#418701

Topic: Basics of Straight Lines

Find the slope of the line, which makes an angle of 30^o with the positive direction of y-axis measured anticlockwise.

Solution

If a line makes an angle of 30° with the positive direction of the y-axis measured anticlockwise, then the angle made by the line with the positive direction of the x-axis measured anticlockwise is $90^\circ + 30^\circ = 120^\circ$.

Thus, the slope of the given line is

$\tan 120^\circ = \tan(180^\circ - 60^\circ) = -\tan 60^\circ = -\sqrt{3}$

#418730

Topic: Basics of Straight Lines

Without using distance formula, show that points $(2, 1), (4, 0), (3, 3)$ and $(3, 2)$ are the vertices of a parallelogram

Solution

Let points $(2, 1), (4, 0), (3, 3)$ and $(-3, 2)$ be respectively denoted by A, B, C and D.

Now, slope of $AB = \dfrac{0+1}{4+2} = \dfrac{1}{6}$

Slope of $CD = \dfrac{2-3}{-3-3} = \dfrac{-1}{-6} = \dfrac{1}{6}$

$\Rightarrow$ slope of $AB =$ slope of CD

$\Rightarrow$ AB and CD are parallel to each other

Also, slope of $BC = \dfrac{3-0}{3-4} = \dfrac{3}{-1} = -3$

Slope of $AD = \dfrac{2+1}{-3+2} = \dfrac{3}{-1} = -3$

$\Rightarrow$ slope of $BC =$ slope of AD

$\Rightarrow$ BC and AD are parallel to each other.

Therefore, both are pairs of opposite sides of quadrilateral $ABCD$ are parallel.

Hence $ABCD$ is parallelogram.

#418765

Topic: Basics of Straight Lines

The slope of a line is double of the slope of another line. If tangent of the angle between them is $\dfrac{1}{3}$, find the slopes of the lines.

Solution

Let m_1 and m be the slopes of two given line such that $m_1 = 2m$.

We know that if θ is the angle between the lines l_1 and l_2 with slopes m_1 and m_2,

then $\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|$

It is given that the tangent of the angle between the two lines is $\dfrac{1}{3}$

$\therefore \dfrac{1}{3} = \left|\dfrac{m - 2m}{1 + (2m)m}\right|$

$\Rightarrow \dfrac{1}{3} = \left|\dfrac{-m}{1 + 2m^2}\right|$

$\Rightarrow \dfrac{1}{3} = \dfrac{-m}{1 + 2m^2}$ or $\dfrac{1}{3} = -\left(\dfrac{-m}{1 + 2m^2}\right) = \dfrac{m}{1 + 2m^2}$

$\Rightarrow 2m^2 + 3m + 1 = 0$ or $2m^2 - 3m + 1 = 0$

$\Rightarrow m = -1, -\dfrac{1}{2}$ or $m = 1, \dfrac{1}{2}$

Corresponding $m_1 = -2, -1, 2, 1$

#418800

Topic: Basics of Straight Lines

A line passes through $\left(x_1, y_1\right)$ and (h, k). If slope of the line is m, show that $k - y_1 = m\left(h - x_1\right)$

Solution

The slope of line passing through (x_1, y_1) and (h, k) is $m = \dfrac{k - y_1}{h - x_1}$.

Thus required equation is given by,

$\Rightarrow k - y_1 = m\left(h - x_1\right)$

#418814

Topic: Various Forms of Equation of Line

If three points $(h, 0)$, (a, b) and $(0, k)$ lie on a line, show that $\dfrac{a}{h} + \dfrac{b}{k} = 1$.

Solution

If the points $A(h, 0)$, $B(a, b)$ and $C(0, k)$ lie on line, then slope of AB = slope of BC

$\dfrac{b - 0}{a - h} = \dfrac{k - b}{0 - a}$

$\Rightarrow \dfrac{b}{a - h} = \dfrac{k - b}{-a}$

$\Rightarrow -ab = (k - b)(a - h)$

$\Rightarrow -ab = ka - kh - ab + bh$

$\Rightarrow ka + bh = kh$

On dividing both sides by kh, we obtain

$\dfrac{ka}{kh} + \dfrac{bh}{kh} = \dfrac{kh}{kh}$

$\Rightarrow \dfrac{a}{h} + \dfrac{b}{k} = 1$

#418882

Topic: Basics of Straight Lines

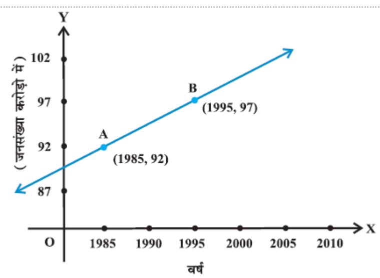

Fig 10.10

Consider the following population and year graph, find the slope of the line AB and using it, find what will be the population in the year 2010?

Solution

Since line AB passes through point $A(1985, 92)$ and $B(1995, 97)$, its slope is $\frac{97-92}{1995-1985} = \frac{5}{10} = \frac{1}{2}$.

Now, let y be the population in the year 2010. Then according to the given graph line AB must pass through point $C(2010, y)$.

$\therefore$ slope of AB = slope of BC

$\Rightarrow \frac{1}{2} = \frac{y-97}{2010-1995}$

$\Rightarrow \frac{1}{2} = \frac{y-97}{15}$

$\Rightarrow \frac{15}{2} = y - 97$

$\Rightarrow y - 97 = 7.5$

$\Rightarrow y = 7.5 + 97 = 104.5$

Thus, the slope of line AB is $\frac{1}{2}$, while in the year 2010 the population will be 104.5 crores.

