NCERT SOLUTIONS
CLASS-IX MATHS
CHAPTER-1 NUMBER SYSTEM

1) Prove that zero as a rational number. Write zero in *p*/*q*, where *p* and *q* are integers and *q* ≠ 0?

Answer:

Yes. Zero is a rational number and it can be represented as p/q

2) In between 3 and 4, find six rational numbers.

Answer:
There are an infinite number of rational numbers between the numbers 3 and 4.
They can be represented as

$\frac{24}{8}\,and\,\frac{32}{8}$ respectively.

Therefore, six rational numbers between 3 and 4 are
$\frac{25}{8}, \frac{26}{8}, \frac{27}{8}, \frac{28}{8}, \frac{29}{8}, \frac{30}{8}$

3) In between 3/5 and 4/5, find five rational numbers.

Answer:
There are an infinite number of rational numbers between the numbers $\frac{3}{5}\,and\,\frac{4}{5}$.

$\frac{3}{5} = \frac{3\times6}{5\times6} = \frac{18}{30}$

$\frac{4}{5} = \frac{4\times6}{5\times6} = \frac{24}{30}$

therefore there are infinite numbers of rational number

$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$

4) State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

The statement is true since the collection of whole numbers contains all natural numbers.

(ii) Every integer is a whole number.

No, the statement is false, as integers may be negative but whole numbers are always positive.

(iii) Every rational number is a whole number.

No, the statement is false, as rational numbers may be fractional but whole numbers may not be.

5) State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

The statement is true since the collection of real numbers is made up of rational and irrational numbers.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.
No, the statement is false since as per the rule, a negative number could not be expressed as square roots.

(iii) Every real number is an irrational number.

No, the statement is false, the real numbers include both irrational and rational numbers. Therefore, every real number cannot be an irrational number.

6) Prove that the square roots of all positive integers irrational. If not, give an example of the square root of a number that is supposed to be a rational number.

Answer:
No, the square roots of all positive integers are not irrational. For example $\sqrt{4}$ = 2.

7) Show how $\sqrt{5}$ can be represented on the number line.

Answer:

Step 1: Let AB be length 2 unit length line on a number line.
Step 2: At B, draw a perpendicular line BC of length 1 unit. Join CA.
Step 3: Now, ABC is a right angled triangle. Applying Pythagoras theorem,
$AB^2 + BC^2 = CA^2$
$2^2 + 1^2 = CA^2$
⇒CA^2 = 5
⇒ CA = $\sqrt{5}$
Thus, CA is a line of length $\sqrt{5}$ unit.
Step 4: Taking CA as a radius and A as a centre draw an arc touching the number line. The point at which number line get intersected by arc is at $\sqrt{5}$ distance from 0 because it is a radius of the circle whose centre was A.
Thus, $\sqrt{5}$ is represented on the number line as shown in the figure.

8) Write the following questions in decimal form and mention what kind of decimal expansion each question has:
(i) $\frac{36}{100}$

= 0.36 (Terminating)

(ii) $\frac{1}{11}$

0.09090909… = 0.9 (Non terminating and repeating)

(iii) $4\frac{1}{8}$
= 33/8 = 4.125 (Terminating)

(iv) $\frac{3}{13}$
= 0.230769230769… = 0.230769 (Non terminating and repeating)

(v) $\frac{2}{11}$
= 0.181818181818… = 0.18 (Non terminating and repeating)

(vi) $\frac{329}{400}$
= 0.8225 (Terminating)

9) We know that 1/7 = 0.142857. Could we predict what the decimal expansion of 2/7, 3/7, 4/7, 5/7, 6/7 are without actually doing the long division? If possible, how?
Answer:
Yes. We can be done this by:

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = \overline{285714}\ \frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = \overline{428571}\ \frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = \overline{571428}$$
$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = \overline{714285}\ \frac{6}{7} = 6 \times \frac{1}{7} = 3 \times 0.\overline{142857} = \overline{857142}$$

10) Express the following in the form *p*/*q*where *p*and *q* are integers and *q* ≠ 0.

(i) 0. $\overline{6}$

(ii) $0.4\overline{7}$

(iii) $0.\overline{001}$

Answer:

(i) $0.\overline{6}$ = 0.666…
Let x = 0.666…
$10x$ = 6.666…
$10x = 6 + x$
$9x = 6$
$x = 2/3$

(ii) $0.4\overline{7}$

= 0.4777…
= 4/10 + 0.777/10
Let x = 0.777…
$10x$ = 7.777…
$10x = 7 + x$

$x = 7/9$

4/10 + 0.777…/10 = 4/10 + 7/90

= 36/90 +7/90 = 43/90

(iii) $0.\overline{001}$

= 0.001001…

Let x = 0.001001…

$1000x$ = 1.001001…
$1000x = 1 + x$
$999x = 1$

$x = 1/999$

11) Express 0.99999…in the form *p*/*q*.

