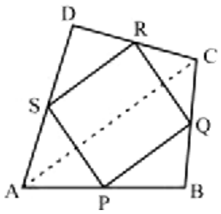
#463888

Topic: Theorems of Triangles



$ABCD$ is a quadrilateral in which P, Q, R and S are mid-points of the sides. AB, BC, CD and DA. AC is a diagonal. Show that:

(i) $SR \parallel AC$ and $SR = \dfrac{1}{2}AC$

(ii) $PQ = SR$

(iii) $PQRS$ is a parallelogram

Solution

(i) In $\triangle ACD$, we have S is the mid-point of AD and R is the mid- point of CD.

Then $SR \parallel AC$

Using Mid point theorem $SR = \dfrac{1}{2}AC$

(ii) In $\triangle ABC$,

P is the mid-point of the side AB and Q is the mid-point of the side BC.

Then, $PQ \parallel AC$

and using Mid point Theorem

$PQ = \dfrac{1}{2}AC$

Thus, we have proved that :

$PQ \parallel AC$ and $SR \parallel AC$

$\Rightarrow PQ \parallel SR$

Also $PQ = SR = \dfrac{1}{2}AC$

(iii) Since $PQ = SR$ and $PQ \parallel SR$

One pair of opposite sides are equal and parallel.

$\Rightarrow PQRS$ is a parallelogram.

#464971

Topic: Theorems of Triangles

In a right angled triangle ABC. $\angle B = 90^o$.

(i) If AB = 6 cm , BC= 8 cm, find AC.

(ii) If AC = 13 cm, BC= 5 cm. find AB.

Solution

i) In $\triangle ABC, \angle B = 90^o$

$\therefore$ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 6^2 + 8^2$

$\therefore AC^2 = 36 + 64 = 100$

$\therefore AC = 10$cm

ii) In $\triangle ABC, \angle B = 90^o$

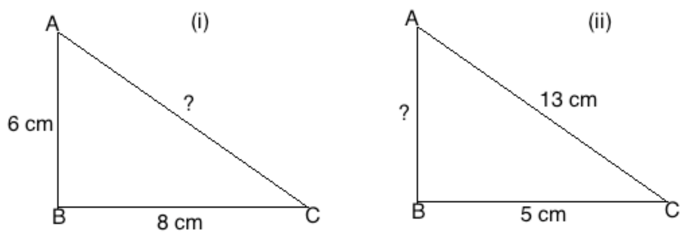
$\therefore$ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$13^2 = AB^2 + 5^2$

$\therefore AB^2 = 169 - 25 = 144$

$\therefore AB = 12$cm



#465414
Topic: Similar Triangles

Fill in the blanks using the correct word given in brackets :

(i) All circles are ________. (congruent, similar)

(ii) All squares are _________. (similar, congruent)

(iii) All ________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ________ and (b) their corresponding sides are _______. (equal, proportional)

Solution

Two figures that have the same shape are said to be similar.

When two figures are similar, the ratios of the lengths of their corresponding sides are equal.

(i) All circles are similar.

Since they have same shape.

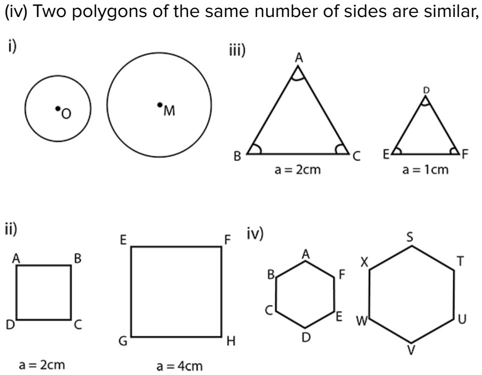
(ii) All square are similar.

Since the ratios of the lengths of their corresponding sides are equal.

(iii) All equilateral triangles are similar.

Since the ratios of the lengths of their corresponding sides are equal.

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are equal and (b) their corresponding sides are proportional.



#465415
Topic: Similar Triangles

Give two different examples of pair of:

(i) similar figures. (ii) non-similar figures.

Solution

(i) Similar figures :

1. Two equilateral triangles of sides 5 cm and 6 cm each.

2. Two circle of different diameter and centre.

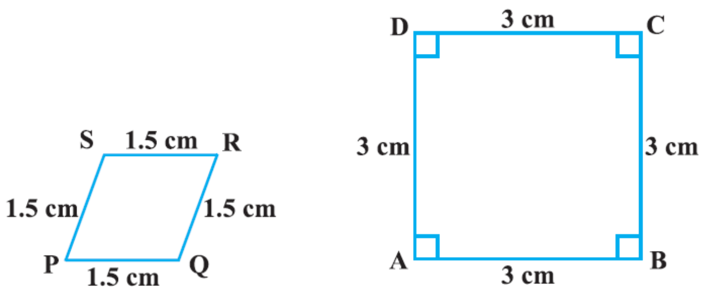
(ii) Non-similar figures :

1. A square and a triangle.

2. A circle and a quadrilateral.

This is one of the various possible solutions as this question might have several possible answers.

#465416
Topic: Similar Triangles



State whether the following quadrilaterals are similar or not:

Solution

From the given two figures,

$\angle SPQ$ is not equal to $\angle DAB$

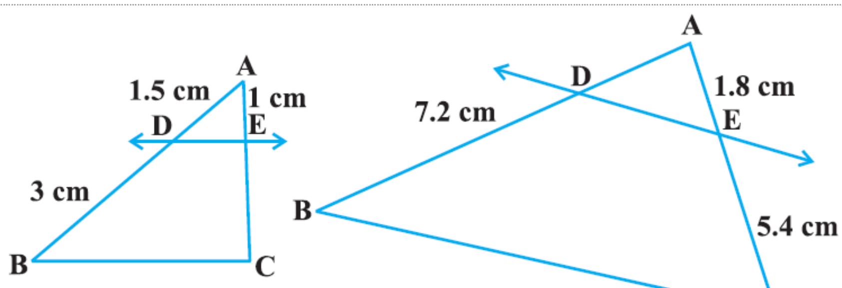
$\angle PQR$ is not equal to $\angle ABC$

$\angle QRS$ is not equal to $\angle BCD$

$\angle RSP$ is not equal to $\angle CDA$

Hence, the quadrilaterals are not similar.

#465417
Topic: Theorems of Triangles



(i) (ii)

In Fig., (i) and (ii), $DE \mid\mid BC$. Find EC in (i) and AD in (ii).

Solution

(i) Given : $DE \parallel BC$ in $\triangle$ ABC,

Using Basic proportionality theorem,

$\therefore \dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\Rightarrow \dfrac{1.5}{3} = \dfrac{1}{EC}$

$\Rightarrow EC = \dfrac{3}{1.5}$

$EC = 3 \times \dfrac{10}{15} = 2$ cm

$EC = 2$ cm.

(ii) In $\triangle ABC$, $DE \parallel BC$ (Given)

Using Basic proportionality theorem,

$\therefore \dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\Rightarrow \dfrac{AD}{7.2} = \dfrac{1.8}{5.4}$

$\Rightarrow AD = 1.8 \times \dfrac{7.2}{5.4} = \dfrac{18}{10} \times \dfrac{72}{10} \times \dfrac{10}{54} = \dfrac{24}{10}$

$\Rightarrow AD = 2.4$cm

So, $AD = 2.4$ cm

#465418

Topic: Theorems of Triangles

E and F are points on the sides PQ and PR respectively of a $\triangle PQR$. For each of the following cases, state whether $EF \mid\mid QR$:

(i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm

(ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm

(iii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.36$ cm

Solution

E and F are two points on side PQ and PR in $\triangle PQR$.

(i) PE = 3.9 cm, EQ = 3 cm and PF = 3.6 cm, FR = 2.4 cm

Using Basic proportionality theorem,

$\therefore \dfrac{PE}{EQ} = \dfrac{3.9}{3} = \dfrac{39}{30} = \dfrac{13}{10} = 1.3$

$\dfrac{PF}{FR} = \dfrac{3.6}{2.4} = \dfrac{36}{24} = \dfrac{3}{2} = 1.5$

$\dfrac{PE}{EQ} \neq \dfrac{PF}{FR}$

So, EF is not parallel to QR.

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

Using Basic proportionality theorem,

$\therefore \dfrac{PE}{QE} = \dfrac{4}{4.5} = \dfrac{40}{45} = \dfrac{8}{9}$

$\dfrac{PF}{RF} = \dfrac{8}{9}$

$\dfrac{PE}{QE} = \dfrac{PF}{RF}$

So, EF is parallel to QR.

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

Using Basic proportionality theorem,

$EQ = PQ - PE = 1.28 - 0.18 = 1.10$ cm

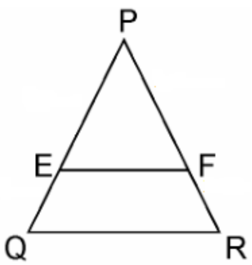
$FR = PR - PF = 2.56 - 0.36 = 2.20$ cm

$\dfrac{PE}{EQ} = \dfrac{0.18}{1.10} = \dfrac{18}{110} = \dfrac{9}{55}$... (i)

$\dfrac{PE}{FR} = \dfrac{0.36}{2.20} = \dfrac{36}{220} = \dfrac{9}{55}$ ... (ii)

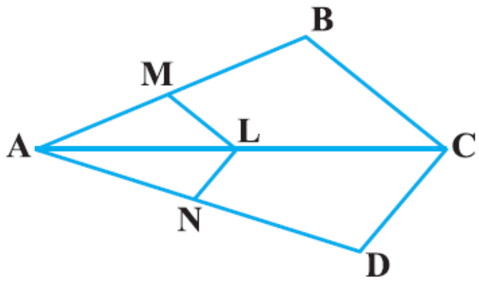
$\therefore \dfrac{PE}{EQ} = \dfrac{PF}{FR.}$

So, EF is parallel to QR.



#465419

Topic: Theorems of Triangles



In Fig., if $LM \mid\mid CB$ and $LN \mid\mid CD$, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.

