SOLUTIONS
CLASS-IX MATHS
CHAPTER-2 POLYNOMIALS

Exercise – 1

Q.1.Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:

(i)$4x^2 - 3x + 7$

Answer: It is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Answer: It is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Answer: It is not a polynomial since the power of the variable is not a whole number.

(iv) $y + \frac{2}{y}$

Answer: It is not a polynomial since the power of the variable is not a whole number.

(v) $x^{10} + y^3 + t^{50}$

Answer: It is a polynomial in three variables.

Q.2. Write the coefficients of a^2 in each of the following:

(i) $2 + a^2 + a$

Answer: Coefficient of a^2 is 1.

(ii) $2 – a^2 + a^3$

Answer: Coefficient of a^2 is -1.

(iii) $\frac{\pi}{2}x^2 + x$

Answer: Coefficient of a^2 is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x - 1$

Answer: Coefficient of a^2 is 0.

Q.3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Answer: $3x^{35} + 5 \; and \; 4x^{100}$

Q.4. Write the degree of each of the following polynomials:

Q.4. Write the degree of each of the following polynomials.

(i) $5a^3 + 4a^2 + 7a$

Answer : Degree is 3.
(ii) $3 - b^2$

Answer : Degree is 2.
(iii) $5t - \sqrt{8}$

Answer : Degree is 1.

(iv)8

Answer : No Degree.

Q.5. Classify the following as linear, quadratic and cubic polynomial.

(i) $a^2 + a$

Answer: Quadratic Polynomial

(ii)$a - a^3$

Answer: Cubic Polynomial

(iii) $y + y^2 + 4$

Answer: Quadratic Polynomial

(iv)1 + x

Answer: Linear Polynomial

(v) 3a

Answer: Linear Polynomial

(vi) a^2

Answer: Quadratic Polynomial
(vii) $6a^3$

Answer: Cubic Polynomial

Exercise – 2

Q.1. Find the value of the polynomial at $f(x) = 5a - 4a^2 + 3$ ***at***

(i) a= 0

(ii) a = – 1

(iii) a = 2

Answer:

Let $f(x) = 5a - 4a^2 + 3$

(i) When a=0

$f(0) = 5(0) + 4(0)^2 + 3 = 3$

(ii) When a=-1

$f(a) = 5a + 4a^2 + 3$
$f(-1) = 5(-1) + 4(-1)^2 + 3 = -5 – 4 + 3 = -6$

(iii) When a=2

$f(a) = 5a + 4a^2 + 3$
$f(2) = 5(2) + 4(2)^2 + 3 = 10 – 16 + 3 = -3$

Q.2. Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$

Answer: $p(y) = y2 – y + 1$
$\therefore p(0) = (0)^2 - (0) + 1 = 1$
$p(1) = (1)^2 – (1) + 1 = 1$
$p(2) = (2)^2 – (2) + 1 = 3$

(ii) $p(a) = 2 + a + 2a^2 - a^3$

Answer: $p(a) = 2 + a + 2a^2 - a^3$
$\therefore p(0) = 2 + 0 + 2(0)^2 – (0)^3 = 2$
$p(1) = 2 + 1 + 2(1)^2 – (1)^3 = 2 + 1 + 2 – 1 = 4$
$p(2) = 2 + 2 + 2(2)^2 – (2)^3 = 2 + 2 + 8 – 8 = 4$

(iii) $p(x) = x^3$

Answer: $p(x) = x3$
$\therefore p(0) = (0)^3 = 0$
$p(1) = (1)^3 = 1$
$p(2) = (2)^3 = 8$

(iv) $p(a) = (a - 1)(a + 1)$

Answer: $p(a) = (a – 1)(a + 1)$
$\therefore p(0) = (0 – 1)(0 + 1) = (-1)(1) = – 1$
$p(1) = (1 – 1)(1 + 1) = 0(2) = 0$
$p(2) = (2 – 1)(2 + 1) = 1(3) = 3$

Q.3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1, x = -\frac{1}{3}$

Answer: For, $x = -\frac{1}{3}$

$p(x) = 3x + 1$
$\therefore p(-\frac{1}{3}) = 3(-\frac{1}{3}) + 1 = -1 + 1 = 0$

$\therefore -\frac{1}{3}$ is a zero of p(x).

(ii) $p(x) = 5x - \pi, x = \frac{4}{5}$

Answer: For, $x = \frac{4}{5}$

$p(x) = 5x - \pi$
$\therefore p(\frac{4}{5}) = 5(\frac{4}{5}) - \pi = 4 - \pi$

$\therefore \frac{4}{5}$ is not a zero of p(x).

