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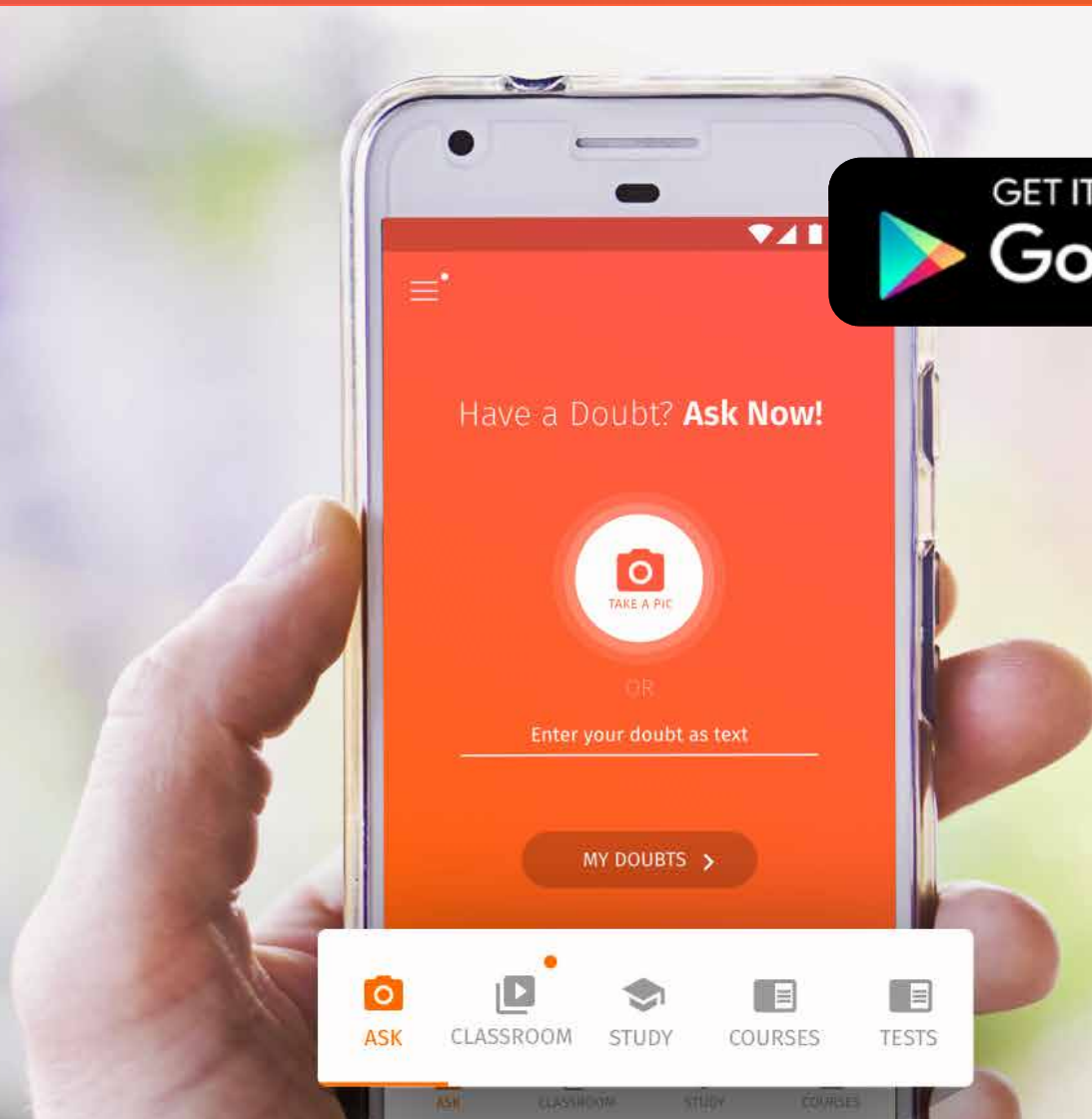
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Question 1:

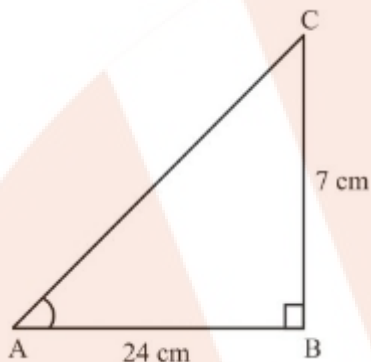
In $\triangle ABC$ right angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine

(i) $\sin A$, $\cos A$

(ii) $\sin C$, $\cos C$

Solution 1:

Let us draw a right angled triangle ABC



Given,

- $AB = 24$ cm
- $BC = 7$ cm
- $\sin A = ?$
- $\cos A = ?$
- $\sin C = ?$
- $\cos C = ?$
- $AC = ?$

We know that by Pythagoras theorem for $\triangle ABC$,

$$AC^2 = AB^2 + BC^2$$

$$= 24^2 + 7^2 \text{ (By Substituting the values)}$$

$$= 576 + 49$$

$$= 625 \text{ cm}^2$$

$$\therefore \text{Hypotenuse, } AC = 25 \text{ cm}$$

$$(i) \sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7}{25}$$

$$(ii) \cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$$

$$(iii) \sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{24}{25}$$

$$(iv) \cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7}{25}$$

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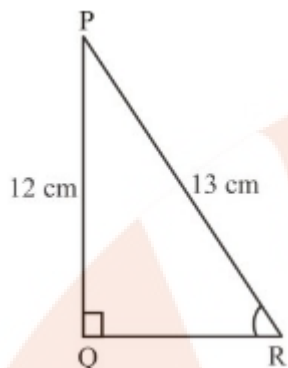
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Question 2:

In the given figure find $\tan P - \cot R$



Solution 2:

From the above Figure,

Given

- $PQ = 12$ cm
- $PR = 13$ cm
- $QR = ?$
- $\tan P - \cot R = ?$

We know that by applying Pythagoras theorem for ΔPQR ,

$$PR^2 = PQ^2 + QR^2$$

$$13^2 = 12^2 + QR^2 \text{ (By Substituting the values)}$$

$$169 = 144 + QR^2$$

$$QR^2 = 169 - 144$$

$$QR^2 = 25 \text{ cm}^2$$

$$QR = 5 \text{ cm}$$

Hence,

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

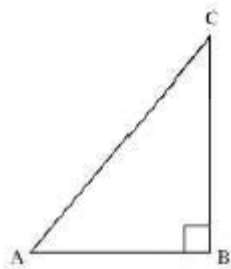
$$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$$

Question 3:

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Solution 3:

From the figure,



Let $\triangle ABC$ is a right-angled triangle

Given,

- $\sin A = \frac{3}{4}$,
- We know that $\text{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}}$
 - Hence with respect to angle A,
 - $BC = 3$
 - $AC = 4$
 - $AB = ?$
- $\cos A = ?$
- $\tan A = ?$

By Applying Pythagoras theorem in $\triangle ABC$, we get

$$AC^2 = AB^2 + BC^2$$

$$4^2 = AB^2 + 3^2$$

$$16 - 9 = AB^2$$

$$AB^2 = 7$$

$$AB = \sqrt{7}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$\cos A = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

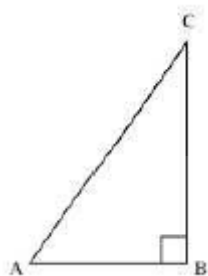
$$\tan A = \frac{3}{\sqrt{7}}$$

Question 4:

Given $15 \cot A = 8$. Find $\sin A$ and $\sec A$.

Solution 4:

From the Figure,



Let ABC be the right-angled triangle,
Given,

- $15 \cot A = 8$
- $\sin A = ?$
- $\sec A = ?$

$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

From Given,

$$15 \cot A = 8$$

$$\cot A = \frac{8}{15} \quad (\text{By Transposing})$$

$$\frac{AB}{BC} = \frac{8}{15}$$

By applying Pythagoras theorem in $\triangle ABC$, we get

$$AC^2 = AB^2 + BC^2$$

$$= 8^2 + 15^2$$

$$= 64 + 225$$

$$= 289$$

$$AC = 17$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15}{17}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle A} = \frac{AC}{AB}$$

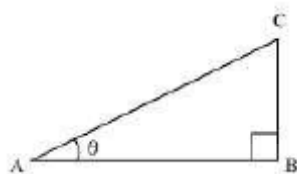
$$= \frac{17}{8}$$

Question 5:

Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Solution 5:

From the Figure,



Let $\triangle ABC$ be a right-angle triangle
Given,

$$\sec \theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle \theta}$$

$$= \frac{13}{12} = \frac{AC}{AB}$$

Hence

- $AC = 13$
- $AB = 12$
- $BC = ?$
- $\sin \theta = ?$
- $\cos \theta = ?$
- $\tan \theta = ?$
- $\cot \theta = ?$
- $\operatorname{cosec} \theta = ?$

By applying Pythagoras theorem in $\triangle ABC$, we obtain

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$13^2 = 12^2 + BC^2$$

$$169 = 144 + BC^2$$

$$25 = BC^2$$

$$BC = 5$$

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5}{13}$$

$$\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{12}{13}$$

$$\tan \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Side adjacent to } \angle \theta} = \frac{BC}{AB} = \frac{5}{12}$$

$$\cot \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{AB}{BC} = \frac{12}{5}$$

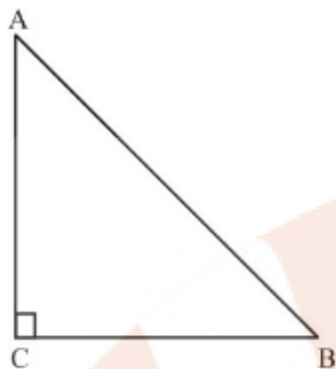
$$\operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle \theta} = \frac{AC}{BC} = \frac{13}{5}$$

Question 6:

If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Solution 6:

From the Figure,



Given

- Let $\triangle ABC$ be a right Angled Triangle
- $\angle A$ and $\angle B$ are Acute Angles
- $\cos A = \cos B$

To Prove:

$$\angle A = \angle B$$

Proof:

In the Right Angled Triangle ABC,

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AC}{AB}$$

$$\cos B = \frac{\text{Side adjacent to } \angle B}{\text{Hypotenuse}} = \frac{BC}{AB}$$

Since we know $\cos A = \cos B$

$$\frac{AC}{AB} = \frac{BC}{AB}$$

Hence by observation,

$$AC = BC$$

Hence, $\angle A = \angle B$ (Angles opposite to the equal sides of the triangle).