Solution

In △ABC,

$LM \parallel BC$

∴ By proportionality theorem,

$\frac{AM}{AB} = \frac{AL}{AC}$............(1)

Similarly,

In △ADC,

$LN \parallel CD$

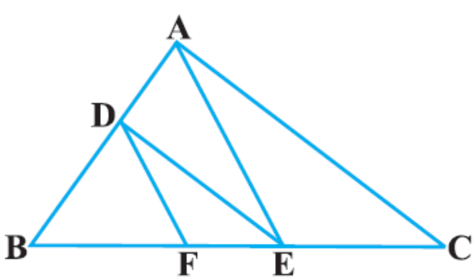
∴ By proportionality theorem,

$\frac{AN}{AD} = \frac{AL}{AC}$............(2)

∴ from (1) and (2),

$\frac{AM}{AB} = \frac{AN}{AD}$

#465420
Topic: Theorems of Triangles



In Fig., $DE \mid\mid AC$ and $DF \mid\mid AE$. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

Solution

In $\triangle ABC$,

$DE \parallel AC$

$\therefore$ By proportionality theorem,

$\frac{BD}{DA} = \frac{BE}{EC}$............(1)

Similarly,

In $\triangle ABE$,

$DF \parallel AE$

$\therefore$ By proportionality theorem,

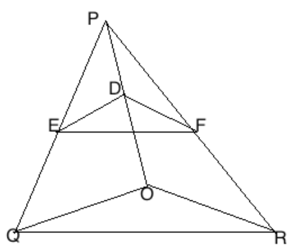
$\frac{BD}{DA} = \frac{BF}{FE}$............(2)

$\therefore$ from (1) and (2),

$\frac{BE}{EC} = \frac{BF}{FE}$.

#465421

Topic: Theorems of Triangles



In Fig., $DE \mid\mid OQ$ and $DF \mid\mid OR$. Show that $EF \mid\mid QR$.

Solution

In $\triangle POQ$,

$DE \parallel OQ$

$\therefore$ By basic proportionality theorem,

$\frac{PE}{EQ} = \frac{PD}{DO}$............(1)

Similarly,

In $\triangle POR$,

$DF \parallel OR$

$\therefore$ By basic proportionality theorem,

$\frac{PD}{DO} = \frac{PF}{FR}$............(2)

$\therefore$ from (1) and (2),

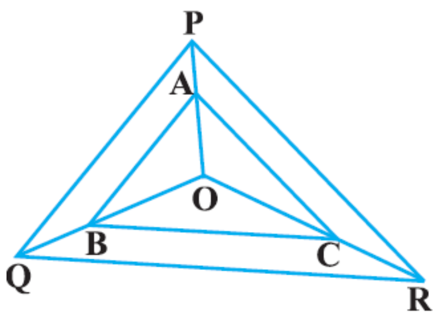
$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore$ By converse of Basic Proportionality Theorem,

$EF \parallel QR$

#465422

Topic: Theorems of Triangles



In fig., A, B and C are points on OP, OQ and OR respectively such that $AB \mid\mid PQ$ and $AC \mid\mid PR$. Show that $BC \mid\mid QR$.

Solution

In $\triangle POR$,

$PR \parallel AC$

$\therefore$ By basic proportionality theorem,

$\dfrac{PA}{AO} = \dfrac{RC}{CO}$............(1)

Similarly,

In $\triangle POQ$,

$AB \parallel PQ$

$\therefore$ By basic proportionality theorem,

$\dfrac{PA}{AO} = \dfrac{QB}{BO}$............(2)

$\therefore$ From (1) and (2),

$\dfrac{RC}{CO} = \dfrac{QB}{BO}$

$\therefore$ By converse of basic proportionality theorem,

$BC \parallel QR$

#465423

Topic: Theorems of Triangles

Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Theorem 6.1: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Solution

Given:

In $\triangle ABC$, D is midpoint of AB and DE is parallel to BC.

$\therefore\ AD = DB$

To prove:

$AE = EC$

Proof:

Since, $DE \parallel BC$

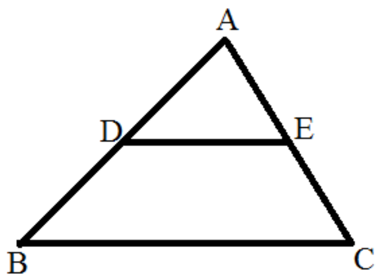
$\therefore$ By Basic Proportionality Theorem,

$\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Since, $AD = DB$

$\therefore\ \dfrac{AE}{EC} = 1$

$\therefore\ AE = EC$



#465424

Topic: Theorems of Triangles

Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Theorem 6.2: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Solution

Given: In $\triangle ABC$, D and E are midpoints of AB and AC respectively,

i.e., $AD = DB$ and $AE = EC$

To Prove: $DE \parallel BC$

Proof:

Since, $AD = DB$

$\therefore \dfrac{AD}{DB} = 1$............(1)

Also,

$AE = EC$

$\therefore \dfrac{AE}{EC} = 1$............(2)

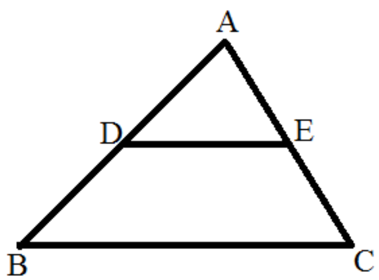
From (1) and (2),

$\dfrac{AD}{DB} = \dfrac{AE}{EC} = 1$

i.e.,$\dfrac{AD}{DB} = \dfrac{AE}{EC}$

$\therefore$ By converse of Basic Proportionality theorem,

$DE \parallel BC$



#465425

Topic: Theorems of Triangles

$ABCD$ is a trapezium in which $AB \mid\mid DC$ and its diagonals intersect each other at the point O. Show that $\dfrac{AO}{BO} = \dfrac{CO}{DO}$.

Solution

Given:

$ABCD$ is a trapezium and $AB \parallel DC$

To Prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction:

Draw $OE \parallel DC$ such that E lies on BC.

Proof:

In $\triangle BDC$,

By Basic Proportionality Theorem,

$\frac{BO}{OD} = \frac{BE}{EC} \ldots\ldots\ldots\ldots (1)$

Now, In $\triangle ABC$,

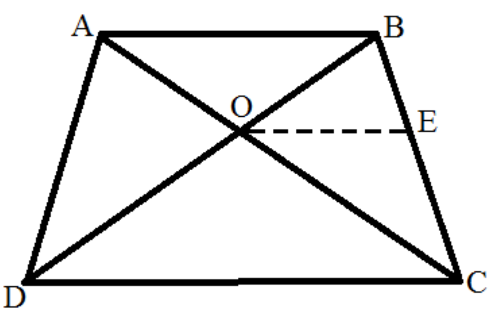
By Basic Proportionality Theorem,

$\frac{AO}{OC} = \frac{BE}{EC} \ldots\ldots\ldots\ldots (2)$

$\therefore$ From (1), and (2),

$\frac{AO}{OC} = \frac{BO}{OD}$

i.e., $\frac{AO}{BO} = \frac{CO}{DO}$



#465426

Topic: Theorems of Triangles

The diagonals of a quadrilateral $ABCD$ intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that $ABCD$ is a trapezium.

Solution

Given:

The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$

i.e.,

$\frac{AO}{CO} = \frac{BO}{DO}$

To Prove: $ABCD$ is a trapezium

Construction:

Draw $OE \parallel DC$ such that E lies on BC.

Proof:

In $\triangle BDC$,

By Basic Proportionality Theorem,

$\frac{BO}{OD} = \frac{BE}{EC}$............ (1)

But,

$\frac{AO}{CO} = \frac{BO}{DO}$ (Given) .... (2)

$\therefore$ From (1) and (2)

$\frac{AO}{CO} = \frac{BE}{EC}$

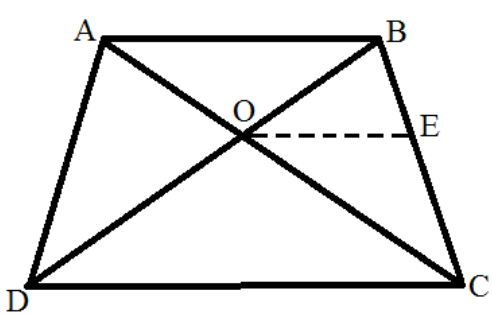
Hence, By Converse of Basic Proportionality Theorem,

$OE \parallel AB$

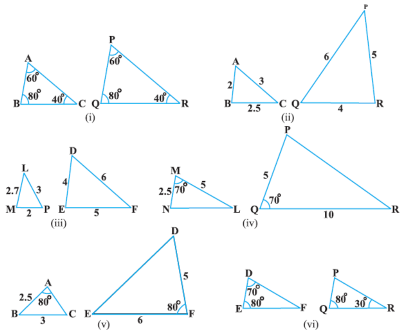
Now Since, $AB \parallel OE \parallel DC$

$\therefore AB \parallel DC$

Hence, $ABCD$ is a trapezium.



#465427
Topic: Theorems of Triangles



State which pairs of triangles in Fig. are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :

Solution

(i)

In $\triangle ABC$ and $\triangle PQR$

$\angle A = \angle P,$

$\angle B = \angle Q,$

$\angle C = \angle R,$

$\therefore$ By AAA criterion of similarity, $\triangle ABC \sim \triangle PQR$

(ii)

In $\triangle ABC$ and $\triangle QRP$

$\frac{AB}{QR} = \frac{BC}{RP} = \frac{AC}{QP} = \frac{1}{2}$

$\therefore$ By SSS criterion of similarity, $\triangle ABC \sim \triangle QRP$

(iii)

In $\triangle LMP$ and $\triangle DEF$

$\frac{LM}{DE} = \frac{2.7}{4}, \frac{LP}{DF} = \frac{1}{2}$

The sides are not in the equal ratios, Hence the two triangles are not similar.

(iv)

In $\triangle MNL$ and $\triangle QPR$

$\angle M = \angle Q,$

$\frac{MN}{QP} = \frac{ML}{QR} = \frac{1}{2}$

$\therefore$ By SAS criterion of similarity, $\triangle MNL \sim \triangle QPR$

(v)

In $\triangle ABC$ and $\triangle EFD$

$\angle A = \angle F,$

$\frac{AB}{FD} = \frac{BC}{FD} = \frac{1}{2}$

$\therefore$ By SAS criterion of similarity, $\triangle ABC \sim \triangle EFD$

(vi)

In $\triangle DEF$ and $\triangle PQR$

Since, sum of angles of a triangle is 180^o , Hence, $\angle F = 30^o$ and $\angle P = 70^o$

$\angle D = \angle P,$

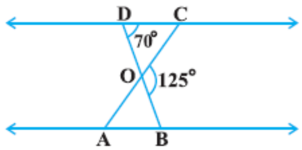
$\angle E = \angle Q,$

$\angle F = \angle R,$

$\therefore$ By AAA criterion of similarity, $\triangle DEF \sim \triangle PQR$

#465428

Topic: Similar Triangles



In Fig, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^o$ and $\angle CDO = 70^o$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.

