#429764

Topic: Measurement of Angles

Find the radian measures corresponding to the following degree measures:

(i) 25°

(ii) $-47^\circ 30'$

(iii) 240°

(iv) 520°

Solution

We know that , $180^\circ = \pi$ radian

(i) $25^\circ = \frac{\pi}{180} \times 25 = \frac{5\pi}{36}$ radian

(ii) $-47^\circ 30' = -47\frac{1}{2}$ degree $[\because 1^\circ = 60'] = -\frac{95}{2}$ degree

$\therefore -\frac{95}{2}\ degree = \frac{\pi}{180} \times \left(-\frac{95}{2}\right) radian = \left(-\frac{19}{36 \times 2}\right)\pi\ radian = -\frac{19}{72}\pi$ radian

(iii) $\therefore 240^\circ = \frac{\pi}{180} \times 240 = \frac{4}{3}\pi$ radian

(iv) $\therefore 520^\circ = \frac{\pi}{180} \times 520\ radian = \frac{26}{9}\pi$ radian

#429767

Topic: Measurement of Angles

Find the degree measures corresponding to the following radian measures $\left(Use\ \pi = \frac{22}{7}\right)$.

(i) $\frac{11}{16}$

(ii) -4

(iii) $\frac{5\pi}{3}$

(iv) $\frac{7\pi}{6}$

Solution

We know that π radian $= 180^\circ$

(i) $\therefore \frac{11}{16}\ radian = \frac{180}{\pi} \times \frac{11}{16}\ degree = \frac{45 \times 11 \times 7}{22 \times 4}\ degree = 39\frac{3}{8}$ degree

$= 39^\circ + \frac{3 \times 60}{8}$ minutes , $[\because 1^\circ = 60']$

$= 39^\circ + 22' + \frac{1}{2}$ minutes

$= 39^\circ 22' 30''$, $[\because 1' = 60'']$

(ii) $\therefore -4\ radian = \frac{180}{\pi} \times (-4)\ degree = \frac{180 \times 7(-4)}{22}\ degree = -229\frac{1}{11}$ degree

$= -229^\circ + \frac{1 \times 60}{11}$ minutes , $[\because 1^\circ = 60']$

$= -229^\circ + 5' + \frac{5}{11}$ minutes

$= -229^\circ 5' 27''$, $[\because 1' = 60'']$

(iii) $\therefore \frac{5\pi}{3}\ radian = \frac{180}{\pi} \times \frac{5\pi}{3}\ degree$

$= 300^\circ$

(iv) $\therefore \frac{7\pi}{6}\ radian = \frac{180}{\pi} \times \frac{7\pi}{6}\ degree = 210^\circ$

#429771

Topic: Measurement of Angles

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution

Number of revolutions made by the wheel in 1 minute $= 360$

$\therefore$ Number of revolutions made by the wheel in 1 second $= \frac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of $6 \times 2\pi$ radian, i.e., 12π radian

Thus, in one second, the wheel turns an angle of 12π radian.

#429772

Topic: Measurement of Angles

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm. $\left(Use\ \pi = \frac{22}{7}\right)$

Solution

We know that in a circle of radius r unit, if an arc of length l unit

subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$

Therefore, for $r = 100$ cm, $l = 22$ cm, we have

$$\theta = \frac{22}{100}\ radian = \frac{180}{\pi} \times \frac{22}{100}\ degree = \frac{180 \times 7 \times 22}{22 \times 100}\ \text{degree}$$

$$= \frac{126}{10}\ degree = 12\frac{3}{5}\ degree = 12^\circ 36', [\because 1^\circ = 60']$$

#429773

Topic: Measurement of Angles

In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Solution

Diameter of the circle $= 40$ cm

$\therefore$ Radius (r) of the circle $= \frac{40}{2} = 20$ cm

Let AB be a chord (length = 20 cm) of the circle.

In $\triangle OAB$, OA = OB = Radius of circle = 20 cm

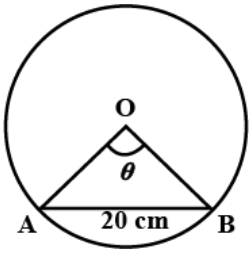
Also, AB = 20 cm

Thus, $\triangle OAB$ is an equilateral triangle.

$\therefore \theta = 60^\circ = \frac{\pi}{3}$ radian

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$

$\therefore \frac{\pi}{3} = \frac{arc\ AB}{20} \Rightarrow arc\ AB = \frac{20}{3}\pi$ cm



#429774

Topic: Measurement of Angles

If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Solution

Let the radii of the two circles be r_1 and r_2. Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r_1, while let an arc of length l subtend an angle of 75° at the centre of the circle of radius r_2.