Question 7:

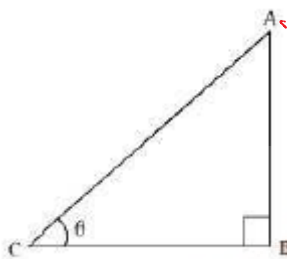
If $\cot \theta = \frac{7}{8}$, evaluate

(i) $\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$

(ii) $\cot^2 \theta$

Solution 7:

From the Figure,



Given,

Let $\triangle ABC$ be a right triangle ABC,

$$\cot \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Side opposite to } \angle \theta} = \frac{BC}{AB} = \frac{7}{8}$$

Hence,

- $BC = 7$
- $AB = 8$
- $AC = ?$

By applying Pythagoras theorem in $\triangle ABC$, we obtain

$$AC^2 = AB^2 + BC^2$$

$$= 8^2 + 7^2$$

$$= 64 + 49$$

$$= 113$$

By Taking the Square roots,

$$AC = \sqrt{113}$$

$$\sin \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{8}{\sqrt{113}}$$

$$\cos \theta = \frac{\text{Side adjacent to } \angle \theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{\sqrt{113}}$$

$$(i) \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{(1 - \sin^2 \theta)}{(1 - \cos^2 \theta)}$$

$$= \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

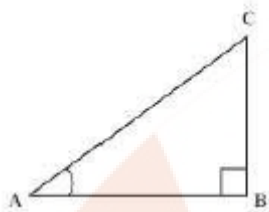
$$(ii) \cot^2 \theta = (\cot \theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

Question 8:

If $3 \cot A = 4$, Check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not

Solution 8:

From the figure,



Given,

Let $\triangle ABC$ be a right angled triangle.

- $3 \cot A = 4$

Hence, $\cot A = \frac{4}{3}$

We know that,

$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A} = \frac{AB}{BC} = \frac{4}{3}$$

Hence, $AB = 4$ and $BC = 3$. $AC = ?$

By applying the Pythagoras Theorem in $\triangle ABC$,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= 4^2 + 3^2$$

$$= 16 + 9$$

$$= 25$$

$$AC = 5$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A} = \frac{BC}{AB} = \frac{3}{4}$$

By substituting the above values of trigonometric functions in the LHS of the Equation,

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

$$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

By substituting the above values of trigonometric functions in the RHS of the Equation

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\therefore \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

Hence it is proved.

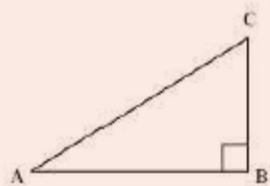
Question 9:

In $\triangle ABC$, right angled at B. If $\tan A = \frac{1}{\sqrt{3}}$, find the value of

- (i) $\sin A \cos C + \cos A \sin C$
- (ii) $\cos A \cos C - \sin A \sin C$

Solution 9:

From the figure,



Given,

Let $\triangle ABC$ be a right angled triangle

$$\tan A = \frac{1}{\sqrt{3}}$$

- $\frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Hence,

- $BC = 1$
- $AB = \sqrt{3}$
- $AC = ?$

By applying Pythagoras Theorem for $\triangle ABC$,

$$AC^2 = AB^2 + BC^2$$

$$= (\sqrt{3})^2 + 1^2$$

$$= 3 + 1 = 4$$

$$AC = 2$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}$$

(i) $\sin A \cos C + \cos A \sin C$

By substituting the values of the trigonometric functions below in the equation below,

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

(ii) $\cos A \cos C - \sin A \sin C$

By substituting the values of the trigonometric functions below in the equation below,

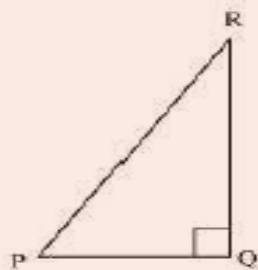
$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Question 10:

In $\triangle PQR$, right angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

Solution 10:

From the figure,



Given

Let $\triangle PQR$ be a right angled triangle

- $PR + QR = 25$ cm
- $PQ = 5$ cm
- $\sin P = ?$
- $\cos P = ?$
- $\tan P = ?$
- $PR = ?$

Therefore, $QR = 25 - x$

By applying Pythagoras theorem in $\triangle PQR$, we obtain

$$PR^2 = PQ^2 + QR^2$$

$$x^2 = 5^2 + (25 - x)^2$$

$$x^2 = 25 + 625 + x^2 - 50x$$

$$50x = 650$$

$$\text{Hence, } x = 13$$

$$\text{Therefore, } PR = 13 \text{ cm}$$

$$QR = (25 - 13) \text{ cm} = 12 \text{ cm}$$

By Substituting the values of the obtained above in the trigonometric functions below,

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Question 11:

State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A .

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A .

(iv) $\cot A$ is the product of $\cot$ and A

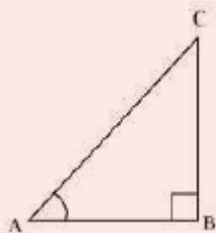
(v) $\sin \theta = \frac{4}{3}$, for some angle θ

Solution 11:

(i) False, because sides of a right angled triangle may have any length, So $\tan A$ may have any value.

(ii) $\sec A = \frac{12}{5}$

True, as the value of $\sec A > 1$,



$$\sec A = \frac{1}{\cos A} = \frac{1}{\frac{\text{Side of Adjacent } \angle A}{\text{Hypotenuse}}} = \frac{\text{Hypotenuse}}{\text{Side of Adjacent } \angle A}$$

As Hypotenuse is the largest Side, $\sec A > 1$

(iii) Abbreviation used for cosecant of $\angle A$ is cosec A . And $\cos A$ is the abbreviation used for cosine of $\angle A$. Hence, the given statement is false.

(iv) $\cot A$ is not the product of $\cot$ and A . It is the cotangent of $\angle A$. 'Cot' separated from 'A' has no meaning. Hence, the given statement is false.