Answer:

Let x = 0.9999…
$10x$ = 9.9999…
$10x = 9 + x$
$9x = 9$
$x = 1$
The difference between 1 and 0.999999 is 0.000001 which is negligible. Thus, 0.999 is too much near 1, Therefore, 1 as the answer can be justified.

12) What can be the maximum number of digits in the repeating block of digits in the decimal expansion of 1/17? Explain the answer.

Answer:

$\frac{1}{17} = 0.\overline{0588235294117647}$
There are 16 digits in the repeating block of the decimal expansion of $\frac{1}{17}$

=$0.\overline{0588235294117647}$

13) Look at several examples of rational numbers in the form *p*/*q* (*q* ≠ 0) where *p* and *q* are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property *q* must satisfy?

Answer:

We observe that when q is 2, 4, 5, 8, 10… Then the decimal expansion is terminating. For example:
1/2 = $0.\overline{5}$, denominator $q = 2^1$
7/8 = $0.\overline{875}$, denominator $q = 2^3$
4/5 = $0.\overline{8}$, denominator $q = 5^1$

We can observe that the terminating decimal may be obtained in the situation where prime factorization of the denominator of the given fractions has the power of only 2 or only 5 or both.

14) Write three numbers whose decimal expansions are non-terminating non-recurring.

Answer:

Three numbers with decimal expansions that are non-terminating non-recurring are:

0.303003000300003…

0.505005000500005…

0.720720072000720000720000…

15) Find three different irrational numbers between the rational numbers 5/7 and

9/11.

Answer:

5/7 = $0.\overline{714285}$

9/11 = $0.\overline{81}$

Three different irrational numbers are:

0.73073007300073000073…

0.75075007300075000075…

0.76076007600076000076…

16) Classify the following numbers as rational or irrational according to their type:

(i) $\sqrt{23}$

(ii) $\sqrt{225}$

(iii) 0.3796

(iv) 7.478478

(v) 1.101001000100001…

Answer:

(i) $\sqrt{23}$= 4.79583152331…

Since the number is non-terminating non-recurring therefore, it is an irrational number.

(ii) $\sqrt{225}$= 15 = 15/1
Since the number is rational number as it can represented in *p*/*q* form.

(iii) 0.3796

Since the number is terminating therefore, it is an rational number.

(iv) 7.478478 = 7.478

Since the this number is non-terminating recurring, therefore, it is a rational number.

(v) 1.101001000100001…

Since the number is non-terminating non-repeating, therefore, it is an irrational number.

17) Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

(iii) $2\sqrt{7}/7\sqrt{7}$

(iv) $1/\sqrt{2}$

(v) 2π

Answer

(i) $2 - \sqrt{5}$ = 2 – 2.2360679… = – 0.2360679…

Since the number is is non-terminating non-recurring therefore, it is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23} = 3 = 3/1$
Since the number is rational number as it can represented in *p*/*q* form.

(iii) $2\sqrt{7}/7\sqrt{7}$ = 2/7
Since the number is rational number as it can represented in *p*/*q* form.

(iv) $1/\sqrt{2} = \sqrt{2}/2$ = 0.7071067811…
Since the number is is non-terminating non-recurring therefore, it is an irrational number.

(v) $2\pi = 2 \times 3.1415\ldots = 6.2830\ldots$
Since the number is is non-terminating non-recurring therefore, it is an irrational number.

18) Simplify each of the following expressions:

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$

(ii) (3 + $\sqrt{3}$) (3 – √3)

(iii) ($\sqrt{5}$ + $\sqrt{2}$)^2

(iv) ($\sqrt{5}$ – $\sqrt{2}$) ($\sqrt{5}$ + $\sqrt{2}$)

Answer:

(i) (3 + $\sqrt{3}$) (2 + $\sqrt{2}$)

3 × 2 + 2 + $\sqrt{3}$ + 3$\sqrt{2}$ + √3 ×$\sqrt{2}$

6 + 2$\sqrt{3}$+3$\sqrt{2}$+ √6

(ii) (3 + $\sqrt{3}$) (3 – $\sqrt{3}$)

= $3^2 - (\sqrt{3})^2$
= 9 – 3

= 6

(iii) ($\sqrt{5}$ + $\sqrt{2}$) ^{2}

= $(\sqrt{5})^2$+$(\sqrt{2})^2$ + 2 ×√5 × √2
= 5 + 2 + 2 × $\sqrt{5}$× 2

= 7 +2 + $+\sqrt{10}$

(iv) ($\sqrt{5}$– $\sqrt{2}$) ($\sqrt{5}$+ $\sqrt{2}$)

= $(\sqrt{5})^2$– $(\sqrt{2})^2$

= 5 – 2

= 3.

