Unit 9: Real Numbers

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Lesson 1: Introduction to Real Numbers

Topic 1: Variables and Expressions

Learning Objectives

- Evaluate expressions with one variable for given values for the variable.
- Evaluate expressions with two variables for given values for the variables.

Topic 2: Integers

Learning Objectives

- Locate integers on a number line.
- Find the absolute value of a given number.
- Find the opposite of a given number.

Topic 3: Rational and Real Numbers

Learning Objectives

- Identify the subset(s) of the real numbers that a given number belongs to.
- Locate points on a number line.
- Compare rational numbers.
- Identify rational and irrational numbers.

Lesson 2: Operations with Real Numbers

Topic 1: Adding Integers

Learning Objectives

- Add two or more integers with the same sign.
- Add two or more integers with different signs.

Topic 2: Adding Real Numbers

Learning Objectives

- Add two or more real numbers with the same sign.
- Add two or more real numbers with different signs.
- Simplify by using the identity property of 0.
- Solve application problems requiring the addition of real numbers.

Topic 3: Subtracting Real Numbers

Learning Objectives

- Subtract two or more real numbers.
- Simplify combinations that require both addition and subtraction of real numbers.
- Solve application problems requiring subtraction of real numbers.

Topic 4: Multiplying and Dividing Real Numbers

Learning Objectives

- Multiply two or more real numbers.
- Simplify by using the identity property of 1.
- Divide real numbers.
- Solve application problems requiring the multiplication or division of real numbers.

Lesson 3: Properties of Real Numbers

Topic 1: Associative, Commutative and Distributive Properties for Real Numbers

Learning Objectives

- Identify and use the commutative properties for addition and multiplication.
- Identify and use the associative properties for addition and multiplication.
- Identify and use the distributive property.

Lesson 4: Simplifying Expressions

Topic 1: Order of Operations

Learning Objectives

- Use the order of operations to simplify expressions.
- Simplify expressions containing absolute values.

Unit 9: Real Numbers

Instructor Notes

The Mathematics of Real Numbers

This unit provides a thorough grounding in the use and manipulation of real numbers. Students will learn the definitions of irrational and rational numbers and their subsets. They'll practice various techniques for carrying out operations and get comfortable applying the commutative, associative, and distributive properties to real number expressions.

The course steps into algebra in this unit as well, introducing variables and teaching students how to evaluate one and two variable expressions.

Teaching Tips: Challenges and Approaches

The math in this unit isn't difficult, but a few of the ideas will challenge students. Some will come into the course thinking that algebra is both hard and pointless. Others will be intimidated by the language used in rules and definitions. You will have to provide reassurance along with math skills. Ease them into an algebraic way of thinking with simple, confidence-building problems and real world examples. Translate formal rules into everyday terms.

Numbers and Variables

This unit is all about values—rational and irrational, absolute and opposite, constant and variable. It's easy for students to get confused by the different sets and definitions. Illustrate these ideas with word problems about real-world situations as much as possible. For example, when introducing the idea of variables, start with a very simple example, like the one below where test scores are represented by the letter s:

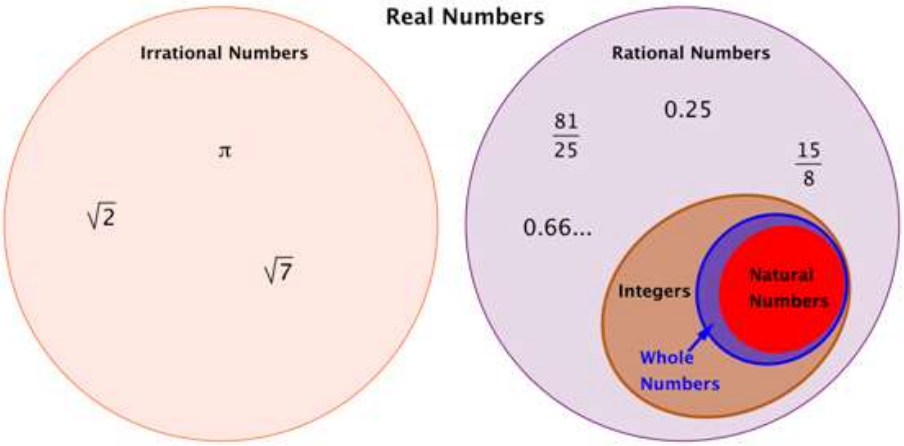
[From Lesson 1, Topic 1, Presentation]