Now, $60^\circ = \frac{\pi}{3}$ radian and $75^\circ = \frac{5\pi}{12}$ radian

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then

$\theta = \frac{l}{r}$ or $l = r\theta$

$\therefore l = \frac{r_1\pi}{3}$ and $l = \frac{r_2 5\pi}{12}$

$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$

$\Rightarrow r_1 = \frac{r_2 5}{4}$

$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$

#429775

Topic: Measurement of Angles

Find the angle in radian though which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm

(ii) 15 cm

(iii) 21 cm

Solution

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then $\theta = \frac{l}{r}$.

It is given that $r = 75$ cm

(i) $l = 10$ cm

$\therefore \theta = \frac{10}{75} = \frac{2}{15}$ radian

(ii) $l = 15$ cm

$\therefore \theta = \frac{15}{75} = \frac{1}{5}$ radian

(iii) $l = 21$ cm

$\therefore \theta = \frac{21}{75} = \frac{7}{25}$ radian

#429930

Topic: Trigonometric Ratios of Any Angle

Find the value of other five trigonometric ratios:

$\cos x = -\frac{1}{2}$, x lies in third quadrant.

Solution

$\cos x = -\frac{1}{2}$

$\therefore \sec x = \frac{1}{\cos x} = -2$

Now $\because \sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x \Rightarrow \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$

Since x lies in third quadrant, the value of $\sin x$ will be negative.

$\therefore \sin x = -\frac{\sqrt{3}}{2}$

$\csc x = \frac{1}{\sin x} = -\frac{2}{\sqrt{3}}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \sqrt{3}$

$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}$

#429937

Topic: Trigonometric Ratios of Any Angle

Find the value of other five trigonometric ratios:

When $\sin x = \frac{3}{5}$, and x lies in second quadrant.

Solution

$\sin x = \frac{3}{5}$

$\Rightarrow \csc x = \frac{1}{\sin x} = \frac{5}{3}$

$\because \sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x = 1 - \frac{9}{25} \Rightarrow \cos^2 x = \frac{16}{25}$

$\Rightarrow \cos x = \pm\frac{4}{5}$

Since x lies in second quadrant, the value of $\cos x$ will be negative

$\therefore \cos x = -\frac{4}{5}$

$\therefore \sec x = \frac{1}{\cos x} = -\frac{5}{4}$

$\therefore \tan x = \frac{\sin x}{\cos x} = \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{4}$

$\therefore \cot x = \frac{1}{\tan x} = -\frac{4}{3}$

#429942

Topic: Trigonometric Ratios of Any Angle

Find the value of other five trigonometric ratios:

$\cot x = \frac{3}{4}$, x lies in third quadrant.

Solution

$\cot x = \frac{3}{4}$

$\Rightarrow \tan x = \frac{1}{\cot x} = \frac{4}{3}$

Now $\because 1 + \tan^2 x = \sec^2 x$

$\Rightarrow \sec^2 x = 1 + \frac{16}{9} \Rightarrow \sec^2 x = \frac{25}{9}$

$\sec x = \pm \frac{5}{3}$

Since x lies in third quadrant, the value of $\sec x$ will be negative.

$\therefore \sec x = -\frac{5}{3}$

$\cos x = \frac{1}{\sec x} = -\frac{3}{5}$

$\tan x = \frac{\sin x}{\cos x}$

$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(-\frac{3}{5}\right) = -\frac{4}{5}$

$\therefore \csc x = \frac{1}{\sin x} = -\frac{5}{4}$

#429948

Topic: Trigonometric Ratios of Any Angle

Find the value of other five trigonometric ratios:

$\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Solution

$\sec x = \frac{13}{5}$

$\Rightarrow \cos x = \frac{1}{\sec x} = \frac{5}{13}$

Now $\because \sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x = 1 - \frac{25}{169} = \frac{144}{169}$

$\Rightarrow \sin x = \pm \frac{12}{13}$

Since x lies in fourth quadrant, the value of $\sin x$ will be negative.

$\therefore \sin x = -\frac{12}{13}$

$\csc x = \frac{1}{\sin x} = -\frac{13}{12}$

$\tan x = \frac{\sin x}{\cos x} = \frac{\left(-\frac{12}{13}\right)}{\left(\frac{5}{13}\right)} = -\frac{12}{5}$

$\cot x = \frac{1}{\tan x} = -\frac{5}{12}$

#429954

Topic: Trigonometric Ratios of Any Angle

Find the value of other five trigonometric ratios:

$\tan x = -\frac{5}{12}$, x lies in second quadrant.