#419022

Topic: Various Forms of Equation of Line

Find equation of the line passing through the point $(4, 3)$ with slope $\frac{1}{2}$

Solution

We know that the equation of line passing through point (x_0, y_0) whose slope is m is

$(y - y_0) = m(x - x_0)$

Thus the equation of line passing through the point $(-4, 3)$ whose slope is $\frac{1}{2}$ is,

$(y-3) = \frac{1}{2}(x+4)$

$\Rightarrow 2(y-3) = (x+4)$

$\Rightarrow x - 2y + 10 = 0$

#419034

Topic: Various Forms of Equation of Line

Find equation of the line passing through $(0, 0)$ with slope m.

Solution

We know that, the equation of line passing through point (x_0, y_0) whose slope is m is

$(y - y_0) = m(x - x_0)$

Thus the equation of the line passing through the point $(0, 0)$ with slope m is

$(y-0) = m(x-0) \Rightarrow y = mx$

#419152

Topic: Various Forms of Equation of Line

Find the equation of the line passing through $(2, 2\sqrt{3})$ and inclined with the x-axis at an angle of 75°

Solution

We have, $m = \tan 75°$

$\Rightarrow m = \tan(45° + 30°) = \dfrac{\tan 45° + 30°}{1 - \tan 45°.\tan 30°}$

$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - 1\frac{1}{\sqrt{3}}} = \dfrac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

Thus equation of line passing through $(2, 2\sqrt{3})$ and inclined with the x-axis at an angle of $75°$ is given by,

$(y - 2\sqrt{3}) = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}(x - 2)$

$(y - 2\sqrt{3})(\sqrt{3} - 1) = (\sqrt{3} + 1)(x - 2)$

$y(\sqrt{3} - 1) - 2\sqrt{3}(\sqrt{3} - 1) = x(\sqrt{3} + 1) - 2(\sqrt{3} + 1)$

$(\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 2\sqrt{3} + 2 - 6 + 2\sqrt{3}$

$(\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 4\sqrt{3} - 4$

i.e; $(\sqrt{3} + 1)x - (\sqrt{3} - 1)y = 4(\sqrt{3} - 1)$

#419173

Topic: Various Forms of Equation of Line

Find the equation of the line intersecting the y-axis at a distance of 2 units above the origin and making an angle of $30°$ with positive direction of the x-axis.

Solution

We know that if a line with slope m makes y-intercept c, then equation of line is given as $y = mx + c$.

Here. we have given, $c = 2$ and $m = \tan 30° = \dfrac{1}{\sqrt{3}}$

Thus the required equation of the given line is $y = \dfrac{1}{\sqrt{3}}x + 2$

$\Rightarrow y = \dfrac{x + 2\sqrt{3}}{\sqrt{3}}$

$\Rightarrow x - \sqrt{3}y + 2\sqrt{3} = 0$

#419320

Topic: Various Forms of Equation of Line

Find the equation of the line perpendicular distance from the origin is 5 units and the angle made by the perpendicular with the positive x-axis is $30°$.

Solution

We know that, If p is the length of the normal from origin to a line and w is the angle made by the normal with the positive direction of x-axis then the equation of the line is given by $x\cos w + y\sin w = p$.

Here, $p = 5$ units and $w = 30°$

Thus, the required equation of the given line is

$x\cos 30° + y\sin 30° = 5$

$\Rightarrow x\dfrac{\sqrt{3}}{2} + y.\dfrac{1}{2} = 5$

$\Rightarrow \sqrt{3}x + y = 10$

#419338

Topic: Various Forms of Equation of Line

The vertices of PQR are $P(2, 1)$, $Q(-2, 3)$ and $R(4, 5)$. Find equation of the median through the vertex R.

Solution

Let L be the mid-point of PQ.

Thus by mid-point formula, the coordinates of point L is given by,

$$\left(\frac{2-2}{2}, \frac{1+3}{2}\right) = (0, 2)$$

Therefore, the equation of RL (median through vertex R) is given by,

$$(y-5) = \frac{2-5}{0-4}(x-4)$$

$$\Rightarrow y-5 = \frac{-3}{-4}(x-4)$$

$$\Rightarrow 4(y-5) = 3(x-4)$$

$$\Rightarrow 4y-20 = 3x-12$$

$$\Rightarrow 3x-4y+8 = 0$$

#419358

Topic: Various Forms of Equation of Line

Find the equation of the line passing through $(3, 5)$ and perpendicular to the line through the points $(2, 5)$ and $(3, 6)$.

Solution

The slope of line joining points $(2, 5)$ and $(3, 6)$ is $m = \dfrac{6-5}{-3-2} = \dfrac{1}{-5}$

We know that, the two non-vertical lines are perpendicular to each other only if and only if their slopes are negative reciprocals of each other.

Therefore, slope of line perpendicular to the line through the points $(2, 3)$ and $(-3, 6)$

$$= -\frac{1}{m} = -\left(\frac{1}{\frac{-1}{5}}\right) = 5$$

Now equation of the line passing through points $(-3, 5)$ whose slope is 5 is $(y-5) = 5(x+3)$.

$$\Rightarrow y-5 = 5x+15$$

$$\Rightarrow 5x-y+20 = 0$$

#419379

Topic: Basics of Straight Lines

A line perpendicular to the line segment joining the points $(1, 0)$ and $(2, 3)$ divides it in the ratio $1:n$. Find the equation of the line.

