
#418217

Topic: Linear Combination of Vectors

Show that the points $(-2, 3, 5), (1, 2, 3)$ and $(7, 0, -1)$ are collinear

Solution

Let points $(-2, 3, 5), (1, 2, 3)$ and $(7, 0, -1)$ be denoted by P, Q and R respectively.

Points P, Q and R are collinear if they lie on a line.

$PQ = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$

$= \sqrt{(3)^2 + (-1)^2 + (-2)^2}$

$= \sqrt{9+1+4}$

$= \sqrt{14}$

$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$

$= \sqrt{(6)^2 + (-2)^2 + (-4)^2}$

$= \sqrt{36+4+16}$

$= \sqrt{56}$

$= 2\sqrt{14}$

$PR = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$

$= \sqrt{(9)^2 + (-3)^2 + (-6)^2}$

$= \sqrt{81+9+36}$

$= \sqrt{126}$

$= 3\sqrt{14}$

Here $PQ + QR = \sqrt{14} + 2\sqrt{14}$

$= 3\sqrt{14}$

$= PR$

$\Rightarrow PQ + QR = PR$

Hence points $P(-2, 3, 5), Q(1, 2, 3)$ and $R(7, 0, -1)$ are collinear

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#424873

Topic: Lines

Show that points

A (a, b + c), B (b, c + a), C (c, a + b) are collinear

Solution

Area of triangle ABC is,

$$\Delta = \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & b+c & 1 \\ a+b+c & c+a & 1 \\ a+b+c & a+b & 1 \end{vmatrix} \text{Using } C_1 \to C_1 + C_2$$

$$= (a+b+c)\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix} = 0$$

Since $C_1 = C_2$

Hence points A, B, and C are collinear.

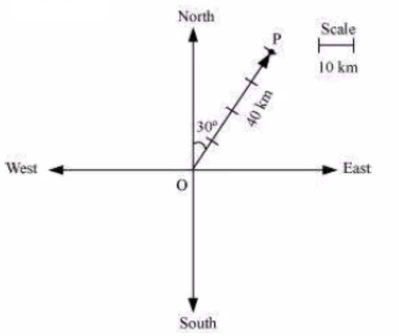
#428689

Topic: Introduction

Represent graphically a displacement of $40\ km, 30^o$ east of north.

Solution

Here, vector $\vec{OP}$ represents the displacement of 40 km,30^o East of North.



#428692

Topic: Introduction

Classify the following measures as scalars and vectors.

(i) $10\ kg$

(ii) $2\ metres$ north-west

(iii) 40^o

(iv) $40\ watt$

(v) 10^{-19} coluomb

(vi) $20\ m/s^2$

Solution

(i) $10\ kg$

"kg" is unit of mass which is a scalar quantity because it involves only magnitude.

(ii) 2 meters north-west is a vector quantity as it involves both magnitude and direction.

(iii) 40^o is a scalar quantity as it involves only magnitude.

(iv) 40 watts a scalar quantity as it involves only magnitude.

(v) 10^{-19} coulomb is a scalar quantity as it involves only magnitude.

(vi) 20m/s^2 is a vector quantity as it involves magnitude as well as direction.

#428706

Topic: Introduction

Classify the following as scalar and vector quantities.

(i) Time period

(ii) Distance

(iii) Force

(iv) Velocity

(v) Work done

Solution

(i) Time period is a scalar quantity as it involves only magnitude.

(ii) Distance is a scalar quantity as it involves only magnitude.

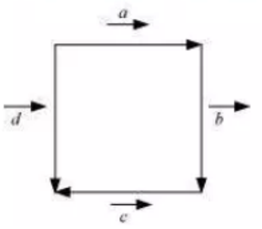
(iii) Force is a vector quantity as it involves both magnitude and direction.

(iv) Velocity is a vector quantity as it involves both magnitude as well as direction.

(v) Work done is a scalar quantity as it involves only magnitude.

#428722

Topic: Types of Vectors



In Figure, identify the following vectors.

(i) Coinitial

(ii) Equal

(iii) Collinear but not equal

Solution

(i)Vectors $\vec{a}$ and $\vec{d}$ are coinitial because they have the same initial point.

(ii) Vectors $\vec{b}$ and $\vec{d}$ are equal because they have the same magnitude and direction.

(iii) Vectors $\vec{a}$ and $\vec{b}$ are collinear but not equal as they are parallel, their directions are not the same.

Vectors $\vec{b}$ and $\vec{d}$ are collinear and equal vectors as they are parallel and their directions are same.

#428723

Topic: Types of Vectors

Answer the following as true or false.

(i) $\vec{a}$ and $-\vec{a}$ are collinear

(ii) Two collinear vectors are always equal in magnitude.

(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.

Solution

(i)

Two vectors are said to be be collinear vectors if they are parallel to the same line. In other words, any two parallel vectors are collinear.

Since, the negative of $\vec{a}$ i.e. $-\vec{a}$ is a vector having same magnitude but opposite direction.

So, $\vec{a}$, $-\vec{a}$ are collinear vectors.

(iii)

Given statement is "False".

As collinear vectors are those vectors that are parallel to the same line.