Solution

$\tan x = -\frac{5}{12}$

$\Rightarrow \cot x = \frac{1}{\tan x} = -\frac{12}{5}$

Now using, $1 + \tan^2 x = \sec^2 x$

$\Longrightarrow \sec^2 x = 1 + \frac{25}{144} = \frac{169}{144}$

$\Longrightarrow \sec x = \pm\frac{13}{12}$

Since x lies in second quadrant, the value of $\sec x$ will be negative.

$\therefore \sec x = -\frac{13}{12}$

$\cos x = \frac{1}{\sec x} = -\frac{12}{13}$

$\tan x = \frac{\sin x}{\cos x}$

$\Longrightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$

$\therefore \csc x = \frac{1}{\sin x} = \frac{13}{5}$

#429959

Topic: Trigonometric Functions

Find the value of $\sin 765^\circ$

Solution

It is known that the values of $\sin x$ repeat after an interval of 2π or 360°

$\therefore \sin 765^\circ = \sin(2 \times 360^\circ + 45^\circ)$

$= \sin 45^\circ$

$= \frac{1}{\sqrt{2}}$

#429960

Topic: Trigonometric Functions

Find the value of $\csc(-1410^\circ)$

Solution

It is known that the values of $\csc x$ repeat after an interval of 2π or 360°.

$\therefore \csc(-1410^\circ) = \csc(-1410^\circ + 4 \times 360^\circ)$

$= \csc(-1410^\circ + 1440^\circ)$

$= \csc 30^\circ$

$= 2$

#429961

Topic: Trigonometric Functions

Find the value of $\tan\frac{19\pi}{3}$

Solution

It is known that the values of $\tan x$ repeat after an interval of π or 180°

$\therefore \tan\frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan\left(6\pi + \frac{\pi}{3}\right)$

$= \tan\frac{\pi}{3}$

$= \tan 60^\circ$

$= \sqrt{3}$

#429962

Topic: Trigonometric Functions

Find the value of $\sin\left(-\frac{11\pi}{3}\right)$

Solution

It is known that the values of $\sin x$ repeat after an interval of 2π or 360°.

$\therefore \sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2 \times 2\pi\right)$

$= \sin\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2}$

#429963

Topic: Trigonometric Functions

Find the value of $\cot\left(-\frac{15\pi}{4}\right)$

Solution

It is known that the values of $\cot x$ repeat after an interval of π or 180°.

$\therefore \cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right)$

$= \cot\frac{\pi}{4}$

$= 1$

#431788

Topic: Trigonometric Equations

Find the principal and general solutions of the following equations:

(i) $\tan x = \sqrt{3}$

(ii) $\sec x = 2$

(iii) $\cot x = -\sqrt{3}$

(iv) $cosec x = -2$

Solution

(i) $\tan x = \sqrt{3} = \tan\frac{\pi}{3} = \tan\frac{4\pi}{3}$

Hence principal solution of $\tan x = \sqrt{3}$

are $\frac{\pi}{3}$ and $\frac{4\pi}{3}$

$\tan x = \sqrt{3} = \tan\frac{\pi}{3}$

Hence general solution is given by,

$x = n\pi + \frac{\pi}{3}$ where $n \in Z$

(ii) $\sec x = 2 = \sec\frac{\pi}{3} = \sec\frac{5\pi}{3}$

Hence principal solution of $\sec x = 2$

are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

$\sec x = 2 = \sec\frac{\pi}{3}$

Hence general solution is given by,

$x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

(iii) $\cot x = -\sqrt{3} = \cot\frac{5\pi}{6} = \cot\frac{11\pi}{6}$

Hence principal value of $\cot x = -\sqrt{3}$

are $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

$\cot x = -\sqrt{3} = \cot\frac{5\pi}{6}$

Hence general solution is given by,

$x = n\pi + \frac{5\pi}{6}$, where $n \in Z$

(iv) $\csc x = -2 = \csc\frac{7\pi}{6} = \csc\frac{11\pi}{6}$

Hence principal solution of $\csc x = -2$

are $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$

$\csc x = -2 = \csc\frac{7\pi}{6}$

Hence general solution is given by,

$x = n\pi + (-1)^n\frac{7\pi}{6}$, where $n \in Z$

#431834

Topic: Domain and Range of Trigonometric functions

$\tan x = -\frac{4}{3}$, x in quadrant II. Find the value of $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$

Solution

As $\tan x = -\frac{4}{3}, \frac{\pi}{2} < x < \pi$

i.e x lies in $2nd$ quadrant

Hence $\tan x = -\frac{4}{3} \Rightarrow \sin x = \frac{4}{\sqrt{4^2+3^2}} = \frac{4}{5}$

