#465242

How many tangents can a circle have?

Solution

Circle is the locus of points equidistant from a given point, which is the center of the circle and tangent is the line which intersect circle at one point only

As circle has infinite points So there will be infinite tangents can be drawn on these points which touches at only one point

#465245

Fill in the Blanks:

(i) A tangent to a circle intersects it in _________ point(s).

(ii) A line intersecting a circle in two points is called a ________.

(iii) A circle can have _______ parallel tangents at the most.

(iv) A common point of a tangent to a circle and the circle is called ______.

Solution

(i) One point

Circle is the locus of points equidistant from a given point, the center of the circle, and nd Tangent is the line which intersect circle at **one point** only

(ii) Secant

A line which intersect circle in two points is called a chord if this line passes through center then it is called **secant. (Points are A and B)**

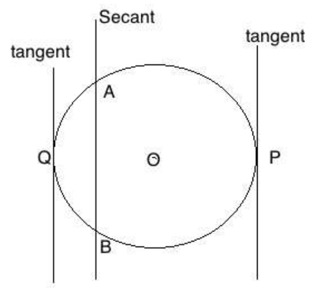
(iii) Two

As circle has infinite points, so there will be infinite tangents can be drawn on these points which touches at only one point. (P and Q)

So there will be infinite pairs of tangents which are parallel.

(iv) Point of Contact

The common point of a tangent to a circle and the circle is called point of **contact**. (P or Q in the given figure)



#465259

A tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q so that OQ$=12$ cm. Length PQ is:

A 12 cm

B 13 cm

C 8.5 cm

D $\sqrt{119}$ cm

Solution

$OP = 5$ cm and $OQ = 12$ cm

To find: PQ

By using Pythagoras Theorem, we have

$OP^2 + PQ^2 = OQ^2$

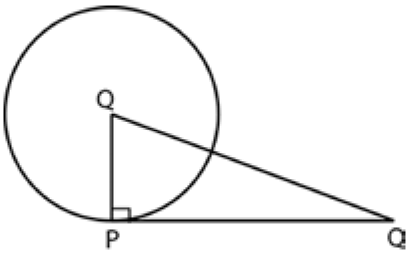
Putting given values in above equation

$5^2 + PQ^2 = 12^2$

$\implies PQ^2 = 144 - 25$

$\implies PQ^2 = 119$

$PQ = \sqrt{119}$ cm



#465264

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is.

A 7 cm

B 12 cm

C 15 cm

D 24.5 cm

Solution

According to question.

Given: $PQ = 24 \;\; OQ = 25$

By using Pythagoras Theorem, we have

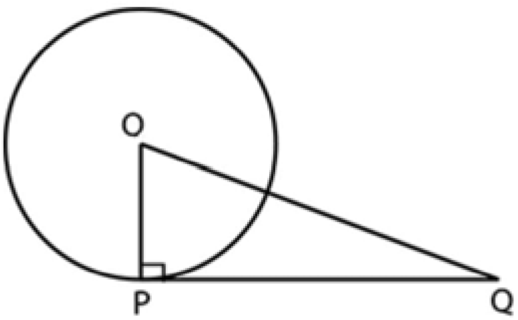
$OP^2 + PQ^2 = OQ^2$ (OP is the radius of circle)

$OP^2 + 24^2 = 25^2$

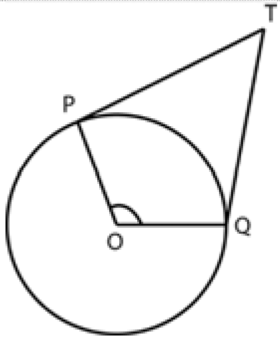
$\implies OP^2 = 625 - 576$

$\implies OP^2 = 49$

$OP = \sqrt{49} = 7$ cm



#465295



In Figure If TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^o$, the $\angle\ PTQ$ is equal to.

A 60^o

B 70^o

C 80^o

D 90^o

Solution

Join PQ in given figure then we have two triangles ΔOPQ and Δ PTQ,

In ΔOPQ ,$OP = OQ$ radius of triangle, and in Δ PTQ, $PT = TQ$ (Property of circle, two tangents drawn from a point to a circle are equal), so Both triangles are isosceles triangle

Now in ΔOPQ, by Angle sum property of a triangle

$\implies \angle OPQ + \angle OQP + \angle POQ = 180^o$

$\angle OPQ = \angle OQP = 35^\circ$

Since PT is tangent $\therefore \angle OPT = 90^\circ$

$\therefore \angle QPT = 90^\circ - 35^\circ = 55^\circ = \angle PQT$

Now Δ PTQ,

$\angle PTQ + \angle TPQ + \angle TQP = 180^o$

$\implies \angle PTQ = 180^\circ - 110^\circ = 70^\circ$

#465296

If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80^o, then $\angle POA$ is equal to.

