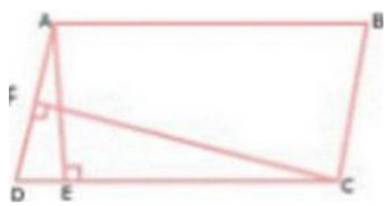
NCERT SOLUTIONS
CLASS-IX MATHS
CHAPTER-9 AREAS OF PARALLELOGRAMS AND TRIANGLES

Exercise-9.2

Q.1.In the given figure, PQRS is a parallelogram, $PE \perp SR \, and \, RF \perp PS$..If PQ = 16 cm, PE = 8 cm & RF = 10 cm. Calculate AD.

Solution:



Area of the parallelogram $PQRS = PQRS = RS \times PE$ ————(1)

$= 16 \times 8 \, cm^2 (\, because PQ = RS, PQRS \, is \, a \, parallelogram)$

$= 128 \, cm^2$

Now, area of parallelogram PQRS=$PS \times RF$ ————(2)

$= PS \times 10 \, cm^2$

From equation (1) and (2), we get

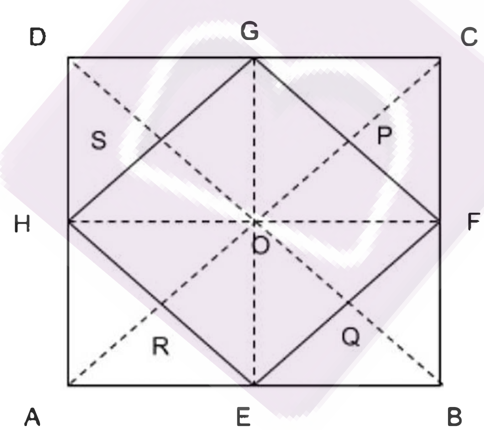
$PS \times 10 = 128$

$\Rightarrow PS = \frac{128}{10}$

$\Rightarrow PS = 12.8 \, cm$

Q.2.If E, F, G and H are the mid-points respectively, of the sides of a parallelogram ABCD show that
$Area(EFGH) = \frac{1}{2} Area(ABCD)$

Solution:



Lets join HF.

In the parallelogram, i.e ABCD, AD = BC and AD || BC (because in a parallelogram the opposite sides are equal and parallel)

$AB = CD$ (opposite sides are equal)

$\Rightarrow \frac{1}{2} AD = \frac{1}{2} BC$

$And\ AH \,||\, \bar{B}F$

$\Rightarrow AH = BF\, and\, AH||BF($ $because$The mid point of AD and BC are H and F)

$therefore$ ABFH is a parallelogram.

Since ΔHEF and parallelogram ABFH are between the same parallel lines AB and HF, and are on the same base HF.

$therefore$ Area (ΔHEF)= $\frac{1}{2}Area(ABFH)$ ... (1)

Similarly, we can prove that,

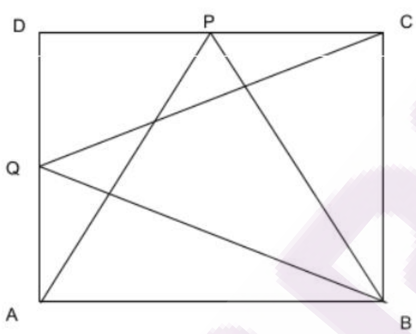
$Area(\Delta HGF) = \frac{1}{2}Area(HDFC)$ ... (2)

Add Equation (1) and Equation (2), we obtain

$Area(\Delta HEF) + Area(\Delta HGF) = \frac{1}{2}Area(ABFH) + \frac{1}{2}Area(HDCF)$
$= \frac{1}{2}[Area(ABFH) + Area(HDCF)]$
$\Rightarrow Area(EFGH) = \frac{1}{2}(ABCD)$

Q.3. DC and AD are two sides on which P and Q are two points lying respectively of a parallelogram ABCD. Prove that $area(APB) = area(BQC)$

Solution:



It is observed that, ΔBQC and parallelogram ABCD are between the same parallel lines AD and BC and lie on the same base BC.

$therefore Area(\Delta BQC) = \frac{1}{2}Area(ABCD)$ ...(1)

Similarly,we can say that ΔAPB and parallelogram ABCD lie between the same lines AB and DC that are parallel and on the same base AB .

$therefore Area(\Delta APB) = \frac{1}{2}Area(ABCD)$ ... (2)

Equating both the equations, i.e $equation(1) and equation(2)$,we get

$Area(\Delta BQC) = Area(\Delta APB)$

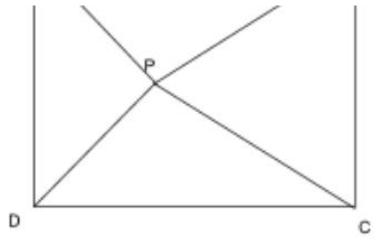
Hence, proved.

Q.4. In the given figure, $in\, the\, interior\, of\, a\, parallelogram\, ABCD$, $there\, exist\, a\, point\, P$. ***Show that***

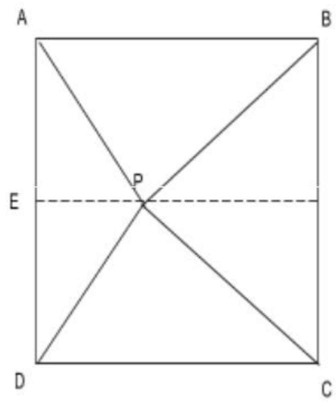
(i) $area(APB) + area(PCD) = \frac{1}{2}area(ABCD)$

(ii) $area(APD) + area(PBC) = area(APB) + area(PCD)$

[Hint: Draw a line i.e. parallel to AB, through P]



Solution:



A line segment EF is drawn, parallel to line segment AB and passing through point P.