(iii) $p(x) = x^2 - 1, x = 1, -1$

Answer: For, $x = 1, -1$

$p(x) = x^2 - 1$
$\therefore p(1) = x^1 - 1 = 1 - 1 = 0$
$p(-1) = x^{-1} - 1 = 1 - 1 = 0$

$\therefore 1, -1$ are zeros of p(x).

(iv) $p(x) = (x + 1)(x - 2), x = -1, 2$

Answer: For, $x = -1, 2$

$p(x) = (x + 1)(x - 2)$
$\therefore p(-1) = (-1 + 1)(-1 - 2)$
$= ((0)(-3))$
$= 0$
$p(2) = (2 + 1)(2 - 2)$
$= (3)(0) = 0$

$\therefore -1, 2$ are zeros of p(x).

(v) $p(x) = x^2, x = 0$

Answer: For, $x = 0$

$p(x) = 0^2 = 0$

$\therefore 0$ is a zero of p(x).

(vi) $p(x) = lx + m, x = -\frac{m}{l}$

Answer: For, $x = -\frac{m}{l}$

$p(x) = lx + m$
$\therefore p(-\frac{m}{l}) = l(-\frac{m}{l}) + m = -m + m = 0$

$\therefore -\frac{m}{l}$ is a zero of p(x).

(vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Answer: For, $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

$p(x) = 3x^2 - 1$

$\therefore p(-\frac{1}{\sqrt{3}}) = 3(-\frac{1}{\sqrt{3}})^2 - 1$

$= 3(\frac{1}{3}) - 1 = 1 - 1 = 0$

$\therefore p(\frac{2}{\sqrt{3}}) = 3(\frac{2}{\sqrt{3}})^2 - 1$

$= 3(\frac{4}{3}) - 1 = 4 - 1 = 3 \neq 0$

$\therefore -\frac{1}{\sqrt{3}}$ is a zero of p(x) but $\frac{2}{\sqrt{3}}$ is not a zero of p(x).

(viii) $p(x) = 2x + 1, x = \frac{1}{2}$

Answer: For, $x = \frac{1}{2}$

$p(x) = 2x + 1$

$\therefore p(\frac{1}{2}) = 2(\frac{1}{2}) + 1 = 1 + 1 = 2 \neq 0$

Misplaced & is not a zero of p(x).

Q.4. Find the zero of the polynomial in each of the following cases:

(i)p(x) = x + 5

Answer: $p(x) = x + 5$

$\Rightarrow x + 5 = 0 \Rightarrow x = -5$

$\therefore$-5 is a zero polynomial of the polinomila p(x).

(ii) p(x) = x – 5

Answer: $p(x) = x - 5$

$\Rightarrow x - 5 = 0 \Rightarrow x = 5$

$\therefore$5 is a zero polynomial of the polinomila p(x).

(iii) p(x) = 2x + 5

Answer: $p(x) = 2x + 5$

$\Rightarrow 2x + 5 = 0 \Rightarrow 2x = -5$

$\Rightarrow x = \frac{-5}{2}$

$\therefore x = \frac{-5}{2}$ is a zero polynomial of the polynomial p(x).

(iv)p(x) = 3x – 2

Answer: $p(x) = 3x – 2$

$\Rightarrow 3x - 2 = 0 \Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$

$\therefore x = \frac{2}{3}$ is a zero polynomial of the polynomial p(x).

(v)p(x) = 3x

Answer: $p(x) = 3x$

$\Rightarrow 3x = 0 \Rightarrow x = 0$

$\therefore$0 is a zero polynomial of the polynomial p(x).

Exercise – 3

Q.1.Find the remainder when $x^3 + 3x^2 + 3x + 1$ ***is divided by***

(i) x+1

Answer:

$x + 1 = 0 \;\therefore\; Remainder = p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1 + 3 - 3 + 1 = 0$

$\Rightarrow x = -1$

(ii) $x - \frac{1}{2}$

Answer:

$x - \frac{1}{2} = 0 \therefore\; Remainder(\frac{1}{2})^3 + 3(\frac{1}{2})^2 + 3(\frac{1}{2}) + 1$

$\Rightarrow x = \frac{1}{2} \quad = \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 = \frac{27}{8}$

(iii) x

Answer:

$\therefore\; Remainder = (0)^3 + 3(0)^2 + 3(0) + 1$

$= 1$

(iv) $x + \pi$

Answer:

$x + \pi = 0 \;\therefore\; Remainder = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1 = -\pi^3 + 3\pi^2 - 3\pi + 1$

$\Rightarrow x = -\pi$

(v) 5+2x

Answer:

$5 + 2x = 0 \qquad \therefore\; Remainder = (-\frac{5}{2})^3 + 3(-\frac{5}{2})^2 + 3(-\frac{5}{2}) + 1 = -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = -\frac{27}{8}$

$\Rightarrow 2x = -5 \Rightarrow x = -\frac{5}{2}$

Q.2.Find the remainder when $x^3 - ax^2 + 6x - a$ ***is divided by x-a.***

Answer:

Let $p(x) = x^3 - ax^2 + 6x - a$

$x - a = 0$

$\therefore x = a \; Remainder = (a)^2 - a(a^2) + 6(a) - a$

$= a^3 - a^3 + 6a - a$

$= 5a$

Q.3.Check whether 7+3x is a factor of $3x^3+7x$.