Then work up to more complicated problems that show how writing a general expression with a variable is more efficient than having to come up with unique expressions for all possible situations:

Example		
Problem	**John is planning a rectangular garden that is 2 feet wide. He hasn't decided how long to make it, but he's considering 4 feet, 5 feet, and 6 feet. He wants to put a short fence around the garden. Using x to represent the length of the rectangular garden, he will need $x + x + 2 + 2$, or $2x + 4$, feet of fencing.** **How much fencing will he need for each possible garden length? Evaluate the expression when $x = 4$, $x = 5$, and $x = 6$ to find out.**	
	$2x + 4$ $2(4) + 4$ $8 + 4$ 12	For $x = 4$, substitute 4 for x in the expression. Evaluate by multiplying and adding.
	$2x + 4$ $2(5) + 4$ $10 + 4$ 14	For $x = 5$, substitute 5 for x. Evaluate by multiplying and adding.
	$2x + 4$ $2(6) + 4$ $12 + 4$ 16	For $x = 6$, substitute 6 for x and evaluate.
Answer	John needs 12 feet of fencing when $x = 4$, 14 feet when $x = 5$, and 16 feet when $x = 6$.	

[From Lesson 1, Topic 1, Topic Text]

When discussing the various kinds of real numbers, sketches and number lines will help students see how the different sets are related:



[From Lesson 1, Topic 3, Topic Text]

A good way to ensure understanding is to state a series of numbers and have the students respond with the categories they belong to.

Translating Rules

Students need to know the formal rules for adding, subtracting, multiplying and dividing real (signed) numbers. But when they read the rules for the first time, you'll see a lot of puzzled faces. Consider the rule for adding real numbers:

> When the signs are the same, add the absolute values of the addends and use the same sign. When the signs are different, subtract the absolute values and use the same sign as the addend with the greater absolute value.

What makes these rules confusing is the use of the words “absolute values”. These rules have to be stated but students should be shown how to add using informal terms that are more easily understood.

Example

Work through the following four examples:

(+3) + (+4)
(-3) + (-4)
(+3) + (-4)
(-3) + (+4)

When adding +3 and +4, tell your students that since both numbers are positive, add the 3 and 4 and your answer is positive.

When adding -3 and -4, tell your students that since both numbers are negative, add the 3 and 4 and your answer is negative.

When adding +3 and -4, -3 and +4, remind your students how to add two numbers with different signs (one positive and one negative):

- Find the difference of their absolute values, subtracting the smaller number from the larger one.
- Give the sum the same sign as the number with the greater absolute value.

Therefore, when adding +3 and -4, the answer is negative because 4 is greater than 3 and the 4 is negative.

When adding -3 and +4, the answer is positive because 4 is greater than 3 and the 4 is positive.

Work through several examples like this, and students will understand how to add signed numbers. Another way to reinforce the rules of addition with signed numbers is to refer to examples involving money.

When tackling subtraction problems, show students how to convert them into equivalent addition problems. Using a trick such as “keep-change-opposite” will help a student remember how to do the conversion.

Example
Solve:

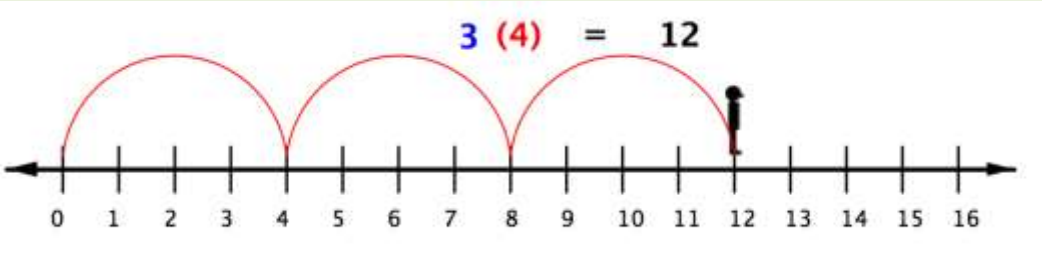
$$-4 - (-3)$$

keep the -4, **change** the subtraction sign to an addition sign, take the **opposite** of -3