In the parallelogram ABCD,

$AB||EF$ ...(1) (as ABCD is a parallelogram)

$therefore AD \,||\, BC (opposite\ sides\ of\ a\ parallelogram)$

$\Rightarrow AE \,||\, BF$ ...(2)

By equating , Equation (1) and Equation(2) , we get,

$AB \,||\, EF\ and\ AE \,||\, BF$

Therefore, quad ABFE is a parallelogram.

Misplaced & $therefore Area(\Delta APB) = \frac{1}{2} Area(ABFE)$ ...(3)

Similarly ,for ΔPCD and parallelogram EFCD,

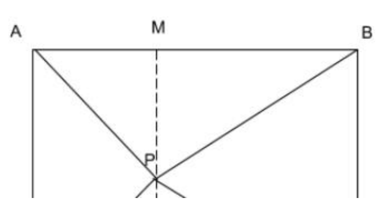
$Area(\Delta PCD) = \frac{1}{2} Area(EFCD)$ ...(4)

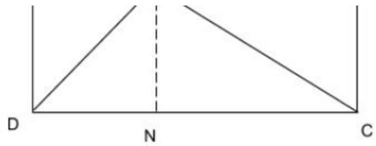
Add equation(3) and equation(4), we get,

$Area(\Delta APB) + Area(\Delta PCD) = \frac{1}{2}[Area(ABEF) + Area(EFCD)]$

$Area(\Delta APB) + Area(\Delta PCD) = \frac{1}{2} Area(ABCD)$ ...(5)

(ii)





A line segment MN is drawn,parallel to line segment AD and passing through point P.

In the parallelogram ABCD,

$MN||AD$ ...(6) (as ABCD is a parallelogram)

$therefore AB \,||\, DC (opposite\ sides\ of\ a\ parallelogram)$

$\Rightarrow AM \,||\, DN$ ...(7)

By equating , Equation (6) and Equation(7) , we get,

$MN \,||\, AD\, and\, AM \,||\, DN$

Therefore , quad AMND $AMND\ is\ a\ parallelogram.$

It can be said that ΔAPD and parallelogram AMND are between $the\ same\ parallel\ lines\ AD\ and\ MN$ and lying on the same base AD.

$therefore Area(\Delta APD) = \frac{1}{2} Area(AMND)$ ...(8)

Similarly ,for ΔPCB and parallelogram MNCB,

$Area(\Delta PCB) = \frac{1}{2} Area(MNCB)$ ...(9)

Add equation(8) and equation(9), we get,

$Area(\Delta APD) + Area(\Delta PCB) = \frac{1}{2}[Area(AMND) + Area(MNCB)]$

$Area(\Delta APD) + Area(\Delta PCB) = \frac{1}{2} Area(ABCD)$ ...(10)

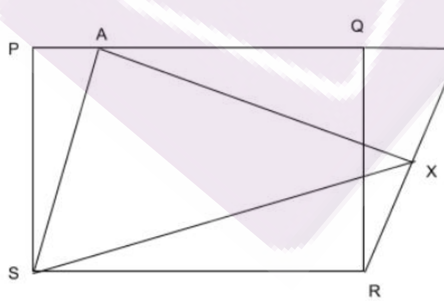
Now compare equation(5) with equation(10), we get

$Area(\Delta APD) + Area(\Delta PBC) = Area(\Delta APB) + Area(\Delta PCD)$

Q.5. In the figure given below X is a point on the side BR and ABRS and PQRS are parallelograms. Prove that

(i) area (PQRS) = area (ABRS)

(ii) $area(\Delta PXS) = \frac{1}{2} area(PQRS)$



Solution:

(i)It can be said that $parallelogram\ PQRS$ and the parallelogram ABRS lie $in\ between\ the\ same\ parallel\ lines\ SR\ and\ PB$ and also, on the same base SR.

$therefore Area(PQRS) = Area(ABRS)$ ...(1)

$therefore Area(PQRS) = Area(ABRS)$ ...(1)

(ii)Consider ΔAXS and parallelogram ABRS.

As these lie on the same base and are between the same parallel lines AS and BR,

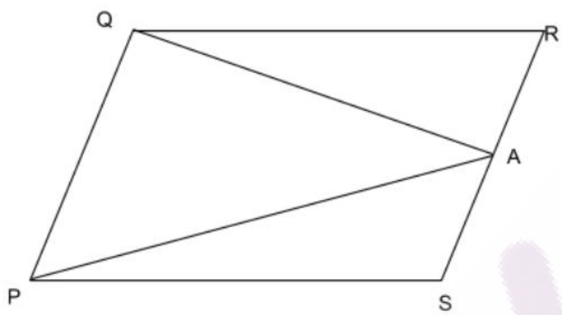
$therefore \frac{1}{2} Area(\Delta AXS) = Area(ABRS)$ ... (2)

By equating , equation (1) and equation(2) ,we get

$Area(\Delta AXS) = \frac{1}{2} Area(PQRS)$

Q.6. A farmer had a field and that was in a parallelogram shape PQRS. He took a point A on RS and he joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should he do it?