(v) $\sin \theta = \frac{4}{3}$

We know that in a right-angled triangle,

$$\sin \theta = \frac{\text{Side opposite to } \angle \theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Also, the value of Sine should be less than 1 always. Therefore, such value of $\sin \theta$ is not possible. Hence, the given statement is false

Exercise (8.2)

Question 1:

Evaluate the following

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ - \cot 45^\circ}$

(v) $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sec^2 30^\circ + \cos^2 30^\circ}$

Solution 1:

We know that,

Exact Values of Trigonometric Functions				
Angle (θ)		$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not Defined

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

(By Substituting the Values taken from the chart above)

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$



$$(ii) 2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \quad (\text{By Substituting the Values taken from the chart above})$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

$$(iii) \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} \quad (\text{By Substituting the Values taken from the chart above})$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{2} \times 2 + (\sqrt{3}+1)}$$

$$= \frac{\sqrt{3}}{\sqrt{2} \times 2 + (\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \quad (\text{By multiplying \& dividing by } \sqrt{3}-1)$$

$$= \frac{\sqrt{3}(\sqrt{3}-1)}{\sqrt{2} \times 2 + (\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3-\sqrt{3}}{2\sqrt{2}((\sqrt{3})^2-1^2)}$$

$$= \frac{3-\sqrt{3}}{2\sqrt{2}(3-1)} = \frac{3-\sqrt{3}}{4\sqrt{2}}$$

(iv)

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}} \quad (\text{By Substituting the Values taken from the chart above})$$

$$= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}} = \frac{(3\sqrt{3}-4)}{(3\sqrt{3}+4)}$$

$$= \frac{(3\sqrt{3}-4)(3\sqrt{3}-4)}{(3\sqrt{3}+4)(3\sqrt{3}-4)} = \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2 - (4)^2} \quad (\text{By Using } (a+b)(a-b) = a^2 - b^2)$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

(v)

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

(By Substituting the Values taken from the chart above)

$$= \frac{5\left(\frac{1}{2}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{15 + 64 - 12}{4} = \frac{67}{12}$$

Question 2:

Choose the correct option and justify your choice.

(i) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \underline{\hspace{2cm}}$

- (A). $\sin 60^\circ$
- (B). $\cos 60^\circ$
- (C). $\tan 60^\circ$
- (D). $\sin 30^\circ$

(ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \underline{\hspace{2cm}}$

- (A). $\tan 90^\circ$
- (B). 1
- (C). $\sin 45^\circ$
- (D). 0

(iii) $\sin 2A = 2 \sin A$ is true when $A = \underline{\hspace{2cm}}$

- (A). 0°
- (B). 30°
- (C). 45°
- (D). 60°

(iv) $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \underline{\hspace{2cm}}$

- (A). $\cos 60^\circ$
- (B). $\sin 60^\circ$
- (C). $\tan 60^\circ$
- (D). $\sin 30^\circ$

Solution 2:

We know that,

Exact Values of Trigonometric Functions				
Angle ()		sin()	cos()	tan()
Degrees	Radians			
0°	0	0	1	0
30°				
45°				1
60°				
90°		1	0	Not Defined

$$(i) \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \underline{\hspace{2cm}}$$

$$= \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 + \left(\frac{1}{\sqrt{3}} \right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

(By Substituting the Values taken from the chart above)

$$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

$$(ii) \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \underline{\hspace{2cm}}$$

$$\frac{1 - (1)^2}{1 + (1)^2} = \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0$$

(By Substituting the Values taken from the chart above)

Hence, (D) is correct.

(iii) Out of the given alternatives, only $A = 0^\circ$ is correct.

$$\text{As } \sin 2A = \sin 0^\circ = 0$$

(By Substituting the Values taken from the chart above)

$$2 \sin A = 2 \sin 0^\circ = 2(0) = 0$$

Hence, (A) is correct.

$$(iv) \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \underline{\hspace{2cm}}$$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

(By Substituting the Values taken from the chart above)

Out of the given alternatives, only $\tan 60^\circ = \sqrt{3}$
Hence, (C) is correct.

Question 3:

If and; $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$

$0^\circ < A + B \leq 90^\circ$, $A > B$ find A and B.

Solution 3:

We know that,

Exact Values of Trigonometric Functions				
Angle ()		sin()	cos()	tan()
Degrees	Radians			
0°	0	0	1	0
30°				
45°				1
60°				
90°		1	0	Not Defined

$$\tan(A + B) = \sqrt{3}$$

$$\Rightarrow \tan(A + B) = \tan 60^\circ \quad (\text{By Substituting the Values taken from the chart above})$$

$$\Rightarrow A + B = 60^\circ \quad \dots\dots\dots \text{Equation (1)}$$

$$\tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan(A - B) = \tan 30^\circ \quad (\text{By Substituting the Values taken from the chart above})$$

$$\Rightarrow A - B = 30^\circ \quad \dots\dots\dots \text{Equation (2)}$$

On adding both Equation (1) & Equation (2), we obtain

$$A + B + A - B = 60^\circ + 30^\circ$$

$$2A = 90^\circ$$

$$\Rightarrow A = 45^\circ$$

By substituting the value of A in Equation (1), we obtain

$$45^\circ + B = 60^\circ$$

$$B = 15^\circ$$

Therefore, $\angle A = 45^\circ$ and $\angle B = 15^\circ$

Question 4:

State whether the following are true or false. Justify your answer.

- (i) $\sin(A + B) = \sin A + \sin B$
- (ii) The value of $\sin \theta$ increases as θ increases
- (iii) The value of $\cos \theta$ increases as θ increases
- (iv) $\sin \theta = \cos \theta$ for all values of θ
- (v) $\cot A$ is not defined for $A = 0^\circ$

Solution 4:

We know that,

Exact Values of Trigonometric Functions				
Angle ()		$\sin()$	$\cos()$	$\tan()$
Degrees	Radians			
0°	0	0	1	0
30°				
45°				1
60°				
90°		1	0	Not Defined

(i) $\sin(A + B) = \sin A + \sin B$

- For the purpose of verification, Take $A = 30^\circ$ and $B = 60^\circ$

By substituting the values in LHS,

$$\sin(A + B) = \sin(30^\circ + 60^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

By substituting the values in RHS,

$$\sin A + \sin B = \sin 30^\circ + \sin 60^\circ$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

Clearly, $\sin(A + B) \neq \sin A + \sin B$

Hence, the given statement is false.

