Introduction

Sets and the Real Number System

Sets: Basic Terms and Operations

Definition (Set)

A set is a **well-defined** collection of objects. The objects which form a set are called its **members or Elements.**

Examples:

a) The set of Students in MTH 101C

b) The set of counting numbers less than 10.

Description of Sets:

There are two ways a set may be described; namely, 1) **Listing Method** and 2) **Set Builder Method.**

1) Listing Method: In this method **all** or **partial** members of the set are listed.

Examples:

a) Let R be the set of Natural number less than 10.

$\boldsymbol{R} = \{1, \ 2, \ 3, \ 4, \ 5, \ 6, \ 7, \ 8, \ 9\}$, complete listing

b) Let H be the set of counting numbers less than 1000

$\boldsymbol{H} = \{1, \ 2, \ 3, \ . \ . \ ., \ 999\}$, Partial listing

c) Let **N** be the set of Natural Numbers

$\boldsymbol{N} = \{1, \ 2, \ 3, \ . \ . \ . \ \}$, Partial listing

Definition: (Empty Set)

A set containing no element is called an **empty set** or a **null set.** Notations $\{ \ \}$ $\boldsymbol{or}$ $\emptyset$ denotes empty set.

Example: The set of natural numbers less than 1

2) Set Builder Method: In this method the set is described by listing the properties that describe the elements of the set.

Examples:

a) **S** be the set of students in this class, then using set builder S can be describes as

$$\boldsymbol{S} = \{\boldsymbol{x} \mid \boldsymbol{x} \ \boldsymbol{is} \ \boldsymbol{a} \ \boldsymbol{student} \ \boldsymbol{in} \ \boldsymbol{Math} \ \mathbf{1111} \ \boldsymbol{class}\}$$

b) **N** be the set of natural numbers

$$\boldsymbol{N} = \{\boldsymbol{n} \mid \boldsymbol{n} \ \boldsymbol{is} \ \boldsymbol{a} \ \boldsymbol{natural} \ \boldsymbol{number}\}$$

Note: Set-Builder form has two parts

1) A variable $\boldsymbol{x}, \ \boldsymbol{n}, \ etc.$ representing **any elements** of the set.

2) A **property** which **defines** the elements of the set

A set can be described using **the listing** or **set builder method.** For example, consider the set of Natural numbers:

$\boldsymbol{N} = \{1,\ 2,\ 3,\ .\ .\ .\ \}$, **Partial Listing**

$\boldsymbol{N} = \{\, \boldsymbol{n} \mid \ \boldsymbol{n\ is\ a\ natural\ number}\ \}$, **Set- Builder method**

Examples:

Describe the following sets using Listing method (if possible).

a) $P = \{\, n \mid \ n\ is\ a\ natural\ number\ less\ than\ 8\ \}$

b) $S = \{\, x \mid x\ \ is\ \ a\ natural\ \ number\ whose\ \ square\ is\ less\ than\ 25\}$

c) $R = \{x \mid x\ is\ a\ real\ number\ between\ 0\ and\ 2\ \}$

Notations: If a is an element of a set S, we write $a \in S$.

If a is not an element of a set S, we write $a \notin S$.

Examples:

Let $S = \{1,\ 2,\ 3,\ 4,\ 5,\ \ 6,\ 7,\ 8,\ 9\,\}$, then $9 \in S$ and $0 \notin S$.

Definition: (Equal Sets)

Two sets are said to be equal if they contain the same elements.

Examples:

a) $A = \{\, a,\ \ b,\ \ c,\ \ d\,\}$ and $B = \{\, d,\ \ b,\ \ c,\ \ a\,\}$ are equal sets

b) Let, $\boldsymbol{M}\ = The$ set of natural numbers 1 through 100 and

$\boldsymbol{P}\ = The$ set of counting numbers less than 101.

$\boldsymbol{M}$ and $\boldsymbol{P}$ are equal sets

Subsets

Definition: (**Subset**)

A set **A** is said to be a subset of a set **B** if every element of set **A** is also an element of set **B**.

Examples:

1) Let $A = \{1,\ 2,\ 3\,\}\ and\ \ B = \{\, a,\ 1,\ 2,\ 3\,\}$. Since every element of set A is also in B

A is a subset of B

Notation: $\boldsymbol{A} \subseteq \boldsymbol{B}$ means **A** is a **subset** of **B**

2) Let $D = \{\, 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ a,\ b,\ c,\ d,\ e,\ g\,\}$. Answer the following as True or False.

a) $\{0,\ g\,\} \subseteq D$ b) $\{0,\ 1,\ 3,\ a\,\} \subseteq D$ c) $\{0,\ 1,\ 6,\ a,\ f\,\} \subseteq D$