Answer:

$7+3x=0 \Rightarrow 3x=-7$ only if 7+3x divides $3x^3+7x$ leaving no reaminder.

Let $p(x)=3x^3+7x$

$7+3x=0 \Rightarrow 3x=-7 \Rightarrow x=-\frac{7}{3} \therefore Remainder=3(-\frac{7}{3})^3+7(-\frac{7}{3})=-\frac{343}{9}-\frac{49}{3}=-\frac{490}{9} \neq 0$

$\therefore$7+3x is not a factor of $3x^3+7x$

Exercise – 4

Q.1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Answer:

Let p(x)= x^3+x^2+x+1

The zero of x+1 is -1.

$p(-1)=(-1)^3+(-1)^2+(-1)+1$
$=-1+1-1+1=0$

$\therefore$By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4+x^3+x^2+x+1$

Answer:

Let p(x)= $x^4+x^3+x^2+x+1$

The zero of x+1 is -1.

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$ $\therefore$By factor theorem, x+1 is a factor of $x^4+x^3+x^2+x+1$
$=1-1+1-1+1=1 \neq 0$

(iii) $x^4+3x^3+3x^2+x+1$

Answer:

Let p(x)= $x^4+3x^3+3x^2+x+1$

The zero of x+1 is -1.

$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$ $\therefore$By factor theorem, x+1 is a factor of $x^4+3x^3+3x^2+x+1$
$=1-3+3-1+1=1 \neq 0$

(iv) $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

Answer:

Let p(x)= $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

The zero of x+1 is -1.

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$

$p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 – x^2 – (2+\sqrt{2})x+\sqrt{2}$

Q.2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x)=2x^3+x^2–2x–1$, ***g(x) = x + 1***

Answer: $p(x)=2x^3+x^2–2x–1$, g(x) = x + 1

g(x)=0

$\Rightarrow x+1=0 \Rightarrow x=-1$

∴Zero of g(x) is -1.

Now, $p(-1)=2(-1)^3+(-1)^2–2(-1)–1$
$=-2+1+2-1=0$

∴By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, ***g(x) = x + 2***

Answer: $p(x)=x^3+3x^2+3x+1$, g(x) = x + 2

g(x)=0

$\Rightarrow x+2=0 \Rightarrow x=-2$

∴Zero of g(x) is -2.

Now, $p(-2)=(-2)^3+3(-2)^2+3(-2)+1$
$=-8+12-6+1=-1 \neq= 0$

∴By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3–4x^2+x+6$, ***g(x) = x – 3***

Answer: $p(x)=x^3–4x^2+x+6$, g(x) = x -3

g(x)=0

$\Rightarrow x-3=0 \Rightarrow x=3$

∴Zero of g(x) is 3.

Now, $p(3)=(3)^3-4(3)^2+(3)+6$
$=27-36+3+6=0$

∴By factor theorem, g(x) is a factor of p(x).

Q.3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

(i)$p(x)=x^2+x+k$

Answer: If x-1 is a factor of p(x), then p(1)=0 |By Factor Theorem $\Rightarrow (1)^2+(1)+k=0 \Rightarrow 1+1+k=0$

$\Rightarrow 2+k=0 \Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Answer: If x-1 is a factor of p(x), then p(1)=0

$\Rightarrow 2(1)^2+k(1)+\sqrt{2}=0 \Rightarrow 2+k+\sqrt{2}=0 \Rightarrow k=-(2+\sqrt{2})$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

Answer: If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$$
$$\Rightarrow k = \sqrt{2} - 1$$

(iv) $p(x) = kx^2 - 3x + k$

Answer: If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$$\Rightarrow k(1)^2 - 3(1) + k = 0 \Rightarrow k - 3 + k = 0$$
$$\Rightarrow 2k - 3 = 0$$
$$\Rightarrow k = \frac{3}{2}$$

Q.4. Factorize:

(i) $12x^2 \quad 7x + 1$

Answer:

$$12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$$
$$= 4x(3x - 1) - 1(3x - 1) = (4x - 1)(3x - 1)$$

Let p(x)= $12x^2 - 7x + 1$

Then p(x)= $12(x^2 - \frac{7}{12}x + \frac{1}{12}) = 12q(x)$

Where q(x)= $x^2 - \frac{7}{12}x + \frac{1}{12}$

By trial, we find that

$$q(\tfrac{1}{3}) = (\tfrac{1}{3})^2 - \tfrac{7}{12}(\tfrac{1}{3}) + \tfrac{1}{12}$$
$$= \frac{4-7+3}{36} = \frac{0}{36} = 0$$

∴By Factor Theorem,

$(x - \frac{1}{3})$ is a factor of q(x).