Solution:



From the figure , it can be observed that point A divides the field into three parts.

The parts which are triangular in shape are – $\Delta PSA, \Delta PAQ, and \Delta QRA$

$Area of \Delta PSA + Area of \Delta PAQ + Area of \Delta QRA = Area\, of\, parallelogram\, PQRS$ ...(i)

We know that if a parallelogram and a triangle are between the same parallels and on the same base , then the area of the triangle becomes half the area of the parallelogram.

$therefore Area(\Delta PAQ) = \frac{1}{2} Area(PQRS)$ ... (ii)

By equation , equation (i) and (ii) , we get

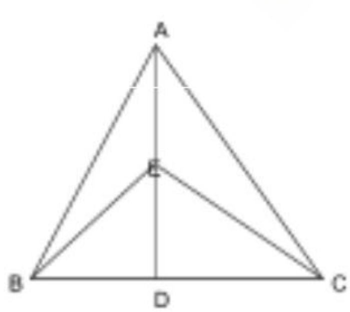
$Area(\Delta PSA) + Area(\Delta QRA) = \frac{1}{2} Area(PQRS)$ ... (iii)

Clearly , it can be said that the farmer should sow wheat in triangular part PAQ and he should sow pulses in other two triangular parts PSA and QRA or wheat in triangular parts PSA and QRA and in triangular part PAQ he should sow pulses.

Exercise – 9.3

Q.1.In the given figure, E is any point on median AD of a ΔABC. Prove that

$Area(ABE) = Area(ACE)$



Solution:

Given: AD is the median of ΔABC.

To Prove: $Area(\Delta ABC) = Area(\Delta ACE)$

Proof: In ΔABC,

AD is a median.

$therefore Area(\Delta ABD) = Area(\Delta ACD)$ ....(i)

In ΔEBC,

ED is a median.

$therefore Area(\Delta EBD) = Area(\Delta ECD)$ ....(ii)

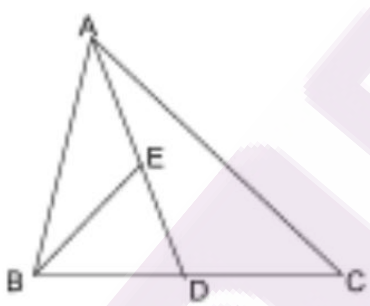
Subtracting equation (ii) from equation (i) , we get

$$Area(\Delta ABD) - Area(\Delta EBD) = Area(\Delta ACD) - Area(\Delta ECD)$$
$$\Rightarrow Area(\Delta ABE) = Area(\Delta ACE)$$

Q.2.In a ΔABC ,E is the mid-point of median AD. Prove that

Misplaced &.

Solution:



Given: E is the mid-point of median AD in ΔABC.

To Prove: $Area(\Delta BED) = Area(\Delta ABC)$

Proof: In ΔABC,

AD is a median.

$therefore Area(\Delta ABD) = Area(\Delta ABC)$ ....(i) ($because$ median of a triangle divides it into two triangles of equal area.)

In ΔABD,

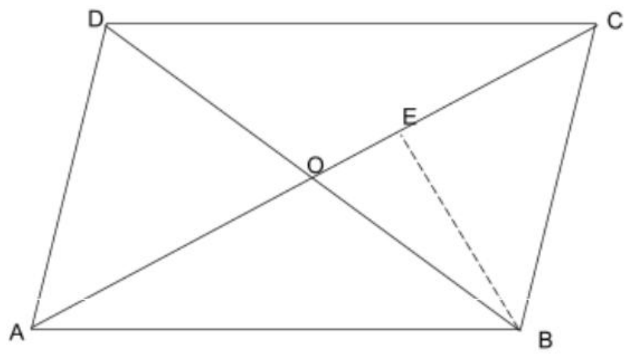
BE is a median.

$therefore Area(\Delta BED) = Area(\Delta BEA)$ = $\frac{1}{2} Area(\Delta ABD)$

$$Area(\Delta BED) = \frac{1}{2} Area(\Delta ABD) = \frac{1}{2} \cdot \frac{1}{2} Area(\Delta ABC)(From(i))) \quad Area(\Delta BED) = \frac{1}{4} Area(\Delta ABC)$$
$$= \frac{1}{4} Area(\Delta ABC)$$

Q.3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Solution:



We know that diagonals of parallelogram bisect each other.

$therefore$O is the mid-point of AC and BD.

BO is the median in ∆ABC. Therefore, it will divide it into two triangles of equal areas.

$therefore \ Area(\Delta AOB) = Area(\Delta BOC)$ ...(i)

In ΔBCD,CO is the median.

$therefore \ Area(\Delta BOC) = Area(\Delta COD)$ ...(ii)

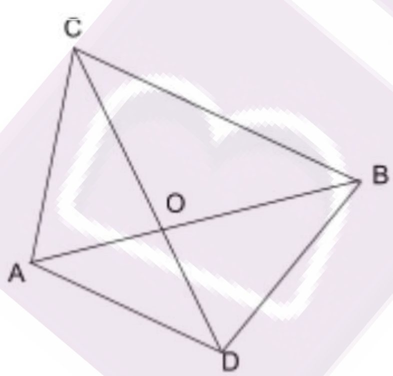
Similarly , $Area(\Delta COD) = Area(\Delta AOD)$ ...(iii)

By Equating equation(i) , (ii) $inline$,and (iii) , we get

$Area(\Delta AOB) = Area(\Delta BOC) = Area(\Delta COD) = Area(\Delta AOD)$

Therefore , it is evident that the diagonals of a parallelogram divide it into four triangles of equal area.