A 50^o

B 60^o

C 70^o

D 80^o

Solution

Given: $\angle APB = 80^\circ$

As we know that sum of $\angle APB + \angle AOB = 180^\circ$(Using Property)

$\angle AOB = 100^\circ$

$\Delta\ AOP \simeq \Delta BOP$ by SSS congruency criteria.

$AO = BO$ (Radius of circle)

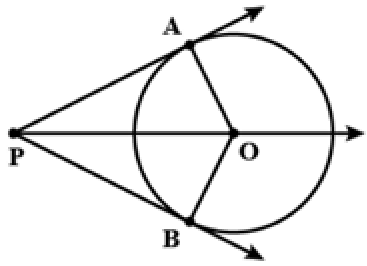
$PO = PO$ (common side of triangle)

$AP = BP$ (tangents form an external point)

$\angle AOP = \angle BOP$ (CPCT)

$\angle POA + \angle POB = \angle AOB = 100^\circ$

$\angle POA = 50^\circ$



#465297

The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Solution

Given: $OA = 5$cm and $AB = 4$cm

ΔAOB is right angle triangle, right angle at B

By using Pythagoras Theorem, we have

$OB^2 + AB^2 = OA^2$

OB is the radius of triangle

$OB^2 + 16 = 25$

$\implies OB^2 = 25 - 16$

$\implies OB^2 = 9$

$\implies OB = \sqrt{9} = 3cm$

Radius of circle is 3 cm

#465298

Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution

$OA = 3$ cm and $OP = 5$ cm

To find: length of PQ

By using Pythagoras Theorem, we have

$OA^2 + AP^2 = OP^2$

$\implies AP^2 = 5^2 - 3^2$

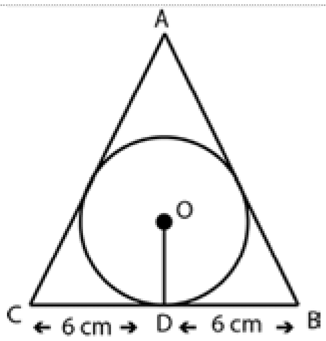
$\implies AP^2 = 25 - 9$

$AP = \sqrt{16} = 4$cm

As we know that perpendicular drawn from center to chord of circle bisects the chord

$\therefore PQ = 2AP = 2 \times 4 = 8$cm

#465313



A $\triangle ABC$ is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of the lengths 8 cm anc cm respectively. Find the sides AB and AC.

Solution

Let the circle touch the side AB and AC of the triangle at point E and F.

In $\triangle ABC$,

$CF = CD = 6$ cm ....Tangents from an external point to a circle are equal

$BE = BD = 8$cm ....Tangents from an external point to a circle are equal

$AE = AF = y$ ....Tangents from an external point to a circle are equal

Now, $AB = AE + EB$ ....$[\because A - E - B]$

$\therefore AB = (x+8)$cm

Similarly, $BC = BD + DC = 14$cm

and $CA = CF + FA = (6+x)$cm

Perimeter of $(\Delta ABC) = AB + BC + AC$

$= x + 8 + 14 + x + 6$

$= 2x + 28$

By Heron's formula,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \dfrac{2x+28}{2} = x + 14$

$A(\Delta ABC) = \sqrt{(x+14)[(x+14)-14][(x+14)-(6+x)][(x+14)-(8+x)]}$

$= \sqrt{(x+14)x \times 8 \times 6}$

$= 4\sqrt{3(x^2+14x)}$

Area of $\Delta OBC = \dfrac{1}{2} \times OD \times BC = \dfrac{1}{2} \times 4 \times 14 = 28$

Area of $\Delta OCA = \dfrac{1}{2} \times OF \times AC = \dfrac{1}{2} \times 4(x+6) = 2x + 12$

Area of $\Delta OAB = \dfrac{1}{2} \times OE \times AB = \dfrac{1}{2} \times 4(x+8) = 2x + 16$