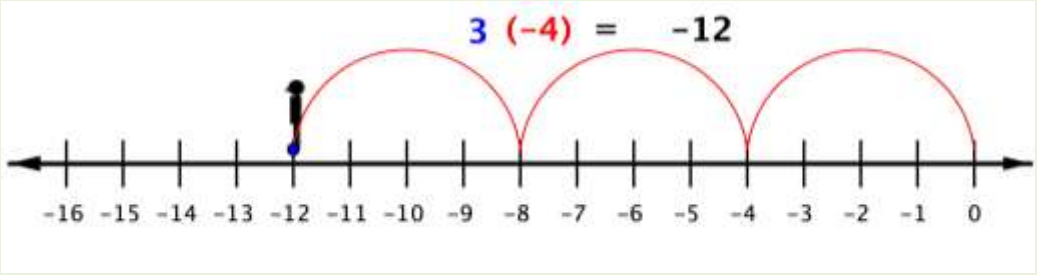
$$-4 + 3 = -1$$

Students generally don’t have a problem with the rules for multiplying and dividing signed numbers. However, it is always a good idea to try to show why the rules are what they are:

With whole numbers, you can think of multiplication as repeated addition. Using the number line, you can make multiple jumps of a given size. For example, the following picture shows the product 3 • 4 as 3 jumps of 4 units each.



So to multiply 3 (−4), you can face left (toward the negative side) and make three "jumps" forward (in a negative direction).



[From Lesson 2, Topic 4, Topic Text]

Since students usually understand the rules for multiplication easily, have them try multiplying more than two signed numbers together. Ask them to solve a problem like (-2)(-3)(4)(-1)(5), then change one of the signs and get them to multiply again. After doing a few similar problems, have the students come up with a rule that determines the sign of the final answer, i.e. if there are an odd number of negative signs when multiplying, the answer is negative; an even number of negative signs means the answer is positive.

Calculators

Once students show they can do operations on real numbers with pencil and paper, we recommend letting them use calculators. However, calculators can lead students astray because they assume that the machine makes no mistakes and its numbers are always right. Be very sure students understand that if they enter the wrong information or hit the wrong key, the answer will still be wrong even if the calculator functions perfectly.

Keep in Mind

Absolute value problems are always a little tricky for some students. They will think $-|-5|$ is 5 because absolute value is always positive. Or they'll confuse this expression with –(-5).

As students work to simplify expressions in this unit, remind them to only combine like terms, i.e. $10x + 5y$ is simplified and does not equal $15xy$.

Most students are already familiar with the associative, commutative, and distributive properties, but they are likely to be a little rusty, as well as unsure how to apply them to variables. Go over these concepts thoroughly—students who don't learn how to use them now will fail when the course moves deeper into algebra.

Identity and inverse properties with identities should be reviewed, too, with particular attention given to explaining why x and $1x$ are the same.

Additional Resources

In all mathematics, the best way to really learn new skills and ideas is repetition. Problem solving is woven into every aspect of this course—each topic includes warm-up, practice, and review problems for students to solve on their own. The presentations, worked examples, and topic texts demonstrate how to tackle even more problems. But practice makes perfect, and some students will benefit from additional work.

A good site for practicing the four basic operations on signed numbers is http://www.thatquiz.org/tq-1/?-jh0v-la-p0.

For more help on evaluating expressions, go to http://www.321know.com/equ723x3.htm and http://www.thatquiz.org/tq-0/?-j104-l1-p0 (select levels 1, 2 and, for a real challenge, 3).

Summary

Traditional approaches to algebra often plunge students into working with abstract notation and symbols without having much idea why or what they are doing. In this unit, students are given a taste of what algebra is about as they explore arithmetic operations and properties of real numbers. This unit prepares students for the leap into solving algebraic equations that comes later in the course.

Instructor Overview
Tutor Simulation: Tricks With Real Numbers

Purpose

This simulation allows students to demonstrate their ability to write real numbers. Students will be asked to apply what they have learned to solve a problem involving:

- Adding and subtracting real numbers
- Multiplying and dividing real numbers
- Order of operations
- Using variables

Problem

Students are presented with the following problem:

> A magician put on a show and convinced people that he could read minds and control people's thoughts. He asked everyone in the room to pick a whole number and, keeping it secret, perform some calculations. Not only does he correctly predict each person's final answer, but he has apparently "controlled" their thoughts and made them all obtain the *same* answer. He repeated this trick several times using different calculations.
>
> You will use your knowledge of real numbers and the order of operations to figure out how he has fooled the audience.

Recommendations

Tutor simulations are designed to give students a chance to assess their understanding of unit material in a personal, risk-free situation. Before directing students to the simulation,

- Make sure they have completed all other unit material.
- Explain the mechanics of tutor simulations.
    - Students will be given a problem and then guided through its solution by a video tutor;
    - After each answer is chosen, students should wait for tutor feedback before continuing;
    - After the simulation is completed, students will be given an assessment of their efforts. If areas of concern are found, the students should review unit materials or seek help from their instructor.
- Emphasize that this is an exploration, not an exam.

