NCERT Solutions for Class 11 Maths Chapter 5

Complex Numbers and Quadratic Equations Class 11

Chapter 5 Complex Numbers and Quadratic Equations Exercise 5.1, 5.2, 5.3, miscellaneous Solutions

Exercise 5.1 : Solutions of Questions on Page Number : **103**

Q1 :

Express the given complex number in the form *a* + *ib*: $(5i)\left(-\frac{3}{5}i\right)$

Answer :

$$\begin{aligned}(5i)\left(\frac{-3}{5}i\right) &= -5\times\frac{3}{5}\times i\times i\\ &= -3i^2\\ &= -3(-1) \qquad \left[i^2=-1\right]\\ &= 3\end{aligned}$$

Q2 :

Express the given complex number in the form *a* + *ib*: $i^9 + i^{19}$

Answer :

$$\begin{aligned}i^9+i^{19} &= i^{4\times2+1}+i^{4\times4+3}\\ &= \left(i^4\right)^2\cdot i+\left(i^4\right)^4\cdot i^3\\ &= 1\times i+1\times(-i) \qquad \left[i^4=1,\ i^3=-i\right]\\ &= i+(-i)\\ &= 0\end{aligned}$$

Q3 :

Express the given complex number in the form *a* + *ib*: i^{-39}

Answer :

$$i^{-39} = i^{-4\times9-3} = \left(i^4\right)^{-9} \cdot i^{-3}$$
$$= (1)^{-9} \cdot i^{-3} \qquad \left[i^4 = 1\right]$$
$$= \frac{1}{i^3} = \frac{1}{-i} \qquad \left[i^3 = -i\right]$$
$$= \frac{-1}{i} \times \frac{i}{i}$$
$$= \frac{-i}{i^2} = \frac{-i}{-1} = i \qquad \left[i^2 = -1\right]$$

Q4 :

Express the given complex number in the form *a* + *ib*: 3(7 + *i*7) + *i*(7 + *i*7)

Answer :

$$3(7+i7)+i(7+i7) = 21+21i+7i+7i^2$$
$$= 21+28i+7\times(-1) \qquad \left[\because i^2 = -1\right]$$
$$= 14+28i$$

Q5 :

Express the given complex number in the form *a* + *ib*: (1 - *i*) - (-1 + *i*6)

Answer :

$$(1-i)-(-1+i6) = 1-i+1-6i$$
$$= 2-7i$$

Q6 :

Express the given complex number in the form *a* + *ib*: $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Answer :

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$
$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$
$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$
$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$
$$=\frac{-19}{5}-\frac{21}{10}i$$

Q7 :

Express the given complex number in the form *a* + *ib*: $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Answer :

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$
$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$
$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$
$$=\frac{17}{3}+i\frac{5}{3}$$

Q8 :

Express the given complex number in the form *a* + *ib*: $(1 - i)^4$

Answer :

$$(1-i)^4 = \left[(1-i)^2\right]^2$$
$$= \left[1^2 + i^2 - 2i\right]^2$$
$$= \left[1 - 1 - 2i\right]^2$$
$$= (-2i)^2$$
$$= (-2i)\times(-2i)$$
$$= 4i^2 = -4 \qquad \left[i^2 = -1\right]$$

Q9 :

Express the given complex number in the form *a* + *ib*: $\left(\frac{1}{3}+3i\right)^3$

Answer :

$$\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$
$$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3}+3i\right)$$
$$= \frac{1}{27} + 27(-i) + i + 9i^2 \qquad \left[i^3 = -i\right]$$
$$= \frac{1}{27} - 27i + i - 9 \qquad \left[i^2 = -1\right]$$
$$= \left(\frac{1}{27} - 9\right) + i(-27+1)$$
$$= \frac{-242}{27} - 26i$$

Q10 :

Express the given complex number in the form *a* + *ib*: $\left(-2-\frac{1}{3}i\right)^3$

Answer :

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad \left[i^3=-i\right]$$
$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad \left[i^2=-1\right]$$
$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$
$$=-\frac{22}{3}-\frac{107}{27}i$$

Q11 :

Find the multiplicative inverse of the complex number 4 - 3*i*

Answer :

Let z = 4 â€“ 3i

Then, $\bar{z}$ = 4 + 3i and $|z|^2=4^2+(-3)^2=16+9=25$

Therefore, the multiplicative inverse of 4 â€“ 3i is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{4+3i}{25}=\frac{4}{25}+\frac{3}{25}i$$

Q12 :

