#459651

Topic: Introduction

Determine order and degree (if defined) of differential equations given in the following.

1. $\frac{d^4y}{dx^4} + \sin(y''') = 0$
2. $y' + 5y = 0$
3. $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$
4. $\left(\frac{d^2y}{dx^2}\right)^2 - \cos\left(\frac{dy}{dx}\right) = 0$
5. $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$
6. $(y''')^2 + (y)^3 + (y)^4 + y^5 = 0$
7. $y + 2y'' + y = 0$
8. $y' + y = e^x$
9. $y'' + (y)2 + 2y = 0$
10. $y'' + 2y + \sin y = 0$

Solution

1. $\frac{d^4y}{dx^4} + \sin(y''') = 0$

Order $= 4$

Degree $= 1$

2. $y' + 5y = 0$

Order $= 1$

Degree $= 1$

3. $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

Order $= 2$

Degree $= 1$

4. $\left(\frac{d^2y}{dx^2}\right)^2 - \cos\left(\frac{dy}{dx}\right) = 0$

Order $= 2$

Degree $= 2$

5. $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$

Order $= 2$

Degree $= 1$

6. $(y''')^2 + (y)^3 + (y)^4 + y^5 = 0$

Order $= 3$

Degree $= 2$

7. $y + 2y'' + y = 0$

Order $= 2$

Degree $= 1$

8. $y' + y = e^x$

Order $= 1$

Degree $= 1$

9. $y'' + (y)2 + 2y = 0$

Order $= 2$

Degree $= 1$

10. $y'' + 2y + \sin y = 0$

Order $= 2$

Degree $= 1$

#459652
Topic: Introduction

The degree of the differential equation

$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is

A 3

B 2

C 1

D Not defined

Solution

The degree of a differential equation is the power of the highest order derivative in the equation. Here it is 3

#459653

Topic: Introduction

The order of the differential equation

$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is

A 2

B 1

C 0

D Not defined

Solution

Order is the highest derivative in the equation. Here it is 2

#459655

Topic: Types of Solution of Differential Equation

Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

1. $y = e^x + 1$: $y'' - y = 0$
2. $y = x^2 + 2x + C$: $y' - 2x - 2 = 0$
3. $y = \cos x + C$: $y' + \sin x = 0$
4. $y = \sqrt{1 + x^2}$: $y' = \frac{xy}{1 + x^2}$
5. $y = Ax$: $xy = y(x \neq 0)$
6. $y = x\sin x$: $xy = y + x\sqrt{x^2y^2}(x \neq 0 \, and \, x > y \, or \, x < y)$
7. $xy = \log y + C$: $y' = \frac{y^2}{1 - xy}(xy \neq 1)$
8. $y - \cos y = x$: $(y\sin y + \cos y + x)y' = y$
9. $x + y = \tan^{-1}y$: $y^2y' + y^2 + 1 = 0$
10. $y = \sqrt{a^2 - x^2}x\epsilon(-a, a)$: $x + y\frac{dy}{dx} = 0(y \neq 0)$

Solution

1) Given, $y = e^x + 1$; $y'' - y = 0$

For ex. $y'' - y = 0 \Rightarrow \frac{d^2y}{dx^2} - y = 0$

$\Rightarrow \frac{d^2y}{dx^2} = y$ ...(1)

For this type of equation solunation is

$y = e^{x} + c \;\; C \in R$

And it satisfies the above eqn.

So, $y = e^{x} + 1$

2) $y' - 2x - 2 = 0$

$\Rightarrow \dfrac{dy}{dx} = 2x + 2$

$\Rightarrow dy = (2x + 2)dx$

Integrating on both sides, we get

$\int dy = \int (2x + 2)dx$

$y = x^2 + 2x + c, C \in R$

3) $y' + \sin x = 0$

$\Rightarrow \dfrac{dy}{dx} + \sin = 0$

$\Rightarrow dy = \sin dx$

Integrating on both side

$\int dy = \int \sin x\, dx$

$\Rightarrow y = \; + \cos x + c \quad \{\int \sin x\, dx = \cot x\}$

4) $y' = \dfrac{xy}{1 + x^2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{xy}{1 = x^2}$