Q.4. In figure , ABC and ABD are two triangles on the same base AB. If line segment CD is bisected by AB at O , show that $Area(\Delta ABC) = Area(\Delta ABD)$.



Solution:

Given: $\Delta ABC \, and \, \Delta DBC$ are on the same base AB. Line segment CD is bisected by AB at O.

To Prove: $Area(\Delta ABC) = Area(\Delta ABD)$

Proof: Line-segment CD is bisected by AB at O.

Misplaced & AO is the median of ∆ACD.

$Area(\Delta ACO) = Area(\Delta ADO)$

BO is the median of ΔBCD

BO is the median of ΔBCD.

$therefore Area(\Delta BCO) = Area(\Delta BDO)$ ...(ii)

Adding (i) and (ii), we get

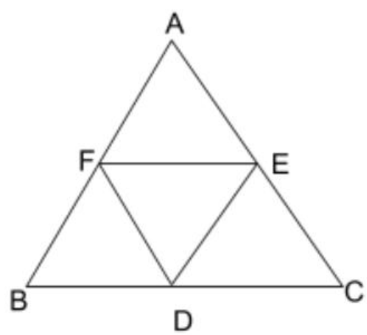
$$Area(\Delta ACO) + Area(\Delta BCO) = Area(\Delta ADO) + Area(\Delta BDO)$$
$$\Rightarrow Area(\Delta ABC) = Area(\Delta ABD)$$

Q.5. D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Prove that
(i) BDEF is a parallelogram

(ii) $Area(\Delta DEF) = \frac{1}{4} Area(\Delta ABC)$

(iii) $Area(BDEF) = \frac{1}{2} Area(\Delta ABC)$

Solution:



Given: D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC.

To Prove: (i) BDEF is a parallelogram

(ii) $Area(\Delta DEF) = \frac{1}{4} Area(\Delta ABC)$

(iii) $Area(BDEF) = \frac{1}{2} Area(\Delta ABC)$

Proof: In ΔABC,

F is the mid-point of side AB and E is the mid-point of side AC.
$therefore$EF || BC | In a triangle, the line segment joining the mid-points of any two sides is parallel to the 3rd side.

$\Rightarrow EF||BD$ ......(1)

Similarly, $EF||BD$ ......(2)

From (1) and (2) , we can say that ,

BDEF is a parallelogram.

(ii)As in (i), we can prove that

AFCE and AFDE are parallelograms.
FD is diagonal of the parallelogram BDEF.

$therefore\ Area(\Delta FBD) = Area(\Delta DEF)$ ....(3)

Similarly , $Area(\Delta DEF) = Area(\Delta FAE)$ .....(4)

$Area(\Delta DEF) = Area(\Delta DCE)$ .....(5)

From equation(3),(4) and (5), we have

$Area(\Delta FBD) = Area(\Delta DEF) = Area(\Delta FAE) = Area(\Delta DCE)$ .....(6)

$therefore\ \Delta ABC$ is divided into four non-overlapping triangles $\Delta FBD, \Delta DEF, \Delta FAE\ and\ \Delta DCE$

$therefore\ Area(\Delta ABC) = Area(\Delta FBD) + Area(\Delta DEF) + Area(\Delta FAE) + Area(\Delta DCE)$

$=4Area(\Delta DEF)$ (From equation(6))

$\Rightarrow Area(\Delta DEF) = \frac{1}{4}Area(\Delta ABC)$ ......(7)

(iii) $Area(BDEF) = Area(\Delta FBD) + Area(\Delta DEF)$ (From equation(3))

$== Area(\Delta DEF) + Area(\Delta DEF)$
$= 2Area(\Delta DEF)$
$= 2.\frac{1}{4}Area(\Delta ABC)(From\ equation(7))$
$= \frac{1}{2}Area(\Delta ABC)$

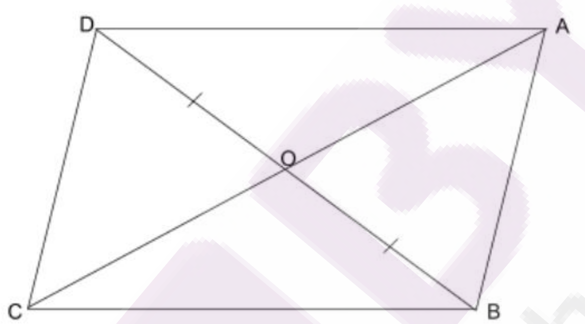
Q.6. In the given figure, diagonals AC and BD of a quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

(i) $Area(\Delta DOC) = Area(\Delta AOB)$

(ii) $Area(\Delta DCB) = Area(\Delta ACB)$

(iii) $DA||CB$ ***or ABCD is a parallelogram.***

[Hint: From D and B, draw perpendiculars to AC]



Solution:

Given: Diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD.