Find the multiplicative inverse of the complex number $\sqrt{5}+3i$

Answer :

Let z = $\sqrt{5}+3i$

$$\text{Then, } \bar{z}=\sqrt{5}-3i \text{ and } |z|^2=\left(\sqrt{5}\right)^2+3^2=5+9=14$$

Therefore, the multiplicative inverse of $\sqrt{5}+3i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{\sqrt{5}-3i}{14}=\frac{\sqrt{5}}{14}-\frac{3i}{14}$$

Q13 :

Find the multiplicative inverse of the complex number -*i*

Answer :

Let z = â€“i

$$\text{Then, } \bar{z}=i \text{ and } |z|^2=1^2=1$$

Therefore, the multiplicative inverse of â€“i is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{i}{1}=i$$

Q14 :

Express the following expression in the form of *a* + *ib*.

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

Answer :

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad \left[i^2=-1\right]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

Exercise 5.2 : Solutions of Questions on Page Number : **108**

Q1 :

Find the modulus and the argument of the complex number $z=-1-i\sqrt{3}$

Answer :

$z=-1-i\sqrt{3}$

Let $r\cos\theta=-1$ and $r\sin\theta=-\sqrt{3}$

On squaring and adding, we obtain

$$(r\cos\theta)^2+(r\sin\theta)^2=(-1)^2+(-\sqrt{3})^2$$
$$\Rightarrow r^2(\cos^2\theta+\sin^2\theta)=1+3$$
$$\Rightarrow r^2=4 \qquad [\cos^2\theta+\sin^2\theta=1]$$
$$\Rightarrow r=\sqrt{4}=2 \qquad [\text{Conventionally, } r>0]$$
$$\therefore \text{Modulus}=2$$
$$\therefore\ 2\cos\theta=-1 \text{ and } 2\sin\theta=-\sqrt{3}$$
$$\Rightarrow \cos\theta=\frac{-1}{2} \text{ and } \sin\theta=\frac{-\sqrt{3}}{2}$$

Since both the values of sin $\theta$and cos θ are negative and sin$\theta$and cos$\theta$are negative in III quadrant,

$$\text{Argument}=-\left(\pi-\frac{\pi}{3}\right)=\frac{-2\pi}{3}$$

Thus, the modulus and argument of the complex number $-1-\sqrt{3}\,i$ are 2 and $\frac{-2\pi}{3}$ respectively.

Q2 :

Find the modulus and the argument of the complex number $z=-\sqrt{3}+i$

Answer :

$$z=-\sqrt{3}+i$$

$$\text{Let } r\cos\theta=-\sqrt{3} \text{ and } r\sin\theta=1$$

On squaring and adding, we obtain

$$r^2\cos^2\theta+r^2\sin^2\theta=(-\sqrt{3})^2+1^2$$
$$\Rightarrow r^2=3+1=4 \qquad [\cos^2\theta+\sin^2\theta=1]$$
$$\Rightarrow r=\sqrt{4}=2 \qquad [\text{Conventionally, } r>0]$$
$$\therefore \text{Modulus }=2$$
$$\therefore\ 2\cos\theta=-\sqrt{3} \text{ and } 2\sin\theta=1$$
$$\Rightarrow \cos\theta=\frac{-\sqrt{3}}{2} \text{ and } \sin\theta=\frac{1}{2}$$
$$\therefore \theta=\pi-\frac{\pi}{6}=\frac{5\pi}{6} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

Thus, the modulus and argument of the complex number $-\sqrt{3}+i$ are 2 and $\frac{5\pi}{6}$ respectively.

Q3 :

Convert the given complex number in polar form: 1 - *i*

Answer :

1 â€“ *i*

Let *r*cos *θ* = 1 and *r*sin *θ* = â€“1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1+1$$
$$\Rightarrow r^2 = 2$$
$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \sqrt{2}\cos\theta = 1 \text{ and } \sqrt{2}\sin\theta = -1$$
$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$
$$\therefore \theta = -\frac{\pi}{4} \qquad [\text{As } \theta \text{ lies in the IV quadrant}]$$
$$\therefore 1-i = r\cos\theta + i\,r\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right) = \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

This is the required polar form.

Q4 :

Convert the given complex number in polar form: - 1 + *i*

Answer :

â€“ 1 + *i*

Let *r*cos *θ* = â€“1 and *r*sin *θ* = 1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$$
$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$$
$$\Rightarrow r^2 = 2$$
$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$
$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$
$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

It can be written,

$$\therefore -1 + i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

Q5 :

Convert the given complex number in polar form: - 1 - *i*

Answer :

â€“ 1 â€“ *i*

Let $r\cos\theta$ = â€“1 and $r\sin\theta$ = â€“1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$$
$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$$
$$\Rightarrow r^2 = 2$$
$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = -1$$
$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$
$$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the III quadrant}]$$

$\therefore -1-i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4} = \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$ This is the required polar form.