#428728

Topic: Operations on Vector

If origin O is in the centre of the triangle ABC and a, b and c are the lengths of the sides, then the force $a_{\overrightarrow{OA}} + b_{\overrightarrow{OB}} + c_{\overrightarrow{OC}} =$

A -1

B 2

[✓ **C**] 0

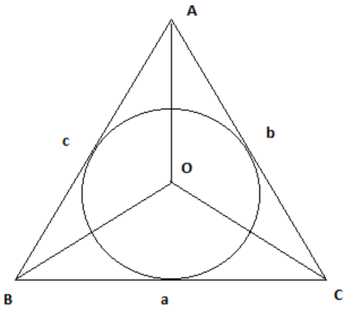
D none of these

Solution

If O is the origin, then tge center of $\triangle ABC$ is given by

$$\frac{a_{\overrightarrow{OA}} + b_{\overrightarrow{OB}} + c_{\overrightarrow{OC}}}{a+b+c} = 0$$

$$\therefore a_{\overrightarrow{OA}} + b_{\overrightarrow{OB}} + c_{\overrightarrow{OC}} = 0$$



#428729

Topic: Vector Component Form

Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k};\ \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k};\ \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Solution

The given vectors are:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k};\ \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k};\ \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Magnitude of these vectors are given by,

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

$$|\vec{b}| = \sqrt{(2)^2 + (-7)^2 + (-3)^2}$$

$$= \sqrt{4 + 49 + 9} = \sqrt{62}$$

$$|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2}$$

$$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$$

#428730

Topic: Types of Vectors

Write two different vectors having same magnitude

Solution

Consider $\vec{a} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$ and $\vec{b} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right)$.

It can be observed that $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

and $|\vec{b}| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$

Hence, $\vec{a}$ and $\vec{b}$ are two different vectors having the same magnitude. The vectors are different because they have different directions.

#428731

Topic: Types of Vectors

Write two different vectors having same direction

Solution

Consider $\vec{p} = \left(\hat{i} + \hat{j} + \hat{k}\right)$ and $\vec{q} = \left(2\hat{i} + 2\hat{j} + 2\hat{k}\right)$

The directions cosines of $\vec{p}$ are given by,

$l = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, m = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}$, and $n = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}$.

The direction cosines of $\vec{q}$ are given by

$l = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, m = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$, and $n = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$.

The direction cosines of $\vec{p}$ and $\vec{q}$ are the same. Hence, the two vectors have the same direction.

#428732

Topic: Types of Vectors

Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal

Solution

The two vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ will be equal if their corresponding components are equal.

Hence, the required values of x and y are 2 and 3 respectively.

#428734

Topic: Vector Component Form

Find the scalar and vector components of the vector with initial point $(2, 1)$ and terminal point $(-5, 7)$

Solution

The vector with the initial point $P(2, 1)$ and terminal point $Q(-5, 7)$ can be given by,

$\vec{PQ} = (-5 - 2)\hat{i} + (7 - 1)\hat{j}$

$\Rightarrow \vec{PQ} = -7\hat{i} + 6\hat{j}$

Hence, the required scalar components are -7 and 6 while the vector components are $-7\hat{i}$ and $6\hat{j}$.

#428737

Topic: Operations on Vector

Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Solution

The given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

$\therefore \vec{a} + \vec{b} + \vec{c} = (1 - 2 + 1)\hat{i} + (-2 + 4 - 6)\hat{j} + (1 + 5 - 7)\hat{k}$

$= 0 \cdot \hat{i} - 4\hat{j} - 1 \cdot \hat{k}$

$= -4\hat{j} - \hat{k}$

#428738

Topic: Vector Component Form

Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

Solution

The unit vector $\hat{a}$ in the direction of vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is given by $\hat{a} = \frac{a}{|a|}$.

$|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$

$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$

#428740

Topic: Vector Component Form

Find the unit vector in the direction of vector $\vec{PQ}$, where P and Q are the points $(1, 2, 3)$ and $(4, 5, 6)$, respectively

Answer required

Solution

The given points are $P(1, 2, 3)$ and $Q(4, 5, 6)$.

$\therefore \vec{PQ} = (4 - 1)\hat{i} + (5 - 2)\hat{j} + (6 - 3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$\left|\vec{PQ}\right| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9 + 9 + 9} = \sqrt{27} = 3\sqrt{3}$

Hence, the unit vector in the direction of $\vec{PQ}$ is

$$\frac{\vec{PQ}}{\left|\vec{PQ}\right|} = \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

#428860

Topic: Vector Component Form

For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$

Solution

The given vectors are $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$.

$\therefore \vec{a} + \vec{b} = (2 - 1)\hat{i} + (-1 + 1)\hat{j} + (2 - 1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$

$\left|\vec{a} + \vec{b}\right| = \sqrt{1^2 + 1^2} = \sqrt{2}$

Hence, the unit vector in the direction of $\left(\vec{a} + \vec{b}\right)$ is

$$\frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \frac{\hat{i} + \hat{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}.$$

#428863

Topic: Vector Component Form

Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units

Solution

Let $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$

$\therefore |\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25 + 1 + 4} = \sqrt{30}$

$\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$

Hence, the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units is given by,

$$8\hat{a} = 8\left(\frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}\right) = \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

#428865

Topic: Linear Combination of Vectors

Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear

Solution

Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

It is observed that $\vec{b} = 4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$

$\therefore \vec{b} = \lambda\vec{a}$, where $\lambda = -2$

Hence, the given vectors are collinear.

#428875

Topic: Section Formula

Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio $2:1$,

(i) internally

(ii) externally

Solution

The position vector of point R dividing the line segment joining two points P and Q in the ratio $m:n$ is given by:

i. Internally:

$$\frac{m\vec{Q} + n\vec{P}}{m+n}$$

ii. Externally:

$$\frac{m\vec{Q} - n\vec{P}}{m-n}$$

Position vectors of P and Q are given as:

$\vec{OP} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{OQ} = -\hat{i} + \hat{j} + \hat{k}$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio $2:1$ is given by,

$$\vec{OR} = \frac{2(-\hat{i} + \hat{j} + \hat{k}) + 1(\hat{i} + 2\hat{j} - \hat{k})}{2+1} = \frac{-2\hat{i} + 2\hat{j} + \hat{k} + (\hat{i} + 2\hat{j} - \hat{k})}{3}$$

$$= \frac{-\hat{i} + 4\hat{j} + \hat{k}}{3} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}$$

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio $2:1$ is given by,

$$\vec{OR} = \frac{2(-\hat{i} + \hat{j} + \hat{k}) - 1(\hat{i} + 2\hat{j} - \hat{k})}{2-1} = (-2\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k})$$

$$= -3\hat{i} + 3\hat{k}$$

#429149

Topic: Section Formula

Find the position vector of the mid point of the vector joining the points $P(2, 3, 4)$ and $Q(4, 1, 2)$.