And $\cos x = -\frac{5}{\sqrt{4^2+3^2}} = -\frac{3}{5}$

Now using $1-\cos x = 2\sin^2\frac{x}{2} \Rightarrow \sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$, we get

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}} = \pm\sqrt{\frac{8}{10}}$

As $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ and $sine$ is positive in $1st$ quadrant

Then $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

Using $1+\cos x = 2\cos^2\frac{x}{2} \Rightarrow \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

We get, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}} = \pm\sqrt{\frac{2}{10}}$

$\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ and cos is positive $1st$ quadrant

$\therefore \cos\frac{x}{2} = \frac{1}{\sqrt{5}}$

Using $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \Rightarrow \tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

We get, $\tan\frac{x}{2} = \pm\sqrt{\frac{1-\left(-\frac{3}{5}\right)}{1+\left(-\frac{3}{5}\right)}} = \pm\sqrt{4}$

As $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ and tan is positive in $1st$ quadrant

$\therefore \tan\frac{x}{2} = 2$

#431838

Topic: Domain and Range of Trigonometric functions

$\cos x = -\frac{1}{3}$, x in quadrant III. Find the value of $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$

Solution

$\cos x = -\frac{1}{3}, \pi < x < \frac{3\pi}{2}$

i.e. x lies in $3rd$ quadrant

Using $1-\cos x = 2\sin^2\frac{x}{2} \Rightarrow \sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

We get, $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\left(-\frac{1}{3}\right)}{2}} = \pm\sqrt{\frac{4}{6}}$

As $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ and sin is positive in $2nd$ quadrant

$\therefore \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{5}}$

Using $1+\cos x = 2\cos^2\frac{x}{2} \Rightarrow \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

we get, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\left(-\frac{1}{3}\right)}{2}} = \pm\sqrt{\frac{1}{3}}$

As $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ and cos is negative in $2nd$ quadrant

$\therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{3}}$

Using $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \Rightarrow \tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

We get $\tan\frac{x}{2} = \pm\sqrt{\frac{1-\left(-\frac{1}{3}\right)}{1+\left(-\frac{1}{3}\right)}} = \pm\sqrt{2}$

As $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$ and tan is negative in $2nd$ quadrant

$\therefore \tan\frac{x}{2} = -\sqrt{2}$

#431842

Topic: Domain and Range of Trigonometric functions

$\sin x = \frac{1}{4}$, x in quadrant II. Find the value of $\sin\frac{x}{2}$

Solution

$\sin x = \frac{1}{4}, x$ in $2nd$ quadrant

$\Rightarrow \frac{\pi}{2} < x < \pi$ and cos is negative in $2nd$ quadrant

Using $1 - \cos x = 2\sin^2\frac{x}{2} \Rightarrow \sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

We get, $\sin\frac{x}{2} = \pm\sqrt{\frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2}} = \pm\sqrt{\frac{4+\sqrt{15}}{8}}$

As $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ and sin is positive in $1st$ quadrant

$\therefore \sin\frac{x}{2} = \frac{\sqrt{8+2\sqrt{15}}}{4}$

Using $1 + \cos x = 2\cos^2\frac{x}{2} \Rightarrow \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

We get $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2}} = \pm\sqrt{\frac{4-\sqrt{15}}{8}} = \pm\frac{\sqrt{8-2\sqrt{15}}}{4}$

As $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ and cos is positive in first quadrant

$\therefore \cos\frac{x}{2} = \frac{\sqrt{8-2\sqrt{15}}}{4}$

Using $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} \Rightarrow \tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

We get $\tan\frac{x}{2} = \pm\sqrt{\frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{1+\left(-\frac{\sqrt{15}}{4}\right)}} = \pm\sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}}}$

As $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ and tan is positive in first quadrant

$\therefore \tan\frac{x}{2} = 4 + \sqrt{15}$

#446718

Topic: Trigonometric Equations

Prove that:

$sin^2\frac{\pi}{6} + cos^2\frac{\pi}{3} - tan^2\frac{\pi}{4} = -\frac{1}{2}$

Solution

$L.H.S = sin^2\frac{\pi}{6} + cos^2\frac{\pi}{3} - tan^2\frac{\pi}{4} = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - 1^2 = \frac{-1}{2}$

Therefore, L.H.S = R.H.S

Hence proved.