Solution

Since, $\angle COD + \angle COB = 180^o$

$\therefore \ \angle COD = 180^o - 125^o = 55^o$

Since, $\angle COD + \angle ODC + \angle DCO = 180^o$

$\therefore \ \angle DCO = 180^o - 70^o - 55^o = 55^o$

Since, $\angle DCO = \angle OAB =$ Alternate angles

$\therefore \ \angle OAB = 55^o$

#465429

Topic: Theorems of Triangles

Diagonals AC and BD of a trapezium $ABCD$ with $AB \mid \mid DC$ intersect each other at the point O. Using a similarity criterion for two triangles, show that $\dfrac{OA}{OC} = \dfrac{OB}{OD}$.

Solution

Given:

ABCD is a trapezium with $AB \parallel DC$.

O is the point of intersection of two diagonals

To Prove:

$\dfrac{OA}{OC} = \dfrac{OB}{OD}$

Proof:

In $\triangle AOB$ and $\triangle DOC$

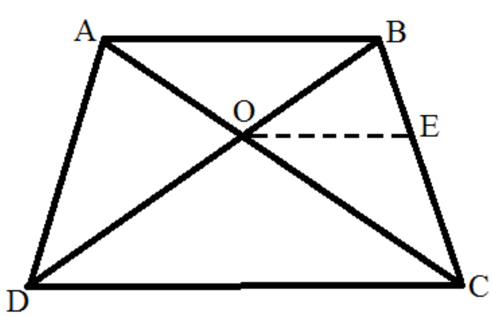
$\angle BAO = \angle OCD$ (Alternate Angles)

$\angle ABO = \angle ODC$ (Alternate Angles)

$\angle AOB = \angle DOC$ (Vertically opposite angles)

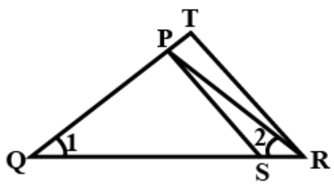
$\therefore$ By AAA criterion of similarity, $\triangle AOB \sim \triangle DOC$

$\therefore \ \dfrac{OA}{OC} = \dfrac{OB}{OD}$ (Corresponding Sides of Similar Triangles)



#465430

Topic: Theorems of Triangles



In Fig., $\dfrac{QR}{QS} = \dfrac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\triangle PQS \sim \triangle TQR$.

Solution

In $\triangle PQR$,

Since, $\angle 1 = \angle 2$

$\therefore$ $PR = PQ$ (Opposite sides of equal angles are equal) $.....(1)$

In $\triangle PQS$ and $\triangle TQR$,

$\dfrac{QR}{QS} = \dfrac{QT}{PR}....(Given)$

i.e., $\dfrac{QR}{QS} = \dfrac{QT}{PQ}....(From\ 1)$

Also, $\angle Q$ is common

$\therefore$ By SAS criterion of similarity, $\triangle PQS \sim \triangle TQR$.

#465431

Topic: Similar Triangles

S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$. Show that $\triangle RPQ \sim \triangle RTS$.

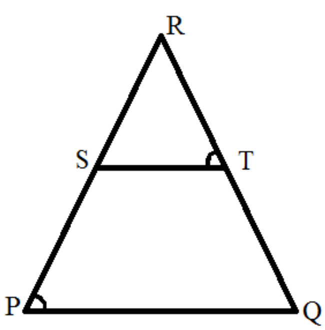
Solution

In $\triangle RPQ$ and $\triangle RTS$

$\angle R$ is common

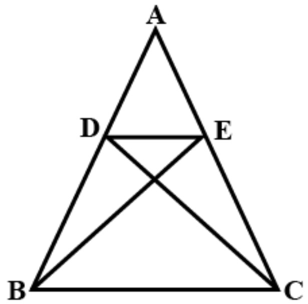
$\angle RTS = \angle P$ (Given)

Hence, By AA criterion of similarity, $\triangle RPQ \sim \triangle RTS$



#465432

Topic: Theorems of Triangles



In Fig., if $\triangle ABE \cong \triangle ACD$, show that $\triangle ADE \sim \triangle ABC$.

Solution

Since, $\triangle ABE \cong \triangle ACD$

$\therefore$ $AB = AC$.........(1)

Also, $AE = AD$.........(2)

From (1) and (2),

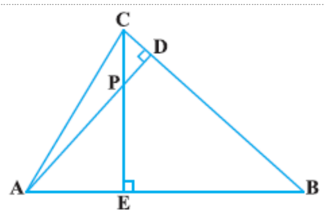
$\dfrac{AB}{AD} = \dfrac{AC}{AE}$

$\angle A$ is Common

$\therefore$ By SAS Criterian of Similarity, $\triangle ADE \sim \triangle ABC$

#465434

Topic: Theorems of Triangles



In Fig., altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that:

(i) $\triangle AEP \sim \triangle CDP$

(ii) $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$

(iv) $\triangle PDC \sim \triangle BEC$

Solution

In $\triangle$ AEP and $\triangle$ CDP,

$\angle APE = \angle CPD$ (Vertically opposite angle)

$\angle AEP = \angle CDP = 90^o$

$\therefore$ By AA criterion of similarity, $\triangle$ AEP $\sim$ $\triangle$ CDP

In $\triangle ABD$ and $\triangle CBE$

$\angle ADB = \angle CEB = 90^o$

$\angle B$ is common

$\therefore$ By AA criterion of similarity, $\triangle ABD$ $\sim$ $\triangle CBE$

In $\triangle AEP$ and $\triangle ADB$

$\angle AEP = \angle ADB = 90^o$

$\angle A$ is common

$\therefore$ By AA criterion of similarity, $\triangle AEP$ $\sim$ $\triangle ADB$

$\triangle PDC$ and $\triangle BEC$

$\angle PDC = \angle BEC = 90^o$

$\angle C$ is common

$\therefore$ By AA criterion of similarity, $\triangle PDC$ $\sim$ $\triangle BEC$

#465435

Topic: Theorems of Triangles

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$.

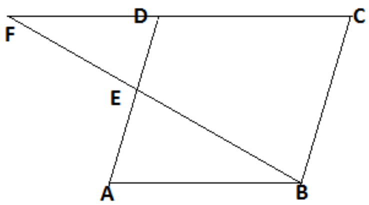
Solution

In $\triangle ABE$ and $\triangle CFB$,

$\angle ABE = \angle CFB$ (Alternate angles)

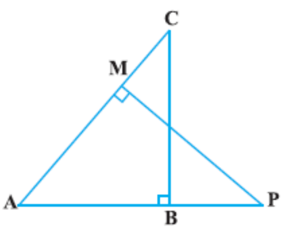
$\angle BAE = \angle BCF$ (opposite angles of a parallelogram)

$\therefore$ By AA criterion of similarity, $\triangle ABE \sim \triangle CFB$



#465436

Topic: Theorems of Triangles



In Fig., ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

(i) $\triangle ABC \sim \triangle AMP$

(ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$

Solution

In $\triangle ABC$ and $\triangle AMP$,

$\angle ABC = \angle AMP = 90^o$

$\angle A$ is common

$\therefore$ By AA criterion of similarity, $\triangle ABC \sim \triangle AMP$

$\therefore \dfrac{CA}{PA} = \dfrac{BC}{MP}$ (Corresponding Sides of Similar Triangles)

#465438

Topic: Similar Triangles

CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle EFG$ respectively. If $\triangle ABC \sim \triangle FEG$, show that:

(i) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$

(ii) $\triangle DCB \sim \triangle HGE$

(iii) $\triangle DCA \sim \triangle HGF$

Solution

In $\triangle ABC$ and $\triangle FEG$,

`$\triangle ABC \sim FEG$

$\therefore \angle ACB = \angle EGF$ (Corresponding angles of similar triangles)

Since, DC and GH are bisectors of $\angle ACB$ and $\angle EGH$ respectively.

$\therefore \angle ACB = 2\angle ACD = 2\angle BCD$

And $\angle EGF = 2\angle FGH = 2\angle HGE$

$\therefore \angle ACD = \angle FGH$ and $\angle DCB = \angle HGE$ .... (1)

Also $\angle A = \angle F$ and $\angle B = \angle E$ .... (2)

In $\triangle ACD$ and $\triangle FGH$,

$\angle A = \angle F$ (From 2)

$\angle ACD = \angle FGH$ (From 1)

$\therefore$ By AA criterion of similarity $\triangle ACD \sim \triangle FGH$

$\triangle DCA \sim \triangle HGF$ [(i) and (iii) proved]

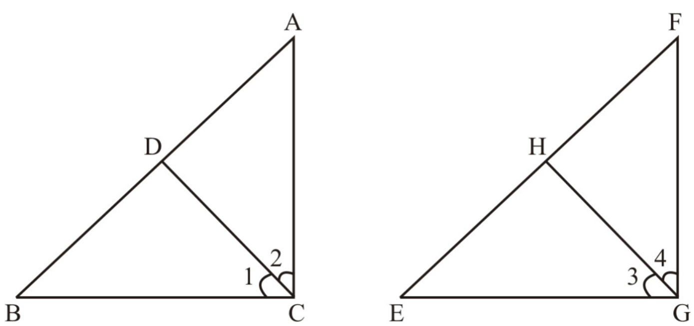
$\therefore \dfrac{CD}{GH} = \dfrac{AC}{FG}$ (Corresponding Sides of Similar Triangles)

In $\triangle DCB$ and $\triangle HGE$,

$\angle B = \angle E$ (From 2)

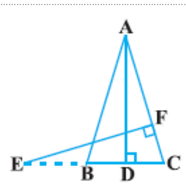
$\angle DCB = \angle HGE$ (From 1)

$\therefore$ By AA criterion of similarity $\triangle DCB \sim \triangle HGE$ [(ii) proved]



#465439

Topic: Theorems of Triangles



In Fig., E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.