Solution

According to the section formula, the coordinates of the points that divides the line segment joining the points $(1, 0)$ and $(2, 3)$ in the ratio $1:n$ is given by $\left\{\dfrac{n(1)+1(2)}{1+n}, \dfrac{n(0)+1(3)}{1+n}\right\}$

$$= \left\{\frac{n+2}{n+1}, \frac{3}{n+1}\right\}$$

The slope of the line joining the points $(1, 0)$ and $(2, 3)$ is $m = \dfrac{3-0}{2-1} = 3$.

We know that two non-vertical lines are perpendicular to each other if and only if their slpoes are negative reciprocals of each other.

Therefore, slope of the line that is perpendicular to the line joining the points $(1, 0)$ and $(2, 3)$ is $= -\dfrac{1}{m} = -\dfrac{1}{3}$

Now the equation of the line passing through $= \left\{\dfrac{n+2}{n+1}, \dfrac{3}{n+1}\right\}$ and whose slope is $-\dfrac{1}{3}$ is given by, $\left\{y - \dfrac{3}{n+1}\right\} = \dfrac{-1}{3}\left\{x - \dfrac{n+2}{n+1}\right\}$

$$\Rightarrow 3[(n+1)y-3] = -[x(n+1)-(n+2)]$$

$$\Rightarrow 3(n+1)y-9 = -(n+1)x+n+2$$

$$\Rightarrow (1+n)x+3(1+n)y = n+11$$

#419392

Topic: Various Forms of Equation of Line

Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point $(2, 3)$.

Solution

The equation of a line in the intercept form is $\frac{x}{a}+\frac{y}{b}=1$ ..(i)

where a and b are the intercepts on x and y axis respectively.

It is given that the line cuts off equal intercepts on both the axes. This means that $a=b$.

Accordingly, equation(i) reduces to,

$\frac{x}{a}+\frac{y}{b}=1 \Rightarrow x+y=a$ ...(ii)

Since the given line passes through point $(2,3)$, equation(ii) reduces to $2+3=a \Rightarrow a=5$

On substituting the value of a in equation (ii), we obtain

$x+y=5$ which is the required equation of the line.

#419537

Topic: Various Forms of Equation of Line

Find equation of the line passing through the point $(2,2)$ and cutting off intercepts on the axes whose sum is 9.

Solution

The equation of a line in the intercept form is

$\frac{x}{a}+\frac{y}{b}=1$.......(1)

where a and b are intercepts on x and y axes respectively.

it is given that $a+b=9 \Rightarrow b=9-a$.......(2)

From equation (1), we obtain

$\frac{x}{a}+\frac{y}{9-a}=1$

Also it is passing through $(2,2)$

$\frac{2}{a}+\frac{2}{9-a}=1$

$\Rightarrow 2\left\{\frac{1}{a}+\frac{1}{9-a}\right\}=1$

$\Rightarrow 2\left\{\frac{9-a+a}{a(9-a)}\right\}=1$

$\Rightarrow \frac{18}{9a-a^2}=1$

$\Rightarrow 18=9a-a^2$

$\Rightarrow a^2-6a-3a+18=0$

$\Rightarrow a(a-6)-3(a-6)=0$

$\Rightarrow (a-6)(a-3)=0$

$\Rightarrow a=6$ or $a=3$

If $a=3 \Rightarrow b=9-3=6$, then the equation of the line is

$\frac{x}{3}+\frac{y}{6}=1 \Rightarrow 2x+y-6=0$

And if $a=6 \Rightarrow b=9-6=3$, then the equation of the line is

$\frac{x}{6}+\frac{y}{3}=1 \Rightarrow x+2y-6=0$

#419540

Topic: Various Forms of Equation of Line

Find equation of the line through the point $(0,2)$ making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

Solution

The slope of the line making an angle $\frac{2\pi}{3}$ with the positive x-axis is $m = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

Now, the equation of the line passing through point $(0, 2)$ and having a slope $-\sqrt{3}$ is $(y-2) = -\sqrt{3}(x-0)$

$y - 2 = -\sqrt{3x}$

i.e; $\sqrt{3x} + y - 2 = 0$

The slope of line parallel to line $\sqrt{3x} + y - 2 = 0$ is $-\sqrt{3}$

It is given that the line parallel to line $\sqrt{3x} + y - 2 = 0$ crosses the y-axis 2 units below the origin

i.e;it passes through point $(0, -2)$

Hence, the equation of the line passing point $(0, -2)$ and having a slope $-\sqrt{3}$ is,

$y - (-2) = -\sqrt{3}(x-0)$

$\Rightarrow y + 2 = -\sqrt{3}x$

$\Rightarrow \sqrt{3}x + y + 2 = 0$

#419550

Topic: Various Forms of Equation of Line

The perpendicular from the origin to a line meets it at the point $(2, 9)$, find the equation of the line.

Solution

The slope of joining the origin $(0, 0)$ and point $(-2, 9)$ is,

$m_1 = \frac{9-0}{-2-0} = -\frac{9}{2}$

Accordingly the slope of the line perpendicular to the line joining the origin and point $(-2, 9)$ is,

$m_2 = -\frac{1}{m_1} = -\frac{1}{\left(-\frac{9}{2}\right)} = \frac{2}{9}$

Thus the equation of the line passing through point $(-2, 9)$ and having a slope m_2 is,

$(y-9) = \frac{2}{9}(x+2)$

$\Rightarrow 9y - 81 = 2x + 4 \Rightarrow 2x - 9y + 85 = 0$

#419571

Topic: Various Forms of Equation of Line

The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs 14/litre and 1220 litres of milk each week at Rs 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?