Similarly, by trial, we find that

$$q(\tfrac{1}{4}) = (\tfrac{1}{4})^2 - \tfrac{7}{12}(\tfrac{1}{4}) + \tfrac{1}{12}$$
$$= \frac{3-7+4}{18} = \frac{0}{48} = 0$$

∴By Factor Theorem,

$(x - \frac{1}{4})$ is a factor of q(x).

Therefore, $12x^2 - 7x + 1 = 12(x - \frac{1}{3})(x - \frac{1}{4})$

$$= 12(\tfrac{3x-1}{3})(\tfrac{4x-1}{4}) = (3x - 1)(4x - 1)$$

(ii)$2x^2 + 7x + 3$

Answer:

$$2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$$
$$= 2x(x + 3) + 1(x + 3)$$
$$= (x + 3)(2x + 1)$$

Let p(x)= $2x^2+7x+3$

Then p(x)= $2(x^2+\frac{7}{2}x+\frac{3}{2})=2q(x)$
Where q(x)= $x^2+\frac{7}{2}x+\frac{3}{2}$

By trial, we find that

$q(-3)=(-3)^2-\frac{7}{2}(-3)+\frac{3}{2}$
$=9-\frac{21}{2}+\frac{3}{2}=0$

∴By Factor Theorem,

$(x-3), i.e. (x+3)$ is a factor of q(x).

Similarly, by trial, we find that

$q(-\frac{1}{2})=(-\frac{1}{2})^2+\frac{7}{12}(-\frac{1}{2})+\frac{3}{2}$
$=\frac{1}{3}-\frac{7}{4}+\frac{3}{4}=0$

∴By Factor Theorem,

$(x-(-\frac{1}{2})), i.e. (x+\frac{1}{2})\ is\ a\ factor\ of\ q(x).$
$Therefore, 2x^2+7x+3=2(x+3)(x+\frac{1}{2})$
$=2(x+3)(\frac{2x+1}{2})$
$=(x+3)(2x+1)$

By Simpler method,

(iii) \(6x^{2}+5x-6 =6x^{2}+ 9x – 4x – 6

=3x (2x + 3) – 2 (2x + 3) = (2x + 3) (3x – 2)\)

(iv) $\boldsymbol{3x^2-x-4}$ **=** $3x^2-x-4$
$=3\times 2-4x+3x-4$
$=x(3x-4)+1(3x-4)$
$=(3x-4)(x+1)$

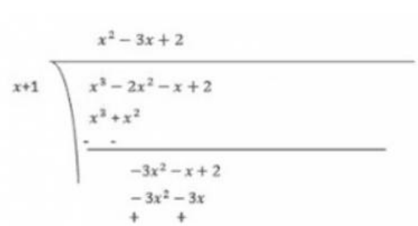
Q.5. Factorize:

(i) x^3-2x^2-x+2

Answer: Let $p(x)=x^3-2x^2-x+2$
Factors of 2 are ±1 and ± 2
By trial method, we find that
p(1) = 0
So, (x+1) is factor of p(x)
Now,
$p(x)=x^3-2x^2-x+2$

$p(-1)=(-1)^3-2(-1)^2-(-1)+2=-1-1+1+2=0$

Therefore, (*x*+1) is the factor of *p*(*x*)



$$\begin{array}{l} 2x+2 \\ 2x+2 \\ -\quad - \\ \hline 0 \end{array}$$

Now, Dividend = Divisor × Quotient + Remainder

$$(x+1)(x^2 - 3x + 2)$$
$$= (x+1)(x^2 - x - 2x + 2)$$
$$= (x+1)(x(x-1) - 2(x-1))$$
$$= (x+1)(x-1)(x+2)$$

(ii) $x^3 - 3x^2 - 9x - 5$

Answer: Let p(x) = $x^3 - 3x^2 - 9x - 5$

Factors of 5 are ±1 and ±5

By trial method, we find that

p(5) = 0

So, (x-5) is factor of p(x)