Solution

The position vector of mid-point R of the vector joining points $P(2, 3, 4)$ and $Q(4, 1, -2)$ is given by,

$$\vec{OR} = \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2} = \frac{(2+4)\hat{i} + (3+1)\hat{j} + (4-2)\hat{k}}{2}$$

$$= \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}$$

#429153

Topic: Vector Component Form

Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right angled triangle.

Solution

Position vectors of points A, B and C are respectively given as:

$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\therefore \overrightarrow{AB} = \vec{b} - \vec{a} = (2 - 3)\hat{i} + (-1 + 4)\hat{j} + (1 + 4)\hat{k} = -\hat{i} - 3\hat{j} + 5\hat{k}$

$\overrightarrow{BC} = \vec{c} - \vec{b} = (1 - 2)\hat{i} + (-3 + 1)\hat{j} + (-5 - 1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$

$\overrightarrow{CA} = \vec{a} - \vec{c} = (3 - 1)\hat{i} + (-4 + 3)\hat{j} + (-4 + 5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$

$\therefore \left|\overrightarrow{AB}\right|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$

$\left|\overrightarrow{BC}\right|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$

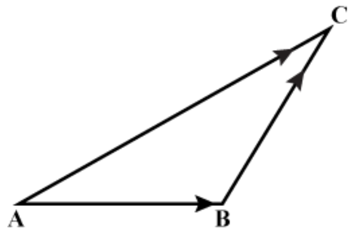
$\left|\overrightarrow{CA}\right|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$

$\therefore \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{CA}\right|^2 = 36 + 6 = 41 = \left|\overrightarrow{BC}\right|^2$

Hence, ABC is a right-angled triangle.

#429162

Topic: Operations on Vector



In triangle ABC, which of the following is not true?

A $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

B $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

[✓ C] $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

D $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

Solution

On applying the triangle law of addition in the given triangle, we have:

$\vec{AB} + \vec{BC} = \vec{AC} \ldots\ldots\ldots(1)$

$\Rightarrow \vec{AB} + \vec{BC} = -\vec{CA}$

$\Rightarrow \vec{AB} + \vec{BC} + \vec{CA} = \vec{0} \ldots\ldots\ldots(2)$

$\therefore$ The equation given in alternative A is true.

$\vec{AB} + \vec{BC} = \vec{AC}$

$\Rightarrow \vec{AB} + \vec{BC} - \vec{AC} = \vec{0}$

$\therefore$ The equation given in alternative B is true.

From equation (2), we have:

$\vec{AB} - \vec{CB} + \vec{CA} = 0$

$\therefore$ The equation given in alternative D is true.

Now, consider the equation given in alternative C:

$\vec{AB} + \vec{BC} - \vec{CA} = \vec{0}$

$\Rightarrow \vec{AB} + \vec{BC} = \vec{CA} \ldots\ldots\ldots(3)$

From equations (1) and (3), we have:

$\vec{AC} = \vec{CA}$

$\Rightarrow \vec{AC} = -\vec{AC}$

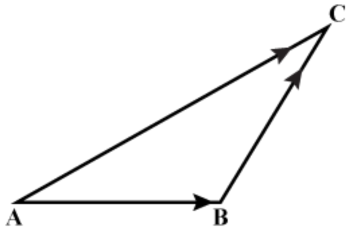
$\Rightarrow \vec{AC} + \vec{AC} = \vec{0}$

$\Rightarrow 2\vec{AC} = \vec{0}$

$\Rightarrow \vec{AC} = \vec{0}$, which is not true.

Hence, the equation given in alternative C is incorrect.

The correct answer is C.



#429163

Topic: Linear Combination of Vectors

If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect?

A $\vec{b} = \lambda\vec{a}$, for some scalar λ

B $\vec{a} = \pm\vec{b}$

C the respective components of $\vec{a}$ and $\vec{b}$ are proportional

D both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes

Solution

If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then they are parallel.

Therefore, we have:

$\vec{b} = \lambda\vec{a}$ (For some scalar λ)

If $\lambda = \pm 1$, then $\vec{a} = \pm\vec{b}$

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$\vec{b} = \lambda\vec{a}$

$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda(a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = \lambda(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$$

$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$

$\Rightarrow b_1 = \lambda a_1, b_1 = \lambda a_2, b_3 = \lambda a_3$

$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$

Thus, the respective components of $\vec{a}$ and $\vec{b}$ are proportional.

However, vectors $\vec{a}$ and $\vec{b}$ can have different directions.

Hence, the statement given in D is incorrect.

The correct answer is D.

#429165

Topic: Applications of Dot Product

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2, respectively having $\vec{a}\cdot\vec{b} = \sqrt{6}$

Answer required

Solution

It is given that,

$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and, $\vec{a}\cdot\vec{b} = \sqrt{6}$

Now, we know that $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore \sqrt{6} = \sqrt{3} \times 2 \times \cos\theta$

$\Rightarrow \cos\theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{4}$

Hence, the angle between the given vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$.

#429169

Topic: Applications of Dot Product

Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

Solution

Let the given vectors are $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$

Now, $\vec{a}\cdot\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k})(3\hat{i} - 2\hat{j} + \hat{k})$

$= 1.3 + (-2)(-2) + 3.1$

$= 3 + 4 + 3 = 10$

Also, we know that $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore 10 = \sqrt{14}\sqrt{14}\cos\theta$

$\Rightarrow \cos\theta = \frac{10}{14}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{5}{7}\right)$

#429171

Topic: Applications of Dot Product

Find the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$.

Solution

Let $\vec{a}=\hat{i}-\hat{j}$ and $\vec{b}=\hat{i}+\hat{j}$.