Solution

Assuming selling price per litre along the x-axis and demand along the y-axis. we have two points $(14, 980)$ and $(16, 1220)$ in the XY plane that satisfy the linear relationship between selling price and demand.

Therefore, the linear relationship between selling price per litre and demand is the equation of the line passing through points $(14, 980)$ and $(16, 1220)$, which is given by,

$y - 980 = \frac{1220-980}{16-14}(x-14)$

$y - 980 = \frac{240}{2}(x-14)$

$y - 980 = 120(x-14)$

i.e $y = 120(x-4) + 980$

when $x =$ Rs 17/litre,

$y = 120(17-14) + 980$

$\Rightarrow y = 120 \cdot 3 + 980 = 360 + 980 = 1340$

Thus the owner of the milk store could sell 1340 litres milk weekly at Rs 17/litre

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#419575

Topic: Various Forms of Equation of Line

A rod of length 12 cm moves with it ends always touching the coordinate axes. Dertermine the equation of the locus of a point P on the rod which is 3 cm from the end in contact with the x-axis

Solution

Let AB be the rod making an angle θ with positive direction of x-axis and $P(x, y)$ be the point on it such that $AP = 3cm$

Now, $PB = AB - AP = (12 - 3)cm = 9cm \quad (AB = 12cm)$

Draw $PQ \perp OY$ and $PR \perp OX$

In $\triangle PBQ$,

$\cos\theta = \frac{PQ}{PB} = \frac{x}{9}$

In $\triangle PRA$,

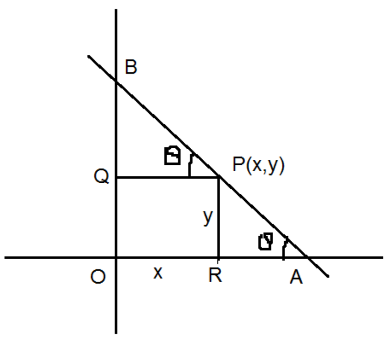
$\sin\theta = \frac{PR}{PA} = \frac{y}{3}$

Since $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\frac{y}{3}\right)^2 + \left(\frac{x}{9}\right)^2 = 1$

$\Rightarrow \frac{x^2}{81} + \frac{y^2}{9} = 1$

Thus the equation of the locus of point P on the rod is $\frac{x^2}{81} + \frac{y^2}{9} = 1$



#419584

Topic: Various Forms of Equation of Line

$P(a, b)$ is the mid-point of a line segment between axes. Show that equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$

Solution

Let the coordinates of A and B be $(0, y)$ and $(x, 0)$ respectively.

Since $P(a, b)$ is the midpoint of AB

$\left\{\frac{0+x}{2}, \frac{y+0}{2}\right\} = (a, b)$

$\Rightarrow \left(\frac{x}{2}, \frac{y}{2}\right) = (a, b)$

$\Rightarrow \frac{x}{2} = a$ and $\frac{y}{2} = b$

$\therefore x = 2a$ and $y = 2b$

Thus the respective coordinates of A and B are $(0, 2b)$ and $(2a, 0)$

The equation of the line passing through points $(0, 2b)$ and $(2a, 0)$ is,

$(y - 2b) = \frac{(0 - 2b)}{(2a - 0)}(x - 0)$

$\Rightarrow y - 2b = \frac{-2b}{2a}(x)$

$\Rightarrow a(y - 2b) = -bx$

$\Rightarrow ay - 2ab = -bx$

$\Rightarrow bx + ay = 2ab$

On dividing both sides by ab, we obtain

$\frac{bx}{ab} + \frac{ay}{ab} = \frac{2ab}{ab}$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 2$

Thus equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$

#419607

Topic: Various Forms of Equation of Line

By using the concept of equation of a line, prove that the three points (3, 0),(-2, -2) and (8, 2) are collinear.

Solution

The equation of the line passing through points $(3, 0)$ and $(-2, -2)$ is

$(y - 0) = \frac{(-2 - 0)}{(-2 - 3)}(x - 3)$

$\Rightarrow y = \frac{-2}{-5}(x - 3)$

$\Rightarrow 2x - 5y = 6$......(1)

It is observed that point $(8, 2)$ also satisfying eqn. (1)

Hence given points are collinear.

#419663

Topic: Various Forms of Equation of Line

Reduce the following equations into slope - intercept form and find their slopes and the y - intercepts.

(i)$x + 7y = 0$

(ii)$6x + 3y - 5 = 0$

(iii)$y = 0$

Solution

(i) The given equation is $x + 7y = 0$

It can be written as

$y = -\frac{1}{7}x + 0$

This equation is of the form y $= mx + c$, where $m = -\frac{1}{7}$ and $c = 0$

Therefore equation (1) is in the slope-intercept form,where the slope and the y-intercept are $-\frac{1}{7}$ and 0 respectively.

(ii) The given equation is $6x + 3y - 5 = 0$ it can be written as

$y = \frac{1}{3}(-6x + 5)$

$y = 2x + \frac{5}{3}$

This equation is of the form y $= mx + c$,where $m = -2$ and $c = \frac{5}{3}$

Therefore equation (2) is in the slope intercept form,where the slope and the y-intercept are -2 and $\frac{5}{3}$ respectively.