Now,

p(x) = $x^3 - 3x^2 - 9x - 5$

p(5) = $(5)^3 - 3(5)^2 - 9(5) - 5 = 125 - 75 - 45 - 5 = 0$

Therefore, (x-5) is the factor of p(x)

$$\begin{array}{r|l} & x^2+2x+1 \\ \hline x-5 & x^3-3x^2-9x-5 \\ & x^3-5x^2 \\ & -\quad + \\ \hline & 2x^2-9x-5 \\ & 2x^2-10x \\ & -\quad + \\ \hline & x-5 \\ & x-5 \\ & -\quad + \\ \hline & 0 \end{array}$$

Now, Dividend = Divisor × Quotient + Remainder

$$(x-5)(x^2 + 2x + 1)$$
$$= (x-5)(x^2 + x + x + 1)$$
$$= (x-5)(x(x+1) + 1(x+1))$$
$$= (x-5)(x+1)(x+1)$$

(iii) $x^3 + 13x^2 + 32x + 20$

Answer: Let p(x) = $x^3 + 13x^2 + 32x + 20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

p(-1) = 0

So, (x+1) is factor of p(x)

Now,

p(x) = $x^3 + 13x^2 + 32x + 20$

p(-1) = $(-1)^3 + 13(-1)^2 + 32(-1) + 20 = -1 + 13 - 32 + 20 = 0$

Therefore, (x+1) is the factor of p(x)

$$\begin{array}{r|l} & x^2+12x+20 \\ \hline x+1 & x^3+13x^2+32x+20 \end{array}$$

$x^3 + x^2$
$-\quad -$

$12x^2 + 32x + 20$
$12x^2 + 12x$
$-\quad -$

$20x + 20$
$20x + 20$
$-\quad -$

0

Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
&(x+1)(x^2+12x+20) \\
&=(x+1)(x^2+2x+10x+20) \\
&=(x-5)x(x+2)+10(x+2) \\
&=(x-5)(x+2)(x+10)
\end{aligned}
$$

(iv) $2y^3 + y^2 – 2y – 1$
Answer: Let p(y) = $2y^3 + y^2 – 2y – 1$

Factors of ab = $2 \times (-1)$= -2 are ±1 and ±2
By trial method, we find that
p(1) = 0
So, (y-1) is factor of p(y)
Now,
p(y) = $2y^3 + y^2 – 2y – 1$

p(1) = $2(1)^3 + (1)^2 – 2(1) – 1 = 2 + 1 - 2 = 0$

Therefore, (y-1) is the factor of p(y)

$2y^2 + 3y + 1$

$y-1 \;\big)\; 2y^3 + y^2 - 2y - 1$
$2y^3 - 2y^2$
$-\quad +$

$3y^2 - 2y - 1$
$3y^2 - 3y$
$-\quad +$

$y - 1$
$y - 1$
$-\quad +$

0

Now, Dividend = Divisor × Quotient + Remainder

$$
\begin{aligned}
&(y-1)(2y^2+3y+1) \\
&=(y-1)(2y^2+2y+y+1) \\
&=(y-1)(2y(y+1)+1(y+1)) \\
&=(y-1)(2y+1)(y+1)
\end{aligned}
$$

Exercise – 5

Q.1.Use suitable identities to find the following products:

(i)(x + 4) (x + 10)

Answer: $(x+4)(x+10) = x^2 + (4+10)x + (4 \times 10)$
$= x^2 + 14x + 40$

(ii)(x + 8) (x – 10)

Answer: $(x+8)(x-10) = x^2 + (8+(-10))x + (8 \times (-10))$
$= x^2 + (8-10)x - 80$
$= x^2 - 2x - 80$

(iii)(3x + 4) (3x – 5)

Answer: $(3x+4)(3x-5) = (3x)^2 + 4 + (-5)3x + 4 \times (-5)$
$= 9x^2 + 3x(4-5) - 20$
$= 9x^2 - 3x - 20$

(iv)$(y^2 + \frac{3}{2})(y^2 - \frac{3}{2})$

Answer: $(y^2 + \frac{3}{2})(y^2 - \frac{3}{2}) = (y^2)^2 - (\frac{3}{2})^2$
$= y^4 - \frac{9}{4}$

Q.2. Evaluate the following products without multiplying directly:

(i)103 × 107

Answer: $103 \times 107 = (100+3) \times (100+7)$
$= (100)^2 + (3+7)(100 + (3 \times 7))$
$= 10000 + 1000 + 21$
$= 11021$

(ii) 95 × 96

Answer: $95 \times 96 = (90+5) \times (90+6)$
$= (90)^2 + 90(5+6) + (5 \times 6)$
$= 8100 + 990 + 30 = 9120$

(iii) 104 × 96

Answer: $104 \times 96 = (100+4) \times (100-4)$
$= (100)^2 - (4)^2$
$= 10000 - 16$
$= 9984$

Q.3. Factorize the following using appropriate identities:

(i)$9x^2 + 6xy + y^2$

Answer: $9x^2 + 6xy + y^2$
$= (3x)^2 + (2 \times 3x \times y) + y^2$
$= (3x+y)^2$
$= (3x+y)(3x+y)$

(ii) $4y^2 - 4y + 1$

Answer: $4y^2 - 4y + 1$
$= (2y)^2 - (2 \times 2y \times 1) + 1^2$
$= (2y-1)^2$
$= (2y-1)(2y-1)$

(iii) $x2-\frac{y^2}{100}$

Answer: $x^2-(\frac{y}{10})^2=(x-\frac{y}{10})(x+\frac{y}{10})$

Q.4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$

(ii) $(2x-y+z)^2$

(iii) $(-2x+3y+2z)^2$

(iv) $[\frac{1}{4}a-\frac{1}{2}b+1]^2$

Answer:

(i) $(x+2y+4z)^2$

Using identity,

$$(x+2y+4z)^2$$
$$=x^2+(2y)^2+(4z)^2+(2\times x\times 2y)+(2\times 2y\times 4z)+(2\times 4z\times x)$$
$$=x^2+4y^2+16z2+4xy+16yz+8xz$$

(ii) $(2x-y+z)^2$

Using identity,

$$(2x-y+z)^2$$
$$=(2x)^2+(-y)^2+z^2+(2\times 2x\times -y)+(2\times -y\times z)+(2\times z\times 2x)$$
$$=4x^2+y^2+z^2-4xy-2yz+4xz$$

(iii) $(-2x+3y+2z)^2$

Using identity,

$$(-2x+3y+2z)^2$$
$$=(-2x)^2+(3y)^2+(2z)^2+(2\times -2x\times 3y)+(2\times 3y\times 2z)+(2\times 2z\times -2x)$$
$$=4x^2+9y^2+4z^2-12xy+12yz-8xz$$

(iv) $[\frac{1}{4}a-\frac{1}{2}b+1]^2$

Using identity,

$$[\frac{1}{4}a-\frac{1}{2}b+1]^2=(\frac{1}{4}a)^2+(-\frac{1}{2}b)^2+1^2+(2\times\frac{1}{4}a\times-\frac{1}{2}b)+(2\times-\frac{1}{2}b\times 1)+(2\times 1\times\frac{1}{4}a)$$
$$=\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$$

Q.5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Answer:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

Answer:

$$4x^2+9y^2+16z^2+12xy-24yz-16xz$$
$$=(2x)^2+(3y)^2+(-4z)^2+(2\times 2x\times 3y)+(2\times 3y\times -4z)+(2\times -4z\times 2x)$$
$$=(2x+3y-4z)^2$$
$$=(2x+3y-4z)(2x+3y-4z)$$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

(ii) $2x^2 + y^2 + 8z^2 – 2\sqrt{2}xy + 4\sqrt{2}yz – 8xz$

Answer:

$$
\begin{aligned}
&2x^2 + y^2 + 8z^2 – 2\sqrt{2}xy + 4\sqrt{2}yz – 8xz \\
&= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + (2 \times -\sqrt{2}x \times y) + (2 \times y \times 2\sqrt{2}z) + (2 \times 2\sqrt{2}z \times -\sqrt{2}x) \\
&= (-\sqrt{2}x + y + 2\sqrt{2}z)^2 \\
&= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)
\end{aligned}
$$

Q.6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $[\frac{3}{2}x+1]^3$

(iv) $[x-\frac{2}{3}y]^3$

Answer:

(i) $(2x+1)^3$

$$
\begin{aligned}
&(2x+1)^3 = (2x)^3 + 1^3 + (3 \times 2x \times 1)(2x+1) \\
&= 8x^3 + 1 + 6x(2x+1) \\
&= 8x^3 + 12x^3 + 6x + 1
\end{aligned}
$$

(ii) $(2a-3b)^3$

$$
\begin{aligned}
&(2a-3b)^3 = (2a)^3 - (3b)^3 – (3 \times 2a \times 3b)(2a – 3b) \\
&= 8a^3 – 27b^3 – 18ab(2a – 3b) \\
&= 8a^3 – 27b^3 – 36a^2b + 54ab^2
\end{aligned}
$$

(iii) $[\frac{3}{2}x+1]^3$

$$
\begin{aligned}
&[\tfrac{3}{2}x+1]^3 = (\tfrac{3}{2}x)^3 + 1^3 + (3 \times \tfrac{3}{2}x \times 1)(\tfrac{3}{2}x+1) \\
&= \tfrac{27}{8}x^3 + 1 + \tfrac{9}{2}x(\tfrac{3}{2}x+1) \\
&= \tfrac{27}{8}x3 + 1 + \tfrac{27}{4}^2 + \tfrac{9}{2}x \\
&= \tfrac{27}{8}x3 + \tfrac{27}{4}^2 + \tfrac{9}{2}x + 1
\end{aligned}
$$