Q6 :

Convert the given complex number in polar form: -3

Answer :

â€“3

Let *r*cos *θ* **=** â€“3 and *r*sin *θ* = 0

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 9$$
$$\Rightarrow r^2 = 9$$
$$\Rightarrow r = \sqrt{9} = 3 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore 3\cos\theta = -3 \text{ and } 3\sin\theta = 0$$
$$\Rightarrow \cos\theta = -1 \text{ and } \sin\theta = 0$$
$$\therefore \theta = \pi$$
$$\therefore -3 = r\cos\theta + ir\sin\theta = 3\cos\pi + i3\sin\pi = 3\left(\cos\pi + i\sin\pi\right)$$

This is the required polar form.

Q7 :

Convert the given complex number in polar form: $\sqrt{3}+i$

Answer :

$\sqrt{3}+i$

Let *r*cos *θ* = $\sqrt{3}$ and *r*sin *θ* = 1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 3 + 1$$
$$\Rightarrow r^2 = 4$$
$$\Rightarrow r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore 2\cos\theta = \sqrt{3} \text{ and } 2\sin\theta = 1$$
$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$
$$\therefore \theta = \frac{\pi}{6} \qquad [\text{As } \theta \text{ lies in the I quadrant}]$$
$$\therefore \sqrt{3} + i = r\cos\theta + ir\sin\theta = 2\cos\frac{\pi}{6} + i2\sin\frac{\pi}{6} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

This is the required polar form.

Q8 :

Convert the given complex number in polar form: *i*

Answer :

i

Let $r\cos\theta = 0$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = 0^2 + 1^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$$
$$\Rightarrow r^2 = 1$$
$$\Rightarrow r = \sqrt{1} = 1 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \cos\theta = 0 \text{ and } \sin\theta = 1$$
$$\therefore \theta = \frac{\pi}{2}$$
$$\therefore i = r\cos\theta + ir\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

This is the required polar form.

Exercise 5.3 : Solutions of Questions on Page Number : **109**

Q1 :

Solve the equation $x^2 + 3 = 0$

Answer :

The given quadratic equation is $x^2 + 3 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 1$, $b = 0$, and $c = 3$

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = 0^2 â€“ 4 × 1 × 3 = â€“12

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{\pm\sqrt{-12}}{2\times 1} = \frac{\pm\sqrt{12}\,i}{2} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{\pm 2\sqrt{3}\,i}{2} = \pm\sqrt{3}\,i$$

Q2 :

Solve the equation $2x^2 + x + 1 = 0$

Answer :

The given quadratic equation is $2x^2 + x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 2$, $b = 1$, and$c = 1$

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = 1^2 â€“ 4 × 2 × 1 = 1 â€“ 8 = â€“7

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1\pm\sqrt{-7}}{2\times 2} = \frac{-1\pm\sqrt{7}\,i}{4} \qquad \left[\sqrt{-1} = i\right]$$

Q3 :

Solve the equation $x^2 + 3x + 9 = 0$

Answer :

The given quadratic equation is $x^2 + 3x + 9 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 1$, $b = 3$, and $c = 9$

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = 3^2 â€“ 4 × 1 × 9 = 9 â€“ 36 = â€“27

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-3\pm\sqrt{-27}}{2(1)}=\frac{-3\pm3\sqrt{-3}}{2}=\frac{-3\pm3\sqrt{3}\,i}{2}\qquad\left[\sqrt{-1}=i\right]$$

Q4 :

Solve the equation $-x^2 + x - 2 = 0$

Answer :

The given quadratic equation is â€“x^2 + x â€“ 2 = 0

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

a = â€“1, b = 1, and c = â€“2

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = 1^2 â€“ 4 × (â€“1) × (â€“2) = 1 â€“ 8 = â€“7

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\times(-1)}=\frac{-1\pm\sqrt{7}\,i}{-2}\qquad\left[\sqrt{-1}=i\right]$$

Q5 :

Solve the equation $x^2 + 3x + 5 = 0$

Answer :

The given quadratic equation is $x^2 + 3x + 5 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 1$, $b = 3$, and $c = 5$

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = 3^2 â€“ 4 × 1 × 5 =9 â€“ 20 = â€“11