(iii) The given equation is $y = 0$.

It can be written as

$y = 0.x + 0$

This equation is of the form $y = mx + c$,where $m = 0$ and $c = 0$.

Therefore equation (3) is in the slope-intercept form,where the slope and y-intercept are 0 and 0 respectively.

#419675

Topic: Various Forms of Equation of Line

Reduce the following equations into intercept form and find their intercepts on

the axes.

(i)$3x + 2y - 12 = 0$

(ii)$4x - 3y = 6$

(iii)$3y + 2 = 0$

Solution

(i)The equation is $3x + 2y - 12 = 0$

It can be written as

$3x + 2y = 12$

$\frac{3x}{12} + \frac{2y}{12} = 1$

i.e; $\frac{x}{4} + \frac{y}{6} = 1$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = 4$ and $b = 6$.

Therefore equation (1) is in the intercept form,where the intercepts on the x and y axes are 4 and 6 respectively.

(ii)The given equation is $4x - 3y = 6$

It can be written as

$\frac{4x}{6} - \frac{3y}{6} = 1$

$\frac{2x}{3} - \frac{y}{2} = 1$

i.e; $\frac{x}{\left(\frac{3}{2}\right)} + \frac{y}{(-2)} = 1$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = \frac{3}{2}$ and $b = -2$

Therefore equation (2) is in the intercept form,where the intercepts on the x and y axes are $\frac{3}{2}$ and -2 respectively.

(iii)The given equation is $3y + 2 = 0$

It can be written as

$3y = -2$

i.e; $\frac{y}{\left(-\frac{2}{3}\right)} = 1$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, where $a = \infty$ and $b = -\frac{2}{3}$

Therefore, equation (3) is in the intercept form, where the intercept on the y-axis is $-\frac{2}{3}$ and it has no intercept on the x-axis.

#419694

Topic: Various Forms of Equation of Line

Reduce the following equations into normal form. Find their perpendicular distances

from the origin and angle between perpendicular and the positive x-axis.

(i) $x - \sqrt{3}y + 8 = 0$

(ii) $y - 2 = 0$

(iii) $x - y = 4$

Solution

(i) The given equation is

$x-\sqrt{3}y+8=0$

which can be written as

$x-\sqrt{3}y=-8$

$\Rightarrow -x+\sqrt{3}y=8$

On dividing both sides by $\sqrt{(-1)^2+(\sqrt{3})^2}=\sqrt{4}=2$, we get

$-\frac{x}{2}+\frac{\sqrt{3}}{2}y=\frac{8}{2}$

$\Rightarrow \left(-\frac{1}{2}\right)x+\left(\frac{\sqrt{3}}{2}\right)y=4$

$\Rightarrow x\cos 120^\circ+y\sin 120^\circ=4$

which is the normal form.

On comparing it with the normal form of equation of line

$x\cos\alpha+y\sin\alpha=p$, we get

$\alpha=120^\circ$ and $p=4$

So, the perpendicular distance of the line from the origin is 4 and the angle between the perpendicular and the positive x-axis is 120°

(ii) The given equation is

$y-2=0$

which can be written as $0.x+1.y=2$

On dividing both sides by $\sqrt{0^2+1^2}=1$,we get $0.x+1.y=2$

$\Rightarrow x\cos 90^\circ+y\sin 90^\circ=2$

which is the normal form.

On comparing it with the normal form of equation of line $x\cos\alpha+y\sin\alpha=p$,we get

$\alpha=90^\circ$ and $p=2$

So, the perpendicular distance of the line from the origin is 2 and the angle between the perpendicular and the positive x--axis is 90°

(iii) The given equation is $x-y=4$

which can be written as

$1.x+(-1)y=4$

On dividing both sides by $\sqrt{1^2+(-1)^2}=\sqrt{2}$,we get

$\frac{1}{\sqrt{2}}x+\left(-\frac{1}{\sqrt{2}}\right)y=\frac{4}{\sqrt{2}}$

$\Rightarrow x\cos\left(2\pi-\frac{\pi}{4}\right)+y\sin\left(2\pi-\frac{\pi}{4}\right)=2\sqrt{2}$ (Since, cosine is positive and sine is negative in fourth quadrant)

$\Rightarrow x\cos 315^\circ+y\sin 315^\circ=2\sqrt{2}$

which is the normal form.

On comparing it with the normal form of equation of line $x\cos\alpha+y\sin\alpha=p$,we get

$\alpha=315^\circ$ and $p=2\sqrt{2}$

So, the perpendicular distance of the line from the origin is $2\sqrt{2}$, and the angle between the perpendicular and the positive x-axis is 315°

#419759

Topic: Basics of Straight Lines

If the equation of line parallel to the line $3x-4y+2=0$ and passing through the point $(-2,3)$ is $3x-4y+k$, value of k is

Solution

Equation of line parallel to $3x-4y+2=0$ is

given by, $3x-4y+k=0$

Given it also passes through $(-2,3)$

$\Rightarrow 3(-2)-4(3)+k=0 \Rightarrow k=18$

Thus equation of required line is, $3x-4y+18=0$

#419886

Topic: Various Forms of Equation of Line

The line through the points $(h,3)$ and $(4,1)$ intersects the line $7x-9y-19=0$ at right angle. Find the value of h.