(ii) The value of $\sin \theta$ increases as θ increases in the interval of $0^\circ < \theta < 90^\circ$

We know that

- $\sin 0^\circ = 0$
- $\sin 30^\circ = \frac{1}{2} = 0.5$

- $\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$
- $\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$
- $\sin 90^\circ = 1$

Hence, the given statement is true.

(iii) We know that,

- $\cos 0^\circ = 1$
- $\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$
- $\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$
- $\cos 60^\circ = \frac{1}{2} = 0.5$

It can be observed that the value of $\cos \theta$ does not increase in the interval of $0^\circ < \theta < 90^\circ$. Hence, the given statement is false.

(iv) $\sin \theta = \cos \theta$ for all values of θ .

This is true when $\theta = 45^\circ$

$$\text{As } \sin 45^\circ = \frac{1}{\sqrt{2}} \text{ and } \cos 45^\circ = \frac{1}{\sqrt{2}}$$

It is not true for all other values of θ .

$$\text{As } \sin 30^\circ = \frac{1}{2} \text{ and } \cos 30^\circ = \frac{\sqrt{3}}{2},$$

Hence, the given statement is false.

(v) $\tan 0^\circ = 0$ and $\cot A$ is not defined for $A = 0^\circ$

$$\text{As, } \cot A = \frac{\cos A}{\sin A} \text{ and } \cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ} = \frac{1}{0} \text{ undefined}$$

Hence, the given statement is true.

Exercise (8.3)

Question 1:

Evaluate:

(I) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(II) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(III) $\cos 48^\circ - \sin 42^\circ$

(IV) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

Solution 1:

$$\begin{aligned}\text{(I)} \quad \frac{\sin 18^\circ}{\cos 72^\circ} &= \frac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ} \\ &= \frac{\cos 72^\circ}{\cos 72^\circ} = 1\end{aligned}$$

(Since $\sin(90^\circ - \theta) = \cos \theta$)

$$\begin{aligned}\text{(II)} \quad \frac{\tan 26^\circ}{\cot 64^\circ} &= \frac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ} \\ &= \frac{\cot 64^\circ}{\cot 64^\circ} = 1\end{aligned}$$

(Since $\tan(90^\circ - \theta) = \cot \theta$)

$$\begin{aligned}\text{(III)} \quad \cos 48^\circ - \sin 42^\circ \\ &= \cos(90^\circ - 42^\circ) - \sin 42^\circ \\ &= \sin 42^\circ - \sin 42^\circ \\ &= 0\end{aligned}$$

(Since $\sin(90^\circ - \theta) = \cos \theta$)

$$\begin{aligned}\text{(IV)} \quad \operatorname{cosec} 31^\circ - \sec 59^\circ \\ &= \operatorname{cosec}(90^\circ - 59^\circ) - \sec 59^\circ \\ &= \sec 59^\circ - \sec 59^\circ \\ &= 0\end{aligned}$$

(Since $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$)

Question 2:

Show that

(I) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(II) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

Solution 2:

(I) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

Taking LHS,

$\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$ -----Equation (1)

We know that $\tan(90^\circ - A) = \tan A$

By manipulating the Equation (1) using the property above,

$$\begin{aligned}
 &= \tan (90^\circ - 42^\circ) \tan (90^\circ - 67^\circ) \tan 42^\circ \tan 67^\circ \\
 &= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ \\
 &= (\cot 42^\circ \tan 42^\circ) (\cot 67^\circ \tan 67^\circ) \text{ (By rearranging)} \\
 &= (1) (1) \quad [\text{As } \cot A \cdot \tan A = 1] \\
 &= 1
 \end{aligned}$$

$$(II) \cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$$

Consider LHS :

$$\begin{aligned}
 &\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ \quad \text{-----Equation (1)} \\
 &= \cos (90^\circ - 52^\circ) \cos (90^\circ - 38^\circ) - \sin 38^\circ \sin 52^\circ \quad [\text{As, } \cos (90^\circ - \theta) = \sin \theta] \\
 &= \sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ \\
 &= 0
 \end{aligned}$$

Question 3:

If $\tan 2A = \cot (A - 18^\circ)$, where $2A$ is an acute angle, find the value of A .

Solution 3:

Given that,

$$\tan 2A = \cot (A - 18^\circ) \quad \text{-----Equation (1)}$$

We know that $\tan 2A = \cot (90^\circ - 2A)$ by substituting this in Equation (1)

$$\cot (90^\circ - 2A) = \cot (A - 18^\circ)$$

Hence by Equating,

$$90^\circ - 2A = A - 18^\circ$$

$$A + 2A = 90^\circ + 18^\circ$$

$$3A = 108^\circ$$

$$A = 36^\circ$$

Question 4:

If $\tan A = \cot B$, prove that $A + B = 90^\circ$

Solution 4:

Given,

$$\tan A = \cot B \quad \text{-----Equation (1),}$$

We know that $\tan A = \cot (90^\circ - A)$ by substituting this in Equation (1)

$$\tan A = \tan (90^\circ - B)$$

By Equating,

$$A = 90^\circ - B$$

$$A + B = 90^\circ \text{ (By Transposing)}$$

Question 5:

If $\sec 4A = \operatorname{cosec} (A - 20^\circ)$, where $4A$ is an acute angle, find the value of A .