Now, projection of vector $\vec{a}$ and $\vec{b}$ is given by,

$$\frac{1}{|\vec{b}|}(\vec{a}\cdot\vec{b})=\frac{1}{\sqrt{1+1}}\{1.1+(-1)(1)\}=\frac{1}{\sqrt{2}}(1-1)=0$$

Hence, the projection of vector $\vec{a}$ on $\vec{b}$ is 0.

#429174

Topic: Applications of Dot Product

Find the projection of the vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$.

Solution

Let $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ and $\hat{b}=7\hat{i}-\hat{j}+8\hat{k}$.

Now, projection of vector $\vec{a}$ on $\vec{b}$ is given by,

$$\frac{1}{|\vec{b}|}(\vec{a}\cdot\vec{b})=\frac{1}{\sqrt{7^2+(-1)^2+8^2}}\{1(7)+3(-1)+7(8)\}=\frac{7-3+56}{\sqrt{49+1+64}}=\frac{60}{\sqrt{114}}$$

#429180

Topic: Dot Product

Show that each of the given three vectors is a unit vector: $\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k}),\ \frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k}),\ \frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})$.

Also, show that they are mutually perpendicular to each other.

Solution

Let $\vec{a}=\frac{1}{7}(2\hat{i}+3\hat{j}+6\hat{k})=\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$,

$\vec{b}=\frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k})=\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}$,

and $\vec{c}=\frac{1}{7}(6\hat{i}+2\hat{j}-3\hat{k})=\frac{6}{7}\hat{i}+\frac{2}{7}\hat{j}-\frac{3}{7}\hat{k}$.

$$|\vec{a}|=\sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{6}{7}\right)^2}=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=1$$

$$|\vec{b}|=\sqrt{\left(\frac{3}{7}\right)^2+\left(-\frac{6}{7}\right)^2+\left(\frac{2}{7}\right)^2}=\sqrt{\frac{9}{49}+\frac{36}{49}+\frac{9}{49}}=1$$

$$|\vec{c}|=\sqrt{\left(\frac{6}{7}\right)^2+\left(\frac{2}{7}\right)^2+\left(-\frac{3}{7}\right)^2}=\sqrt{\frac{36}{49}+\frac{4}{49}+\frac{9}{49}}=1$$

Thus, each of the given three vectors is a unit vector.

$$\vec{a}\cdot\vec{b}=\frac{2}{7}\times\frac{3}{7}+\frac{3}{7}\times\left(\frac{-6}{7}\right)+\frac{6}{7}\times\frac{2}{7}=\frac{6}{49}-\frac{18}{49}+\frac{12}{49}=0$$

$$\vec{b}\cdot\vec{c}=\frac{3}{7}\times\frac{6}{7}+\left(\frac{-6}{7}\right)\times\frac{2}{7}+\frac{2}{7}\times\left(\frac{-3}{7}\right)=\frac{18}{49}-\frac{12}{49}-\frac{6}{49}=0$$

$$\vec{c}\cdot\vec{a}=\frac{6}{7}\times\frac{2}{7}+\frac{2}{7}\times\frac{3}{7}+\left(\frac{-3}{7}\right)\times\frac{6}{7}=\frac{12}{49}+\frac{6}{49}-\frac{18}{49}=0$$

Hence, the given three vectors are mutually perpendicular to each other.

#429185

Topic: Applications of Dot Product

Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})=8$ and $|\vec{a}|=8|\vec{b}|$

Solution

$(\vec{a} \cdot \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$

$\Rightarrow \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b} = 8$$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow (8|\vec{b}|)^2 - |\vec{b}|^2 = 8,\ [\because |\vec{a}| = 8|\vec{b}|]$

$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow 63|\vec{b}|^2 = 8$

$\Rightarrow |\vec{b}|^2 = \dfrac{8}{63}$

$\Rightarrow |\vec{b}| = \sqrt{\dfrac{8}{63}}$ [Magnitude of a vector is non-negative]

$\therefore |\vec{b}| = \dfrac{2\sqrt{2}}{3\sqrt{7}}$ and $|\vec{a}| = \dfrac{16\sqrt{2}}{3\sqrt{7}}$

#429188

Topic: Dot Product

Evaluate the product $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$.

Solution

$(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$

$= 3\vec{a} \cdot 2\vec{a} + 3\vec{a} \cdot 7\vec{b} - 5\vec{b} \cdot 2\vec{a} - 5\vec{b} \cdot 7\vec{b}$$

$= 6\vec{a} \cdot \vec{a} + 21\vec{a} \cdot \vec{b} - 10\vec{a} \cdot \vec{b} - 35\vec{b} \cdot \vec{b}$

$= 6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$

#429189

Topic: Applications of Dot Product

Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude and such that the angle between them is 60° and their scalar product is $\dfrac{1}{2}$.

Solution

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

It is given that $|\vec{a}| = |\vec{b}|, \vec{a} \cdot \vec{b} = \dfrac{1}{2}$, and $\theta = 60^\circ$............... (1)

We know that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\therefore \dfrac{1}{2} = |\vec{a}||\vec{a}|\cos 60^\circ$ [Using (1)]

$\Rightarrow \dfrac{1}{2} = |\vec{a}|^2 \times \dfrac{1}{2}$

$\Rightarrow |\vec{a}|^2 = 1$

$\Rightarrow |\vec{a}| = |\vec{b}| = 1$

#429195

Topic: Dot Product

Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$

Answer required

Solution

$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$

$\Rightarrow \vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{x} - \vec{a} \cdot \vec{a} = 12$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$

$\Rightarrow |\vec{x}|^2 - 1 = 12,\ [\because |\vec{a}| = 1$ as $\vec{a}$ is a unit vector]

$\Rightarrow |\vec{x}|^2 = 13$

$\therefore |\vec{x}| = \sqrt{13}$

#429200

Topic: Applications of Dot Product

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is a perpendicular to $\vec{c}$, then find the value of λ.