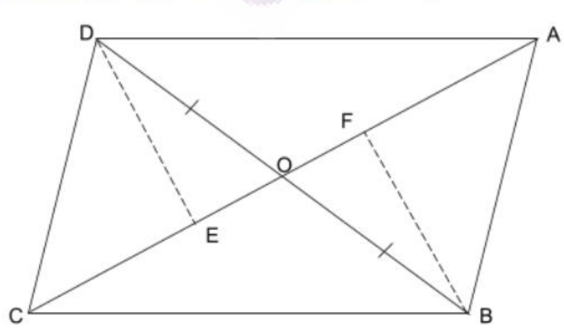
To Prove: If AB = CD, then

(i) $Area(\Delta DOC) = Area(\Delta AOB)$

(ii) $Area(\Delta DCB) = Area(\Delta ACB)$

(iii) $DA||CB$ or ABCD is a parallelogram.

Construction: Draw $DE \perp AC and BF \perp AC$.



Proof: In $\Delta DON \: and \: \Delta BOM$,

$\perp DNO = \perp BMO$(By Construction)

$\perp DON = \perp BOM$ (vertically opposite angles)

OD = OB (Given)

By AAS congruence rule,

$\Delta DON \perp \Delta BOM$

$\perp DN = \perp BM$ ....(1)

We know that congruent triangles have equal areas.

$Area(\Delta DON) = Area(\Delta BOM)$ .....(2)

In $\Delta DNC \: and \: \Delta BMA$,

$\perp DNC = \perp BMA$ (By Construction)

$CD = AB$ (Given)

$DN = BM$ [using equation(1)]

$\Delta DNC \sim \Delta BMA$ (RHS congruence rule)

$Area(\Delta DNC) = Area(\Delta BMA)$ .....(3)

Add equation(2) and (3) , we get

$Area(\Delta DON) + Area(\Delta DNC) = Area(\Delta BOM) + Area(\Delta BMA) \: therefore Area(\Delta DOC) = Area(\Delta AOB)$

(ii)We got ,

$Area(\Delta DOC) = Area(\Delta AOB)$

Now, add $Area(\Delta OCB)$ on both the sides.

$\Rightarrow Area(\Delta DOC) + Area(\Delta OCB) = Area(\Delta AOB) + Area(\Delta OCB) \; \Rightarrow Area(\Delta DCB) = Area(\Delta ACB)$

(iii)We got ,

$Area(\Delta DCB) = Area(\Delta ACB)$

If two triangles have the same base and equal areas , then these will lie between the same parallels.

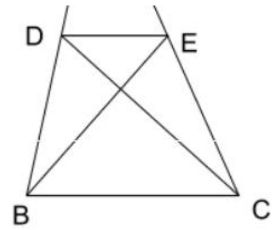
$DA || CB$ ......(4)

In quadrilateral ABCD , one pair of opposite sides is equal $(AB = CD)$and the other pair of opposite sides is parallel $(DA || CB)$.

$therefore ABCD is a parallelogram.$

Q.7. D and E are points on sides AB and AC respectively of △ABC such that $Area(\Delta DBC) = Area(\Delta EBC)$ ***Prove that DE || BC.***

Solution:



Given : D and E are points on sides AB and AC respectively of ∆ABC such that $Area(\Delta DBC) = Area(\Delta EBC)$.

To Prove: DE || BC

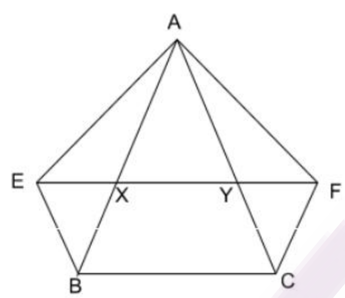
Proof: As (ΔDBC) and (ΔEBC) have equal area and are on the same base.

$therefore$ (ΔDBC) and (ΔEBC) will lie between the same parallel lines.

$therefore$ DE || BC

Q.8. XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and E respectively , show that $Area(\Delta ABE) = Area(\Delta ACF)$

Solution:



Given: XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and E respectively.

To Prove: $Area(\Delta ABE) = Area(\Delta ACF)$

Proof: XY||BC(given)

And CF||BX ($because$ CF||AB(given))

$therefore$BCFX is a parallelogram.

BC= XF

BC=XY+YF .....(1)

Again ,

XY||BC ($because$ BE||AC)

And BE||CY

$therefore$BCYE is a parallelogram

$therefore$BC=YE ($because$opposite sides of a parallelogram are equal)

$\Rightarrow BC = XY + XE$ .....(2)

From (1) and (2) ,

XY+YF=XY+XE

$\Rightarrow YF = XE$

$\Rightarrow XE = YF$

$therefore \Delta AXE \; and \; \Delta AYF$ have equal bases(XE=YF) on the same line EF and have common vertex A.

$therefore$Their altitudes are also the same.

$therefore Area(\Delta AXE) = Area(\Delta AFY)$ ....(4)

$therefore \;\; therefore \Delta BXE \; and \; \Delta BXE$ have equal bases(XE=YF) on the same line EF and are between the same parallels EF and BC(XY||BC).