$\Rightarrow \dfrac{dy}{y} = \dfrac{x\, dx}{1 + x^2}$

$\Rightarrow \int \dfrac{dy}{y} = \int \dfrac{xdx}{1 + x^2}$

Let $x^2 + 1 = t$

$2xdx = \; dt \pm xdx = \dfrac{dt}{2}$

$\Rightarrow \ln(y) = \dfrac{1}{2} \int \dfrac{dt}{(1pt)}$

$\Rightarrow \ln(y) = \dfrac{1}{2}[\ln(1 + 2)] + c, \; c \in R$

$\Rightarrow$ taking $c = 0$

$\ln(y) = \dfrac{1}{2}\ln(1 + 1) = \ln(1 + 1)^{1/2}$

$y = (1 + 1)^{1/2}$

$y = \sqrt{1 + x^2}$

5) $y = Ax; xy = y(x \neq 0)$

$x\frac{dy}{dx} = y \Rightarrow \frac{dy}{y} = \frac{dx}{x} \Rightarrow \int\frac{dy}{y} = \int\frac{dx}{x}$

$\ln(y) = \ln(x) + \ln(A) \Rightarrow \ln(y) = \ln(Ax), A \in R$

$\Rightarrow y = Ax$

9) $x + y = \tan^{-1}y; y^2y' + y^2 + 1 = 0$

$y^2y^1 = -(1 + y^2)$

$y^1 = \frac{-(1 + y^2)}{y^2}$

$\frac{y^2y^1}{1 + y^2} = (-1)$

$\int\frac{y^2}{1 + y^2}dy = \int - dx$

$\Rightarrow \int\frac{1 + y^2 - 1}{1 + y^2}dy = -\int dx$

$= \int dy - \int\frac{1}{1 + y^2}dy = -\int dx$

$\Rightarrow y - \tan^{-1}(y) = -x + c$

$\Rightarrow x + y = \tan^{-1}(y) + c$

10) $y = \sqrt{a^2 - x^2}x\epsilon(-a, a); x + y\frac{dy}{dx} = 0(y \neq 0)$

$x + y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$

$\int ydy = \int - x\,dx$

$\frac{y^2}{2} = \int - xdx$

$y^2 = a - x^2$

$y = \sqrt{a^2 - x^2}, x\epsilon(-a, a)$

#459658

Topic: Introduction

The number of arbitrary constants in the general solution of a differential equation of fourth order are

A 0

B 2

C 3

[✓ **D**] 4

Solution

The number of arbitrary constants in a solution of a differential equation of order n is equal to its order.

So, here it is 4.

#459659

Topic: Introduction

The number of arbitrary constants in the particular solution of a differential equation of third order are:

A 3

B 2

C 1

D 0

Solution

The number of arbitrary constants in a solution of a differential equation of order n is equal to its order.

So here it will be 3

#459661

Topic: Formation of Differential Equation

Form the differential equation of the family of circles touching the y-axis at origin.

Solution

The system of circles touching y axis at origin will have centres on x axis. Let $(a, 0)$ be the centre of the circle. Then the radius of the circle should be a units, since the circle should touch y axis at origin.

Equation of a circle with centre at $(a, 0)$ and radius a is

$x^2 + y^2 - 2ax = 0$

The above equation represents the family of circles touching y axis at origin. Here a is an arbitrary constant.

In order to find the differential equation of system of circles touching y axis at origin, eliminate the arbitrary constant from equation.

Differentiating equation with respect to x, we get

$2a = 2(x + y\frac{dy}{dx})$

Replacing $2a$ of the above equation, we get

$x^2 + y^2 - 2(x + y\frac{dy}{dx})x$

Which is $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

#459662

Topic: Formation of Differential Equation

Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

Solution

Equation of such parabola is given by $x^2 = 4ay$

In order to make a differential equation, we just have to remove a

Differentiating both the sides we get $2x = 4ay'$

$a = \frac{x}{2y'}$

Putting this in original equation of parabola, we get

$xy' - 2y = 0$

#459663

Topic: Formation of Differential Equation

Form the differential equation of the family of ellipses having foci on y-axis and center at origin.