Unit 9: Real Numbers

Instructor Overview
Puzzle: Absolutely!

Objectives

Absolutely! is a number game designed to reinforce the meaning of absolute value, the value of a number without regard to its sign. In order to solve the puzzles, students must take the absolute value of numbers, and demonstrate that they understand tricky concepts such as the differences between a negative number, the absolute value of a negative number, and the opposite of the absolute value of a negative number.

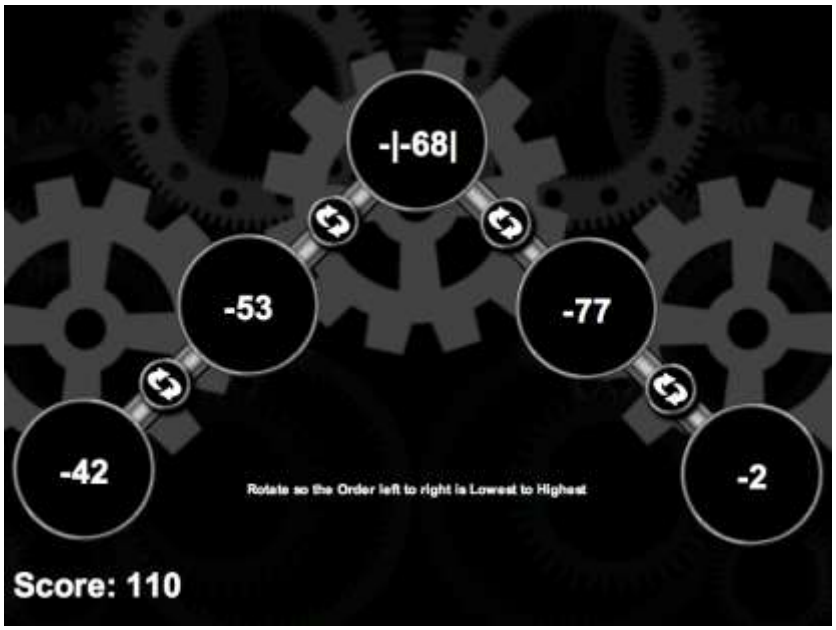


Figure 1. *Absolutely!* asks players to arrange a sequence of numbers in order from smallest to largest.

Description

Each puzzle in the game consists of a short sequence of numbers. Numbers include positive and negative integers as well as numbers with negative signs within and without the absolute value symbol. Players earn points by correctly rearranging the numbers in each series in order of value from smallest to largest. They do this by clicking on a pivot point between numbers to move them up and down the sequence. The game has 3 levels of difficulty, and each level has 10 puzzles. Level 1 puzzles have 3 numbers each, Level 2 puzzles have 4 numbers, and Level

3 puzzles contain 5 numbers per sequence. Every time a puzzle opens, the sequences are randomly generated, so the game can be played over and over with new problems each time.

Absolutely! can be shown and even played in class, as well as worked independently by individuals and small groups.

Unit 9: Real Numbers

Instructor Overview
Project: Planning A Trip

Student Instructions

Introduction

There are many things to consider when planning a trip. In this project, you will explore how to plan for a trip and determine the actual cost of traveling to and from your vacation destination.

Task

Working together with your group, you will explore the rate at which you must travel to arrive at your vacation destination, a reasonable plan for travel, and costs associated with travel. Finally, using all that your group has learned, you will make an itinerary for each day of your trip. You will create a poster presentation of your trip to serve as an advertisement, so your group can recommend it to others.

Instructions

Solve each problem in order and save your work along the way, as you will create a professional advertisement at the conclusion of the project. If required, round to the hundredths place (two decimal places).

- First problem:
    - Your group will be taking a road trip. You need to determine a destination at a distance of between 1000-1500 miles. You should go to a website, such as www.randmcnally.com/ *or* www.mapquest.com/, to find a map, driving directions, and estimated travel time. Print these resources as they will be needed in your project. Record the information in the table below.

      Hint: If your estimated time is in hours and minutes, divide the minute portion by 60, so it is part of an hour, and then add that to the number of hours.