Solution

The given vectors are $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 3\hat{i} + \hat{j}$.

Now,

$\vec{a} + \lambda\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k}) = (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$

If $(\vec{a} + \lambda\vec{b})$ is perpendicular to $\vec{c}$, then

$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$.

$\Rightarrow [(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{k} + (3 + \lambda)\hat{k}] \cdot (3\hat{i} + \hat{j}) = 0$

$\Rightarrow (2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

$=> \ -\lambda + 8 = 0$

$\Rightarrow \lambda = 8$

Hence, the required value of λ is 8.

#429204

Topic: Applications of Dot Product

Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two nonzero vectors $\vec{a}$ and $\vec{b}$.

Solution

$(|\vec{a}|\vec{b} + |\vec{b}|\vec{a}) \cdot (|\vec{a}|\vec{b} - |\vec{b}|\vec{a})$

$= |\vec{a}|^2\vec{b} \cdot \vec{b} - |\vec{a}||\vec{b}|\vec{b} \cdot \vec{a} + |\vec{b}||\vec{a}|\vec{a} \cdot \vec{b} - |\vec{b}|^2\vec{a} \cdot \vec{a}$

$= |\vec{a}|^2|\vec{b}|^2 - |\vec{b}|^2|\vec{a}|^2 = 0$

Hence, $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ and $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$ are perpendicular to each other.

#429218

Topic: Dot Product

If $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$, then what can be concluded about the vector $\vec{b}$?

Solution

It is given that $\vec{a} \cdot \vec{a} = 0$ and $\vec{a} \cdot \vec{b} = 0$.

Now,

$\vec{a} \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0$

$\therefore \vec{a}$ is a zero vector.

Hence, vector $\vec{b}$ satisfying $\vec{a} \cdot \vec{b} = 0$ can be any vector.

#429219

Topic: Dot Product

If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

Solution

We have $|\vec{a}| = 1, |\vec{b}| = 1, |\vec{c}| = 1$

Also $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

Squaring we get,

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\Rightarrow 1 + 1 + 1 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$

$\therefore \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$

#429221

Topic: Dot Product

If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b} = 0$. But the converse need not be true. Justify your answer with an example.

Solution

Consider $\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$.

Then,

$\vec{a} \cdot \vec{b} = 2.3 + 4.3 + 3(-6) = 6 + 12 - 18 = 0$

We now observe that:

$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$

$\therefore \vec{a} \neq \vec{0}$

$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$

$\therefore \vec{b} \neq \vec{0}$

Hence, the converse of the given statement need not be true.

#429224

Topic: Applications of Dot Product

If the vertices A, B, C of a triangle ABC are $(1, 2, 3), (-1, 0, 0), (0, 1, 2)$, respectively then find $\angle ABC$. [$\angle ABC$ is the angle between the vectors $\vec{BA}$ and $\vec{BC}$] .

Solution

The vertices of ΔABC are given as $A(1, 2, 3), B(-1, 0, 0)$, and $C(0, 1, 2)$.

Also, it is given that $\angle ABC$ is the angle between the vectors $\vec{BA}$ and $\vec{BC}$.

$\vec{BA} = \{1 - (-1)\}\hat{i} + (2 - 0)\hat{j} + (3 - 0)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{BC} = \{0 - (-1)\}\hat{i} + (1 - 0)\hat{j} + (2 - 0)\hat{k} = \hat{i} + \hat{j} + 2\hat{k}$

$\therefore \vec{BA} \cdot \vec{BC} = (2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 2 \times 1 + 2 \times 1 + 3 \times 2 = 2 + 2 + 6 = 10$

$|\vec{BA}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{4 + 4 + 9} = \sqrt{17}$

$|\vec{BC}| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$

Now, it is known that:

$\vec{BA} \cdot \vec{BC} = |\vec{BA}||\vec{BC}|\cos(\angle ABC)$

$\therefore 10 = \sqrt{17} \times \sqrt{6}\cos(\angle ABC)$

$\Rightarrow \cos(\angle ABC) = \frac{10}{\sqrt{17} \times \sqrt{6}}$

$\Rightarrow \angle ABC = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$

#429226

Topic: Linear Combination of Vectors

Show that the points $A(1, 2, 7), B(2, 6, 3)$ and $C(3, 10, -1)$ are collinear .

Solution

The given points are $A(1, 2, 7), B(2, 6, 3)$ and $C(3, 10, -1)$.

$\therefore \vec{AB} = (2 - 1)\hat{i} + (6 - 2)\hat{j} + (3 - 7)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$

$\vec{BC} = (3 - 2)\hat{i} + (10 - 6)\hat{j} + (-1 - 3)\hat{k} = \hat{i} + 4\hat{j} = 4\hat{k}$

$\vec{AC} = (3 - 1)\hat{i} + (10 - 2)\hat{j} + (-1 - 7)\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k}$

$|\vec{AB}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$

$|\vec{BC}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$

$|\vec{AC}| = \sqrt{2^2 + 8^2 + 8^2} = \sqrt{4 + 64 + 64} = \sqrt{132} = 2\sqrt{33}$

$\therefore |\vec{AC}| + |\vec{AB}| + |\vec{BC}|$

Hence, the given points A, B, and C are collinear.

#429230

Topic: Vector Component Form

Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle .

Solution

Let vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ be position vectors of points A, B and C respectively.

i.e., $\vec{OA} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

Now, vectors $\vec{AB}$, $\vec{BC}$, and $\vec{AC}$ represent the sides of ΔABC.

$\therefore \vec{AB} = (1 - 2)\hat{i} + (-3 + 1)\hat{j} + (-5 - 1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$

$\vec{BC} = (3 - 1)\hat{i} + (-4 + 3)\hat{j} + (-4 + 5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{AC} = (2 - 3)\hat{i} + (-1 + 4)\hat{j} + (1 + 4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$

$|\vec{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1 + 4 + 36} = \sqrt{41}$

$|\vec{BC}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$

$|\vec{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1 + 9 + 25} = \sqrt{35}$

$\therefore |\vec{BC}|^2 + |\vec{AC}|^2 = 6 + 35 = 41 = |\vec{AB}|^2$

Hence, ΔABC is a right-angled triangle.

