Notes on Sets

A *set* is a collection of distinct objects, none of which is the set itself.

The objects in a set are called the *elements* of the set. We use capital letters for sets and lowercase letters for elements of sets.

Sets can be described in different methods:

1. in *words*, describing the elements

2. by listing the elements in between curly brackets (roster method)

Example. S is the set of colors of regular M & Ms.
In roster method, $S = \{$red, orange, yellow, green, blue, brown $\}$

Example. Let A be the set of letters in the alphabet. Express A in roster method.
$A = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$.

Example. Describe the set $D = \{$Canada, Mexico, USA$\}$ in words.
D is the set of countries in the North American continent.

If a is an element of a set S, we write $a \in S$. We list the elements in a set within curly brackets, $\{\}$.
Example. Let S be the set given by $S = \{1, 2, 3, 4, 5\}$. We can say $2 \in S$, but $7 \notin S$.

Note the order in which we list the elements of a set does not matter; S could be written as $\{2, 3, 4, 1, 5\}$ and would still denote the same set.

Example. $D =$ the set of countries in the North American continent. Mexico $\in D$, Canada $\in D$, but Spain $\notin D$.
Example. L is the set of letters in the word "superstition." List the elements of L.
$L = \{s, u, p, e, r, t, i, o, n\}$
Note that each element is listed only once, even if it "counts" toward the set multiple times.
$S = \{1, 2, 3, 4, 5\} = \{1, 2, 1, 3, 5, 4, 2\}$. We will
Example. Let $\emptyset$ denote the set with no elements, that is, $\emptyset = \{\}$. This will be a very important set!

Definition 0.1. Let B and S be sets. We say B is a *subset* of S, and write $B \subset S$, if every element of B is also an element of S.

Example. Let $B = \{1, 2, 4\}$, $S = \{1, 2, 3, 4, 5\}$. Then $B \subset S$ because $1, 2$, and 4 are all elements of S. The set with one element, 2 is a subset of B and a subset of S because 2 is an element in both B and S. Also, S is a subset of itself, because everything in S is in S! And $\emptyset$ is a subset of every set, because since it has no elements, all its elements are in every set.
Example. The set of vowels, $\{a, e, i, o, u\}$ is a subset of the alphabet set A defined above.

Example. Let us consider the set $F = \{a, b, c\}$. List all the subsets of F.
Ans: $\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$.

Q: How many subsets are there for a set S with n elements?
A: 2^n. There are 2^n subsets of a set S with n elements.
How do we see this is true?
For each element in the set, you can choose whether or not to include it in any given subset. So for each element, there are 2 choices, include, or don't. We have n elements so the number of subsets will be $2 \cdot 2 \cdot 2 \cdots 2 \cdot 2 = 2^n$ total subsets.

Example 0.2. Let $S = \{1, 2, 3, 4, 5\}$. How many subsets does S have?
There are 5 elements in the set, so it has $2^5 = 32$ subsets.

Definition 0.3. Let S and T be sets that are both subsets of a larger, universal set U. Then we define the *intersection* of S and T to be the set $S \cap T$ made up of all elements of U that are both in S and in T.

We define the *union* of S and T to be the set $S \cup T$ made up of all elements of U that are in either S or in T or in both.

We define the *complement* of S in U to be the set $\bar{S}$ made up of all elements in U that are NOT in S. Notice that $S \cup \bar{S} = U$ and $S \cap \bar{S} = \emptyset$.

Example Let $T = \{1,3,5,7\}$, $S = \{1,2,3,4,5\}$ be considered as subsets of $U = \{1,2,3,4,5,6,7,8\}$. Then

$S \cap T = \{1,3,5\}$ and $S \cup T\{1,2,3,4,5,7\}$.

The complement of S in U is $\bar{S} = \{6,7,8\}$. The complement of T in U is $\bar{T} = \{2,4,6,7\}$

$\bar{S} \cap \bar{T} = \{6,7\}$

$\bar{S} \cup \bar{T} = \{2,4,6,7,8\}$

$\bar{S} \cup T = \{1,3,5,6,7,8\}$

$S \cap \bar{T} = \{2,4\}$

$S \cap \bar{S} = \emptyset$

$S \cup \bar{S} = \{1,2,3,4,5,6,7,8\} = U$

These last two statements are always true, no matter what the set S is.

We say two sets are *equal* and write $S = T$, if $S \subset T$ and $T \subset S$.

Example $\{0,2,4,6,8\} = \{8,0,6,2,4\}$