Solution 5:

Given,

$$\sec 4A = \operatorname{cosec} (A - 20^\circ) \text{ -----Equation (1),}$$

We know that $\sec A = \operatorname{Cosec} (90-A)$ by substituting this in Equation (1)

$$\operatorname{cosec} (90^\circ - 4A) = \operatorname{cosec} (A - 20^\circ)$$

By Equating,

$$90^\circ - 4A = A - 20^\circ$$

$$110^\circ = 5A \text{ (By Transposing)}$$

$$A = 22^\circ$$

Question 6:

If A , B and C are interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Solution 6:

We know that for a triangle ABC ,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + \angle C = 180^\circ - \angle A \quad \text{(By Transposing)}$$

Dividing both the sides by 2

$$\frac{\angle B + \angle C}{2} = 90^\circ - \frac{\angle A}{2}$$

Applying Sine Angle on both the sides,

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$$

$$= \cos\left(\frac{A}{2}\right)$$

Question 7:

Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .

Solution 7:

$$\sin 67^\circ + \cos 75^\circ$$

$$\text{Since, } \cos (90 - \theta) = \sin \theta \text{ and } \sin (90 - \theta) = \cos \theta$$

$$= \sin (90^\circ - 23^\circ) + \cos (90^\circ - 15^\circ)$$

$$= \cos 23^\circ + \sin 15^\circ$$

Exercise (8.4)

Question 1:

Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.

Solution 1:

Consider a $\triangle ABC$ with $\angle B = 90^\circ$

Using the Trigonometric Identity,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

(By taking reciprocal both the sides)

$$\sin^2 A = \frac{1}{1 + \cot^2 A} \quad \left(\text{As } \frac{1}{\operatorname{cosec}^2 A} = \sin^2 A\right)$$

Therefore,

$$\sin A = \pm \frac{1}{\sqrt{1 + \cot^2 A}}$$

For any sine value with respect to an angle in a triangle, sine value will never be negative. Since, sine value will be negative for all angles greater than 180° .

$$\text{Therefore, } \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

$$\text{We know that, } \tan A = \frac{\sin A}{\cos A}$$

$$\text{However, Trigonometric Function, } \cot A = \frac{\cos A}{\sin A}$$

$$\text{Therefore, Trigonometric Function, } \tan A = \frac{1}{\cot A}$$

$$\text{Also, } \sec^2 A = 1 + \tan^2 A \quad (\text{Trigonometric Identity})$$

$$= 1 + \frac{1}{\cos^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Question 2:

Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

Solution 2:

We know that,

$$\text{Trigonometric Function, } \cos A = \frac{1}{\sec A} \quad \dots \text{Equation (1)}$$

Also,

$$\sin^2 A + \cos^2 A = 1 \quad (\text{Trigonometric identity})$$

$$\sin^2 A = 1 - \cos^2 A \quad (\text{By transposing})$$

Using value of $\cos A$ from Equation (1) and simplifying further,

$$\begin{aligned}\sin A &= \sqrt{1 - \left(\frac{1}{\sec A}\right)^2} \\ &= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A} \quad \dots \text{Equation (2)}\end{aligned}$$

$$\tan^2 A + 1 = \sec^2 A \quad (\text{Trigonometric identity})$$

$$\tan^2 A = \sec^2 A - 1 \quad (\text{By transposing})$$

Trigonometric Function,

$$\tan A = \sqrt{\sec^2 A - 1} \quad \dots \text{Equation (3)}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}} \quad \dots (\text{By substituting Equations (1) and (2)})$$

$$\begin{aligned}&= \frac{1}{\sqrt{\sec^2 A - 1}} \\ \operatorname{cosec} A &= \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}} \quad \dots (\text{By substituting Equation (2) and simplifying})\end{aligned}$$

Question 3:

Evaluate

$$(i) \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$$

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

Solution 3:

$$\begin{aligned}(i) & \frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ} \\ &= \frac{[\sin(90^\circ - 27^\circ)]^2 + \sin^2 27^\circ}{[\cos(90^\circ - 73^\circ)]^2 + \cos^2 73^\circ} \\ &= \frac{[\cos 27^\circ]^2 + \sin^2 27^\circ}{[\sin 73^\circ]^2 + \cos^2 73^\circ} \quad (\sin(90^\circ - \theta) = \cos \theta \text{ \& } \cos(90^\circ - \theta) = \sin \theta)\end{aligned}$$

$$\begin{aligned}&= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ} \\ &= \frac{1}{1} \quad (\text{By Identity } \sin^2 A + \cos^2 A = 1) \\ &= 1\end{aligned}$$

$$(ii) \sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$$

$$\begin{aligned}
 &= (\sin 25^\circ) \{ \cos(90^\circ - 25^\circ) \} + \cos 25^\circ \{ \sin(90^\circ - 25^\circ) \} & (\sin(90^\circ - \theta) = \cos \theta \text{ \& } \cos(90^\circ - \theta) = \sin \theta) \\
 &= (\sin 25^\circ) (\sin 25^\circ) + \cos 25^\circ (\cos 25^\circ) \\
 &= \sin^2 25^\circ + \cos^2 25^\circ \\
 &= 1 & (\text{By Identity } \sin^2 A + \cos^2 A = 1)
 \end{aligned}$$

Question 4:

Choose the correct option. Justify your choice.