#446719

Topic: Trigonometric Equations

Prove that:

$2sin^2\frac{\pi}{6} + cosec^2\frac{7\pi}{6}cos^2\frac{\pi}{3} = \frac{3}{2}$

Solution

LHS $= 2sin^2\frac{\pi}{6} + cosec^2\frac{7\pi}{6}cos^2\frac{\pi}{3}$

$= 2sin^2\frac{\pi}{6} + cosec^2\left(\pi + \frac{\pi}{6}\right)cos^2\frac{\pi}{3}$

$= 2sin^2\frac{\pi}{6} + cosec^2\left(-\frac{\pi}{6}\right)cos^2\frac{\pi}{3}$ ....As θ lies in the 3rd quadrant, θ is negative.

$= 2\left(\frac{1}{2}\right)^2 + (-2)^2 \times \left(\frac{1}{2}\right)^2$

$= 2 \times \frac{1}{4} + 1 = \frac{3}{2}$

= RHS

Hence proved.

#446720

Topic: Trigonometric Equations

Prove that:

$cot^2\frac{\pi}{6} + cosec\frac{5\pi}{6} + 3tan^2\frac{\pi}{6} = 6$

Solution

$LHS = cot^2\frac{\pi}{6} + cosec\frac{5\pi}{6} + 3tan^2\frac{\pi}{6}$

$= \sqrt{3}^2 + cosec\left(\pi - \frac{\pi}{6}\right) + 3\frac{1}{\sqrt{3}^2} = 4 + cosec\frac{\pi}{6} = 6$

Hence proved

#446721

Topic: Trigonometric Equations

Prove that:

$2sin^2\frac{3\pi}{4} + 2cos^2\frac{\pi}{4} + 2sec^2\frac{\pi}{3} = 10$

Solution

$LHS = 2sin^2\frac{3\pi}{4} + 2cos^2\frac{\pi}{4} + 2sec^2\frac{\pi}{3}$

$= 2\sin^2\left(\pi - \frac{\pi}{4}\right) + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2 = 2\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8 = 10 = RHS$

#446723

Topic: Trigonometric Functions

Find the value of:

(i) $\sin 75^o$

(ii) $\tan 15^o$

Solution

i) $\sin 75^o = \sin(90^o - 15^o) = \cos 15^o$ $\{\sin(90^o - A) = \cos A\}$

using identity $\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$

put $A = 30^o$ in above identity

Therefore, $\cos 15^o = \sqrt{\frac{1+\cos 30^o}{2}} = \frac{\sqrt{\sqrt{3}+2}}{2}$

ii) $\tan 15^o = \sqrt{\sec^2 15^o - 1} = \sqrt{\frac{1}{\cos^2 15^o} - 1} = \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$ from(i)

#446725

Topic: Compound Angles

Prove the following:

$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x+y)$$

Solution

$$LHS = \cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \cos\left(\frac{\pi}{4} - x + \frac{\pi}{4} - y\right)$$

Using identity $\cos A\cos B - \sin A\sin B = \cos(A+B)$

$$= \cos\left(\frac{\pi}{2} - (x+y)\right) = \sin(x+y) = RHS$$

Hence proved

#446726

Topic: Compound Angles

Prove the following:

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Solution

use identity

$$\tan(A \pm B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$LHS = \frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \frac{\frac{1+\tan x}{1-\tan x}}{\frac{1-\tan x}{1+\tan x}}$$

$$LHS = \left(\frac{1+\tan x}{1-\tan x}\right)^2 = RHS$$

Hence proved

#446727

Topic: Trigonometric Equations

Prove the following:

$$\frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = -\cot^2 x$$

Solution

$$LHS = \frac{cos(\pi + x)cos(-x)}{sin(\pi - x)cos\left(\frac{\pi}{2} + x\right)} = \frac{(-\cos x)(\cos x)}{(\sin x)(\sin x)}$$

$$= -\frac{\cos^2 x}{\sin^2 x} = -\cot^2 x = RHS$$

Hence proved

#446728

Topic: Trigonometric Equations

Prove the following:

$$cos\left(\frac{3\pi}{2} + x\right)cos(2\pi + x)\left[cot\left(\frac{3\pi}{2} - x\right) + cot(2\pi + x)\right] = 1$$

Solution

$$LHS = cos\left(\frac{3\pi}{2} + x\right)cos(2\pi + x)\left[cot\left(\frac{3\pi}{2} - x\right) + cot(2\pi + x)\right]$$

$$= \sin x\cos x[\tan x + \cot x]$$

$$= \frac{\sin x\cos x\left(\sin^2 x + \cos^2 x\right)}{\sin x\cos x} = 1 = RHS$$