Solution

The slope of line passing through points $(h,3)$ and $(4,1)$ is,

$m_1=\frac{1-3}{4-h}=\frac{-2}{4-h}$

and the slope of line $7x-9y-19=0$ is $m_2=\frac{7}{9}$

It is given that the two lines are perpendicular

$\therefore m_1\cdot m_2=-1$

$\Rightarrow\left(\frac{-2}{4-h}\right)\cdot\left(\frac{7}{9}\right)=-1$

$\Rightarrow\frac{-14}{36-9h}=-1$

$\Rightarrow 9h=36-14$

$\Rightarrow h=\frac{22}{9}$

#419891

Topic: Various Forms of Equation of Line

Prove that the line through the point $(x_1,y_1)$and parallel to the line $Ax+By+C=0$ is $A(x-x_1)+B(y-y_1)=0$

Solution

The slope of line $Ax+By+C=0$ is $m=-\frac{A}{B}$

It is known that parallel lines have the same slope

$\therefore$ slope of other line $=m=-\frac{A}{B}$

Thus the equation of the line passing through point(x_1,y_1) and having a slope $m=-\frac{A}{B}$ is,

$y-y_1=m(x-x_1)$

$\Rightarrow y-y_1=-\frac{A}{B}(x-x_1)$

$\Rightarrow B(y-y_1)=A(x-x_1)$

$\Rightarrow A(x-x_1)+B(y-y_1)=0$

Hence, the line through point (x_1,y_1) and parallel to line $Ax+By+C=0$ is $A(x-x_1)+B(y-y_1)=0$

#419892

Topic: Various Forms of Equation of Line

Two lines passing through the point $(2,3)$ intersects each other at an angle of 60°. If slope of one line is 2, find equation of the other line.

Solution

It is given that the slope of the first line $m_1 = 2$

Let the slope of the other line be m_2

The angle between the two lines is $60\ °$

$\therefore \tan 60° = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \sqrt{3} = \left|\dfrac{2 - m_2}{1 + 2m_2}\right|$

$\Rightarrow \sqrt{3} = \pm\left(\dfrac{2 - m_2}{1 + 2m_2}\right)$

$\Rightarrow \sqrt{3} = \dfrac{2 - m_2}{1 + 2m_2} or \sqrt{3} = -\left(\dfrac{2 - m_2}{1 + 2m_2}\right)$

$\Rightarrow \sqrt{3}(1 + 2m_2) = 2 - m_2 or \sqrt{3}(1 + 2m_2) = -(2 - m_2)$

$\Rightarrow \sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2 or \sqrt{3} + 2\sqrt{3m_2} - m_2 = -2$

$\Rightarrow m_2 = \dfrac{2 - \sqrt{3}}{(2\sqrt{3} + 1)} or m_2 = \dfrac{-(2 + \sqrt{3})}{2\sqrt{3} - 1}$

The equation of line passing through point $(2, 3)$ and having a slope of $\dfrac{(2 - \sqrt{3})}{2\sqrt{3} + 1}$ is

$(y - 3) = \dfrac{2 - \sqrt{3}}{2\sqrt{3} + 1}(x - 2)$

$(2\sqrt{3} + 1)y - 3(2\sqrt{3} + 1) = (2 - \sqrt{3})x - 2(2 - \sqrt{3})$

$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}$

In this case the equation of the other line is $(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}$

similarly for another one.

#419926

Topic: Basics of Straight Lines

Find the equation of the right bisector of the line segment joining the points $(3, 4)$ and $(1, 2)$.

Solution

The right bisector of a line segment bisects the line segment at $90°$.

The end-points of the line segment are given as $A(3, 4)$ and $B(-1, 2)$.

Accordingly, mid-point of $AB = \left(\dfrac{3 - 1}{2}, \dfrac{4 + 2}{2}\right) = (1, 3)$

and slope of $AB = \dfrac{2 - 4}{-1 - 3} = \dfrac{-2}{-4} = \dfrac{1}{2}$

$\therefore$ slope of line perpendicular to $AB = -\dfrac{1}{\left(\frac{1}{2}\right)} = -2$

Thus equation of the line passing through $(1, 3)$ and having a slope of -2 is given by,

$(y - 3) = -2(x - 1)$

$\Rightarrow y - 3 = -2x + 2 \Rightarrow 2x + y = 5$

Thus, the required equation of the line is $2x + y = 5$.

#419938

Topic: Various Forms of Equation of Line

The perpendicular from the origin to the line $y = mx + c$ meets it at the point $(-1, 2)$. Find the values of m and c.

Solution

The given equation of line is $y = mx + c$.

It is given that the perpendicular from the origin meets the given line at $(-1, 2)$.

Therefore, the line joining the points$(0, 0)$ and $(-1, 2)$ is perpendicular to the given line.

$\therefore$ slope of the line joining $(0, 0)$ and $(-1, 2)$ is $= \frac{2}{-1} = -2$

The slope of the given line is m.

$\therefore m \times -2 = -1$ [the two lines are perpendicular]

$\Rightarrow m = \frac{1}{2}$

Since point $(-1, 2)$ lies on the given line, it satisfies the equation $y = mx + c$

$\therefore 2 = m(-1) + c$

$\Rightarrow 2 = \frac{1}{2}(-1) + c$

$\Rightarrow c = 2 + \frac{1}{2} = \frac{5}{2}$

Thus, the respective values of m and c are $\frac{1}{2}$ and $\frac{5}{2}$.