(iv) $[x-\frac{2}{3}y]^3$

$$
\begin{aligned}
&[x – \tfrac{2}{3}y]^3 = (x)^3 – (\tfrac{2}{3}y)^3 – (3 \times x \times \tfrac{2}{3}y)(x – \tfrac{2}{3}y) \\
&= (x)^3 – \tfrac{8}{27}y^3 – 2xy(x – \tfrac{2}{3}y) \\
&= (x)^3 – \tfrac{8}{27}y3 – 2x^2y + \tfrac{4}{3}xy^2
\end{aligned}
$$

Q.7. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Answer:

(i) $(99)^3 = (100 – 1)^3$

$(100 – 1)^3 = (100)^3 – 1^3 – (3 \times 100 \times 1)(100 – 1)$

= 1000000 – 1 – 300(100 – 1)

= 1000000 – 1 – 30000 + 300 = 970299

(ii) $(102)^3 = (100+2)^3$
$(100+2)^3 = (100)^3 + 2^3 + (3 \times 100 \times 2)(100+2)$
= 1000000 + 8 + 600(100 + 2)

= 1000000 + 8 + 60000 + 1200= 1061208

(iii) $(998)^3 = (1000–2)^3$

$= (1000)^3 – 2^3 – (3 \times 1000 \times 2)(1000–2)$

= 1000000000 – 8 – 6000(1000 – 2)

= 1000000000 – 8- 6000000 + 12000 = 994011992

Q.8. Factorise each of the following:
(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 – b^3 – 12a^2b + 6ab^2$

(iii)27 – 125a³ – 135a + 225a²

(iv) $64a^3 – 27b^3 – 144a^2b + 108ab^2$

(v) $27p^3 – \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Answer:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$8a^3 + b^3 + 12a^2b + 6ab^2$
$= (2a)^3 + b^3 + 3(2a)^2b + 3(2a)(b)^2$
$= (2a+b)^3$
$= (2a+b)(2a+b)(2a+b)$
(ii) $8a^3 – b^3 – 12a^2b + 6ab^2$

$8a^3 – b^3 - 12a^2b + 6ab^2$
$= (2a)^3 – b^3 – 3(2a)^2b + 3(2a)(b)^2$
$= (2a–b)^3$
$= (2a–b)(2a–b)(2a–b)$
(iii)27 – 125a³ – 135a + 225a²

$27 – 125a^3 – 135a + 225a^2$
$= 3^3 – (5a)^3 – 3(3)^2(5a) + 3(3)(5a)^2$
$= (3–5a)^3$
$= (3–5a)(3–5a)(3–5a)$

(iv) $64a^3 – 27b^3 – 144a^2b + 108ab^2$

$64a^3 – 27b^3 – 144a^2b + 108ab^2$
$= (4a)^3 – (3b)^3 – 3(4a)^2(3b) + 3(4a)(3b)^2$
$= (4a–3b)^3$
$= (4a–3b)(4a–3b)(4a–3b)$
(v) $27p^3 – \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - (\frac{1}{6})^3 - 3(3p)^2(\frac{1}{6}) + 3(3p)(\frac{1}{6})^2$

$= (3p - \frac{1}{6})^3$

$= (3p - \frac{1}{6})(3p - \frac{1}{6})(3p - \frac{1}{6})$

Q.9. Verify:

(i) $x^3 + y^3$

$= (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3$

$= (x - y)(x^2 + xy + y^2)$

Answer:

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$We\ know\ that,$

$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

$\Rightarrow x^3 + y^3 = (x + y)[(x + y)^2 - 3xy] Taking\ (x + y)\ common$

$\Rightarrow x^3 + y^3 = (x + y)[(x^2 + y^2 + 2xy) - 3xy]$

$\Rightarrow x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

(ii) $x^3 - y^3$

$= (x - y)(x^2 + xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$We\ know\ that,$

$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$\Rightarrow x^3 - y^3 = (x - y)^3 + 3xy(x - y)$

$\Rightarrow x^3 - y^3 = (x - y)[(x - y)^2 + 3xy] Taking\ (x + y)\ common$

$\Rightarrow x^3 - y^3 = (x - y)[(x^2 + y^2 - 2xy) + 3xy]$

$\Rightarrow x^3 + y^3 = (x - y)(x^2 + y^2 + xy)$

Q.10. Factorize each of the following:

(i) $27y^3 + 125z^3$

(ii) $64m^3 – 343n^3$

Answer:

(i) $27y^3 + 125z^3$

$$27y^3 + 125z^3$$
$$= (3y)^3 + (5z)^3$$
$$= (3y + 5z)[(3y)^2 – (3y)(5z) + (5z)^2]$$
$$= (3y + 5z)(9y^2 – 15yz + 25z)^2$$

(ii) $64m^3 – 343n^3$

$$64m^3 – 343n^3$$
$$= (4m)^3 – (7n)^3$$
$$= (4m + 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$$
$$= (4m + 7n)(16m^2 + 28mn + 49n)^2$$

Q.11. Factorise : $27x^3 + y^3 + z^3 – 9xyz$

Answer:

$$27x^3 + y^3 + z^3 – 9xyz$$
$$= (3x)^3 + y^3 + z^3 – 3(3x)(y)(z)$$
$$= (3x + y + z)(3x)^2 + y^2 + z^2 – 3xy – yz – 3xz$$
$$= (3x + y + z)(9x^2 + y^2 + z^2 – 3xy – yz – 3xz)$$

Q.12. Verify that:

$$x^3 + y^3 + z^3 – 3xyz = \frac{1}{2}(x + y + z)[(x – y)^2 + (y – z)^2 + (z – x)^2]$$

Answer:

We know that,
$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – xz)$$
$$\Rightarrow x^3 + y^3 + z^3 – 3xyz = \frac{1}{2} \times (x + y + z)[2(x^2 + y^2 + z^2 – xy – yz – xz)]$$
$$= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 – 2xy – 2yz – 2xz)$$
$$= \frac{1}{2}(x + y + z)[(x^2 + y^2 - 2xy) + (y^2 + z^2 – 2yz) + (x^2 + z^2 – 2xz)]$$
$$= \frac{1}{2}(x + y + z)[(x – y)^2 + (y – z)^2 + (z – x)^2]$$

Q.13. If x + y + z = 0, show that $x^3 + y^3 + z^3 - 3xyz.$

Answer:

We know that,
$x^3 + y^3 + z^3 = 3xyz$= *(x + y + z)(x² + y² + z² – xy – yz – xz)*
Now put (x + y + z) = 0,
$$x^3 + y^3 + z^3 = 3xyz = (0)(x^2 + y^2 + z^2 – xy – yz – xz)$$
$$\Rightarrow x^3 + y^3 + z^3 – 3xyz = 0$$

Q.14. Without actually calculating the cubes, find the value of each of the following:
(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Answer:

(i) $(-12)^3 + (7)^3 + (5)^3$

$(-12)^3 + (7)^3 + (5)^3 = 0 + 3(-12)(7)(5)$ $\quad (Since\ (-12) + (7) + (5) = 0) Using\ Identity$

$(-12)^3 + (7)^3 + (5)^3 = 0 + 3(-12)(7)(5)\ldots\ldots\ldots\ldots\ldots\ldots(Since. (-12) + (7) + (5) = 0) Using\ Identity$
$= -1260$
(ii) $(28)^3 + (-15)^3 + (-13)^3$
x + *y* + *z* = 28 – 15 – 13 = 0

$(28)^3 + (-15)^3 + (-13)^3 = 0 + 3(28)(-15)(-13) = 16380$

Q.15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area : $25a^2 – 35a + 12$

(ii) Area : $35y^2 + 13y – 12$

Answer:

(i) Area : $25a^2 – 35a + 12$
$25a^2 – 35a + 12$
$= 25a^2 – 15a - 20a + 12$
$= 5a(5a – 3) – 4(5a – 3)$
$= (5a – 4)(5a – 3)$

Possible expression for length = *5a – 4*
Possible expression for breadth = *5a – 3*

(ii) Area : $35y^2 + 13y – 12$

$35y^2 + 13y – 12$
$= 35y^2 – 15y + 28y – 12$
$= 5y(7y – 3) + 4(7y – 3)$
$= (5y + 4)(7y – 3)$

Possible expression for length = *(5y + 4)*
Possible expression for breadth = *(7y – 3)*

Q.16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume : $3x^2 – 12x$
(ii) Volume : $12ky^2 + 8ky – 20k$

Answer:

(i) Volume : $3x^2 – 12x$
$3x^2 – 12x$
= 3x(x – 4)
Possible expression for length = *3*
Possible expression for breadth = *x*
Possible expression for height = *(x – 4)*

(ii) Volume : $12ky^2 + 8ky – 20k$

$12ky^2 + 8ky – 20k$
$= 4k(3y^2 + 2y – 5)$
$= 4k(3y^2 + 5y – 3y – 5)$
$= 4k[y(3y + 5) – 1(3y + 5)]$
$= 4k(3y + 5)(y – 1)$

Possible expression for length = *4k*
Possible expression for breadth = *(3y +5)*

Possible expression for breadth = *(3y +5)*
Possible expression for height = *(y -1)*