#418690

Topic: Fundamental Principles of Counting

In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Solution

In the given word MISSISSIPPI, there are total 11 letters.

I appears 4 times, S appears 4 times, P appears 2 times.

Therefore, number of distinct permutations of the letters in the given word

$$= \frac{11!}{4! \times 4! \times 2!}$$
$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4 \times 3 \times 2 \times 1 \times 2 \times 1}$$
$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1 \times 2 \times 1}$$
$$= 34650$$

There are 4 $I's$ in the given word. When they occur together, they are considered as a single object . This single object (letter) together with the remaining 7 objects(letters) w give 8 objects.

These objects in which there are 4 $S's$ and 2 $P's$ can be arranged in $\frac{8!}{4!2!}$ ways i.e.,840 ways.

Number of arrangements where all $I's$ occur together $= 840$

Thus, number of distinct permutations of the letters in $MISSISSIPPI$ in which four $I's$ do not come together $= 34650 - 840 = 33810$

#448656

Topic: Fundamental Principles of Counting

How Many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the word $DAUGHTER$?

Solution

Total number of letters are 8

Total vowels $= 3$ and consonants $= 5$

Now we have to make a word using five letters from given word.

Here we have to choose 2 out of three vowels and 3 out of 5 consonants which can further be arranged in $5!$ ways.

$3C_2 \times 5C_3 \times 5!$

#448657

Topic: Permutations

How many words, with or without meaning, can be formed using all the letters of the word EQUATION at a time so that the vowels and consonants occur together?

Solution

Here we have 8 letters and these letters are to be arranged such that all the vowels and consonants remains together

So there are two packets one is of vowels and another one is of consonants, both of them can be arranged in 2! ways.

Now let's go inside the packet so vowels can be arranged in 5! ways and consonants can be arranged in 3! ways.

So total ways $= 2! \times 3! \times 5!$

#448658

Topic: Combination

A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of:

(i) exactly 3 girls?

(ii) at least 3 girls?

(iii) at most 3 girls?

Solution

(i) exactly 3 girls

This means you have to choose 3 girls from 4 and 4 boys from 9 to form a committee of 7. This can be done in

$4C_3 \times 9C_4$

(ii) At least 3 girls

This means girls can either be 3 or 4

So first choose 3 girls and 4 boys or 4 girls and 3 boys

Total ways = $4C_3 \times 9C_4$ + $4C_4 \times 9C_3$

(iii) At most 3 girls

This means we have to choose 1 or 2 or 3 girls

So this can be done in

$4C_1 \times 9C_6$ + $4C_2 \times 9C_5$ + $4C_3 \times 9C_4$ ways.

#448659

Topic: Fundamental Principles of Counting

If the different permutations of all the letter of the words $EXAMINATION$ are listed as in a dictionary, how many words are there in this list before the first word starting wi

E?

Solution

Arranging the words in dictionary order gives us

$AAEIINNMOTX$

Hence consider the words starting with A

$A----------$

The remaining 10 places can be arranged in $\dfrac{10!}{2!.2!2!}$ (as there are 2A,2I,2N)

The next letter that comes is E.

Hence the $4,53,601^{st}$ word will be starting with E.

#448662

Topic: Permutations

How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated?

Solution

Total number of digits are 6 and the number should be divisible by 10. Which means you should have a 0 at the end.

So one out of 6 places is fixed. Now we have 5 digits which needs to be arranged at 5 different places and this can be done in 5! ways.

#448664

Topic: Permutations

The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

Solution

We need 2 vowels and 2 consonants. So first we need to choose them and then they can be arranged.

Choosing = $5C_2 \times^{21} C_2$

They can be arranged in $4!$ ways.

So total number of words are $4! \times 5C_2 \times^{21} C_2$

#448666

Topic: Permutations

In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 question in all, selecting at least 3 from each part. In how many ways can a student select the questions?

Solution

Student has to do 8 questions such that he does at least 3 questions from both the sets

How can he go about (Set I, Set II) $=(3,5)$ or $(4,4)$ or $(5,3)$

So, total number of ways of doing this is given by

$5C_3 \times 7C_5$ + $5C_4 \times 7C_4$ + $5C_5 \times 7C_3 = 420$

#448668

Topic: Permutations

Determine the number of 5-card combination out of a deck of 52 cards if each selection of 5 cards has exactly one king.

Solution

In a deck, there are 4 kings and we need to select 5 cards so that the selection has exactly 1 king.

So, 1 king can be selected in $4C_1$

And remaining 4 cards can be selected in $48C_4$ ways

So total ways to select 5 cards are $4C_1 \times 48C_4 = 194580$

#448670

Topic: Permutations

It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Solution

$\underline{\text{MWMWMWMWM}}$

$1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9$

Since women has to occupy even places so first of all arrange 5 men in the row which can be done in 5 ways and then put women on the places between each men and then arrange these women which can be done in $4!$ ways

Hence total number of ways $= 4!5!$

#448671

Topic: Permutations

From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that their all of them will join or none of them will join. In how many ways can the excursion party be chosen?

Solution

We have 2 cases

1) when the group of 3 decides to join

Then we already have those 3 and we need to select 7 more from 22 and that can be done in $^{22}C_7$ ways

Other case is

2) When the group of 3 decides not to join

Then we have to choose 10 from remaining 22 and that can be done in $^{22}C_{10}$ ways

So total number of ways are $^{22}C_7$ + $^{22}C_{10}$ ways

#448672

Topic: Permutations

In how many ways can be letters of word ASSASSINATION be arranged so that all the S's are together?

Solution

Considering all the S' together we get

$AAANNIITO(SSSS)$

The above alphabets in which repetition of A is 3 times, N is 2 times and I is 2 times,can be arranged in

$\dfrac{10!}{3!2!2!}$ .... where all the S' remain as a pack.

$= 151200$ ways.