    - Your group will need to decide the type of vehicle that you want to use for travel. If there are four or five people, make sure there is enough room for yourselves and your luggage. Use a website such as www.fueleconomy.gov to find the EPA Estimated Fuel Economy for your vehicle and the fuel tank size. Since most of your driving will be on a highway, record the highway fuel economy for your vehicle in the table below.

Beginning Trip Information

Starting Location of Trip	
Vacation Destination	
Total Miles	
Estimated Time to Destination (in hours)	
Vehicle Type	
Estimated Fuel Economy (miles per gallon for highway driving)	
Size of Fuel Tank (in gallons)	
Number of People taking the Trip	

- Based on the estimated time to the destination shown in your table, calculate the average rate at which you must travel to arrive at your destination.
  Hint: To find the average rate of your travel, you will need to use the formula $r = \frac{D}{t}$, where D is the distance, r is the rate (speed), and t is the time.

- Many times when you travel by car, there is road construction or slow traffic. You want to determine how long it would take you to arrive at your destination if your average rate is different from what you calculated. Calculate your estimated time to your destination based on the rates given in the table below, comparing each value to that for 55 miles per hour to see how much time you save by driving faster.

Hint: You will need to use the formula $t = \frac{D}{r}$ to calculate the time to destination for each average rate.

Changing the Average Rate

Average Rate (in miles per hour)	Time to Destination (in hours)	Comparison of Time to that for 55 miles per hour (Time – Time for 55 mph)
55		0
60		
65		
70		

- Using your results from changing the average rate, make a statement describing the effect of increasing your average rate. Be sure to comment on the significance of the sign of your numbers in the third column.

- Second problem:
  - With a long trip, it is wise to have a plan for how far or long you want to travel in one day. Many people will try to travel 500 miles in a day. You need to determine the number of days you will be traveling and how long you will be driving each day if 500 miles per day is your goal.

Average Rate (in miles per hour)	Time Driving Per Day (in hours)
55	
60	
65	
70	

Number of Days of Travel _____

- Some drivers prefer to only have 8 hours of driving in a day. You need to determine how many days and how far you will be driving each day if 8 hours per day is your goal.

  Hint: To find the distance traveled, use the formula $D = \frac{r}{t}$. Then calculate the number of days by dividing the total distance by the distance traveled per day. Number of Days = $\frac{\text{Total Distance of Trip}}{\text{Distance Traveled Per Day}}$

Goal of 8 Hours of Driving Time in a Day

Average Rate (in miles per hour)	Distance Traveled per day (in miles)	Number of Days of Travel
55		
60		
65		
70		

- Your group needs to use your calculations to make a decision whether you want to travel either 500 miles per day or a total of 8 hours of driving time. You should think about what your goal is in terms of how far or long to drive per day and a reasonable rate of travel. Be able to defend your choices.
- Based on your decision for travel, determine, the number of days it will take you to reach your destination, how many days you want to stay at your destination, and the number of days for your return trip home (it should be the same number of days as it took to reach your destination). Then, calculate the total number of days you will be on vacation.
  Decision for travel (including average rate):

  Number of Days to Reach Destination _____

  Number of Days Staying at Destination _____

  Number of Days to Return Home _____

  Total Number of Days on Vacation: _____

- Third Problem:
  - You will need to rent the vehicle that you have decided to travel with. The car rental place has two different offers:

Offer #1: $10 per day, first 100 miles are free, after the first 100 miles, they charge $0.20 per mile.

Offer #2: $25 per day, all miles are charged at the rate of $0.15 per mile, and if you keep the car for more than 1 week, they take $50 off your total bill.

Write an expression for computing the cost of the car rental for each of the above options. Use *d* to represent the number of days and *m* to represent the number of miles driven.

Offer #1

Scenario	Expression for Cost
Under 100 Miles Driven	
Over 100 Miles Driven	

Offer #2

Scenario	Expression for Cost
Under 1 Week	
Over 1 Week	

- For your trip, calculate the cost for each offer and determine the best offer for your vacation.

Finally, determine the per day cost of renting the car.

Hint: You will need to calculate the cost for the entire trip meaning that you will have to double your mileage. Then, estimate how many miles you think you will travel at your destination. Finally, calculate the total mileage of the trip. Use this number and the number of days of travel from Problem #2, to calculate the cost of renting a car.

To find the per day cost of renting the car, use the formula

$$\text{Per Day Cost of Renting Car} = \frac{\text{Total Cost of Car Rental}}{\text{Total Number of Days}}$$

- You will also have to purchase gas for the entire trip. You will need to calculate the amount and cost of fuel based on your mileage and estimated fuel efficiency from Problem #1. Use $2.70 per gallon for the cost of gas.