Hence proved

#446729

Topic: Compound Angles

Prove the following:

$$sin(n + 1)x\, sin(n + 2)x + cos(n + 1)x\, cos(n + 2)x = cos\, x$$

Solution

Use $cosAcosB + sinAsinB = cos(A - B)$

$$LHS = sin(n + 1)x\, sin(n + 2)x + cos(n + 1)x\, cos(n + 2)x = \cos[(n + 2)x - (n + 1)x]$$

$$= \cos x = RHS$$

Hence proved

#446730

Topic: Compound Angles

Prove the following:

$$cos\left(\frac{3\pi}{4} + x\right) - cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2}\, sin\, x$$

Solution

$$LHS = cos\left(\frac{3\pi}{4} + x\right) - cos\left(\frac{3\pi}{4} - x\right)$$

$$= cos\left(\frac{3\pi}{4}\right)cosx - sin\left(\frac{3\pi}{4}\right)sinx - \left[cos\left(\frac{3\pi}{4}\right)cosx + sin\left(\frac{3\pi}{4}\right)sinx\right]$$

$$= -2sin\left(\frac{3\pi}{4}\right)sinx = -\sqrt{2}\sin x = RHS$$

Hence, proved

#446731

Topic: Trigonometric Equations

Prove the following:

$$sin^2 6x - sin^2 4x = sin\, 2x\, sin\, 10x$$

Solution

LHS $= \sin^2 6x - \sin^2 4x$

$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$ ... [$\because a^2 - b^2 = (a+b)(a-b)$]

We know that, $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\therefore$ LHS $= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right)\sin\left(\frac{6x-4x}{2}\right)\right]$

$= \left[2\sin\left(\frac{10x}{2}\right)\cos\left(\frac{2x}{2}\right)\right]\left[2\cos\left(\frac{10x}{2}\right)\sin\left(\frac{2x}{2}\right)\right]$

$= 2\sin 5x\cos x \times 2\cos 5x\sin x$

$= 2\sin 5x\cos 5x \times 2\sin x\cos x$

$= \sin 10x \times \sin 2x$ ...[$\because \sin 2\theta = 2\sin\theta\cos\theta$]

$=$ RHS

Hence proved.

#446732

Topic: Trigonometric Equations

Prove the following:

$cos^2 2x - cos^2 6x = sin 4x\, sin 8x$

Solution

$LHS = cos^2 2x - cos^2 6x$

$= (\cos 2x - \cos 6x)(\cos 2x + \cos 6x)$

$= \left(-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right)\right)\left(-2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right) = (2\sin 4x\cos 4x)(2\sin 2x\cos 2x) = \sin 4x\sin 8x$

#446733

Topic: Trigonometric Equations

Prove the following:

$sin 2x + 2\, sin 4x + sin 6x = 4cos^2 x\, sin 4x$

Solution

$LHS = sin 2x + 2\, sin 4x + sin 6x = sin 2x + sin 6x + 2\, sin 4x$

$= 2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right) + 2\, sin 4x = 2\, sin 4x[1 + \cos 2x] = 4cos^2 x\, sin 4x = RHS$

Hence proved

#446734

Topic: Trigonometric Equations

Prove the following:

$cot 4x(sin 5x + sin 3x) = cot x(sin 5x - sin 3x)$

Solution

$$LHS = cot\,4x(sin\,5x + sin\,3x) = \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$= 2\cos 4x\cos x = \frac{\cos x}{\sin x}(2\cos 4x\sin x) = cot\,x(sin\,5x - sin\,3x)$$

Hence proved

#446735

Topic: Trigonometric Equations

Prove the following:

$$\frac{cos\,9x - cos5x}{sin\,17x - sin\,3x} = -\frac{sin\,2x}{cos\,10x}$$

Solution

$$LHS = \frac{cos\,9x - cos5x}{sin\,17x - sin\,3x} = \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}$$

$$= -\frac{\sin 7x sin\,2x}{cos\,10x\sin 7x}$$

$$= -\frac{sin\,2x}{cos\,10x} = RHS$$

Hence proved

#446736

Topic: Trigonometric Equations

Prove the following:

$$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

Solution

$$LHS = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \tan 4x = RHS$$

Hence proved.