Solution

Since, $AB = AC$

$\therefore \angle B = \angle C$........... (1)

In $\triangle ABD$ and $\triangle ECF$

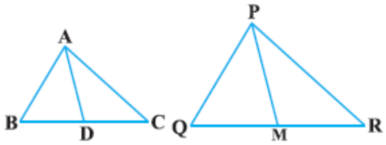
$\angle B = \angle C$ (From 1)

$\angle ADB = \angle EFC = 90^o$

$\therefore$ By AA Criterion of Similarity, $\triangle ABD \sim \triangle ECF$

#465440

Topic: Theorems of Triangles



Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see Fig.). Show that $\triangle ABC \sim \triangle PQR$.

Solution

Since AD and PM are medians of $\triangle ABC$ and $\triangle PQR$,

$\therefore\ BD = \frac{1}{2}BC$ and $QM = \frac{1}{2}QR$. . . . . . . (1)

Given that,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. . . . . (2)

$\therefore$ From (1) and (2),

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$. . . . . (3)

In $\triangle ABD$ and $\triangle PQM$

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\therefore$ By SSS criterian of proportionality $\triangle ABD \sim \triangle PQM$

$\therefore\ \angle B = \angle Q$ (Corresponding Sides of Similar Triangles) . . . . (4)

In $\triangle ABC$ and $\triangle PQR$

$\frac{AB}{PQ} = \frac{BC}{QR}$ (From 2)

$\angle B = \angle Q$ (From 4)

$\therefore$ By SAS criterian of proportionality $\triangle ABC \sim \triangle PQR$

#465441

Topic: Similar Triangles

D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.\ CD$.

Solution

In $\triangle ADC$ and $\triangle BAC$

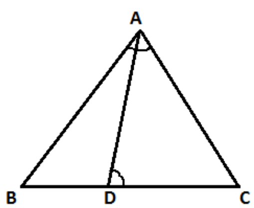
$\angle ADC = \angle BAC$ (Given)

$\angle C$ is Common

$\therefore$ by AA Criterion of Similarity, $\triangle ADC \sim \triangle BAC$

$\frac{AD}{BA} = \frac{DC}{AC} = \frac{AC}{BC}$

$\therefore\ CA^2 = CB.\ CD$



#465443

Topic: Similar Triangles

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$.

Solution

Since AD and PM are medians of $\triangle ABC$ and $\triangle PQR$,

$\therefore\ BD = \dfrac{1}{2}BC$ and $QM = \dfrac{1}{2}QR.\dots\dots.(1)$

Given that,

$\dfrac{AB}{PQ} = \dfrac{AC}{PR} = \dfrac{AD}{PM}$

Hence, $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AD}{PM}.\dots\dots..(2)$

$\therefore$ From (1) and (2),

$\dfrac{AB}{PQ} = \dfrac{BD}{QM} = \dfrac{AD}{PM}.\dots\dots\dots..(3)$

In $\triangle ABD$ and $\triangle PQM$

$\dfrac{AB}{PQ} = \dfrac{BD}{QM} = \dfrac{AD}{PM}$

$\therefore$ By SSS criterian of proportionality $\triangle ABD \sim \triangle PQM$

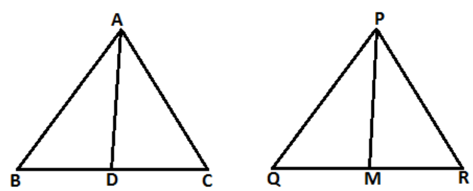
$\therefore\ \angle B = \angle Q$ (Corresponding Sides of Similar Triangles) $\dots\dots\dots\dots.(4)$

In $\triangle ABC$ and $\triangle PQR$

$\dfrac{AB}{PQ} = \dfrac{BC}{QR}$ (From 2)

$\angle B = \angle Q$ (From 4)

$\therefore$ By SAS criterian of proportionality $\triangle ABC \sim \triangle PQR$.



#465444

Topic: Similar Triangles

A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Solution

Let AB be the pole and BC be its shadow. At the same time let PQ be the tower and QR be its shadow.

i.e., $AB = 6\ m$, $BC = 4\ m$ and $QR = 28\ m$

Practically when sunlight falls on pole AB, then the shadow BC is created. The same is with the case of Tower PQ. But in this case, the angle of elevation of shadow with the sun will be the same in both the cases i.e.,

$\angle C = \angle R$.......... (1)

In $\triangle ABC$ and $\triangle PQR$

$\angle B = \angle Q = 90^o$

$\angle C = \angle R$

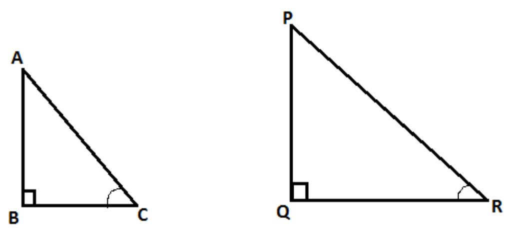
$\therefore$ By AA Criterion of Similarity $\triangle ABC \sim \triangle PQR$

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR}$

$\therefore \dfrac{6}{PQ} = \dfrac{4}{28}$

$\therefore PQ = 42\ m$

So, the height of tower is 42 m.



#465445

Topic: Similar Triangles

If AD and PM are medians of triangles ABC and PQR, respectively where $\triangle ABC \sim \triangle PQR$, prove that $\dfrac{AB}{PQ} = \dfrac{AD}{PM}$.

Solution

Since,

$\triangle ABC \sim \triangle PQR$

$\therefore \dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$

But, $BC = 2BD$ and $QR = 2QM$

Hence, $\dfrac{AB}{PQ} = \dfrac{BD}{QM} = \dfrac{AC}{PR}$

In $\triangle ABD$ and $\triangle PQM$,

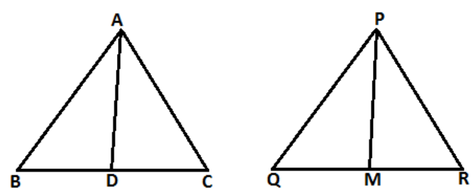
$\dfrac{AB}{PQ} = \dfrac{BD}{QM}$

and $\angle B = \angle Q$

By SAS similarity,

$\triangle ABD \sim \triangle PQM$,

$\therefore \dfrac{AB}{PQ} = \dfrac{AD}{PM}$



#465446
Topic: Similar Triangles

Let $\triangle ABC \sim \triangle DEF$ and their areas be, respectively, $64cm^2$ and $121cm^2$. If $EF = 15.4cm$, find BC.

Solution

Construction:

Draw AO perpendicular to BC and DP perpendicular to EF

Also,

$\triangle ABC \sim \triangle DEF$

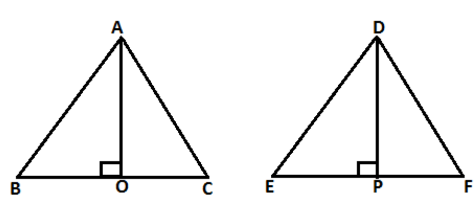
$\therefore \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF} = \dfrac{AO}{DP}$

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle DEF)} = \dfrac{\frac{1}{2} \times BC \times AO}{\frac{1}{2} \times EF \times DP}$

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle DEF)} = \dfrac{BC^2}{EF^2}$

$\therefore \dfrac{64}{121} = \dfrac{BC^2}{15.4^2}$

$\therefore BC = 11.2\ cm$



#465448

Topic: Similar Triangles

Diagonals of a trapezium $ABCD$ with $AB \mid\mid DC$, intersect each other at the point O. If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD.

Solution

Construction:

Draw OD perpendicular to DC and OP perpendicular to AB.

In $\triangle AOB$ and $\triangle DOC$

$\angle CDO = \angle OBA$ (Alternate Angles)

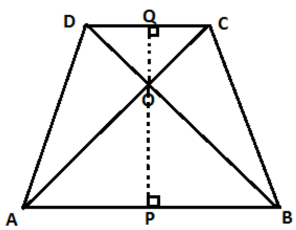
$\angle DCO = \angle OAB$ (Alternate Angles)

$\angle DOC = \angle AOB$ (Vertically opposite angles)

$\therefore$ By AAA Criterion of Similarity $\triangle AOB \sim \triangle DOC$

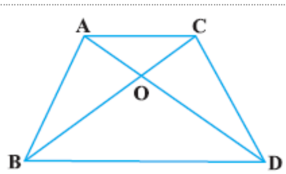
$\therefore \dfrac{AB}{DC} = \dfrac{QO}{PO}.....(1)$

$\therefore \dfrac{A(\triangle AOB)}{A(\triangle DOC)} = \dfrac{(2DC)^2}{(DC)^2} = \dfrac{4}{1}$



#465449

Topic: Similar Triangles



In Fig., ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\dfrac{ar(ABC)}{ar(DBC)} = \dfrac{AO}{DO}$.

Solution

Construction:

Draw AM perpendicular to BC and DN perpendicular to BC.

Now,

In $\triangle AMO$ and $\triangle DNO$

$\angle AOM = \angle DON$.......opp.angles

$\angle AMO = \angle DNO = 90^o$

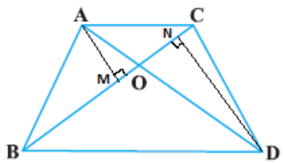
$\therefore$ By AA Criterion of Similarity, $\triangle AMO \sim \triangle DNO$

$\therefore \dfrac{AM}{DN} = \dfrac{AO}{DO}$ (Corresponding Sides of Similar Triangles) . . . . . . (1)

Now,

$$\frac{A(\triangle ABC)}{A(\triangle BDC)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN}$$

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle BDC)} = \dfrac{AO}{DO}$ (From 1)



#465450
Topic: Similar Triangles

If the areas of two similar triangles are equal, prove that they are congruent.