#419991

Topic: Distance of Point from a Line

If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Solution

Equation of line with intercepts a and b is,

$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow bx + ay - ab = 0$....(1)

Now if p is the length of the perpendicular from point $(0, 0)$ to line (1),then,

$p = \frac{|b(0) + a(0) - ab|}{\sqrt{b^2 + a^2}}$

$\Rightarrow p = \frac{|-ab|}{\sqrt{a^2 + b^2}}$

On squaring both sides, we obtain

$p^2 = \frac{|-ab|^2}{(\sqrt{a^2 + b^2})^2}$

$\Rightarrow p^2\left(a^2 + b^2\right) = a^2b^2$

$\Rightarrow \frac{a^2 + b^2}{a^2b^2} = \frac{1}{p^2}$

$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

#420205

Topic: Basics of Straight Lines

Find the values of k for which the line$(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$

(a) Parallel to the x-axis,

(b) Parallel to the y-axis,

(c) Passing through the origin

Solution

The given equation of line is

$(k-3)x-(4-k^2)y+k^2-7k+6=0$

(a) If the given line is parallel to the x-axis, then

slope of the given line = slope of the x-axis $=0$

$\Rightarrow \dfrac{(k-3)}{(4-k^2)}=0$

$\Rightarrow k-3=0$

$\Rightarrow k=3$

Thus, if the given line is parallel to the x-axis, then the value of k is 3.

(b) If the given line is parallel to the y-axis, it is vertical, hence,its slope will be undefined.

The slope of the given line is $\dfrac{(k-3)}{(4-k^2)}$

Now,$\dfrac{(k-3)}{(4-k^2)}$ is undefined at $k^2=4$

$\Rightarrow k=\pm 2$

Thus, if the given line is parallel to the y-axis,then the value of k is ± 2.

(c), If the given line is passing through the origin, then point $(0,0)$ satisfies the given equation of line

$(k-3)(0)-(4-k^2)(0)+k^2-7k+6=0$

$\Rightarrow k^2-7k+6=0 \Rightarrow k^2-6k-k+6=0$

$\Rightarrow (k-6)(k-1)=0 \Rightarrow k=1 \; or \; k=6$

Thus, if the given line is passing through the origin, then the value of k is either 1 or 6.

#420326

Topic: Various Forms of Equation of Line

Find the equations of the lines, which cut-off intercepts on the axes whose sum and product are 1 and -6, respectively.

Solution

Let the intercepts cut by the given line on the axes be a and b.

It is given that

$a+b=1$....(1)

and $ab=-6$....(2)

On solving equation (1) and (2), we obtain

$a=3$ and $b=-2$ or $a=-2$ and $b=3$

It is known that the equation of the line whose intercepts on the axes are a and b is $\dfrac{x}{a}+\dfrac{y}{b}=1$ or $bx+ay-ab=0$.

case1: $a=3$ and $b=-2$

In this case the equation of the line is $-2x+3y+6=0 \Rightarrow 2x-3y=6$

case2: $a=-2$ and $b=3$

In this case the equation of line is $3x-2y+6=0 \Rightarrow -3x+2y=6$

Thus, the required equation of the lines are $2x-3y=6$ and $-3x+2y=6$.

#420364

Topic: Distance of Point from a Line

What are the points on the y-axis whose distance from the line $\dfrac{x}{3}+\dfrac{y}{4}=1$ is 4 units.

Solution

Let $(0, b)$ be the point on the y-axis whose distance from line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units.

The given line can be written as $4x + 3y - 12 = 0$....(1)

Thus using distance formula,

$$4 = \frac{|4(0) + 3(b) - 12|}{\sqrt{4^2 + 3^2}}$$

$$\Rightarrow 4 = \frac{|3b - 12|}{5}$$

$$\Rightarrow 20 = |3b - 12|$$

$$\Rightarrow 20 = \pm(3b - 12)$$

$$\Rightarrow 20 = (3b - 12) \text{ or } 20 = -\{3b - 12\}$$

$$\Rightarrow 3b = 20 + 12 \text{ or } 3b = -20 + 12$$

$$\Rightarrow b = \frac{32}{3} \text{ or } b = -\frac{8}{3}$$

Thus, the required points are $\left(0, \frac{32}{3}\right)$ and $\left(0, -\frac{8}{3}\right)$.

#420402

Topic: Distance of Point from a Line

Find perpendicular distance from the origin to the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$

Solution

The equation of the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$ is given by,

$$y - \sin\theta = \frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}(x - \cos\theta)$$

$$x(\sin\phi - \sin\theta) + y(\cos\phi - \cos\theta) + \cos\theta\sin\phi - \cos\theta\sin\theta - \sin\theta\cos\phi + \sin\theta\cos\theta = 0$$

$$x(\sin\theta - \sin\phi) + y(\cos\phi - \cos\theta) + \sin(\phi - \theta) = 0$$

Therefore, the perpendicular distance (d) of the given line from point $(0, 0)$ is

$$d = \frac{|(0)(\sin\theta - \sin\phi) + (0)(\cos\phi - \cos\theta) + \sin(\phi - \theta)|}{\sqrt{(\sin\theta - \sin\phi)^2 + (\cos\phi - \cos\theta)^2}}$$

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{\sin^2\theta + \sin^2\phi - 2\sin\theta\sin\phi + \cos^2\phi + \cos^2\theta - 2\cos\phi\cos\theta}}$$

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{(\sin^2\theta + \cos^2\theta) + (\sin^2\phi + \cos^2\phi) - 2(\sin\theta\sin\phi + \cos\theta\cos\phi)}}$$

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{2(1 - \cos(\phi - \theta))}}$$

$$= \frac{|\sin(\phi - \theta)|}{\sqrt{2\left\{2\sin^2\left(\frac{\phi - \theta}{2}\right)\right\}}}$$

$$= \frac{|\sin(\phi - \theta)|}{\left|2\sin\left(\frac{\phi - \theta}{2}\right)\right|}$$

#420426

Topic: Various Forms of Equation of Line

Find the equation of a line drawn perpendicular to the line $\frac{x}{4} + \frac{y}{6} = 1$ through the point, where it meets the y-axis.