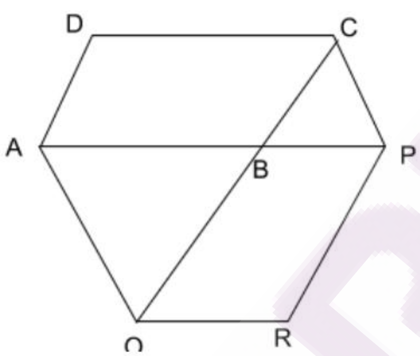
$therefore \;\; Area(\Delta BEX) = Area(\Delta CFY)$ ($because$Two triangles on the same base(or equal bases) and between the same parallels are equal in area)

Add the corresponding sides of (4) and (5), we get

$$Area(\Delta AXE) + Area(\Delta BXE) = Area(\Delta AFY) + Area(\Delta CFY)$$
$$\Rightarrow Area(\Delta ABE) = Area(\Delta ACF)$$

Q.9. The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see the following figure). Show that $Area(Parallelogram ABCD) = Area(Parallelogram PBQR).$

[Hint: Join AC and PQ. Now compare $Area(\Delta ACQ) \; and \; Area(\Delta APQ)$***]***

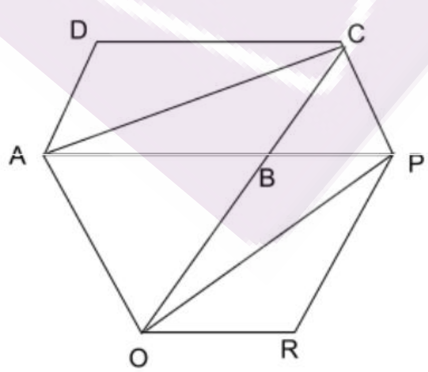


Solution:

Given: The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed.

To Prove: Area(Parallelogram ABCD) = Area(Parallelogram PBQR).

Construction: Join AC and PQ.



Proof: AC is a diagonal of parallelogram ABCD

$therefore Area(\Delta ABC) = \frac{1}{2} Area(parallelogram ABCD)$ ....(1)

PQ is a diagonal of parallelogram BQRP

$therefore Area(\Delta BPQ) = \frac{1}{2} Area(parallelogram BQRP)$ ....(2)

$\Delta ACQ \ and \ \Delta APQ$ are on the same base AQ and between the same parallels AQ and CP.

$therefore Area(\Delta ACQ) = Area(\Delta APQ)$

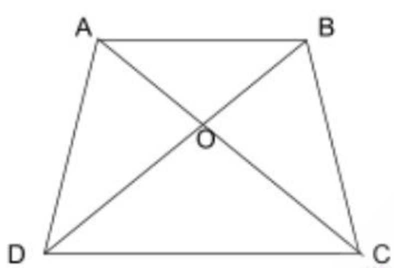
Now , substract $Area(\Delta ABQ)$ from both the sides.

$\Rightarrow Area(\Delta ACQ) - Area(\Delta ABQ) = Area(\Delta APQ) - Area(\Delta ABQ)$
$\Rightarrow Area(\Delta ABC) = Area(\Delta BPQ)$
$\Rightarrow \frac{1}{2} Area(parallelogram \ ABCD) = \frac{1}{2} Area(parallelogram \ PBQR)$

$\Rightarrow Area(parallelogram \ ABCD) = Area(parallelogram \ PBQR)$ [From(1) and (2)]

Q.10. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that $Area(\Delta AOD) = Area(\Delta BOC)$.

Solution:



Given: Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.

To Prove: $Area(\Delta AOD) = Area(\Delta BOC)$.

Proof: $\Delta ABD \ and \ \Delta ABC$ are on the same base AB and between the same parallels AB and DC.

$therefore Area(\Delta ABD) = Area(\Delta ABC)$

Now ,substract $Area(\Delta AOB)$ from both the sides;

$\Rightarrow Area(\Delta ABD) - Area(\Delta AOB) = Area(\Delta ABC) - Area(\Delta AOB)$
$\Rightarrow Area(\Delta AOD) = Area(\Delta BOC)$

Q.11. In the given figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) $Area(\Delta ACB) = Area(\Delta ACF)$

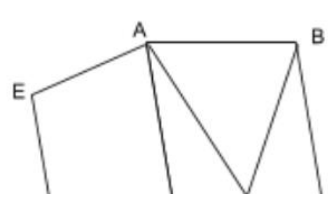
(ii) $Area(AEDF) = Area(ABCDE)$

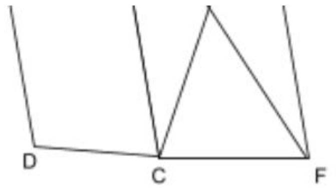
Solution:

Given: ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F.

To Prove: (i) $Area(\Delta ACB) = Area(\Delta ACF)$

(ii) $Area(AEDF) = Area(ABCDE)$





Proof:

(i) $\Delta ACB\ and\ \Delta ACF$ lie on the same base AC and are between the same parallels AC and BF.

$Area(\Delta ACB) = Area(\Delta ACF)$

(ii) $because Area(\Delta ACB) = Area(\Delta ACF)$

Add $Area(AEDC)$ on both the sides, we get

$\Rightarrow Area(\Delta ACB) + Area(AEDC) = Area(\Delta ACF) + Area(AEDC)$
$\Rightarrow Area(ABCDE) = Area(AEDF)$
$\Rightarrow Area(AEDF) = Area(ABCDE)$

Q.12. A villager Ram has a plot of land in the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Ram agrees to the above proposal with the condition that he should be given an equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

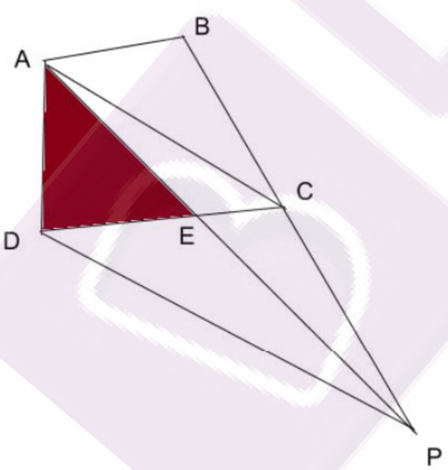
Solution:

Let ABCD be the plot of land in the shape of a quadrilateral.