  Hint: Calculate the fuel needed by determining total mileage divided by fuel efficiency. Number of Gallons of Gas = $\frac{\text{Total Distance of Trip}}{\text{Fuel Efficiency}}$

- The last expenses for your trip are food and lodging. Estimate how much you will spend on food per day. Be reasonable in your estimates. On average lodging is $100 per night for a single room with double beds. If your group is larger than 4 people, you will need to have two rooms. Calculate the daily cost for food and lodging and then the total cost for food and lodging based on the number of days in your trip.

- Finally, determine how much money each person in your group should contribute to the trip.

- Fourth problem:
  - Finally, your group will use all of the information that you have gathered and determine an itinerary for the trip. You will need to calculate how many miles you can drive before stopping for gas. Use the size of your fuel tank and fuel efficiency from Problem #1 to calculate this. Remember you do not want to run out of gas, so leave some gallons of gas in the fuel tank.

  - You will need to use your map to figure out the starting and ending location for each day. Fill out the itinerary chart on the next page.

  - Now, fill out the checkbook for each day of your trip. You should be able to get the beginning balance from Problem #3. Your beginning balance will be positive, but each day's cost should be recorded as a negative number. Record your new balance in the right-hand column for each day.

  - Make a sales-pitch recommending the trip.

Itinerary Chart

	Day__	**Day__**	**Day__**	**Day__**	**Day__**	**Day__**	**Day__**
Goal for the Day (Miles)							
Total Time (Hours)							

Start Location							
Planned Stops							
End Location							
Average Rate of Travel							
Total Daily Cost:							
Gas							
Food							
Lodging							
Car Rental							

Checkbook

		Beginning Balance (Total Money Contributed by All Members of the Group)	**$**
Day Number	Description of Expenses (gas, food, lodging and/or car rental)	Total Cost (these should be negative)	Balance
		$	$
		$	$
		$	$
		$	$
		$	$

		$	$
		$	$
		$	$
		$	$
		$	$
		$	$
		$	$
		$	$

Collaboration

Get together with another group to compare your answers to each of the four problems. Discuss how your group decided to travel, costs for food, number of days at vacation spot and explain your trip itinerary. Some groups may have more than four members, so this should be taken into account.

- What if you wanted to stop at destinations along the way of your trip? How would this affect your plans?
- What is a reasonable amount of time and distance to travel per day?
- What happens if someone wants his or her own lodging? How could you recalculate the price per person?
- Do some Internet research to determine how much it costs to travel in a different way (train, plane, or bus). Determine if another mode of transportation would lower the cost of the trip.

Conclusions

Your advertisement should be a presentation of your trip that would allure other people to join in. It should be on a poster board highlighting the itinerary and costs associated with the trip. It should include all of the mathematics used to solve the four problems above. You may either neatly write out the tables or use software such as Microsoft Word to create a professional computer generated product. You may want to include pictures of the map and destination to make it colorful.

Instructor Notes

Assignment Procedures

Problem 1

An example of a beginning trip information chart is included below.

Beginning Trip Information

Starting Location of Trip	Chicago, IL
Vacation Destination	Yellowstone National Park in Wyoming
Total Miles	1349.5 miles
Estimated Time to Destination (in hours).	20.57 hours
Vehicle Type	Chrysler Town and Country Minivan
Estimated Fuel Economy (miles per gallon for highway driving)	25 miles per gallon
Size of Fuel Tank (in gallons)	20 gallons
Number of People taking the Trip	4 people

Given this information, the trip is 1349.5 miles and the estimated time is 20.57 hours. The average rate would be calculated by 1349.5/20.57. The average rate on the trip would be 65.61 miles per hour.

The table below is filled out based on the distance of 1349.5 miles.

Changing the Average Rate

Average Rate (in miles per hour)	Time to Destination (in hours)	Comparison of Time to that for 55 miles per hour (Time – Time for 55 mph)
55	24.54	0
60	22.49	-2.05
65	20.76	-3.78
70	19.28	-5.26

The reason the answers are negative is because the time to destination decreases as the rate increases.

Problem 2

The table below is based on a trip of 1349.5 miles. The table will be the same for all groups, only the number of days of travel will change.

Goal of 500 Miles in a Day

Average Rate (in miles per hour)	Time Driving Per Day (in hours)
55	9.09
60	8.33
65	7.69
70	7.14

Number of Days of Travel 1349.5/500 = 2.70 days.