#446737

Topic: Trigonometric Equations

Prove the following:

$$\frac{sin\,x - sin\,y}{cos\,x + cos\,y} = tan\frac{x-y}{2}$$

Solution

$$LHS = \frac{sin\,x - siny}{cos\,x + cos\,y} = \frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\frac{x-y}{2} = RHS$$

Hence, proved

#446738

Topic: Trigonometric Equations

Prove the following:

$$\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

Solution

$$LHS = \frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \frac{-2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$$= \tan 2x = RHS$$

Hence, proved

#446739

Topic: Trigonometric Equations

Prove the following:

$$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$$

Solution

$$LHS = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x} = 2\sin x$$

#446740

Topic: Trigonometric Equations

Prove the following:

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

Solution

$$LHS = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{\cos 4x + \cos 2x + \cos 3x}{\sin 4x + \sin 2x + \sin 3x} = \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x} = \cot 3x = RHS$$

Hence proved

#446741

Topic: Compound Angles

Prove the following:

$$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$$

Solution

$x + 2x = 3x$

Taking $\tan$ on both sides

$\tan(x + 2x) = \tan 3x$

$\Rightarrow \dfrac{\tan x + \tan 2x}{1 - \tan x \tan 2x} = \tan 3x$

$\Rightarrow \dfrac{\cot x + \cot 2x}{\cot x \cot 2x - 1} = \dfrac{1}{\cot 3x}$

By rearranging

$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

#446742

Topic: Trigonometric Equations

Prove the following:

$\tan 4x = \dfrac{4 \tan x(1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

Solution

$\tan 4x = \dfrac{2\tan 2x}{1 - \tan^2 2x}$

$= 2\dfrac{\dfrac{2\tan x}{1 - \tan^2 x}}{1 - \left(\dfrac{2\tan x}{1 - \tan^2 x}\right)^2}$

$= \dfrac{4 \tan x(1 - \tan^2 x)}{(1 - \tan^2 x)^2 - 4\tan^2 x} = \dfrac{4 \tan x(1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

Hence proved

#446743

Topic: Trigonometric Equations

Prove the following:

$\cos 4x = 1 - 8 \sin^2 x \cos^2 x$

Solution

$\cos 4x = 2\cos^2 2x - 1 = 2\left(2\cos^2 x - 1\right)^2 - 1 = 1 + 2\left(4\cos^4 x - 4\cos^2 x\right) = 1 - 8\cos^2 x\left(1 - \cos^2 x\right) = 1 - 8\cos^2 x \sin^2 x$

Hence proved

#446744

Topic: Trigonometric Equations

Prove the following:

$\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

Solution

Use $\cos 3x = 4\cos^3 x - 3\cos x$

$LHS = \cos 6x = \cos 3(2x) = 4\cos^3 2x - 3\cos 2x = 4\left(2\cos^2 x - 1\right)^3 - 3\left(2\cos^2 x - 1\right) = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

#446745

Topic: Trigonometric Equations

Find the general solution:

$\cos 4x = \cos 2x$

Solution

$\cos 4x = \cos 2x$

$\Rightarrow 2\cos^2 2x - 1 - \cos 2x = 0 \Rightarrow \cos 2x = -\frac{1}{2}, 1$

$\Rightarrow 2x = 2n\pi \pm \frac{\pi}{3}, 2n\pi$

$\Rightarrow x = n\pi \pm \frac{\pi}{6}, n\pi$

#446746

Topic: Trigonometric Equations

Find the general solution:

$\cos 3x + \cos x - \cos 2x = 0$

Solution

$\cos 3x + \cos x - \cos 2x = 0$

$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \Rightarrow \cos 2x(2\cos x - 1) = 0$

$\Rightarrow \cos 2x = 0, \cos x = \frac{1}{2}$

$\Rightarrow x = \frac{(2n+1)\pi}{4}, x = 2n\pi \pm \frac{\pi}{3}$

#446747

Topic: Trigonometric Equations

Find the general solution:

$\sin 2x + \cos x = 0$

Solution

$\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x\cos x + \cos x = 0$

$\Rightarrow \cos x = 0$ or $\sin x = \frac{-1}{2}$

$\Rightarrow x = \frac{(2n+1)\pi}{2}$ or $x = n\pi - (-1)^n\frac{\pi}{6}$

#446748

Topic: Trigonometric Equations

Find the general solution:

$\sec^2 2x = 1 - \tan 2x$

Solution

$\sec^2 2x = 1 - \tan 2x$

$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$

$\Rightarrow \tan 2x(\tan 2x + 1) = 0$

$\Rightarrow \tan 2x = 0, -1$

$\Rightarrow 2x = n\pi, n\pi - \frac{\pi}{4}$

$\Rightarrow x = \frac{n\pi}{2}, \frac{n\pi}{2} - \frac{\pi}{8}$

#446749

Topic: Trigonometric Equations

Find the general solution:

$\sin x + \sin 3x + \sin 5x = 0$

Solution

$sin\,x + sin\,3x + sin\,5x = 0$

$\Rightarrow sin\,x + sin\,5x + sin\,3x = 0$

$\Rightarrow 2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right) + sin\,3x = 0 \Rightarrow \sin 3x(2\cos 2x + 1) = 0$