19) π is defined as the ratio of the circumference (say c) of a circle to its diameter (D). That is, π = c/d. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer:

There is no contradiction. When we measure a value with a scale, we only obtain an approximate value. We never obtain an exact value. Therefore, we may not realize whether c or d is irrational. The value of π is almost equal to 22/7 or 3.142857…

20) Represent $(\sqrt{9.3})^2$ – on the number line.

Answer:
Step 1: Draw a line segment of 9.3 units. Extend it to C so that BC is forms of 1 unit.
Step 2: Now, AC = 10.3 units. Find the centre of AC and name it as O.
Step 3: Draw a semi-circle with radius OC and centre O.
Step 4: Draw a perpendicular line BD to AC at point B which intersect the semicircle at D. Also, Join OD.
Step 5: Now, OBD is a right angled triangle.
Here, OD = 10.3/2 (radius of semi circle), OC = 10.3/2, BC = 1
OB = OC – BC = (10.3/2) – 1 = 8.3/2
Using Pythagoras theorem,
$OD^2 = BD^2 + OB^2$

$(10.3/2)^2 = BD^2 + (8.3/2)^2 \ (BD)^2 = (10.3/2)^2 + (8.3/2)^2 \ (BD)^2 = (10.3/2 - 8.3/2)(10.3/2 + 8.3/2)$

$\Rightarrow BD^2$ = 9.3
$\Rightarrow BD^2$ = √9.3
Thus, the length of BD is √9.3.
Step 6: Taking BD as radius and B as centre draw an arc which touches the line segment. The point where it touches the line segment is at a distance of √9.3 from O as shown in the figure.

21) Rationalize the denominators of the following:

(i) 1/√7

(ii) 1/√7-√6

(iii) 1/√5+√2

(iv) 1/√7-2

Answer :
(i) $\frac{1}{\sqrt{7}} = \frac{1\times\sqrt{7}}{1\times\sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$
$= \sqrt{7} + \sqrt{6}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$
$= \frac{\sqrt{5}-\sqrt{2}}{3}$

(iv) $\frac{1}{\sqrt{7}-2} = \frac{(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$
$= \frac{\sqrt{7}+2}{3}$

22) Find:
(i) $64^{\frac{1}{2}}$

(ii) $32^{\frac{1}{5}}$

(iii) $125^{\frac{1}{3}}$

Answer:

(i) $64^{\frac{1}{2}}$ = $(2^6)^{\frac{1}{2}}$ =8

(ii) $32^{\frac{1}{5}}$ = $(2^5)^{\frac{1}{5}}$ =2

(iii)$125^{\frac{1}{3}}$ = $(5^3)^{\frac{1}{3}}$ =5

23) Find:

(i) $9^{\frac{3}{2}}$

(ii) $32^{\frac{2}{5}}$

(iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Answer:

$9^{\frac{3}{2}}$ = 3^3 = 27

$32^{\frac{2}{5}}$ = 2^2 =4

$16^{\frac{3}{4}}$ = 2^3 =8

$125^{\frac{-1}{3}}$ = $\frac{1}{125^{\frac{1}{3}}}$ = $\frac{1}{5}$

24)Simplify:

(i) $2^{3/5}.2^{1/5}$

(ii) $(\frac{1}{3})^7$

(iii) $\frac{11^{\frac{1}{2}}}{11^4}$

(iv) $\frac{7^{\frac{1}{2}}}{8^{\frac{1}{2}}}$

Answer

(i) $2^{3/5}.2^{1/5}$ = $2^{\frac{13}{15}}$

(ii) $(\frac{1}{3})^7$ = 3^{-21}

(iii) $\frac{11^{\frac{1}{2}}}{11^4}$ = $11^{\frac{1}{4}}$

(iv) $\frac{7^{\frac{1}{2}}}{8^{\frac{1}{2}}}$ =$56^{\frac{1}{2}}$