(i) $9 \sec 2A - 9 \tan 2A = \underline{\hspace{2cm}}$

- (A) 1
- (B) 9
- (C) 8
- (D) 0

(ii) $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$

- (A) 0
- (B) 1
- (C) 2
- (D) -1

(iii) $(\sec A + \tan A) (1 - \sin A) = \underline{\hspace{2cm}}$

- (A) $\sec A$
- (B) $\sin A$
- (C) $\operatorname{cosec} A$
- (D) $\cos A$

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

- (A) $\sec 2A$
- (B) -1
- (C) $\cot 2A$
- (D) $\tan 2A$

Solution 4:

(i) $9 \sec^2 A - 9 \tan^2 A$

$= 9 (\sec^2 A - \tan^2 A)$ (By taking 9 as common)

$= 9 (1)$ [By the identity, $1 + \sec^2 A = \tan^2 A$, Hence $\sec^2 A - \tan^2 A = 1$]

$= 9$

Hence, alternative (B) is correct.

(ii) $(1 + \tan \theta + \sec \theta) (1 + \cot \theta - \operatorname{cosec} \theta)$ ----- Equation (1)

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

And

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above function in Equation (1),

$$\begin{aligned} &= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \\ &= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \quad (\text{By taking LCM and multiplying}) \\ &= \frac{(\sin \theta + \cos \theta)^2 - (1)^2}{\sin \theta \cos \theta} \quad (\text{Using } a^2 - b^2 = (a + b)(a - b)) \\ &= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \\ &= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta} \quad (\text{Using identify } \sin^2 \theta + \cos^2 \theta = 1) \\ &= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta} = 2 \end{aligned}$$

Hence, alternative (C) is correct.

(iii) $(\sec A + \tan A)(1 - \sin A)$ -----Equation (1)

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

And

$$\sec(x) = \frac{1}{\cos(x)}$$

By substituting the above function in Equation (1),

$$\begin{aligned} &= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A) \\ &= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A) \\ &= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} \quad (\text{By identify } \sin^2 \theta + \cos^2 \theta = 1, \text{ Hence } 1 - \sin^2 \theta = \cos^2 \theta) \\ &= \cos A \end{aligned}$$

Hence, alternative (D) is correct.

$$(iv) \frac{1 + \tan^2 A}{1 + \cot^2 A}$$

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

By substituting the above function in Equation (1),

$$\begin{aligned} \frac{1 + \tan^2 A}{1 + \cot^2 A} &= \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}} \\ &= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} = \frac{1}{\frac{\cos^2 A}{\sin^2 A}} \\ &= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A \end{aligned}$$

Hence, alternative (D) is correct.

Question 5:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

$$(i) (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$(ii) \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$(iv) \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

$$(vi) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$(vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos \theta - \cos^3 \theta} = \tan \theta$$

$$(viii) (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$(ix) (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$(x) \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2$$

Solution 5:

$$(i) (\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

L.H.S = $(\operatorname{cosec} \theta - \cot \theta)^2$ -----Equation (1)

We know that the trigonometric functions,

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above function in Equation (1),

$$= \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2$$

$$= \frac{(1 - \cos \theta)^2}{(\sin \theta)^2} = \frac{(1 - \cos \theta)^2}{\sin^2 \theta}$$

$$= \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}$$

$$= \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)}$$

$$= \frac{1 - \cos \theta}{1 + \cos \theta}$$

= RHS

(By Identity $\sin^2 A + \cos^2 A = 1$ Hence, $1 - \cos^2 A = \sin^2 A$)

[Using $a^2 - b^2 = (a + b)(a - b)$]

$$(ii) \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$\text{L.H.S} = \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1 + \sin)^2}{(1 + \sin)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2 \sin A}{(1 + \sin A)(\cos A)}$$

(Taking LCM and common denominator)

$$\begin{aligned}
 &= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1 + \sin A)(\cos A)} \\
 &\quad \text{(By Identity } \sin^2 A + \cos^2 A = 1) \\
 &= \frac{1 + 1 + 2\sin A}{(1 + \sin A)(\cos A)} = \frac{2 + 2\sin A}{(1 + \sin A)(\cos A)} \\
 &\text{By taking 2 common and simplifying} \\
 &= \frac{2(1 + \sin A)}{(1 + \sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A \\
 &= \text{R.H.S}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad &\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta \\
 \text{LHS} = &\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \text{-----Equation (1)}
 \end{aligned}$$

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

By substituting the above function in Equation (1),

$$\begin{aligned}
 &\frac{\sin \theta}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\cos \theta}{1 - \frac{\sin \theta}{\cos \theta}} \\
 &= \frac{\sin \theta}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\cos \theta}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \quad \text{(By taking LCM and Common denominators)} \\
 &= \frac{\sin^2 \theta}{\sin \theta - \cos \theta} + \frac{\cos^2 \theta}{\sin \theta - \cos \theta} \\
 &= \frac{\sin^2 \theta}{\cos \theta(\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta(\sin \theta - \cos \theta)}
 \end{aligned}$$

Taking $\frac{1}{(\sin \theta - \cos \theta)}$ as common

$$\begin{aligned}
 &= \frac{1}{(\sin \theta - \cos \theta)} \left[\frac{\sin^2 \theta}{\cos \theta} - \frac{\cos^2 \theta}{\sin \theta} \right] \\
 &= \frac{1}{(\sin \theta - \cos \theta)} \left[\frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta} \right]
 \end{aligned}$$

Using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$,

$$\begin{aligned}
 &= \frac{1}{(\sin \theta - \cos \theta)} \left[\frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta} \right] \\
 &= \frac{(1 + \sin \theta \cos \theta)}{(\sin \theta \cos \theta)} \quad (\text{By Identity } \sin^2 A + \cos^2 A = 1) \\
 &= 1 + \sec \theta \operatorname{cosec} \theta \\
 &= \text{R.H.S.}
 \end{aligned}$$

$$(iv) \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

$$\text{L.H.S} = \frac{1 + \sec A}{\sec A} \text{----- Equation (1)}$$

We know that the trigonometric functions,

$$\sec(x) = \frac{1}{\cos(x)}$$

By substituting the above function in Equation (1),

$$\begin{aligned}
 &= \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} \\
 &= \frac{\cos A + 1}{\frac{1}{\cos A}} \\
 &= \frac{\cos A}{\frac{1}{\cos A}} = (\cos A + 1)
 \end{aligned}$$