Solution

Given:

$A(\triangle ABC) = A(\triangle DEF)$

Also, $\triangle ABC \sim \triangle DEF$

To Prove:

$\triangle ABC \cong \triangle DEF$

Construction:

Draw AO Perpendicular to BC and DP Perpendicular to EF

Proof:

Since, $\triangle ABC \sim \triangle DEF$

$\therefore\ \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ (Corresponding Angles of Similar Triangles) $\ldots.(1)$

Also,

$\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$ (Corresponding Sides of Similar Triangles)

In $\triangle AOB$ and $\triangle DPE$

$\angle AOB = \angle DPB = 90^o$

$\angle B = \angle E$ (From 1)

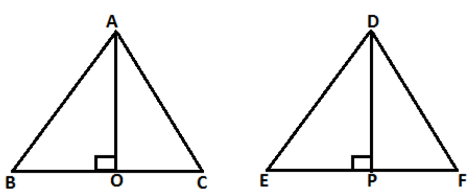
$\therefore$ By AA Criterion of Similarity, $\triangle AOB\ \sim \triangle DPE$

$\therefore\ \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF} = \dfrac{AO}{DP}\ \ldots.(2)$

$$\frac{A(\triangle ABC)}{A(\triangle DEF)} = \frac{\frac{1}{2} \times BC \times AO}{\frac{1}{2} \times EF \times DP}$$

$\therefore\ BC = EF, AB = DE, AC = DF, AO = DP$

$\therefore\ \triangle ABC \cong \triangle DEF$



#465451

Topic: Similar Triangles

D, E and F are respectively the mid-points of sides AB, BC and CA of $\triangle ABC$. Find the ratio of the areas of $\triangle DEF$ and $\triangle ABC$.

Solution

In $\triangle ABC$ D, F and F are the midpoints of sides AB, BC and CA respectively.

$\therefore$ $FE \parallel AB$, $ED \parallel AC$, $FD \parallel BC$

$\therefore$ $\square AFED$, $\square FDBE$, $\square FDEC$ are parallelograms.

In $\triangle ABC$ and $\triangle DEF$

$\angle A = \angle DEF$

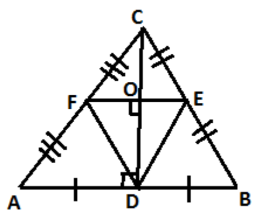
$\angle B = \angle DFE$

$\therefore$ By AA Criterion of Similarity $\triangle ABC \sim \triangle EDF$

$\therefore \dfrac{AB}{FE} = \dfrac{FD}{CB} = \dfrac{DE}{AC} = \dfrac{DO}{DC}.....(1)$

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle DEF)} = \dfrac{\frac{1}{2} \times AB \times DC}{\frac{1}{2} \times EF \times OD}$

$\therefore \dfrac{A(\triangle ABC)}{A(\triangle DEF)} = \dfrac{4}{1}$ (From 1)



#465452

Topic: Similar Triangles

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution

Given:

$\triangle ABC \sim \triangle DEF$

O is a median of BC and P is a median of EF

To Prove:

$\dfrac{A(\triangle ABC)}{A(\triangle DEF)} = \dfrac{(AO)^2}{(DP)^2}$

Proof:

Since, $\triangle ABC \sim \triangle DEF$

$\therefore \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ (Corresponding Angles of Similar Triangles) . . . . (1)

Also,

$\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$ (Corresponding Sides of Similar Triangles) . . . . . . (2)

Since, $BC = 2BO$ and $EF = 2EP$

$\therefore$ Equation (2) can be written as,

$\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF} = \dfrac{BO}{EP}$ . . . . . . (3)

In $\triangle AOB$ and $\triangle DPE$

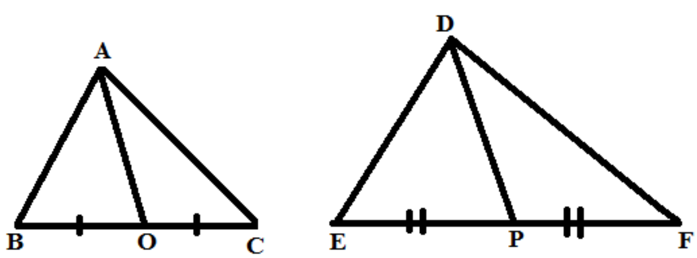
$\angle B = \angle E$ (From 1)

$\dfrac{AB}{DE} = \dfrac{BO}{EP}$ (From 3)

$\therefore$ By SAS Criterion of Similarity, $\triangle AOB \sim \triangle DPE$

$\therefore \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF} = \dfrac{AO}{DP}$ = Ratio of their heights . . . . (4) (Corresponding Sides of Similar Triangles)

$$\frac{A(\triangle ABC)}{A(\triangle DEF)} = \frac{\frac{1}{2} \times BC \times Height}{\frac{1}{2} \times EF \times Height} = \frac{(AO)^2}{(DP)^2}$$



#465453

Topic: Similar Triangles

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Solution

Given:

$ABCD$ is a Square,

DB is a diagonal of square,

$\triangle DEB$ and $\triangle CBF$ are Equilateral Triangles.

To Prove:

$\frac{A(\triangle CBF)}{A(\triangle DEB)} = \frac{1}{2}$

Proof:

Since, $\triangle DEB$ and $\triangle CBF$ are Equilateral Triangles.

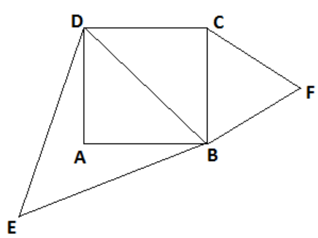
$\therefore$ Their corresponding sides are in equal ratios.

In a Square ABCD, $DB = BC\sqrt{2}$ . . . . . (1)

$$\therefore \frac{A(\triangle CBF)}{A(\triangle DEB)} = \frac{\frac{\sqrt{3}}{4} \times (BC)^2}{\frac{\sqrt{3}}{4} \times (DB)^2}$$

$$\therefore \frac{A(\triangle CBF)}{A(\triangle DEB)} = \frac{\frac{\sqrt{3}}{4} \times (BC)^2}{\frac{\sqrt{3}}{4} \times (BC\sqrt{2})^2} \text{ (From 1)}$$

$$\therefore \frac{A(\triangle CBF)}{A(\triangle DEB)} = \frac{1}{2}$$



#465454

Topic: Similar Triangles

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is:

A $2:1$

B $1:2$

C $4:1$

D $1:4$

Solution

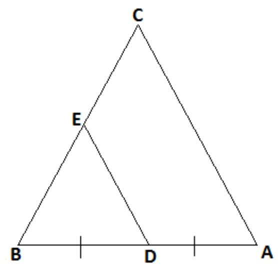
In $\triangle ABC$,

$AB = 2BD \dots . (1)$

$$\therefore \frac{A(\triangle ABC)}{A(\triangle BDE)} = \frac{\frac{\sqrt{3}}{4} \times (AB)^2}{\frac{\sqrt{3}}{4} \times (BD)^2} \dots . (2)$$

From (1) and (2),

$$\frac{A(\triangle ABC)}{A(\triangle BDE)} = \frac{4}{1}.$$



#465455

Topic: Similar Triangles

Sides of two similar triangles are in the ratio $4:9$. Areas of these triangles are in the ratio:

A $2:3$

B $4:9$

C $81:16$

D $16:81$

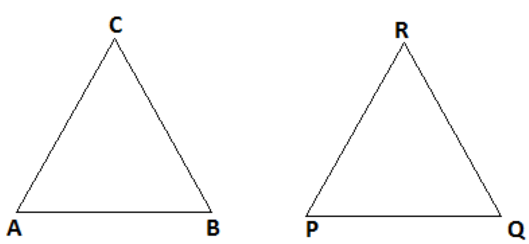
Solution

Since,

$\triangle ABC \sim \triangle PQR$

Also, The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\therefore \frac{A(\triangle ABC)}{A(\triangle PQR} = \frac{4^2}{9^2} = \frac{16}{81}.$$



#465456

Topic: Theorems of Triangles

Sides of triangles are given below. Determine which of them are right triangles.In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Solution

(i)

Since, $(25)^2 = (7)^2 + (24)^2$

Hence, $7cm$, $24cm$ and $25cm$ are the sides of Right Angled Triangle and its Hypotenuse is $25cm$

(ii)

Since, $(8)^2 \neq (3)^2 + (6)^2$

Hence, $8cm$, $6cm$ and $3cm$ do not form Right Angled Triangle.

(iii)

Since, $(100)^2 \neq (50)^2 + (80)^2$

Hence, $100cm$, $50cm$ and $80cm$ do not form Right Angled Triangle.

(iv)

Since, $(13)^2 = (12)^2 + (5)^2$

Hence, $13cm$, $12cm$ and $5cm$ are the sides of Right Angled Triangle and its Hypotenuse is $13cm$.

#465458

Topic: Similar Triangles

PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that $PM^2 = QM.MR$.

Solution

In $\triangle PMR$,

By Pythagoras theorem,

$(PR)^2 = (PM)^2 + (RM)^2 ....(1)$

In $\triangle PMQ$,

By Pythagoras theorem,

$(PQ)^2 = (PM)^2 + (MQ)^2 ....(2)$

In $\triangle PQR$,

By Pythagoras theorem,

$(RQ)^2 = (RP)^2 + (PQ)^2 ....(3)$

$\therefore\ (RM + MQ)^2 = (RP)^2 + (PQ)^2$

$\therefore\ (RM)^2 + (MQ)^2 + 2RM.MQ = (RP)^2 + (PQ)^2 ....(4)$

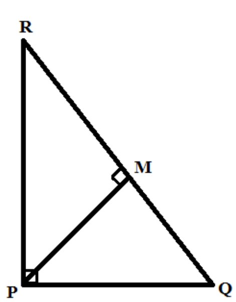
Adding 1) and 2) we get,

$(PR)^2 + (PQ)^2 = 2(PM)^2 + (RM)^2 + (MQ)^2 ...(5)$

From 4) and 5) we get,

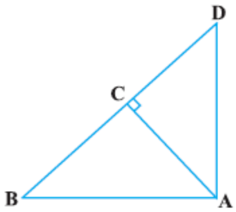
$2RM.MQ = 2(PM)^2$

$\therefore\ (PM)^2 = RM.MQ$



#465459

Topic: Similar Triangles



In Fig., ABD is a triangle right angled at A and AC $\perp$ BD. Show that:

(i) $AB^2 = BC.\ BD$

(ii) $AC^2 = BC.\ DC$

(iii) $AD^2 = BD.\ CD$

Solution

(i) In $\triangle BCA$ and $\triangle BAD$,

$\angle BCA = \angle BAD$ ....Each 90^o

$\angle B$ is common between the two triangles.