Solution

The equation of the given line is $\frac{x}{4} + \frac{y}{6} = 1$.

This equation can also be written as $3x + 2y - 12 = 0$

$\Rightarrow y = \frac{-3}{2}x + 6,$ which is of the form $y = mx + c$

$\therefore$ slope of given line $= -\frac{3}{2}$

$\therefore$ slope of line perpendicular to given line $= -\frac{1}{\left(-\frac{3}{2}\right)} = \frac{2}{3}$

On substituting $x = 0$ in the given equation of line,

we obtain $\frac{y}{6} = 1 \Rightarrow y = 6$

$\therefore$ the given line intersect the y-axis at $(0, 6)$

Hence, the equation of line that has slope $\frac{2}{3}$ and passes through point $(0, 6)$ is

$(y - 6) = \frac{2}{3}(x - 0)$

$\Rightarrow 3y - 18 = 2x \Rightarrow 2x - 3y + 18 = 0$

Thus, the required equation of the line is $2x - 3y + 18 = 0$.

#420452

Topic: Various Forms of Equation of Line

Find the value of p so that the three lines $3x + y - 2 = 0$, $px + 2y - 3 = 0$ and $2x - y - 3 = 0$ may intersect at one point.

Solution

The equation of the given lines are

$3x + y - 2 = 0$...(1)

$px + 2y - 3 = 0$....(2)

$2x - y - 3 = 0$....(3)

On solving equation (1) and (3) we obtain

$x = 1$ and $y = -1$.

Since these three lines may intersect at one point, the point of intersection of lines(1) and (3) will also satisfy lines(2)

$p(1) + 2(-1) - 3 = 0$

$p - 2 - 3 = 0 \Rightarrow p = 5$.

#420771

Topic: Various Forms of Equation of Line

A ray of light passing through the point $(1, 2)$ reflects on the x-axis at point A and the reflected ray passes through the point $(5, 3)$. Find the coordinates of A.

Solution

Let the coordinates of point A be $(a, 0)$.

Draw a line (AL) perpendicular to the x-axis.

We know that angle of incidence is equal to angle of reflection.

Hence, let $\angle BAL = \angle CAL = \theta$

Let $CAX = \phi$

$\therefore \angle OAB = 180^\circ - (\theta + 2\phi) = 180^\circ - \left[\theta + 2(90^\circ - \theta)\right]$

$= 180^\circ - \theta - 180^\circ + 2\theta$

$= \theta$

$\therefore \angle BAX = 180^\circ - \theta$

Now, slope of line $AC = \dfrac{3-0}{5-a}$

$\Rightarrow \tan\theta = \dfrac{3}{5-a}$

Slope of AB

$\Rightarrow \tan(180^\circ - \theta) = \dfrac{2}{1-a}$

$\Rightarrow \tan\theta = \dfrac{2}{1-a}$

$\Rightarrow \tan\theta = \dfrac{2}{a-1}$ ...(2)

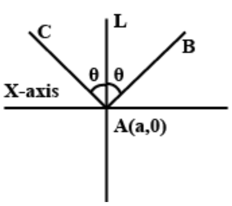
From equation (1) and (2), we obtain

$\dfrac{3}{5-a} = \dfrac{2}{a-1}$

$\Rightarrow 3a - 3 = 10 - 2a$

$\Rightarrow a = \dfrac{13}{5}$

Thus, the coordinates of point A are $\left(\dfrac{13}{5}, 0\right)$.



#459618

Topic: Distance of Point from a Line

Show that the normal at any point θ to the curve

$x = a\cos\theta + a\theta\sin\theta$, $y = a\sin\theta - a\theta\cos\theta$ is at constant distance from the origin.

Solution

First, let us find the slope of the tangent to the curve at a point θ.

$$m_T = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\cos\theta - a\cos\theta + a\theta\sin\theta}{-a\sin\theta + a\sin\theta + a\theta\cos\theta} = \frac{\sin\theta}{\cos\theta}$$

Since, normal is always perpendicular to the tangent, slope of the normal will be

$$m_N = -\frac{\cos\theta}{\sin\theta}$$

Normal passes through the point (x, y) given in the question. Using the equation of the line in slope-point form,

$$\frac{y - a\sin\theta + a\theta\cos\theta}{x - a\cos\theta - a\theta\sin\theta} = -\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow y\sin\theta - a\sin^2\theta + a\theta\sin\theta\cos\theta = a\cos^2\theta + a\theta\sin\theta\cos\theta - x\cos\theta$$

$$\Rightarrow x\cos\theta + y\sin\theta = a(\cos^2\theta + \sin^2\theta) = a$$

Distance of origin from this line will be

$$d = \frac{|0 + 0 - a|}{\sqrt{\sin^2\theta + \cos^2\theta}} = |a|$$

Hence, the distance from origin to the normal at any point θ is a constant.