By taking $1 = 1 - \cos A$ in both denominator and numerator

$$= \frac{(1 - \cos A)(1 + \cos A)}{(1 - \cos A)}$$

By Identity $\sin^2 A + \cos^2 A = 1$

$$\begin{aligned}
 &= \frac{1 - \cos^2 A}{1 - \cos A} = \frac{\sin^2 A}{1 - \cos A} \\
 &= \text{R.H.S}
 \end{aligned}$$

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

Using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

$$\text{L.H.S} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

Dividing both numerator and denominator by $\sin A$

$$\begin{aligned} & \frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A} \\ &= \frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A} \end{aligned}$$

We know that the trigonometric functions,

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

We know that, $1 + \cot^2 A = \operatorname{Cosec}^2 A$

Hence substituting $1 = \cot^2 A - \operatorname{Cosec}^2 A$ in the equation below

$$\begin{aligned} &= \frac{\{(\cot A) - (1 - \operatorname{cosec} A)\} \{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\} \{(\cot A) - (1 - \operatorname{cosec} A)\}} \\ &= \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2} \\ &= \frac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)} \\ &= \frac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 + \operatorname{cosec}^2 A + 2\operatorname{cosec} A} \\ &= \frac{2\operatorname{cosec} A (\operatorname{cosec} A + \cot A) - 2(\cot A - \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A} \\ &= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{1 - 1 + 2\operatorname{cosec} A} \\ &= \frac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)} \end{aligned}$$

$$= \operatorname{cosec} A + \cot A$$

$$= \text{R.H.S}$$

$$(vi) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$\text{L.H.S} = \sqrt{\frac{1 + \sin A}{1 - \sin A}} \text{-----Equation (1)}$$

Multiplying and dividing by $\sqrt{(1 + \sin A)}$

$$\frac{\sqrt{(1+\sin A)(1+\sin A)}}{\sqrt{(1-\sin A)(1+\sin A)}}$$

Using $a^2 - b^2 = (a - b)(a + b)$,

$$\begin{aligned} &= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\sqrt{\cos^2 A}} \\ &= \frac{1+\sin A}{\cos A} = \sec A + \tan A \quad (\text{By separating the denominators}) \\ &= \text{R.H.S} \end{aligned}$$

$$(vii) \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos^3 \theta} = \tan \theta$$

$$\text{L.H.S} = \frac{\sin \theta - 2\sin^3 \theta}{2\cos^3 \theta - \cos \theta}$$

Taking $\sin \theta$ and $\cos \theta$ common in both numerator and denominator respectively.

$$= \frac{\sin \theta(1 - 2\sin^2 \theta)}{\cos \theta(2\cos^2 \theta - 1)}$$

By Identity $\sin^2 A + \cos^2 A = 1$ hence, $\cos^2 A = 1 - \sin^2 A$ and substituting this in the above equation,

$$= \frac{\sin \theta(1 - 2\sin^2 \theta)}{\cos \theta\{2(1 - \sin^2 \theta) - 1\}}$$

$$= \frac{\sin \theta(1 - 2\sin^2 \theta)}{\cos \theta(1 - 2\sin^2 \theta)}$$

$$= \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$(viii) (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$\text{L.H.S} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

By using $(a + b)^2 = a^2 + 2ab + b^2$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$

By rearranging and using $\sec A = 1/\cos A$

$$= (\sin^2 A + \cos^2 A) + (\operatorname{cosec}^2 A + \sec^2 A) + 2\sin A \left(\frac{1}{\sin A} \right) + 2\cos A \left(\frac{1}{\cos A} \right)$$

Hence $(\sin^2 A + \cos^2 A) = 1$ and $(\operatorname{cosec}^2 A + \sec^2 A) = 1$

$$\begin{aligned}
 &= (1) + (1 + \cot^2 A + 1 + \tan^2 A) + (2) + (2) \\
 &= 7 + \tan^2 A + \cot^2 A \\
 &= \text{R.H.S}
 \end{aligned}$$

$$(ix) (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$\text{L.H.S} = (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \text{-----Equation (1)}$$

We know that the trigonometric functions,

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

By substituting the above values in Equation (1)

$$= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right)$$

$$= \frac{(\cos^2 A)(\sin^2 A)}{\sin A \cos A}$$

$$= \sin A \cos A$$

$$\text{R.H.S} = \frac{1}{\tan A \cot A}$$

We know that the trigonometric functions,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

By substituting the above function in RHS

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A$$

By Identity $\sin^2 A + \cos^2 A = 1$

$$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A} = \sin A \cos A$$

Hence, L.H.S = R.H.S

$$(x) \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2$$

Taking LHS, $\left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right)$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{1}{\frac{\cos^2 A}{1}} \times \frac{1}{\sin^2 A}$$

$$= \frac{1}{\cos^2 A} \times \sin^2 A = \tan^2 A$$

Taking RHS, $\left(\frac{1 - \tan A}{1 - \cot A} \right)^2$

$$= \left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}} \right)^2$$

$$= \left(\frac{1 - \tan A}{\frac{\tan A - 1}{\tan A}} \right)^2$$

$$= (-\tan A)^2 = \tan^2 A$$

Hence, L.H.S = R.H.S.

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