#429232

Topic: Operations on Vector

If $\vec{a}$ is a nonzero vector of magnitude $'a'$ and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if

A $\lambda = 1$

B $\lambda = -1$

C $a = |\lambda|$

[✓ **D**] $a = \frac{1}{|\lambda|}$

Solution

Vector $\lambda\vec{a}$ is a unit vector if $|\lambda\vec{a}| = 1$.

Now,

$|\lambda\vec{a}| = 1$

$\Rightarrow |\lambda||\vec{a}| = 1$

$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|}, \ [\lambda \neq 0]$

$\Rightarrow a = \frac{1}{|\lambda|}, \ [\because |\vec{a}| = a]$

Hence, vector $\lambda\vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$

Thus the correct answer is D.

#429235

Topic: Applications of Vector Product

Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

Solution

We have,

$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{array}{c} \begin{matrix} \hat{i} & \hat{j} & \hat{k} \end{matrix} \\ \begin{vmatrix} 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix} \end{array}$$

$= \hat{i}(-14+14) - \hat{j}(2-21) + \hat{k}(-2+21) = 19\hat{j} + 19\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2}$

#429239

Topic: Applications of Vector Product

Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

Solution

We have,

$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore \vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}, \vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{array}{c} \begin{matrix} \hat{i} & \hat{j} & \hat{k} \end{matrix} \\ \begin{vmatrix} 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} \end{array} = \hat{i}(16) - \hat{j}(16) + \hat{k}(-18) = 16\hat{i} - 16\hat{j} - 8\hat{k}$$

$\therefore \left|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\right| = \sqrt{16^2 + (-16)^2 + (-8)^2}$

$= \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2}$

$= 8\sqrt{2^2 + 2^2 + 1} = 8\sqrt{9} = 8 \times 3 = 24$

Hence, the unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is given by the relation.

$$= \pm \frac{(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})}{|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|} = \pm \frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{24}$$

$$= \pm \frac{2\hat{i} - 2\hat{j} - \hat{k}}{3} = \pm \frac{2}{3}\hat{i} \pm \frac{2}{3}\hat{j} \pm \frac{1}{3}\hat{k}$$

#429248

Topic: Applications of Dot Product

If a unit vector $\vec{a}$ makes an angle $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$.

Solution

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

$\therefore \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Since $\vec{a}$ is a unit vector, $|\vec{a}| = 1$.

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have:

$\cos\frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$

$\Rightarrow \frac{1}{2} = a_1 \,[|\vec{a}| = 1]$

$\cos\frac{\pi}{4} = \frac{a_1}{|\vec{a}|}$

$\Rightarrow \frac{1}{\sqrt{2}} = a_2 \,[|\vec{a} = 1]$

Also, $\cos\theta = \frac{a_3}{|\vec{a}|}$.

$\Rightarrow a_3 = \cos\theta$

Now,

$|a| = 1$

$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$

$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1\$$

$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$\therefore a_3 = \cos\frac{\pi}{3} = \frac{1}{2}$

Hence, $\theta = \frac{\pi}{3}$ and the components of $\vec{a}$ are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$.

#429250

Topic: Vector Product

Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

Solution

$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$(\vec{a} - \vec{b}) \times \vec{a} + (\vec{a} - \vec{b}) \times \vec{b}$ [By distributivity of vector product over addition]

$= \vec{a} \times \vec{a} - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{b}$ [Again, by distributivity of vector product over addition]

$= \vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - \vec{0}$

$= 2(\vec{a} \times \vec{b})$

#429255

Topic: Vector Product

Find λ and μ if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$

Solution

$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$\Rightarrow \hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6) = 0\hat{i} + 0\hat{j} + 0\hat{k}$

On comparing the corresponding components, we have:

$6\mu - 27\lambda = 0$

$2\mu - 27 = 0$

$2\lambda - 6 = 0$

Now,

$2\lambda - 6 = 0 \Rightarrow \lambda = 3$

$2\mu - 27 = 0 \Rightarrow \mu = \frac{27}{2}$

Hence, $\lambda = 3$ and $\mu = \frac{27}{2}$.

#429260

Topic: Vector Product

Given that $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$. What can you conclude about the vectors $\vec{a}$ and $\vec{b}$?

Solution

$\vec{a} \cdot \vec{b} = 0$

Then,

(i) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \perp \vec{b}$ (in case $\vec{a}$ and $\vec{b}$ are non-zero)

$\vec{a} \times \vec{b} = 0$

(ii) Either $|\vec{a}| = 0$ or $|\vec{b}| = 0$, or $\vec{a} \parallel \vec{b}$ (in case $\vec{a}$ and $\vec{b}$ are non-zero)

But, $\vec{a}$ and $\vec{b}$ cannot be perpendicular and parallel simultaneously.

Hence, $|\vec{a}| = 0$ or $|\vec{b}| = 0$.