So, $\triangle BCA \sim \triangle BAD$ ...AA test of similarity ....(I)

Hence, $\dfrac{BC}{AB} = \dfrac{AC}{AD} = \dfrac{AB}{BD}$ ...C.S.S.T

And, $\angle BAC = \angle BDA$ ....C.A.S.T ....(II)

So, $\dfrac{BC}{AB} = \dfrac{AB}{BD}$

$\therefore AB^2 = BC \times BD$

Hence proved.

(ii) In $\triangle BCA$ and $\triangle DCA$,

$\angle BCA = \angle DCA$ ....Each 90^o

$\angle BAC = \angle CDA$ ...From (II)

So, $\triangle BCA \sim \triangle ACD$ ...AA test of similarity ....(III)

Hence, $\dfrac{BC}{AC} = \dfrac{AC}{CD} = \dfrac{AB}{AD}$ ...C.S.S.T

So, $\dfrac{BC}{AC} = \dfrac{AC}{CD}$

$\therefore AC^2 = BC \times DC$

Hence proved.

(iii) From (I) and (III), we get

$\triangle BAD \sim \triangle ACD$

Hence, $\dfrac{AB}{AC} = \dfrac{AD}{CD} = \dfrac{BD}{AD}$

So, $AD^2 = BD \times CD$

Hence proved.

In $\triangle DCA$,

By Pythagoras theorem,

$(DA)^2 = (CA)^2 + (CD)^2 \ldots\ldots.(1)$

In $\triangle ABD$,

By Pythagoras theorem,

$(BD)^2 = (AB)^2 + (AD)^2 \ldots\ldots..(2)$

In $\triangle ABC$,

(i)

By Pythagoras theorem,

$(AB)^2 = (AC)^2 + (BC)^2 \ldots\ldots.(3)$

$\therefore\ (AB)^2 = (AD)^2 - (CD)^2 + (BC)^2$ (From 1 and 3)

$\therefore\ (AB)^2 = (BD)^2 - (AB)^2 - (CD)^2 + (BC)^2$

$\therefore\ 2(AB)^2 = (BD)^2 - (BD - BC)^2 + (BC)^2$

$\therefore\ (AB)^2 = BC.\,BD$

(ii)

Adding (1) and (3),

$\therefore\ (AB)^2 + (AD)^2 = 2(AC)^2 + (CD)^2 + (BC)^2$

$\therefore\ (BD)^2 - (AD)^2 + (AD)^2 = 2(AC)^2 + (CD)^2 + (BC)^2$ (From 2)

$\therefore\ (BC + CD)^2 = 2(AC)^2 + (CD)^2 + (BC)^2$

Hence, Solving the above equation we get,

$(AC)^2 = BC.\,DC$

(iII)

Subtracting (1) and (2) we get,

$2(AD)^2 - (BD)^2 = (CA)^2 + (CD)^2 - (AB)^2$

$\therefore\ 2(AD)^2 - (BD)^2 = (CA)^2 + (CD)^2 - (AC)^2 - (BC)^2$

$\therefore\ 2(AD)^2 - (BD)^2 = (BD - BC)^2 - (BC)^2$

Hence, Solving the above equation we get,

$(AD)^2 = BD.\,DC$

#465460

Topic: Theorems of Triangles

ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Solution

In $\triangle ABC$,

By Pythagoras Theorem,

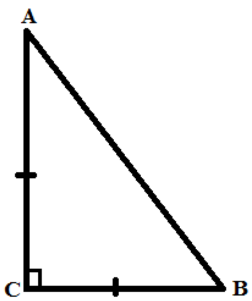
$(AB)^2 = (AC)^2 + (BC)^2 \ldots\ldots (1)$

Since, $\triangle ABC$ is an isosceles triangle,

$\therefore$ $AC = BC \ldots\ldots (2)$

$\therefore$ From (1) and (2),

$(AB)^2 = 2(AC)^2$



#465461

Topic: Theorems of Triangles

ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Solution

Since, $\triangle ABC$ is an isosceles triangle,

$\therefore$ $AC = BC \ldots\ldots (1)$

Also, given that,

$(AB)^2 = 2(AC)^2$

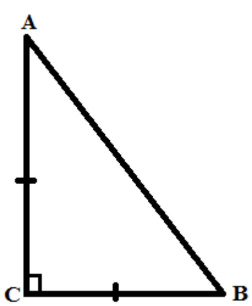
$\therefore$ $(AB)^2 = (AC)^2 + (AC)^2 \ldots\ldots (2)$

$\therefore$ From (1) and (2),

$(AB)^2 = (AC)^2 + (BC)^2$

Hence, By converse of Pythagoras theorem,

$\triangle ABC$ is an isosceles right angles triangle.



#465462

Topic: Theorems of Triangles

ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Solution

Since ABC is an Equilateral Triangle,

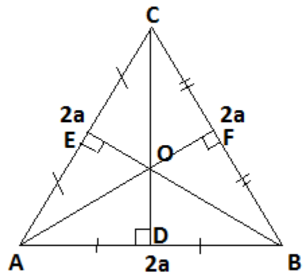
Hence,

$AB = BC = AC = 2a$

and

$CD = BE = AF =$ Altitudes

$\therefore\ CD = BE = AF = \frac{\sqrt{3}}{2} \times 2a = \sqrt{3}a$



#465463

Topic: Theorems of Triangles

Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Solution

Given:

$ABCD$ is a rhombus.

Hence, $AB = BC = CD = AD$

And, AC perpendicular to BD

$DO = \frac{1}{2}DB$ and $AO = \frac{1}{2}AC$

To Prove:

$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

Proof:

In $\triangle AOD$, By Pythagoras Theorem,

$AD^2 = AO^2 + OD^2 \ldots\ldots (1)$

Similarly,

$DC^2 = DO^2 + OC^2 \ldots\ldots (2)$

$BC^2 = OB^2 + OC^2 \ldots\ldots (3)$

$AB^2 = AO^2 + OB^2 \ldots\ldots (4)$

Adding 1, 2, 3, 4 we get,

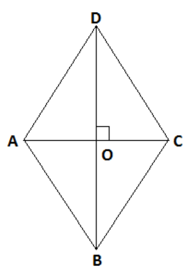
$AB^2 + BC^2 + CD^2 + AD^2 = 2AO^2 + 2BO^2 + 2CO^2 + 2DO^2 \ldots . (5)$

Since, $DO = OB = \frac{1}{2}DB$ and $AO = OC = \frac{1}{2}AC \ldots . (6)$

From 5 and 6,

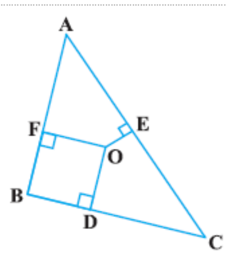
$AB^2 + BC^2 + CD^2 + AD^2 = \frac{AC^2}{2} + \frac{BD^2}{2} + \frac{AC^2}{2} + \frac{BD^2}{2}$

$\therefore\ AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$



#465464

Topic: Theorems of Triangles



In Fig. , O is a point in the interior of a triangle ABC, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that:

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$,

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.

Solution

Construction:

Join AO, BO and OC.

(i)

In $\triangle AOE$

By Pythagoras Theorem,

$AO^2 = AE^2 + OE^2 \ldots\ldots (1)$

In $\triangle AOF$

By Pythagoras Theorem,

$AO^2 = AF^2 + FO^2 \ldots\ldots (2)$

In $\triangle FBO$

By Pythagoras Theorem,

$BO^2 = BF^2 + FO^2 \ldots\ldots (3)$

In $\triangle BDO$

By Pythagoras Theorem,

$BO^2 = BD^2 + OD^2 \ldots\ldots (4)$

In $\triangle DOC$

By Pythagoras Theorem,

$OC^2 = OD^2 + DC^2 \ldots\ldots (5)$

In $\triangle OCE$

By Pythagoras Theorem,

$OC^2 = OE^2 + EC^2 \ldots\ldots (6)$

In $\triangle ABC$

By Pythagoras Theorem,

$AC^2 = AB^2 + BC^2 \ldots\ldots (7)$

Adding 2), 4) and 6) we get,

$AO^2 + BO^2 + OC^2 = AF^2 + FO^2 + BD^2 + OD^2 + OE^2 + EC^2$

$\therefore\ AO^2 + BO^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$

(ii)

Subtraction 2) and 1) we get,

$0 = AF^2 + FO^2 - AE^2 - OE^2$

$\therefore\ AF^2 + FO^2 = AE^2 + OE^2 \ldots\ldots\ldots (8)$

Subtracting 4) and 3) we get,

$0 = BD^2 + OD^2 - BF^2 - FO^2$

$\therefore\ BD^2 + OD^2 = BF^2 + FO^2 \ldots\ldots\ldots (9)$

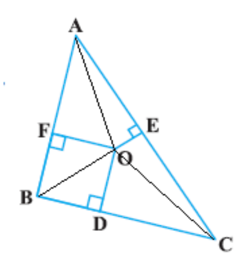
Subtracting 6) and 5) we get,

$0 = OE^2 + EC^2 - OD^2 - DC^2$

$\therefore\ OE^2 + EC^2 = OD^2 + DC^2 \ldots\ldots\ldots\ldots (10)$

Adding 8), 9) and 10) we get,

$AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.



#465465

Topic: Theorems of Triangles

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Solution

In Right angled triangle ABC,

AC is a ladder and $AC = 10m$

Point A is where window is.

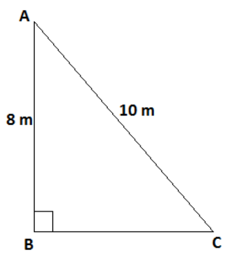
$AB = 8m$

In $\triangle ABC$, Applying Pythagoras Theorem,

$BC^2 = 10^2 - 8^2$

$\therefore\ BC = 6\ m$

Hence, the distance of the foot of the ladder from base of the wall is $6m$.



#465466

Topic: Theorems of Triangles

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Solution

Let $AC = 18m$ be the pole.

$BC = 24m$ is the length of a guy wire and is attached to stake B.

$\therefore$ In $\triangle ABC$

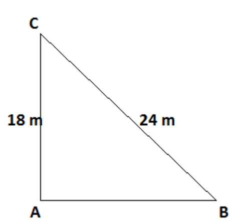
By pythagoras theorem,

$BC^2 = AB^2 + AC^2$

$\therefore\ 24^2 = AB^2 + 18^2$

$\therefore\ AB = 6\sqrt{7}\ m$

Hence, the stake has to be $6\sqrt{7}\ m$ from base A.