$A(\Delta ABC) = A(\Delta OBC) + A(\Delta OCA) + A(\Delta OAB)$

$\therefore 4\sqrt{3(x^2+14x)} = 28 + 2x + 12 + 2x + 16$

$\Rightarrow \sqrt{3(x^2+14x)} = x + 14$

Squaring on both sides we get,

$\Rightarrow 3(x^2+14x) = (x+14)^2$

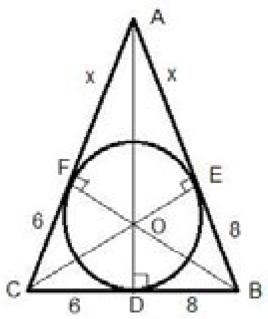
$\Rightarrow 3x = x + 14$

$\Rightarrow 2x = 14$

$\Rightarrow x = 7$

So, $AB = x + 8 = 15$cm.

and $AC = x + 6 = 13$cm.



#465368

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution

PQ and RS are tangents

$OA \perp PQ$

and, $OB \perp RS$.

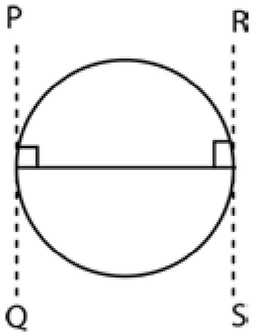
$\angle OAQ = \angle OAP = 90^o$

and, $\angle OBR = \angle OBC = 90^o$

As, $\angle RBO = \angle QAO = 90^o$[Alternate interior angles]

$\angle PAO = \angle SBO = 90^o$[Alternate interior angles]

Therefore, $PQ \parallel SR$



#465370

Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Solution

OP is the radius of circle.

$O'P$ is perpendicular to the tangent at the point of contact and it doesn't pass through the center

$\angle O'PT = 90^o$ ...(1)

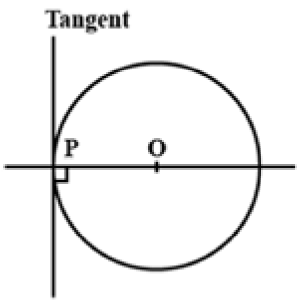
From the property, the radius of the circle is perpendicular to the tangent at the point of contact.

$\angle OPT = 90^o$ ...(2)

(1) and (2) are contradicting each other.

Both (1) and (2) can be true only if O' lies on point O.

Therefore, $O'P$ at the point of contact to the tangent passes through center.



#465372

A quadrilateral $ABCD$ is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$

Solution

$ABCD$ is a quadrilateral circumscribed in a circle

$DR = DS$ (tangents on the circle from same point D) .... (1)

$CR = CQ$(tangents on the circle from same point C) ...... (2)

$BP = BQ$ (tangents on the circle from same point B) .....(3)

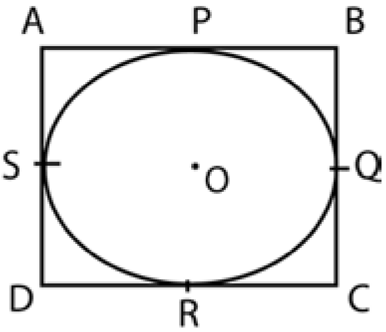
$AP = AS$ (tangents on the circle from same point A) ....(4)

Adding all these equations we get

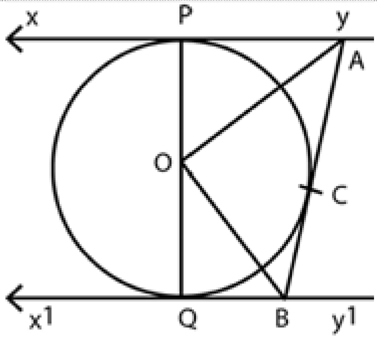
$DR + CR + BP + AP = DS + CQ + BQ + AS$

$(DR + CR) + (BP + AP) = (CQ + BQ) + (DS + AS)$

$CD + AB = AD + BC$



#465374



In Figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y''$ at B. Prove that $AOB = 90^o$.

Solution

Join O and C.