#429265

Topic: Vector Product

Let the vectors $\vec{a}, \vec{b}, \vec{c}$ given as $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then show that $= \vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

Solution

We have,

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{k} + b_3\hat{k}, \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$(\vec{b} + \vec{c}) = (b_1 + c_1)\hat{i} + (b_2 + c_2)\hat{j} + (b_3 + c_3)\hat{k}$

$$\text{Now, } \vec{a} \times (\vec{b} + \vec{c}) \begin{array}{c} \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \end{array} \\ \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix} \end{array}$$

$= \hat{i}[a_2(b_3 + c_3) - a_3(b_2 + c_2)] - \hat{j}[a_1(b_3 + c_3 - a_3(b_1 + c_1)] + \hat{k}[a_1(b_2 + c_2 - a_2(b_1 + c_1)]$

$= \hat{i}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{j}[- a_1b_3 - a_1c_3 + a_3b_1 + a_3c_1] + \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1]. \ldots\ldots\ldots\ldots (1)$

$$\vec{a} \times \vec{b} = \begin{array}{c} \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \end{array} \\ \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \end{array}$$

$= \hat{i}[a_2b_3 - a_3b_2] + \hat{j}[b_1a_3 - a_1b_3] + \hat{k}[a_1b_2 - a_2b_1]. \ldots\ldots\ldots (2)$

$$\vec{a} \times \vec{c} = \begin{array}{c} \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \end{array} \\ \begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \end{array}$$

$= \hat{i}[a_2c_3 - a_3c_2] + \hat{j}[a_3c_1 - a_1c_3] + \hat{k}[a_1c_2 - a_2c_1]. \ldots\ldots\ldots\ldots (3)$

On adding (2) and (3), we get:

$(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$

$= \hat{i}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{j}[b_1a_3 + a_3c_1 - a_1b_3 - a_1c_3] + \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1]. \ldots\ldots\ldots (4)$

Now, from (1) and (4), we have:

$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

Hence, the given result is proved.

#429281

Topic: Vector Product

If either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \times \vec{b} = \vec{0}$. Is the converse true? Justify your answer with an example .

Solution

Take any parallel non-zero vectors so that $\vec{a} \times \vec{b} = \vec{0}$.

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}$.

Then,

$$\vec{a} \times \vec{b} = \begin{array}{c} \begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \end{array} \\ \begin{vmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} \end{array} = \hat{i}(24 - 24) - \hat{j}(16 - 16) + \hat{k}(12 - 12) = 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

It can now be observed that:

$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$

$\therefore \vec{a} \neq \vec{0}$

$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$

$\therefore \vec{b} \neq \vec{0}$

Hence, the converse of the given statement need not be true.

#429317

Topic: Applications of Vector Product

Find the area of the triangle with vertices $A(1, 1, 2)$, $B(2, 3, 5)$ and $C(1, 5, 5)$.

Solution

The vertices of triangle ABC are given as $A(1, 1, 2)$, $B(2, 3, 5)$ and $C(1, 5, 5)$.

The adjacent sides $\vec{AB}$ and $\vec{BC}$ of ΔABC are given as:

$\vec{AB} = (2 - 1)\hat{i} + (3 - 1)\hat{j} + (5 - 2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{BC} = (1 - 2)\hat{i} + (5 - 3)\hat{j} + (5 - 5)\hat{k} = -\hat{i} + 2\hat{j}$

Area of $\Delta ABC = \frac{1}{2}|\vec{AB} \times \vec{BC}|$

$$\vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix}$$

$\therefore |\vec{AB} \times \vec{BC}| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36 + 9 + 16} = \sqrt{61}$

Hence, the area of ΔABC is $\frac{\sqrt{61}}{2}$ square units.

#429320

Topic: Applications of Vector Product

Find the area of the parallelogram whose adjacent sides are determined by the vector $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

Solution

The area of the parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$.

Adjacent sides are given as

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{i}(-1 + 21) - \hat{j}(1 - 6) + \hat{k}(-7 + 2) = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{400 + 25 + 25} = 15\sqrt{2}$

Hence, the area of the given parallelogram is $15\sqrt{2}$ square units.

#429336

Topic: Applications of Vector Product

Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

A $\frac{\pi}{6}$

B ☑ $\frac{\pi}{4}$

C $\frac{\pi}{3}$

D $\frac{\pi}{2}$

Solution

It is given that $|\vec{a}| = 3$ and $|\vec{b}| = \dfrac{\sqrt{2}}{3}$.

We know that $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ and θ is the angle between $\vec{a}$ and $\vec{b}$.

Now, $\vec{a} \times \vec{b}$ is a unit vector if $\left|\vec{a} \times \vec{b}\right| = 1$

$\left|\vec{a} \times \vec{b}\right| = 1$

$\Rightarrow \left||\vec{a}||\vec{b}|\sin\theta\hat{n}\right| = 1$

$\Rightarrow |\vec{a}||\vec{b}||\sin\theta| = 1$

$\Rightarrow 3 \times \dfrac{\sqrt{2}}{3} \times \sin\theta = 1$

$\Rightarrow \sin\theta = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \theta = \dfrac{\pi}{4}$

Hence, $\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is $\dfrac{\pi}{4}$.

The correct answer is B.

#429337

Topic: Applications of Vector Product

Area of a rectangle having vertices A, B, C, and D with position vectors $-\hat{i} + \dfrac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} + \dfrac{1}{2} + 4\hat{k}$, $\hat{i} - \dfrac{1}{2}\hat{j} + 4\hat{k}$ and $-\hat{i} - \dfrac{1}{2}\hat{j} + 4\hat{k}$ respectively is

A $\dfrac{1}{2}$

B 1

[✓ **C**] 2

D 4

Solution

The position vectors of vertices, A, B, C, and D of rectangle $ABCD$ are given as:

$\vec{OA} = -\hat{i} + \dfrac{1}{2}\hat{j} + 4\hat{k}$, $\vec{OB} = \hat{i} + \dfrac{1}{2}\hat{j} + 4\hat{k}$, $\vec{OC} = \hat{i} - \dfrac{1}{2}\hat{j} + 4\hat{k}$, $\vec{OD} = -\hat{i} - \dfrac{1}{2}\hat{j} + 4\hat{k}$

The adjacent sides $\vec{AB}$ and $\vec{BC}$ of the given rectangle are given as:

$$\vec{AB} = (1+1)\hat{i} + \left(\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k} = 2\hat{i}$$

$$\vec{BC} = (1-1)\hat{i} + \left(\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k} = -\hat{j}$$

$$\therefore \vec{AB} \times \vec{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = \hat{k}(-2) = -2\hat{k}$$

$$\left|\vec{AB} \times \vec{AC}\right| = \sqrt{(-2)^2} = 2$$

Now, it is known that the area of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $\left|\vec{a} \times \vec{b}\right|$.