The numbers below are based on a trip of 1349.5 miles. The distance traveled will be the same for all groups, but the number of days of travel will change.

Goal of 8 Hours of Driving Time in a Day

Average Rate (in miles per hour)	Distance Traveled per day(in miles)	Number of Days of Travel
55	440	3.07
60	480	2.81
65	520	2.60
70	560	2.41

Students' answers may vary.

Decision for travel (including average rate): 60 miles per hour average rate and 8 hours per day.

Number of Days to Reach Destination _____2.81_____

Number of Days Staying at Destination _____3_____

Number of Days to Return Home _____2.81_____

Total Number of Days on Vacation: _____9 days_____.

Problem 3

Offer #1

Scenario	Expression for Cost
Under 100 Miles Driven	10d
Over 100 Miles Driven	10d + 0.20(m-100)

Offer #2

Scenario	Expression for Cost
Under 1 Week	25d+0.15m
Over 1 Week	25d+0.15m-50

These calculations are based on 9 days of travel and a total mileage of 3100.

Total mileage was found by 1349.5 x 2 + 400 = 3099, rounded to 3100. The 400 miles is the estimate for driving around Yellowstone.

Offer #1: $10 per day, first 100 miles are free, after the first 100 miles, they charge $0.20 per mile.

Calculations:

Daily Cost: $10 per day x 9 days = $90

Mileage Cost: 3100 miles – 100 free miles = 3000 miles x $0.20 per mile = $600.

Total Cost for Offer #1 is $600+$90 = $690.

Offer #2: $25 per day, all miles are charged at the rate of $0.15 per mile, and if you keep the car for more than 1 week, they take $50 off your total bill.

Calculations:

Daily Cost: $25 per day x 9 days = $225

Mileage Cost: 3100 miles x $0.15 per mile = $465

Total Cost for Offer #2 is $225 + $465 - $50 (since we had the car for 9 days) = $640.

The best offer is Offer #2 for $640.

The per day cost of renting the car is $640/9 = $71.11

The total distance of the trip was 3100 miles and the fuel efficiency for the vehicle was 25 miles per gallon. The total fuel needed would be 3100/25 = 124 gallons, which would cost 124 gallons x $2.70 per gallon = $334.80.

A reasonable estimate for food would be to have a cost (per person) of approximately $7 for breakfast, $10 for lunch, and $15 for dinner, so $32 per person multiplied by the number in the group. If the group contains four members, then the daily total would be $32 x 4 = $128 + $100 for lodging = $228 per day. The trip is for 9 days, so the total cost would be $228 x 9 = $2052.

If the total cost is $2052 and there are 4 members in the group, each person should pay $2052/4 = $513.

Problem 4

Since the fuel tank is 20 gallons and the fuel efficiency is 25 miles per gallon, the vehicle can travel 20 gallons x 25 miles per gallon = 500 miles before running out of gas, so we will stop every 400 miles to fill up with gas.

At this point, the students will have computed much of this information and the answers will vary so widely that an “answer key” will not be helpful.

Recommendations

- Have students work in teams to encourage brainstorming and cooperative learning.
- Assign a specific timeline for completion of the project that includes milestone dates.
- Provide students feedback as they complete each milestone.
- Ensure that each member of student groups has a specific job.

Technology Integration

This project provides abundant opportunities for technology integration, and gives students the chance to research and collaborate using online technology. The students' instructions list several websites that provide information on numbering systems, game design, and graphics.

The following are other examples of free Internet resources that can be used to support this project:

http://www.moodle.org

An Open Source Course Management System (CMS), also known as a Learning Management System (LMS) or a Virtual Learning Environment (VLE). Moodle has become very popular among educators around the world as a tool for creating online dynamic websites for their students.

http://www.wikispaces.com/site/for/teachers or http://pbworks.com/content/edu+overview

Allows you to create a secure online Wiki workspace in about 60 seconds. Encourage classroom participation with interactive Wiki pages that students can view and edit from any computer. Share class resources and completed student work.

http://www.docs.google.com

Allows students to collaborate in real-time from any computer. Google Docs provides free access and storage for word processing, spreadsheets, presentations, and surveys. This is ideal for group projects.

http://why.openoffice.org/

The leading open-source office software suite for word processing, spreadsheets, presentations, graphics, databases and more. It can read and write files from other common office software packages like Microsoft Word or Excel and MacWorks. It can be downloaded and used completely free of charge for any purpose.