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{-11}}{2 \times 1} = \frac{-3 \pm \sqrt{11}\, i}{2} \qquad \left[\sqrt{-1} = i\right]$$

Q6 :

Solve the equation x^2 - x + 2 = 0

Answer :

The given quadratic equation is x^2 â€“ x + 2 = 0

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

a = 1, b = â€“1, and c = 2

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = (â€“1)^2 â€“ 4 × 1 × 2 = 1 â€“ 8 = â€“7

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-1) \pm \sqrt{-7}}{2 \times 1} = \frac{1 \pm \sqrt{7}\, i}{2} \qquad \left[\sqrt{-1} = i\right]$$

Q7 :

Solve the equation $\sqrt{2}x^2 + x + \sqrt{2} = 0$

Answer :

The given quadratic equation is $\sqrt{2}x^2 + x + \sqrt{2} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

a = $\sqrt{2}$, b = 1, and c = $\sqrt{2}$

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = 1^2 â€“ $4 \times \sqrt{2} \times \sqrt{2}$ = 1 â€“ 8 = â€“7

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{-7}}{2 \times \sqrt{2}} = \frac{-1 \pm \sqrt{7}\, i}{2\sqrt{2}} \qquad \left[\sqrt{-1} = i\right]$$

Q8 :

Solve the equation $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Answer :

The given quadratic equation is $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{3}$, $b = -\sqrt{2}$, and $c = 3\sqrt{3}$

Therefore, the discriminant of the given equation is

D = b^2 â€“ $4ac$ = $\left(-\sqrt{2}\right)^2 - 4\left(\sqrt{3}\right)\left(3\sqrt{3}\right) = 2 - 36 = -34$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-\left(-\sqrt{2}\right) \pm \sqrt{-34}}{2 \times \sqrt{3}} = \frac{\sqrt{2} \pm \sqrt{34}\,i}{2\sqrt{3}} \qquad \left[\sqrt{-1} = i\right]$$

Q9 :

Solve the equation $x^2 + x + \frac{1}{\sqrt{2}} = 0$

Answer :

The given quadratic equation is $x^2 + x + \frac{1}{\sqrt{2}} = 0$

This equation can also be written as $\sqrt{2}x^2 + \sqrt{2}x + 1 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{2}$, $b = \sqrt{2}$, and $c = 1$

$$\therefore \text{Discriminant } (D) = b^2 - 4ac = \left(\sqrt{2}\right)^2 - 4 \times \left(\sqrt{2}\right) \times 1 = 2 - 4\sqrt{2}$$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-\sqrt{2}\pm\sqrt{2-4\sqrt{2}}}{2\times\sqrt{2}}=\frac{-\sqrt{2}\pm\sqrt{2\left(1-2\sqrt{2}\right)}}{2\sqrt{2}}$$

$$=\left(\frac{-\sqrt{2}\pm\sqrt{2}\left(\sqrt{2\sqrt{2}-1}\right)i}{2\sqrt{2}}\right)\qquad\left[\sqrt{-1}=i\right]$$

$$=\frac{-1\pm\left(\sqrt{2\sqrt{2}-1}\right)i}{2}$$

Q10 :

Solve the equation $x^2+\frac{x}{\sqrt{2}}+1=0$

Answer :

The given quadratic equation is $x^2+\frac{x}{\sqrt{2}}+1=0$

This equation can also be written as $\sqrt{2}x^2+x+\sqrt{2}=0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{2}$, $b = 1$, and $c = \sqrt{2}$

$$\therefore \text{Discriminant } (D)=b^2-4ac=1^2-4\times\sqrt{2}\times\sqrt{2}=1-8=-7$$

Therefore, the required solutions are

$$\frac{-b\pm\sqrt{D}}{2a}=\frac{-1\pm\sqrt{-7}}{2\sqrt{2}}=\frac{-1\pm\sqrt{7}\,i}{2\sqrt{2}}\qquad\left[\sqrt{-1}=i\right]$$

Exercise Miscellaneous : Solutions of Questions on Page Number : **112**

Q1 :

Evaluate: $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Answer :