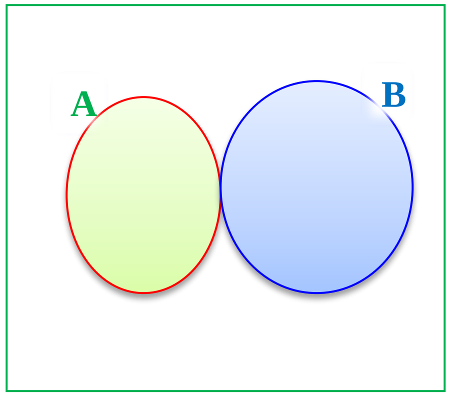
3) Let $\boldsymbol{N} = \{1,\ 2,\ 3,\ \ .\ .\ .\ \}$, $B = \{n \mid n\ is\ an\ odd\ natural\ number\,\}$, and

$C = \{x \mid x\ is\ a\ prime\ number\,\}$. Answer True or False

a) $B \subseteq C$ b) $N \subseteq B$ c) $B \subseteq N$ d) $C \subseteq N$

Pictorial Representation of a Set: Venn Diagrams

Pictorially, a non-empty set is represented by a **circle-like closed figure** inside a **bigger rectangle**. This is called a **Venn diagram. See fig below**



Some properties of subset:

a) **Empty set** is a **subset** of **any set**, that is $\{\,\} \subseteq \boldsymbol{A}$ for any set **A**; thus $\{\,\} \subseteq \{\,\}$

b) **Any set** is a **subset** of **itself**, that is for any set **A**, $\boldsymbol{A} \subseteq \boldsymbol{A}$

c) **A = B**, if and only if $\boldsymbol{A} \subseteq \boldsymbol{B}$ and $\boldsymbol{B} \subseteq \boldsymbol{A}$

Operation on Sets

There are three types of set operations; **Intersection** denoted by $\cap$, **union** denoted by $\cup$, and **complementation**.

Definitions: Let A and be sets

1) **The union** of **A** and **B** is denoted by $\boldsymbol{A} \cup \boldsymbol{B}$ and is defined as the set of all elements that are in **A or B.** That is: $\cup\, \boldsymbol{B} = \{\boldsymbol{x} : \boldsymbol{x} \in \boldsymbol{A} \text{ or } \boldsymbol{x} \in \boldsymbol{B}\}$ **.**

2) **The intersection** of **A** and **B** is denoted by $\boldsymbol{A} \cap \boldsymbol{B}$ and is defined as the set of all elements that are in **A and B.** That is: $\cap\, \boldsymbol{B} = \{\boldsymbol{x} : \boldsymbol{x} \in \boldsymbol{A} \text{ and } \boldsymbol{x} \in \boldsymbol{B}\}$ **.**

3) **The Complement** of **B in A** is denoted by $\boldsymbol{A} - \boldsymbol{B}$ ***or*** $\boldsymbol{A} \setminus \boldsymbol{B}$ and is defined as the set of all elements that are **in A but not in B.** That is: $\boldsymbol{A} \setminus \boldsymbol{B} = \{\boldsymbol{x} : \boldsymbol{x} \in \boldsymbol{A} \text{ and } \boldsymbol{x} \notin \boldsymbol{B}\}$**.**

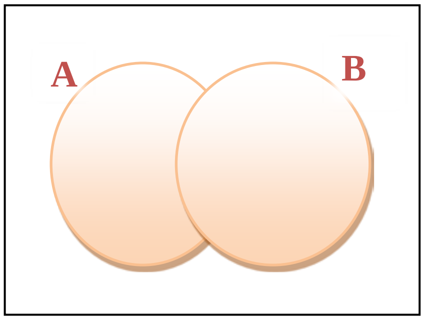
4) The **absolute complement** of set **A** denoted by $\boldsymbol{A}'$ and is defined by:

$$\boldsymbol{A}' = \{\boldsymbol{x} : \boldsymbol{x} \in \boldsymbol{U} \text{ and } \boldsymbol{x} \notin \boldsymbol{A}\}$$, **here U is the universal set**

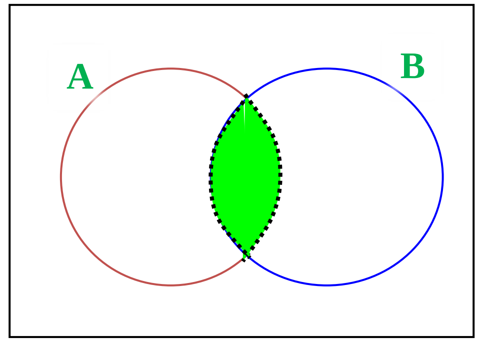
Examples: Venn Diagrams

The Universal Set is represented by a **rectangle.** The shaded regions represent, respectively, the **union**, **intersection** and **complement** of the sets $\boldsymbol{A}$ *and* $\boldsymbol{B}$.