Let the portion ADE be taken over by the Gram Panchayat of the village from one corner D to construct a Health Centre.

Join AC. Draw a line through D parallel to AC to meet BC produced in P.

Then Ram must given the land ECP adjoining his plot so as to form a triangular plot ABP as then.



Proof: $\Delta DAP\ and\ \Delta DCP$ are between the same parallels DP and AC.

$therefore Area(\Delta DAP) = Area(\Delta DCP)$(Two triangles on the same base (or equal bases) and between the same parallels are equal in area)

Substract $Area(\Delta DEP)$ from both the sides.

$\Rightarrow Area(\Delta DAP) - Area(\Delta DEP) = Area(\Delta DCP) - Area(\Delta DEP)$ $Area(\Delta ADE) = Area(\Delta PCE)$

Now , add $Area(ABCE)$ on the both the sides.

$Area(\Delta ADE) = Area(\Delta PCE)$ $\Rightarrow Area(ABCD) = Area(\Delta ABP)$

$\Rightarrow Area(\Delta ADE) + Area(ABCE) = Area(\Delta PCE) + Area(ABCE)$

Q.13. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that $Area(\Delta ADX) = Area(\Delta ACY)$. ***[Hint: Join CX.]***

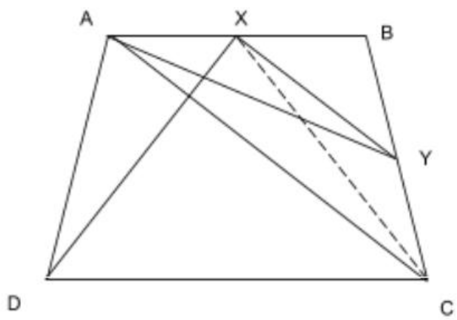
Solution:

Given: ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y.

To Prove: $Area(\Delta ADX) = Area(\Delta ACY)$.

Construction: Join CX

Proof: $\Delta ADX \; and \; \Delta ACX$ are on the same base AX and between the same parallels AB and DC.



$therefore Area(\Delta ADX) = Area(\Delta ACX)$ ....(1)

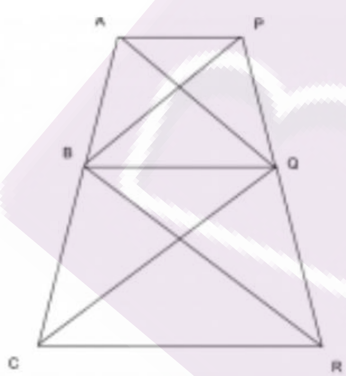
$\Delta ACX \; and \; \Delta ACY$ are on the same base AC and between the same parallels AC and XY.

$therefore Area(\Delta ACX) = Area(\Delta ACY)$ ....(2)

From equation(1) and (2) , we get

$Area(\Delta ADX) = Area(\Delta ACY)$

Q.14. In the given figure, AP || BQ || CR. Prove that $Area(\Delta AQC) = Area(\Delta PBR)$.



Solution: $AP||BQ||CR$

To Prove: $Area(\Delta AQC) = Area(\Delta PBR)$.

Proof: $\Delta BAQ \; and \; \Delta BQR$ are between the same parallels BQ and AP and on the same base BQ.

$Area(\Delta BCQ) = Area(\Delta BPQ)$ ....(1)

$\Delta BCQ \; and \; \Delta BQR$ are between the same parallels BQ and CR and on the same base BQ.

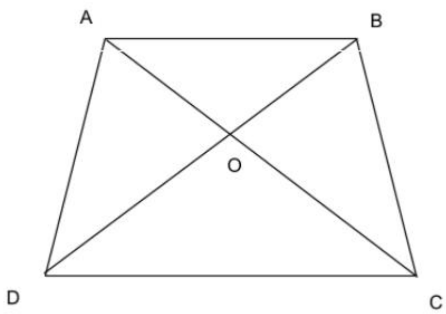
$Area(\Delta BCQ) = Area(\Delta BQR)$ ....(2)

Now , add equation(1) and (2) , we get

$$Area(\Delta BAQ) + Area(\Delta BCQ) = Area(\Delta BPQ) + Area(\Delta BQR)$$
$$\Rightarrow Area(\Delta AQC) = Area(\Delta PBR)$$

Q.15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that $Area(\Delta AOD) = Area(\Delta BOC)$***.Prove that ABCD is a trapezium.***

Solution:



Given: Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that $Area(\Delta AOD) = Area(\Delta BOC)$.

To Prove: ABCD is a trapezium.

Proof: $Area(\Delta AOD) = Area(\Delta BOC)$

Now , add $Area(\Delta AOB)$ on both the sides.

$$Area(\Delta AOD) + Area(\Delta AOB) = Area(\Delta BOC) + Area(\Delta AOB)$$
$$\Rightarrow Area(\Delta ABD) = Area(\Delta ABC)$$

But $\Delta ABD \; and \; \Delta ABC$ are on the same base AB.