Rubric

The project can be given a score of 1 to 4, with 4 being the highest score possible. Evaluation is based on the following criteria:

Score	Content	Presentation/Communication
4	• The solution shows a deep understanding of the problem including the ability to identify the appropriate mathematical concepts and the information necessary for its solution. • The solution completely addresses all mathematical components presented in the task. • The solution puts to use the underlying mathematical concepts upon which the task is designed and applies procedures accurately to correctly solve the problem and verify the results. • Mathematically relevant observations and/or connections are made.	• There is a clear, effective explanation detailing how the problem is solved. All of the steps are included so that the reader does not need to infer how and why decisions were made. • Mathematical representation is actively used as a means of communicating ideas related to the solution of the problem. • There is precise and appropriate use of mathematical terminology and notation. • Your project is professional looking with graphics and effective use of color.
3	• The solution shows that the student has a broad understanding of the problem and the major concepts necessary for its solution. • The solution addresses all of the mathematical components presented in the task. • The student uses a strategy that includes mathematical procedures and some mathematical reasoning that leads to a solution of the problem. • Most parts of the project are correct with only minor mathematical errors.	• There is a clear explanation. • There is appropriate use of accurate mathematical representation. • There is effective use of mathematical terminology and notation. • Your project is neat with graphics and effective use of color.
2	• The solution is not complete indicating that parts of the problem are not understood. • The solution addresses some, but not all of the mathematical components presented in the task. • The student uses a strategy that is partially useful, and demonstrates some evidence of mathematical reasoning. • Some parts of the project may be correct, but major errors are noted and the student could not completely carry out mathematical procedures.	• Your project is hard to follow because the material is presented in a manner that jumps around between unconnected topics. • There is some use of appropriate mathematical representation. • There is some use of mathematical terminology and notation appropriate to the problem. • Your project contains low quality graphics and colors that do not add interest to the project.
1	• There is no solution, or the solution has no relationship to the task. • No evidence of a strategy, procedure, or mathematical reasoning and/or uses a strategy that does not help solve the problem. • The solution addresses none of the	• There is no explanation of the solution, the explanation cannot be understood or it is unrelated to the problem. • There is no use or inappropriate use of mathematical representations (e.g. figures, diagrams, graphs, tables,

	mathematical components presented in the task. • There were so many errors in mathematical procedures that the problem could not be solved.	etc.). • There is no use, or mostly inappropriate use, of mathematical terminology and notation. • Your project is missing graphics and uses little to no color.

Unit 9: Real Numbers

Common Core Standards

Unit 9, Lesson 1, Topic 1: Variables and Expressions

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Unit 9, Lesson 1, Topic 2: Integers

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Unit 9, Lesson 1, Topic 3: Rational and Real Numbers

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.8.NS.**	The Number System
CATEGORY / CLUSTER		Know that there are numbers that are not rational, and approximate them by rational numbers.
STANDARD	**8.NS.1.**	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. (SBAC Summative Assessment Target: 1.01, 3.01)

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.N.**	Number and Quantity
CATEGORY / CLUSTER	**N-RN.**	The Real Number System
STANDARD		Use properties of rational and irrational numbers.
EXPECTATION	**N-RN.3.**	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Unit 9, Lesson 2, Topic 1: Adding Integers

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Unit 9, Lesson 2, Topic 2: Adding Real Numbers

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.N.**	Number and Quantity
CATEGORY / CLUSTER	**N-RN.**	The Real Number System
STANDARD		Use properties of rational and irrational numbers.
EXPECTATION	**N-RN.3.**	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Unit 9, Lesson 2, Topic 3: Subtracting Real Numbers

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.N.**	Number and Quantity
CATEGORY / CLUSTER	**N-RN.**	The Real Number System
STANDARD		Use properties of rational and irrational numbers.
EXPECTATION	**N-RN.3.**	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Unit 9, Lesson 2, Topic 4: Multiplying and Dividing Real Numbers

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.N.**	Number and Quantity
CATEGORY / CLUSTER	**N-RN.**	The Real Number System
STANDARD		Use properties of rational and irrational numbers.
EXPECTATION	**N-RN.3.**	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Unit 9, Lesson 3, Topic 1: Associative, Commutative and Distributive Laws

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.N.**	Number and Quantity
CATEGORY / CLUSTER	**N-RN.**	The Real Number System
STANDARD		Use properties of rational and irrational numbers.

EXPECTATION	**N-RN.3.**	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Unit 9, Lesson 4, Topic 1: Order of Operations

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.