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^{3}$$

$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^{3}$$

$$=\left[\left(i^{4}\right)^{4}\cdot i^{2}+\frac{1}{\left(i^{4}\right)^{6}\cdot i}\right]^{3}$$

$$=\left[i^{2}+\frac{1}{i}\right]^{3} \qquad \left[i^{4}=1\right]$$

$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^{3} \qquad \left[i^{2}=-1\right]$$

$$=\left[-1+\frac{i}{i^{2}}\right]^{3}$$

$$=\left[-1-i\right]^{3}$$

$$=(-1)^{3}\left[1+i\right]^{3}$$

$$=-\left[1^{3}+i^{3}+3\cdot1\cdot i(1+i)\right]$$

$$=-\left[1+i^{3}+3i+3i^{2}\right]$$

$$=-\left[1-i+3i-3\right]$$

$$=-\left[-2+2i\right]$$

$$=2-2i$$

Q2 :

For any two complex numbers z_1 and z_2, prove that

Re (z_1z_2) = Re z_1 Re z_2 - Im z_1 Im z_2

Answer :

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\therefore z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$
$$= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2)$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2 y_1y_2$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad [i^2 = -1]$$
$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$
$$\Rightarrow \operatorname{Re}(z_1z_2) = x_1x_2 - y_1y_2$$
$$\Rightarrow \operatorname{Re}(z_1z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$$

Hence, proved.

Q3 :

Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ **to the standard form.**

Answer :

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$
$$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$
$$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$$
$$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} \qquad [\text{On multiplying numerator and denominator by } (14+5i)]$$
$$= \frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]} = \frac{307+599i}{2(196-25i^2)}$$
$$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}$$

This is the required standard form.

Q4 :

If *x* â€“ *iy* = $\sqrt{\frac{a-ib}{c-id}}$ **prove that** $\left(x^2+y^2\right)^2=\frac{a^2+b^2}{c^2+d^2}$.

Answer :

$$x-iy=\sqrt{\frac{a-ib}{c-id}}$$
$$=\sqrt{\frac{a-ib}{c-id}\times\frac{c+id}{c+id}}\left[\text{On multiplying numerator and denominator by }(c+id)\right]$$
$$=\sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}}$$
$$\therefore (x-iy)^2=\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$
$$\Rightarrow x^2-y^2-2ixy=\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}$$

On comparing real and imaginary parts, we obtain

$$x^2-y^2=\frac{ac+bd}{c^2+d^2},\quad -2xy=\frac{ad-bc}{c^2+d^2} \qquad (1)$$
$$\left(x^2+y^2\right)^2=\left(x^2-y^2\right)^2+4x^2y^2$$
$$=\left(\frac{ac+bd}{c^2+d^2}\right)^2+\left(\frac{ad-bc}{c^2+d^2}\right)^2 \qquad [\text{Using }(1)]$$
$$=\frac{a^2c^2+b^2d^2+2acbd+a^2d^2+b^2c^2-2adbc}{\left(c^2+d^2\right)^2}$$
$$=\frac{a^2c^2+b^2d^2+a^2d^2+b^2c^2}{\left(c^2+d^2\right)^2}$$
$$=\frac{a^2\left(c^2+d^2\right)+b^2\left(c^2+d^2\right)}{\left(c^2+d^2\right)^2}$$
$$=\frac{\left(c^2+d^2\right)\left(a^2+b^2\right)}{\left(c^2+d^2\right)^2}$$
$$=\frac{a^2+b^2}{c^2+d^2}$$

Hence, proved.

Q5 :

Convert the following in the polar form:

(i) $\frac{1+7i}{(2-i)^2}$ **, (ii)** $\frac{1+3i}{1-2i}$

Answer :

(i) Here, $z=\frac{1+7i}{(2-i)^2}$

$$=\frac{1+7i}{(2-i)^2}=\frac{1+7i}{4+i^2-4i}=\frac{1+7i}{4-1-4i}$$

$$=\frac{1+7i}{3-4i}\times\frac{3+4i}{3+4i}=\frac{3+4i+21i+28i^2}{3^2+4^2}$$

$$=\frac{3+4i+21i-28}{3^2+4^2}=\frac{-25+25i}{25}$$

$$=-1+i$$

Let $r\cos\theta$ = â€“1 and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2(\cos^2\theta+\sin^2\theta)=1+1$

$\Rightarrow r^2(\cos^2\theta+\sin^2\theta)=2$

$\Rightarrow r^2=2$ $\quad$ $[\cos^2\theta+\sin^2\theta=1]$

$$\Rightarrow r=\sqrt{2} \qquad [\text{Conventionally, } r>0]$$

$$\therefore \sqrt{2}\cos\theta=-1 \text{ and } \sqrt{2}\sin\theta=1$$

$$\Rightarrow \cos\theta=\frac{-1}{\sqrt{2}} \text{ and } \sin\theta=\frac{1}{\sqrt{2}}$$

$$\therefore \theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in II quadrant}]$$