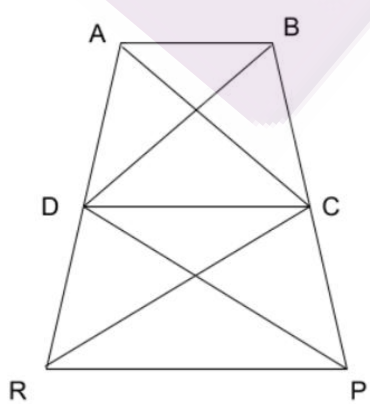
$therefore \Delta ABD \; and \; \Delta ABC$ will have equal corresponding altitudes

and $therefore \Delta ABD \; and \; \Delta ABC$ will lie between the same parallels

$\Rightarrow AB || DC$

$therefore$ ABCD is a trapezium.($because$ A quadrilateral is a trapezium if exactly one pair of opposite sides is parallel)

Q.16. In the given figure , $Area(\Delta DRC) = Area(\Delta DPC) \; and \; Area(\Delta BDP) = Area(\Delta ARC)$***.Show that both the quadrilaterals ABCD and DCPR are trapeziums.***



Solution:

Given: $Area(\Delta DRC) = Area(\Delta DPC)\ and\ Area(\Delta BDP) = Area(\Delta ARC)$

To Prove: both the quadrilaterals ABCD and DCPR are trapeziums.

Proof: $Area(\Delta DRC) = Area(\Delta DPC)$ (given) .....(1)

But $\Delta DRC\ and\ \Delta DPC$ are on the same base DC.

$therefore \Delta DRC\ and \Delta DPC$ will have equal corresponding altitudes.

And $therefore \Delta DRC\ and \Delta DPC$ will lie between the same parallels.

$therefore$DC||RP ($because$A quadrilateral is a trapezium if exactly one pair of opposite sides is parallel.)

$therefore$DCPR is a trapezium.

Again , $Area(\Delta BDP) = Area(\Delta ARC)$ ....(using(1))

$$\Rightarrow Area(\Delta BDC) + Area(\Delta DPC) = Area(\Delta ADC) + Area(\Delta DRC)$$
$$\Rightarrow Area(\Delta BDC) = Area(\Delta ADC)$$

But $\Delta BDC\ and\ \Delta ADC$ are on the same base DC.

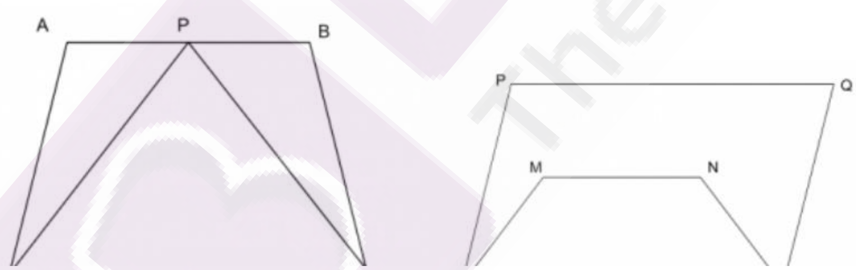
$therefore \Delta BDC\ and\ \Delta ADC$ will have equal corresponding altitudes.

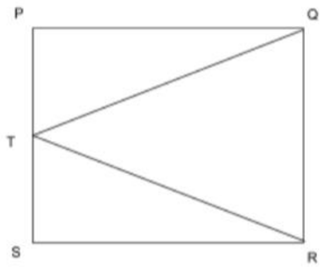
And $\Delta BDC\ and\ \Delta ADC$ will lie between the same parallels.

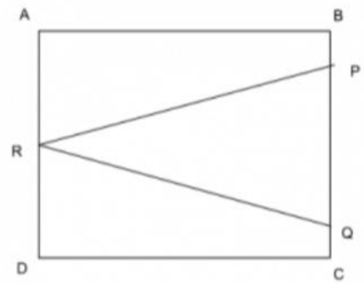
$therefore AB||DC$

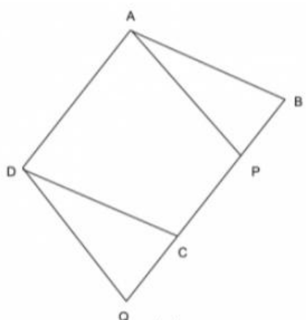
$\Rightarrow ABCD is a trapezium.$ ($because$ A quadrilateral is trapezium if exactly one pair of opposite sides is parallel.)

Q.17.From the following figures, find out which figures lie between the same parallels and same base. If the case is found, then write the common base and two parallels.

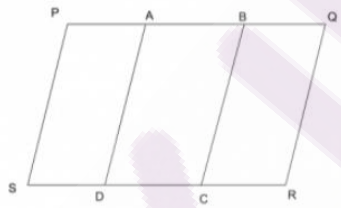








(v)



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Solution:

(I)The figures (quadrilateral APCD and quadrilateral ABCD), and (quadrilateral PBCD and quadrilateral ABCD), lie between the same parallels DC and AB and lie on the same base DC.

(II)The figures (ΔTRQ and parallelogram SRQP), (quads TPQR and parallelogram SRQP), (quad STQR and SRQP), lie on the same base RQ and between the same parallels RQ and SP.

(III)Quads APCD and ABQD lie on the same base AD, and between the same parallels AD and BQ.