Hence, the area of the given rectangle is $\left|\vec{AB} \times \vec{BC}\right| = 2$ square units.

The correct answer is C.

#429338

Topic: Vector Component Form

Write down a unit vector in XY- plane, making an angle of 30° with the positive direction of $x - axis$.

Solution

If $\vec{r}$ is a unit vector in the $XY-plane$, then $\vec{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$.

Here, θ is the angle made by the unit vector with the positive direction of the $x-axis$

Therefore, for $\theta = 30°$:

$\vec{r} = \cos 30° + \sin 30°\hat{j} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

Hence, the required unit vector is $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$.

#429339

Topic: Vector Component Form

Find the scalar components and magnitude of the vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$.

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Solution

The vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ can be obtained by,

$\vec{PQ} =$ Position vector of $Q -$ Position vector of P

$= (x_2 - x_1)\hat{i} + (y_2 - y_1\hat{j} + (z_2 - z_1)\hat{k}$

$\left|\vec{PQ}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Hence, the scalar components and the magnitude of the vector joining the given points are respectively $\left\{(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)\right\}$ and $\sqrt{(x_2 - x_1)^2, (y_2 - y_1)^2, (z_2 - z_1)^2}$.

#429340

Topic: Vector Component Form

A girl walks $4\ km$ towards west, then she walk $3\ km$ in a direction $30°$ east of north and stops. Determine the girl's displacement from her initial point of departure .

Solution

Let O and B be the initial and final positions of the girl respectively.

Then, the girl's position can be shown as:

Now, we have:

$$\vec{OA} = -4\hat{i}$$

$$\vec{AB} = \hat{i}|\vec{AB}|\cos 60^\circ + \hat{j}|\vec{AB}|\sin 60^\circ$$

$$= \hat{i}3 \times \frac{1}{2} + \hat{j}3 \times \frac{\sqrt{3}}{2}$$

$$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

By the triangle law of vector addition, we have:

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$= (-4\hat{i}) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$$

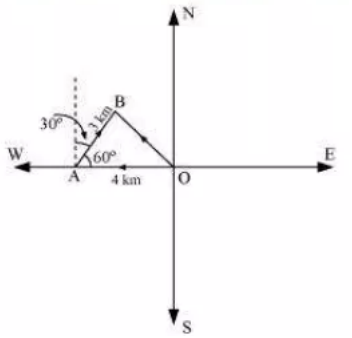
$$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

Hence, the girl's displacement from her initial point of departure is

$$\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}.$$



#429341

Topic: Operations on Vector

If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

Solution

In ΔABC, let $\vec{CB} = \vec{a}$, $\vec{CA} = \vec{b}$, and $\vec{AB} = \vec{c}$ (as shown in the following figure).

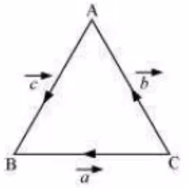
Now, by the triangle law of vector addition, we have $\vec{a} = \vec{b} + \vec{c}$.

It is clearly known that $|\vec{a}|$, $|\vec{b}|$, and $|\vec{c}|$ represent the sides of ΔABC.

Also, it is known that the sum of the lengths of any two sides of a triangle is greater than the third side.

$$\therefore |\vec{a}| < |\vec{b}| + |\vec{c}|$$

Hence, it is not true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$.



#429342

Topic: Vector Component Form

Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector .

Solution

$x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector if $\left|x(\hat{i} + \hat{j} + \hat{k})\right| = 1$.

Now,

$\left|x(\hat{i} + \hat{j} + \hat{k})\right| = 1$

$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1$

$\Rightarrow \sqrt{3x^2} = 1$

$\Rightarrow \sqrt{3}\,|x| = 1$

$\Rightarrow x = \pm \dfrac{1}{\sqrt{3}}$

Hence, the required value of x is $\pm \dfrac{1}{\sqrt{3}}$.

#429343

Topic: Vector Component Form

Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Solution

We have,

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Let $\vec{c}$ be the resultant of $\vec{a}$ and $\vec{b}$.

Then,

$\vec{c} = \vec{a} + \vec{b} = (2 + 1)\hat{i} + (3 - 2)\hat{j} + (- 1 + 1)\hat{k} = 3\hat{i} + \hat{j}$

$\therefore |\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9 + 1} = \sqrt{10}$

$\therefore \hat{c} = \dfrac{\vec{c}}{|\vec{c}|} = \dfrac{(3\hat{i} + \hat{j})}{\sqrt{10}}$

Hence,the vector of magnitude 5 units and parallel to the resultant of vectors $\vec{a}$ and $\vec{b}$ is $\pm 5 \cdot \hat{c} = \pm 5 \cdot \dfrac{1}{\sqrt{10}}(3\hat{i} + \hat{j}) = \pm \dfrac{3\sqrt{10}\hat{i}}{2} \pm \dfrac{\sqrt{10}}{2}\hat{j}$.

#429344

Topic: Vector Component Form

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

Solution

We have,

$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c} = 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k} + 3(\hat{i} - 2\hat{j} + \hat{k}$

$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$

$= 3\hat{i} - \hat{j} + 2\hat{k}$

$\left|2\vec{a} - \vec{b} + 3\vec{c}\right| = \sqrt{3^2 + (- 3)^2 + 2^2} = \sqrt{9 + 9 + 4} = \sqrt{22}$

Hence, the unit vector along $2\vec{a} - \vec{b} + 3\vec{c}$ is

$\dfrac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} = \dfrac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}} = \dfrac{3}{\sqrt{22}}\hat{i} - \dfrac{3}{\sqrt{22}}\hat{j} + \dfrac{2}{\sqrt{22}}\hat{k}$.