$\therefore z=r\cos\theta+i\,r\sin\theta$

$$=\sqrt{2}\cos\frac{3\pi}{4}+i\sqrt{2}\sin\frac{3\pi}{4}=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

(ii) Here, $z=\frac{1+3i}{1-2i}$

$$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$$
$$= \frac{1+2i+3i-6}{1+4}$$
$$= \frac{-5+5i}{5} = -1+i$$

Let $r \cos \theta$ = â€“1 and $r \sin \theta = 1$

On squaring and adding, we obtain

$r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$
$\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 2$

$\Rightarrow r^2 = 2$ $[\cos^2 \theta + \sin^2 \theta = 1]$

$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally, } r > 0]$$
$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$
$$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$
$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in II quadrant}]$$

$\therefore z = r \cos \theta + i\, r \sin \theta$

$$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

Q6 :

Solve the equation $3x^2 - 4x + \frac{20}{3} = 0$

Answer :

The given quadratic equation is $3x^2 - 4x + \frac{20}{3} = 0$

This equation can also be written as $9x^2 - 12x + 20 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 9$, b = â€“12, and $c = 20$

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = (â€“12)^2 â€“ 4 × 9 × 20 = 144 â€“ 720 = â€“576

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-12) \pm \sqrt{-576}}{2 \times 9} = \frac{12 \pm \sqrt{576}\, i}{18} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{12 \pm 24i}{18} = \frac{6(2 \pm 4i)}{18} = \frac{2 \pm 4i}{3} = \frac{2}{3} \pm \frac{4}{3} i$$

Q7 :

Solve the equation $x^2 - 2x + \frac{3}{2} = 0$

Answer :

The given quadratic equation is $x^2 - 2x + \frac{3}{2} = 0$

This equation can also be written as $2x^2 - 4x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2$, b = â€“4, and $c = 3$

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = (â€“4)^2 â€“ 4 × 2 × 3 = 16 â€“ 24 = â€“8

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{-8}}{2 \times 2} = \frac{4 \pm 2\sqrt{2}\, i}{4} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{2 \pm \sqrt{2}\, i}{2} = 1 \pm \frac{\sqrt{2}}{2} i$$

Q8 :

Solve the equation 27x^2 - 10x + 1 = 0

Answer :

The given quadratic equation is 27x^2 â€“ 10x + 1 = 0

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 27$, b = â€“10, and $c = 1$

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = (â€“10)^2 â€“ 4 × 27 × 1 = 100 â€“ 108 = â€“8

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-10) \pm \sqrt{-8}}{2 \times 27} = \frac{10 \pm 2\sqrt{2}\, i}{54} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{5 \pm \sqrt{2}\, i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27} i$$

Q9 :

Solve the equation 21x^2 - 28x + 10 = 0

Answer :

The given quadratic equation is 21x^2 â€“ 28x + 10 = 0

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

a = 21, b = â€“28, and c = 10

Therefore, the discriminant of the given equation is

D = b^2 â€“ 4ac = (â€“28)^2 â€“ 4 × 21 × 10 = 784 â€“ 840 = â€“56

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-28) \pm \sqrt{-56}}{2 \times 21} = \frac{28 \pm \sqrt{56}\, i}{42}$$

$$= \frac{28 \pm 2\sqrt{14}\, i}{42} = \frac{28}{42} \pm \frac{2\sqrt{14}}{42} i = \frac{2}{3} \pm \frac{\sqrt{14}}{21} i$$

Q10 :

If $z_1 = 2 - i,\ z_2 = 1 + i,$ **find** $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$**.**

Answer :

$$z_1 = 2 - i,\ z_2 = 1 + i$$

$$\therefore \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$

$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$

$$= \left|\frac{2}{1-i} \times \frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{1^2 - i^2}\right|$$

$$= \left|\frac{2(1+i)}{1+1}\right| \qquad \left[i^2 = -1\right]$$

$$= \left|\frac{2(1+i)}{2}\right|$$

$$= |1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Thus, the value of $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$ is $\sqrt{2}$.

Q11 :

If $z_1 = 2 - i,\ z_2 = 1 + i,$ **find** $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$.

Answer :

$$z_1 = 2 - i,\ z_2 = 1 + i$$

$$\therefore \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$

$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$

$$= \left|\frac{2}{1-i} \times \frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{1^2 - i^2}\right|$$

$$= \left|\frac{2(1+i)}{1+1}\right| \qquad \left[i^2 = -1\right]$$

$$= \left|\frac{2(1+i)}{2}\right|$$

$$= |1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Thus, the value of $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$ is $\sqrt{2}$.