$\Rightarrow \sin 3x = 0$ or $\cos 2x = \frac{-1}{2}$

$\Rightarrow 3x = n\pi$ or $2x = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = \frac{n\pi}{3}, n\pi \pm \frac{\pi}{6}$

#446750

Topic: Trigonometric Equations

Prove that:

$2\,cos\frac{\pi}{13}cos\frac{9\pi}{13} + cos\frac{3\pi}{13} + cos\frac{5\pi}{13} = 0$

Solution

$LHS = 2\,cos\frac{\pi}{13}cos\frac{9\pi}{13} + cos\frac{3\pi}{13} + cos\frac{5\pi}{13}$

$= cos\frac{10\pi}{13} + cos\frac{8\pi}{13} + cos\frac{3\pi}{13} + cos\frac{5\pi}{13}$

$= \cos\left(\pi - \frac{3\pi}{13}\right) + \cos\left(\pi - \frac{5\pi}{13}\right) + cos\frac{3\pi}{13} + cos\frac{5\pi}{13}$

$= -cos\frac{3\pi}{13} - cos\frac{5\pi}{13} + cos\frac{3\pi}{13} + cos\frac{5\pi}{13} = 0 = RHS$

Hence proved

#446752

Topic: Trigonometric Equations

Prove that:

$(sin\,3x + sin\,x)sin\,x + (cos\,3x - cos\,x)cos\,x = 0$

Solution

$LHS = (sin\,3x + sin\,x)sin\,x + (cos\,3x - cos\,x)cos\,x$

$= \left[2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right]\sin x + \left[-2\sin\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)\right]\cos x$

$\Rightarrow 2[\sin x\cos x\sin 2x - \sin x\cos x\sin 2x] = 0 = RHS$

#446753

Topic: Trigonometric Equations

Prove that:

$(cos\,x + cos\,y)^2 + (sin\,x - sin\,y)^2 = 4\,cos^2\frac{x+y}{2}$

Solution

$LHS = (cos\,x + cos\,y)^2 + (sin\,x - sin\,y)^2$

$= \cos^2 x + \cos^2 y + 2\cos x\cos y + \sin^2 x + \sin^2 y - 2\sin x\sin y$

$= \left(\cos^2 x + \sin^2 x\right) + \left(\cos^2 y + \sin^2 y\right) + 2(\cos x\cos y - \sin x\sin y) = 1 + 1 + 2\cos(x+y)$

$= 2(1 + \cos(x+y)) = 4\,cos^2\frac{x+y}{2} = RHS$

Hence proved

#446754

Topic: Trigonometric Equations

Prove that:

$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\frac{x-y}{2}$$

Solution

$LHS = (\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$= \cos^2 x + \cos^2 y - 2\cos x\cos y + \sin^2 x + \sin^2 y - 2\sin x\sin y$

$= \left(\cos^2 x + \sin^2 x\right) + \left(\cos^2 y + \sin^2 y\right) - 2(\cos x\cos y + \sin x\sin y) = 1 + 1 - 2\cos(x - y)$

$= 2(1 - \cos(x - y)) = 4\sin^2\frac{x-y}{2} = RHS$

Hence proved

#446755

Topic: Trigonometric Equations

Prove that:

$$\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x\cos 2x\sin 4x$$

Solution

$LHS = \sin x + \sin 3x + \sin 5x + \sin 7x = \sin x + \sin 7x + \sin 3x + \sin 5x = 2\sin 4x\cos 3x + 2\sin 4x\cos x = 2\sin 4x(\cos 3x + \cos x) = 4\sin 4x\cos x\cos 2x = RHS$

Hence proved

#446756

Topic: Trigonometric Equations

Prove that:

$$\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$$

Solution

$$LHS = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

$$= \frac{2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right) + 2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)}{2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right) + 2\cos\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)}$$

$$= \frac{\sin 6x\cos x + \sin 6x\cos 3x}{\cos 6x\cos x + \cos 6x\cos 3x} = \tan 6x = RHS$$

Hence proved

#446757

Topic: Trigonometric Equations

Prove that:

$$\sin 3x + \sin 2x - \sin x = 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$$

Solution

$LHS = \sin 3x + \sin 2x - \sin x$

$= 2\sin\left(\frac{3x+2x}{2}\right)\cos\left(\frac{3x-2x}{2}\right) - 2\sin\frac{x}{2}\cos\frac{x}{2}$

$= 2\cos\frac{x}{2}\left(\sin\frac{5x}{2} - \sin\frac{x}{2}\right) = 2\cos\frac{x}{2}\cos\frac{3x}{2}\sin x$

$= 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2} = RHS$