In $\triangle OPA$ and $\triangle OCA$

1) $OP = OC$ [Radius of the circle]

2) $AP = AC$ [Tangents from point A]

3) $AO = AO$ [Common side]

$\triangle OPA \cong \triangle COA$ [sss congruence criterior]

$\angle POA = \angle COA$ [By CPCT]

Similarly, $\triangle OQB \cong \triangle OCB$ [sss congruence criterior]

$\angle QOB = \angle OQB$ [By CPCT]

POQ is a diameter of the circle

$\angle POA + \angle COA + \angle QOB = 180^o$

$2\angle COA + 2\angle COB = 180^o$

$\Rightarrow \angle COA + \angle COB = 90^o$

$\angle AOB = 90^o$

#465377

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Solution

Draw a circle with center O and take a external point P. PA and PB are the tangents.

As radius of the circle is perpendicular to the tangent.

$OA \perp PA$

Similarly $OB \perp PB$

$\angle OBP = 90^o$

$\angle OAP = 90^o$

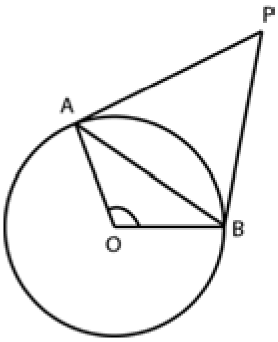
In Quadrilateral OAPB, sum of all interior angles $= 360^o$

$\Rightarrow \angle OAP + \angle OBP + \angle APB = 360^o$

$\Rightarrow 90^o + 90^o + \angle BOA + \angle APB = 360^o$

$\angle BOA + \angle APB = 180^o$

It proves the angle between the two tangents drawn from an external point to a circle supplementary to the angle subtented by the line segment



#465381

Prove that the parallelogram circumscribing a circle is a rhombus.

Solution

Since ABCD is a parallelogram circumscribed in a circle

$AB = CD........(1)$

$BC = AD........(2)$

$DR = DS$ (Tangents on the circle from same point D)

$CR = CQ$(Tangent on the circle from same point C)

$BP = BQ$ (Tangent on the circle from same point B)

$AP = AS$ (Tangents on the circle from same point A)

Adding all these equations we get

$DR + CR + BP + AP = DS + CQ + BQ + AS$

$(DR + CR) + (BP + AP) = (CQ + BQ) + (DS + AS)$

$CD + AB = AD + BC$

Putting the value of equation 1 and 2 in the above equation we get

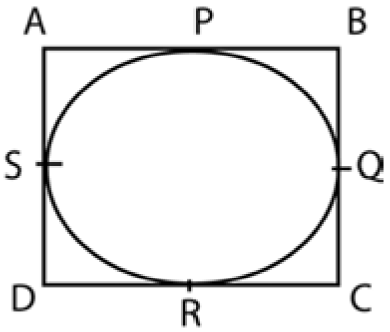
$2AB = 2BC$

$AB = BC...........(3)$

From equation $1, 2,$ and 3 we get

$AB = BC = CD = DA$

$\therefore ABCD$ is a Rhombus



#465382

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution

Let $ABCD$ be the quadrilateral circumscribing a circle at the center O such that it touches the circle at the point P, Q, R, S. Let join the vertices of the quadrilateral $ABCD$ t the center of the circle

In ΔOAP and ΔOAS

$AP = AS$ (Tangents from to same point A)

$PO = OS$ (Radii of the same circle)

$OA = OA$ (Common side)

so, $\Delta OAP = \Delta OAS$ (SSS congruence criterion)

$\therefore \angle POA = \angle AOS$ (CPCT)

$\angle 1 = \angle 8$

Similarly

$\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^0$

$\Longrightarrow (\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^0$

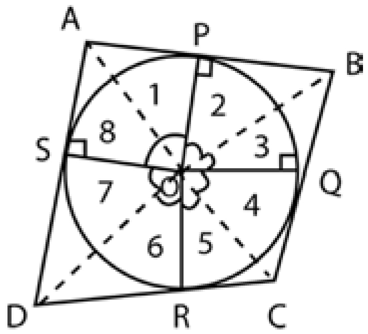
$\Longrightarrow 2(\angle 1) + 2(\angle 2) + 2(\angle 5) + 2(\angle 6) = 360^0$

$\Longrightarrow (\angle 1) + (\angle 2) + (\angle 5) + (\angle 6) = 180^0$

$\therefore \angle AOD + \angle COD = 180^0$

Similarly we can prove $\angle BOC + \angle DOA = 180^0$

Hence proved.



#465384

Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Solution