#465467

Topic: Theorems of Triangles

An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Solution

The first aeroplane leaves an airport and flies due north at a speed of 1000 km per hr.

$\therefore$ Distance travelled in 1.5 hrs.

$= 1000 \times 1.5 = 1500$ km

Similarly, Distance traveled by second aeroplane,

$= 1200 \times 1.5 = 1800$ km

In $\triangle ABC$,

BC is distance travelled by first aeroplane and BA is the distance traveled by second aeroplane.

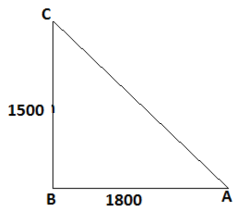
Hence, applying Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

$\therefore AC^2 = 1500^2 + 1800^2$

$\therefore AC = 300\sqrt{61}$ km

Hence, two planes are $300\sqrt{61}$ km apart.



#465468

Topic: Theorems of Triangles

Two poles of heights 6 m and 11 m stand on aplane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Solution

Let, $AE = 11m$ and $BC = 6m$ are the two poles.

E and C are their tops.

Construction:

Join EC

Since, $BC = AD = 6\ m$

and $AB = DC = 12\ m$

$\therefore ED = 5\ m$

In $\triangle EDC$

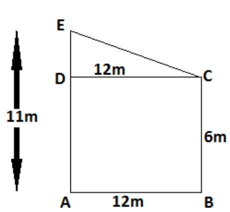
By Pythagoras Theorem,

$EC^2 = ED^2 + DC^2$

$\therefore EC^2 = 169$

$\therefore EC = 13\ m$

Hence, Distance between their tops $= 13\ m$



#465469

Topic: Theorems of Triangles

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Solution

Given:

$\triangle ACB$ is a right angles triangle at C.

Construction:

Join AE, BD and ED.

To Prove:

$AE^2 + BD^2 = AB^2 + DE^2$

Proof:

In $\triangle ACE$

By pythagoras theorem,

$AE^2 = EC^2 + AC^2 \ldots\ldots (1)$

In $\triangle BCD$

By pythagoras theorem,

$BD^2 = BC^2 + CD^2 \ldots\ldots (2)$

In $\triangle ECD$

By pythagoras theorem,

$ED^2 = EC^2 + DC^2 \ldots\ldots (3)$

In $\triangle ABC$

By pythagoras theorem,

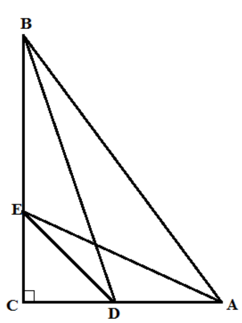
$AB^2 = BC^2 + AC^2 \ldots\ldots (4)$

Adding 1) and 2) we get,

$AE^2 + BD^2 = EC^2 + AC^2 + BC^2 + CD^2 \ldots\ldots (5)$

From 3), 4) and 5) we get,

$AE^2 + BD^2 = AB^2 + DE^2$



#465471

Topic: Theorems of Triangles

In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Solution

$\triangle ABC$ is an Equilateral triangle such that,

$AB = BC = AC$

Construction:

Draw an Altitude AE such that,

E lies on BC and

AE perpendicular to BC

In $\triangle AEB$

By Pythagoras Theorem,

$AB^2 = AE^2 + EB^2 \ldots\ldots (1)$

In $\triangle AED$

By Pythagoras Theorem,

$AD^2 = AE^2 + ED^2 \ldots\ldots (2)$

From 1) and 2)

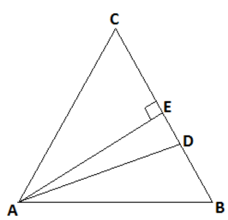
$AD^2 = ED^2 + AB^2 - EB^2 \ldots\ldots (3)$

Since, $EB = \dfrac{1}{2}BC$ and

$ED = \dfrac{BC}{6}$

$\therefore\ AD^2 = \dfrac{AB^2}{36} + AB^2 - \dfrac{9AB^2}{36}$

$\therefore\ 9AD^2 = 7AB^2$



#465472

Topic: Theorems of Triangles

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Solution

Given:

$\triangle ABC$ is an Equilateral Triangle.

$AB = BC = AC$

CD Perpendicular to AB

To Prove:

$3AC^2 = 4CD^2$

Proof:

In $\triangle ADC$

By Pythagoras Theorem,

$AC^2 = AD^2 + DC^2 \ldots\ldots (1)$

In $\triangle BDC$

By Pythagoras Theorem,

$BC^2 = DC^2 + BD^2 \ldots\ldots (2)$

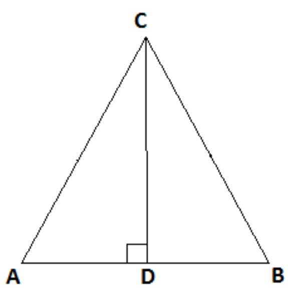
Adding 1) and 2) We get,

$2AC^2 = 2DC^2 + AD^2 + BD^2$ $(Since\ AC = BC)$

Since, $AD = BD = \frac{1}{2}AC$ $(Since\ AC = AB)$

$\therefore\ 2AC^2 = 2DC^2 + \frac{1}{2}AC^2$

$\therefore\ 3AC^2 = 4CD^2$



#465473
Topic: Theorems of Triangles

In $\triangle ABC$, $AB = 6\sqrt{3}cm$, $AC = 12cm$ and $BC = 6cm$. The $\angle B$ is :

A 120^o

B 60^o

$\boxed{\text{C}}$ 90^o

D 45^o

Solution

Given:

In $\triangle ABC$

$AB = 6\sqrt{3}\ cm$

$AC = 12\ cm$

$BC = 6\ cm$

Solution:

$AC^2 = 144$

$AB^2 = 108$

$BC^2 = 36$

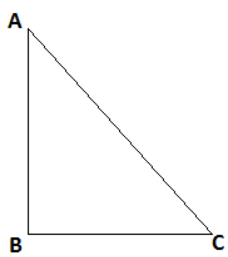
Since,

$AC^2 = AB^2 + BC^2$

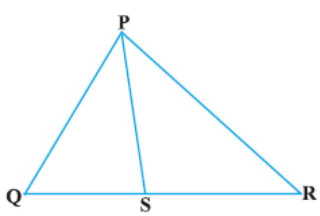
$\therefore$ By Converse of Pythagoras Theorem,

$\triangle ABC$ is an Right Angle Triangle at B.

$\therefore \angle B = 90^o$



#465475
Topic: Theorems of Triangles



In fig., PS is the bisector of $\angle QPR$ of $\triangle PQR$. Prove that $\dfrac{QS}{SR} = \dfrac{PQ}{PR}$.

Solution

Given:

$\angle QPS = \angle RPS$

To Prove:

$\frac{QS}{SR} = \frac{PQ}{PR}$

Construction:

Extend RP to T and

Join QT such that $TQ \parallel PS$

Proof:

Since, $QT \parallel PS$

$\therefore \angle TQP = \angle QPS$ (Alternate Angles)

Also,

$\angle QTP = \angle QPS$ (Corresponding Angles and PS is the bisector of $\angle QPR$ of $\triangle PQR$)

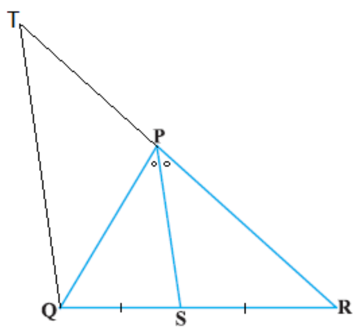
$\therefore \angle TQP = \angle QTP$

$\therefore TP = QP.......(1)$

Since, $QT \parallel PS$, by basic proportionality theorem,

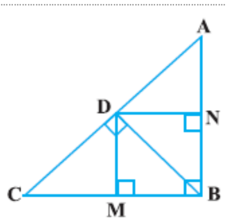
$\therefore \frac{QS}{SR} = \frac{TP}{PR}$

$\therefore \frac{QS}{SR} = \frac{PQ}{PR}$ (From 1)



#465476

Topic: Similar Triangles



In Fig., D is a point on hypotenuse AC of $\triangle ABC$, such that $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$. Prove that :

(i) $DM^2 = DN.MC$

(ii) $DN^2 = DM.AN$

Solution

i) In ΔABC,

$DN \perp AB$ and $BC \perp AB$

So, $DN \parallel BC$ ...(1)

$DM \perp BC$ and $AB \perp BC$

So, $DM \parallel AB$ ...(2)

From (1) and (2),

$\square DMBN$ is a rectangle.

$\therefore BM = DN$

In ΔBMD,

$\angle M + \angle BDM + \angle DBM = 180^o$

$\Rightarrow \angle BDM + \angle DBM = 90^o$ ...(1)

Similarly, in ΔDMC,

$\angle CDM + \angle MCD = 90^o$ ...(2)

We know, $BD \perp AC$ ....given

$\therefore \angle BDM + \angle MDC = 90^o$ ..(3)

From (1) and (3), we get

$\angle BDM + \angle DBM = \angle BDM + \angle MDC$

$\therefore \angle DBM = \angle MDC$ ...(4)

Similarly, $\angle BDM = \angle MCD$ ...(5)

In ΔBMD and ΔDMC,

$\angle BMD = \angle DMC$ ...Each 90^o

$\angle DBM = \angle MDC$ ...From (4)

$\angle BDM = \angle MCD$ ...From (5)

$\Delta BMD \sim \Delta DMC$ ....AAA test of similarity

$\therefore \dfrac{BM}{DM} = \dfrac{MD}{MC}$ ....C.S.S.T.

$\therefore \dfrac{DN}{DM} = \dfrac{DM}{MC}$ ... $\because BM = ND$

$\Rightarrow DM^2 = DN \times MC$

ii) Similarly, we can prove $\Delta DNB \sim \Delta DNA$

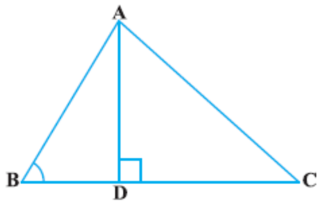
$\dfrac{BN}{DN} = \dfrac{ND}{NA}$

$\dfrac{DM}{DN} = \dfrac{DN}{AN}$ ...[$\because BN = DM$]

$DN^2 = DM \times AN$

#465479

Topic: Theorems of Triangles



In fig., ABC is a triangle in which $\angle ABC < 90^o$ and $AD \perp BC$. Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.