Q12 :

If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{(x^2+1)^2}{(2x+1)^2}$

Answer :

$$a + ib = \frac{(x+i)^2}{2x^2+1}$$

$$= \frac{x^2 + i^2 + 2xi}{2x^2+1}$$

$$= \frac{x^2 - 1 + i2x}{2x^2+1}$$

$$= \frac{x^2 - 1}{2x^2+1} + i\left(\frac{2x}{2x^2+1}\right)$$

On comparing real and imaginary parts, we obtain

$$a = \frac{x^2 - 1}{2x^2 + 1} \text{ and } b = \frac{2x}{2x^2 + 1}$$

$$\therefore a^2 + b^2 = \left(\frac{x^2 - 1}{2x^2 + 1}\right)^2 + \left(\frac{2x}{2x^2 + 1}\right)^2$$

$$= \frac{x^4 + 1 - 2x^2 + 4x^2}{(2x+1)^2}$$

$$= \frac{x^4 + 1 + 2x^2}{(2x^2 + 1)^2}$$

$$= \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$$

$$\therefore a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$$

Hence, proved.

Q13 :

Let $z_1 = 2 - i,\ z_2 = -2 + i$ **. Find**

(i) $\text{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$ **, (ii)** $\text{Im}\left(\frac{1}{z_1 \bar{z}_1}\right)$

Answer :

$z_1 = 2 - i,\ z_2 = -2 + i$

(i) $z_1 z_2 = (2 - i)(-2 + i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$

$$\bar{z}_1 = 2 + i$$

$$\therefore \frac{z_1 z_2}{\bar{z}_1} = \frac{-3 + 4i}{2 + i}$$

On multiplying numerator and denominator by (2 â€“ *i*), we obtain

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{(-3 + 4i)(2 - i)}{(2 + i)(2 - i)} = \frac{-6 + 3i + 8i - 4i^2}{2^2 + 1^2} = \frac{-6 + 11i - 4(-1)}{2^2 + 1^2}$$

$$= \frac{-2 + 11i}{5} = \frac{-2}{5} + \frac{11}{5} i$$

On comparing real parts, we obtain

$$\text{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right) = \frac{-2}{5}$$

(ii) $\frac{1}{z_1\overline{z}_1} = \frac{1}{(2-i)(2+i)} = \frac{1}{(2)^2+(1)^2} = \frac{1}{5}$

On comparing imaginary parts, we obtain

$$\text{Im}\left(\frac{1}{z_1\overline{z}_1}\right) = 0$$

Q14 :

Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$.

Answer :

Let $z = \frac{1+2i}{1-3i}$, then

$$z = \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{1+3i+2i+6i^2}{1^2+3^2} = \frac{1+5i+6(-1)}{1+9}$$

$$= \frac{-5+5i}{10} = \frac{-5}{10} + \frac{5i}{10} = \frac{-1}{2} + \frac{1}{2}i$$

$$\text{Let } z = r\cos\theta + ir\sin\theta$$

$$\text{i.e., } r\cos\theta = \frac{-1}{2} \text{ and } r\sin\theta = \frac{1}{2}$$

On squaring and adding, we obtain

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\Rightarrow r = \frac{1}{\sqrt{2}} \qquad \left[\text{Conventionally, } r > 0\right]$$

$$\therefore \frac{1}{\sqrt{2}}\cos\theta = \frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2}$$
$$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$
$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

Therefore, the modulus and argument of the given complex number are $\frac{1}{\sqrt{2}}$ and $\frac{3\pi}{4}$ respectively.

Q15 :

Find the real numbers *x* and *y* if (*x* - *iy*) (3 + 5*i*) is the conjugate of -6 - 24*i*.

Answer :

Let $z = (x - iy)(3 + 5i)$

$$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x + 5y) + i(5x - 3y)$$
$$\therefore \bar{z} = (3x + 5y) - i(5x - 3y)$$

It is given that, $\bar{z} = -6 - 24i$

$$\therefore (3x + 5y) - i(5x - 3y) = -6 - 24i$$

Equating real and imaginary parts, we obtain

$$3x + 5y = -6 \qquad \text{... (i)}$$
$$5x - 3y = 24 \qquad \text{... (ii)}$$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

$$\begin{array}{r} 9x + 15y = -18 \\ \underline{25x - 15y = 120} \\ 34x \quad = 102 \end{array}$$
$$\therefore x = \frac{102}{34} = 3$$

Putting the value of *x* in equation (i), we obtain

$$3(3) + 5y = -6$$
$$\Rightarrow 5y = -6 - 9 = -15$$
$$\Rightarrow y = -3$$

Thus, the values of *x* and *y* are 3 and â€“3 respectively.