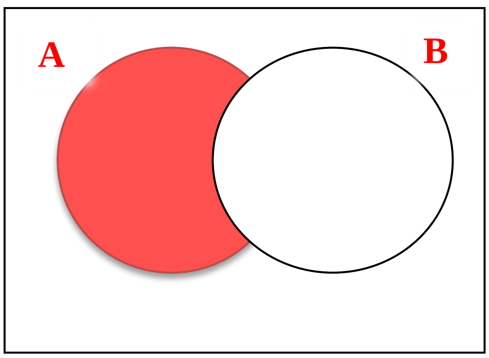
a) $\boldsymbol{A \cup B}$



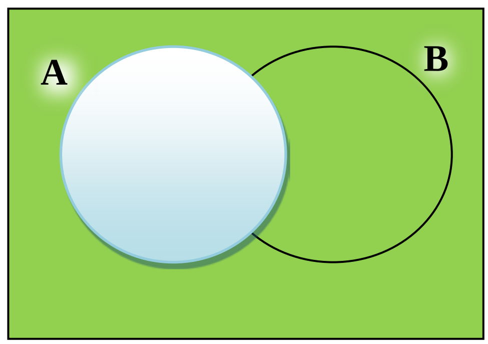
b) $\boldsymbol{A \cap B}$



c) $\boldsymbol{A - B}$



d) $\boldsymbol{A'}$



Examples 1: Let A, B, and C be sets given as follows

$$\boldsymbol{A} = \{-3, -1, 1, 3, 5, 7\}$$

$$\boldsymbol{B} = \{x : x \text{ is an even natural number less than } 6\}$$

$$\boldsymbol{C} = A \text{ set consisting of squares of the first two natural numbers}$$

Compute: a) $\boldsymbol{A \cup B}$ **b)** $\boldsymbol{A \cap B}$ **c)** $\boldsymbol{A - B}$ **d)** $\boldsymbol{B - C}$

e) $(\boldsymbol{A \cup B}) \cup \boldsymbol{C}$ **f)** $\boldsymbol{A} - (\boldsymbol{B \cup C})$ **g)** $(\boldsymbol{A \cap B}) \cup \boldsymbol{C}$

The Real Number System

The Set of Real Numbers **R** is made up **two** disjoint set of Numbers:

- The **Set of Rational Numbers** and
- The **Set of Irrational Numbers**

The Rational Numbers

Definition: (Rational Numbers)

A Rational Number is a number that **can** be written in the form $\boldsymbol{a/b}$**;** $\boldsymbol{a}$ and $\boldsymbol{b}$ **integers,** $\boldsymbol{b \neq 0}$**.**
In other words, a **Rational Number** is a number the **can be written** in a **fraction form**

Examples: Rational Numbers

a) -5, 11, 5/4, 22/7, 111/87, 0, -121, -1/3, 1/3, etc.

b) 0.333…, 5.33, -3.65, 0.242424… $= 0.\overline{24}$, 3.612612612…$= 3.\overline{612}$, etc.

Decimal Representation of a Rational Number

A Rational Number has a **decimal representation** that either **terminates** or **repeats**.

Example 1: Decimal Numbers

a) 23 = 23.0 Terminating decimal

b) 1.253 Terminating decimal

c) 1.333… Repeating Decimal

d) 3.612612612…$= 3.\overline{612}$ Repeating Decimal

e) Any integer is a rational number

Example 2: Write the following numbers in fraction form

a) 1.33

b) 1.333…

c) -2.455

d) $3.\overline{612}$

e) $0.\overline{12}$

Definition: (Irrational Numbers)

An **Irrational Number** is a number that **cannot** be written in the form $\boldsymbol{a/b}$**;** $\boldsymbol{a}$ and $\boldsymbol{b}$ **integers,** $\boldsymbol{b \neq 0}$.

An **Irrational Number Cannot** be **written** in a **fraction form**

Example 3: Examples of Irrational numbers

a) **1.01001000100001…**

b) **0.12345…**

c) **– 4.110111011110…**

d) $\boldsymbol{\pi}$

e) $\sqrt{\mathbf{2}}$

f) $\boldsymbol{e}$

g) $\sqrt[3]{\mathbf{7}}$

Decimal Representation of an Irrational Number

An **Irrational Number** has a **decimal representation** that **neither terminates nor repeats**

Example 4:

a) $\sqrt{2} = 1.41421356237. . .$

b) – 4.110111011110…

c) $e = 2.71828182845 . . .$

d) $\pi = 3.14159265358 \ . . .$

Example 5: Show that $\sqrt{2}$ cannot be written as a fraction.

Important Notations of Set of Numbers

$\mathbb{R}$ **–** Denotes the set of **Real numbers**

$\mathbb{Q}$ **–** Denotes the set of **Rational numbers**

$\mathbb{Z}$ **–** Denotes the set of **Integers**

$\mathbb{W}$**–** Denotes the set of **Whole numbers**

$\mathbb{Z}$ **–** Denotes the set of **Natural numbers**

Summary Chart of the Number Systems