Solution

Proof:

In $\triangle ADC$

By Pythagoras Theorem,

$AC^2 = AD^2 + DC^2 ......(1)$

In $\triangle ABD$

By Pythagoras Theorem,

$AB^2 = AD^2 + BD^2 ......(2)$

Subtracting 1) and 2) we get,

$AC^2 - AB^2 = DC^2 - BD^2$

$\therefore \ AC^2 - AB^2 = DC^2 - (BC - DC)^2$

$\therefore \ AC^2 - AB^2 = 2DC.BC - BC^2$

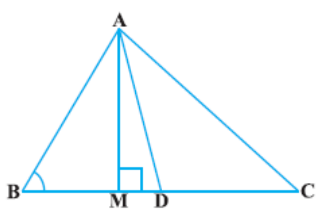
$\therefore \ AC^2 - AB^2 = 2(BC - BD)BC - BC^2$

$\therefore \ AC^2 - AB^2 = -2DB.BC + 2BC^2 - BC^2$

$\therefore \ AC^2 = AB^2 + BC^2 - 2BC.BD$

#465480

Topic: Theorems of Triangles



In fig., AD is a median of a triangle ABC and $AM \perp BC$. Prove that:

(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 BC.DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$

Solution

It is given that

$\angle AMD = 90^0$

Referring to the figure, we can say that

$\angle ADM < 90^0$ and $\angle ADC > 90^0$

Now,

(i)

To prove:

$AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

In ΔADC, $\angle ADC$ ia an obtuse angle.

$\therefore AC^2 = Ad^2 + DC^2 + 2DC.DM$

$\Rightarrow AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + 2.\frac{BC}{2}.DM$

$\Rightarrow AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii)

To prove:

$AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

In ΔABD, $\angle ADM$ is an obtuse angle.

$\therefore AB^2 = AD^2 + BD^2 - 2BD.DM$

$\Rightarrow AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 - 2.\frac{BC}{2}.DM$

$\Rightarrow AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

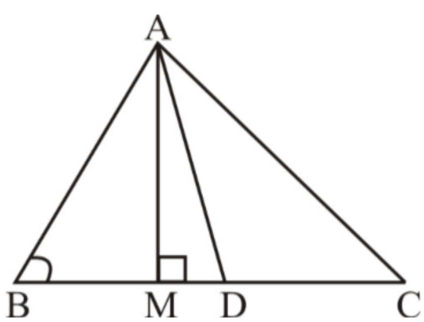
(iii)

To prove:

$AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$

From the result of (i) and (ii), adding those, we get

$AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



#465481

Topic: Theorems of Triangles

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Solution

Let $\square ABCD$ be a parallelogram.

Let its diagonals AC and BD intersect at O.

In $\triangle ABC$,

BO is the median ....Diagonals of a parallelogram bisect each other

$\therefore$ By Apollonius theorem,

$AB^2 + BC^2 = 2OB^2 + 2OA^2$ ....(1)

In $\triangle ADC$,

DO is the median ....Since diagonals bisect each other

$\therefore$ By Apollonius theorem,

$AD^2 + DC^2 = 2OD^2 + 2OC^2$ ....(2)

Adding (1) and (2) we get,

$AB^2 + BC^2 + AD^2 + DC^2 = 2OB^2 + 2OA^2 + 2OD^2 + 2OC^2$

$AB^2 + BC^2 + CD^2 + AD^2 = 2OB^2 + 2OA^2 + 2OB^2 + 2OA^2$

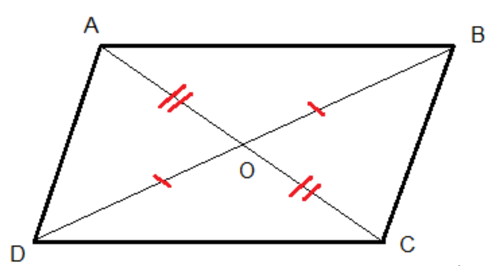
$AB^2 + BC^2 + CD^2 + AD^2 = 4OB^2 + 4OA^2$

$AB^2 + BC^2 + CD^2 + AD^2 = 4\left(\frac{1}{2} \times DB\right)^2 + 4\left(\frac{1}{2} \times CA\right)^2$

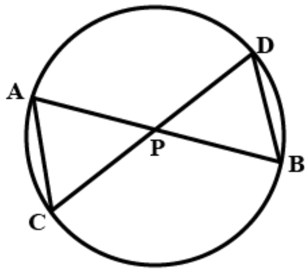
$AB^2 + BC^2 + CD^2 + AD^2 = 4\left(\frac{1}{4} \times DB^2\right) + 4\left(\frac{1}{4} \times CA^2\right)$

$AB^2 + BC^2 + CD^2 + AD^2 = DB^2 + CA^2$

Hence proved.



#465482
Topic: Theorems of Triangles



In Fig., two chords AB and CD intersect each other at the point P. Prove that :

(i) $\triangle APC \sim \triangle DPB$

(ii) $AP. PB = CP. DP$

Solution

(i) Given : In $\triangle APC$ and $\triangle DPB$,

$\angle APC = \angle DPB$ ...[Vert. opp. ∠s]

$\angle CAP = \angle BDP$ ...[Angles subtended by the same arc of a circle are equal]

$\therefore$ By AA-condition of similarity,

$\triangle APC \sim \triangle DPB$

(ii) $\triangle APC \sim \triangle DPB$

So, sides are proportional

$\therefore \dfrac{AP}{DP} = \dfrac{CP}{PB}$

$\Rightarrow AP \times PB = CP \times DP$

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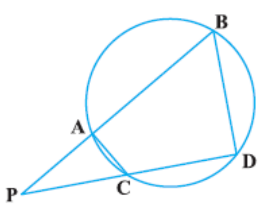
Topic: Theorems of Triangles



In Fig., two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that:

(i) $\triangle PAC \sim \triangle PDB$

(ii) $PA. PB = PC. PD$

Solution

(i) In $\triangle PAC$ and $\triangle PDB$,

$\angle BAC = 180^\circ - \angle PAC$ (linear pairs)

$\angle PDB = \angle CDB = 180^\circ - \angle BAC$

$= 180^\circ - (180^\circ - \angle PAC) = \angle PAC)$

$\angle PAC = \angle PDB$

$\angle APC = \angle BPD$ ...[Common]

$\therefore$ By AA-criterion of similarity,

$\triangle PAC \sim \triangle DPB$

(ii) $\triangle PAC \sim \triangle DPB$

So, sides are proportional

$$\frac{PA}{PD} = \frac{PC}{PB}$$

$$\Rightarrow PA.\ PB = PC.\ PD$$

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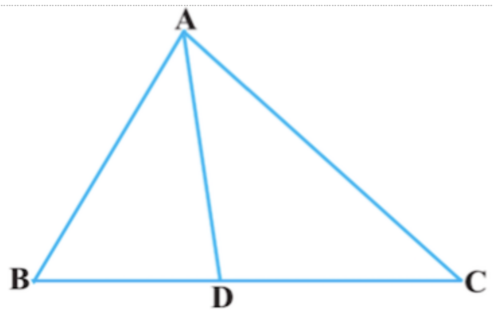
Topic: Theorems of Triangles



In Fig., D is a point on side BC of $\triangle ABC$ such that $\frac{BD}{CD} = \frac{AB}{AC}$.

Prove that AD is the bisector of $\angle BAC$.

Solution

D is a point on BC of ABC.

and $\dfrac{BD}{CD} = \dfrac{AB}{AC}$

Let us construct BA to E such that $AE = AC$. Join CE.

Now, as $AE = AC$,

$\dfrac{BD}{CD} = \dfrac{AB}{AC}$

$\Rightarrow \dfrac{BD}{CD} = \dfrac{AB}{AE}$

Also, $\angle AEC = \angle ACE$ (angles opp. to equal sides of a triangle are equal) .... (i)

By converse of Basic Proportionality Theorem,

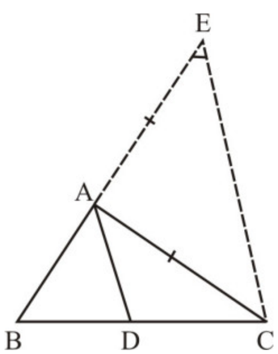
$\angle DAC = \angle ACE$ ...... (ii) ...[Alternate angles]

$\angle BAD = \angle AEC$ .... (iii) ...[Corresponding $\angle$s]

Also, $\angle AEC = \angle ACE$ ...[From (i)]

and $\angle BAD = \angle DAC$ ...[From (ii) and (iii)]

So, AD is the bisector $\angle BAC$.



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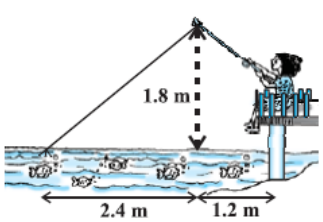
Topic: Theorems of Triangles



Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig.)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?

Solution

Let AB is the height of the tip of the fishing rod from the water surface .Let BC is the horizontal distance of the fly from the tip of the fishing rod.

Then AC is the length of the string.

Then according to the Pythagorean theorem-

$\Rightarrow AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (1.8)^2 + (2.4)^2$

$\Rightarrow AC^2 = 3.24 + 5.76$

$\Rightarrow AC^2 = 9.00$

$\Rightarrow AC = \sqrt{9}m = 3m$

$\therefore$ The length of the string out is 3 m.

She pulls the string at the rate of 5 cm per second .

$\therefore$ She pulls in 12 seconds=$12 \times 5 = 60cm = 0.6m$

Let the fly be at a point of D after 12 seconds

Length of the string out of 12 second is AD.

AD=AC-string pull by Nazima after 12 sec.

$AD = 3 - .6 = 2.4m$

In $\triangle ADB$

$AD^2 = AB^2 + BD^2$

$\Rightarrow BD^2 = AD^2 - AB^2$

$\Rightarrow BD^2 = (2.4)^2 - (1.8)^2$

$\Rightarrow BD^2 = 5.76 - 3.24$

$\Rightarrow BD^2 = 2.52$

$\Rightarrow BD = \sqrt{2.52} = 1.587m$

Horizontal distance to fly = $BD + 1.2$

$\Rightarrow 1.587 + 1.2 \Rightarrow 2.787$ m