Q16 :

Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$**.**

Answer :

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$

$$=\frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}$$

$$=\frac{4i}{2}=2i$$

$$\therefore\left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{2^2}=2$$

Q17 :

If $(x+iy)^3=u+iv$, then show that $\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$**.**

Answer :

$$(x+iy)^3=u+iv$$

$$\Rightarrow x^3+(iy)^3+3\cdot x\cdot iy(x+iy)=u+iv$$

$$\Rightarrow x^3+i^3y^3+3x^2yi+3xy^2i^2=u+iv$$

$$\Rightarrow x^3-iy^3+3x^2yi-3xy^2=u+iv$$

$$\Rightarrow\left(x^3-3xy^2\right)+i\left(3x^2y-y^3\right)=u+iv$$

On equating real and imaginary parts, we obtain

$$u = x^3 - 3xy^2,\ v = 3x^2y - y^3$$

$$\therefore \frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$$

$$= \frac{x\left(x^2 - 3y^2\right)}{x} + \frac{y\left(3x^2 - y^2\right)}{y}$$

$$= x^2 - 3y^2 + 3x^2 - y^2$$

$$= 4x^2 - 4y^2$$

$$= 4\left(x^2 - y^2\right)$$

$$\therefore \frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right)$$

Hence, proved.

Q18 :

If α and ÃŽÂ² are different complex numbers with $|\beta|$ = 1, then find $\left|\frac{\beta - \alpha}{1 - \overline{\alpha}\beta}\right|$.

Answer :

Let α = *a* + *ib* and ÃŽÂ² = *x* + *iy*

It is given that, $|\beta| = 1$

$$\therefore \sqrt{x^2 + y^2} = 1$$

$$\Rightarrow x^2 + y^2 = 1 \qquad \text{... (i)}$$

$$\left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right| = \left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right|$$

$$= \frac{|(x-a)+i(y-b)|}{|(1-ax-by)+i(bx-ay)|} \qquad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$$

$$= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$$

$$= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$$

$$= \frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \qquad [\text{Using } (1)]$$

$$= 1$$

$$\therefore \left|\frac{\beta-\alpha}{1-\overline{\alpha}\beta}\right| = 1$$

Q19 :

Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$ **.**

Answer :

$$|1-i|^x = 2^x$$
$$\Rightarrow \left(\sqrt{1^2+(-1)^2}\right)^x = 2^x$$
$$\Rightarrow \left(\sqrt{2}\right)^x = 2^x$$
$$\Rightarrow 2^{\frac{x}{2}} = 2^x$$
$$\Rightarrow \frac{x}{2} = x$$
$$\Rightarrow x = 2x$$
$$\Rightarrow 2x - x = 0$$
$$\Rightarrow x = 0$$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of non-zero integral solutions of the given equation is 0.

Q20 :

If (a + ib) (c + id) (e + if) (g + ih) = A + iB, then show that

(a^2 + b^2) (c^2 + d^2) (e^2 + f^2) (g^2 + h^2) = A^2 + B^2.

Answer :

$$(a+ib)(c+id)(e+if)(g+ih) = \mathrm{A}+i\mathrm{B}$$
$$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |\mathrm{A}+i\mathrm{B}|$$
$$\Rightarrow |(a+ib)| \times |(c+id)| \times |(e+if)| \times |(g+ih)| = |\mathrm{A}+i\mathrm{B}| \qquad \left[|z_1 z_2| = |z_1||z_2|\right]$$
$$\Rightarrow \sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \sqrt{e^2+f^2} \times \sqrt{g^2+h^2} = \sqrt{\mathrm{A}^2+\mathrm{B}^2}$$

On squaring both sides, we obtain

(a^2 + b^2) (c^2 + d^2) (e^2 + f^2) (g^2 + h^2) = A^2 + B^2

Hence, proved.

Q21 :

If $\left(\frac{1+i}{1-i}\right)^m = 1$ **, then find the least positive integral value of m.**

Answer :

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$$

$$\Rightarrow i^m = 1$$

$$\therefore m = 4k, \text{ where } k \text{ is some integer.}$$

Therefore, the least positive integer is 1.

Thus, the least positive integral value of *m* is 4 (= 4 × 1).