Solution

Equation of such an ellipse is given by $\frac{x^2}{b^2}+\frac{y^2}{a^2}=0$

Here we have a and b as variables, so lets remove them to get the required differential equation.

Differentiating this on both sides, we get

$\frac{2x}{b^2}+\frac{2yy'}{a^2}=0$ --------(1)

Differentiating it again we will get

$\frac{1}{b^2}+(\frac{1}{a^2})[yy''+(y')^2]=0$

$\frac{1}{b^2}=-(\frac{1}{a^2})[yy''+(y')^2]$

Putting this in equation (1), we get

$-[\frac{x}{a^2}][yy''+(y')^2]+\frac{yy'}{a^2}$

This gives $xyy''+x(y')^2-yy'=0$

#459664

Topic: Formation of Differential Equation

Form the differential equation of the family of hyperbolas having foci on x-axis and centre at origin

Solution

Equation of a hyperbola is given by $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Differentiating both the sides, we get

$\frac{2x}{a^2}-\frac{2yy'}{b^2}=0$..........(1)

Again differentiating, we get

$\frac{1}{a^2}-(\frac{1}{b^2})[(y')^2+yy'']$

$\frac{1}{a^2}=(\frac{1}{b^2})[(y')^2+yy'']$

Putting this in equation (1), we get

$(\frac{x}{b^2})[(y')^2+yy'']-\frac{yy'}{b^2}=0$

Multiply by b^2 and after simplification, we get

$xyy''+x(y')^2-yy'=0$

#459665

Topic: Formation of Differential Equation

Form the differential equation of the family of circles having centre on y-axis and radius 3 units

Solution

Here it's given that the circle will have it's center on y axis and radius is 3 units.

So let's assume that the center is at $(0,a)$

Equation is given by $x^2+(y-a)^2=9$

Differentiating it one time, we get

$2x+2yy'-2ay'=0$

from here we can take out a and put it in main equation of circle.

$a=\frac{2x+2yy'}{y'}$

So equation is $x^2+\left(y-\frac{2x+2yy'}{y'}\right)^2=9$

This is the required equation.

#459666

Topic: Types of Solution of Differential Equation

Which of the following differential equations has $y = c_1e^x + c_2e^x$ as the general solution?

A $\frac{d^2y}{dx^2} + y = 0$

[✓ B] $\frac{d^2y}{dx^2} - y = 0$

C $\frac{d^2y}{dx^2} + 1 = 0$

D $\frac{d^2y}{dx^2} - 1 = 0$

Solution

$y = c_1e^x + c_2e^x\frac{dy}{dx} = c_1e^x + c_2e^x\frac{d^2y}{dx^2} = c_1e^x + c_2e^x \therefore \frac{d^2y}{dx^2} - y = 0$

#459668

Topic: Types of Solution of Differential Equation

Which of the following differential equations has y = x as one of its particular solution?

A $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$

B $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$

[✓ C] $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$

D $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$

Solution

$y = x\frac{dy}{dx} = 1\frac{d^2y}{dx^2} = 0$

Now putting these values in given options then we will find that

Option C is correct

$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$

#459739

Topic: Types of Solution of Differential Equation

For each of the differential equations, find the general solution:

1. $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$
2. $\frac{dy}{dx} = \sqrt{4-y^2}(-2 < y < 2)$
3. $\frac{dy}{dx} = 1(y \neq 1)$
4. $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$
5. $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$
6. $\frac{dy}{dx} = (1+x^2)(1+y^2)$
7. $y \log y\, dx - x\, dy = 0$
8. $x^4 \frac{dy}{dx} = -y^5$
9. $\frac{dy}{dx} = \sin^{-1}x$
10. $e^x \tan y\, dx + (1-e^x)\sec^2 y\, dy = 0$

Solution

1) $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

$\int dy = \int\left[\left(\frac{(1-\cos x)}{(1+\cos x)}\right)\right]dx$

$\int dy = \int \frac{2\sin^2(\frac{x}{2})}{2\cos^2(\frac{x}{2})}dx$

$\int dy = \int \tan^2(\frac{x}{2})dx = \int (\sec^2(\frac{x}{2} - 1)dx$

$= \int \sec^2(\frac{x}{2} - 1)dx$

$= 2\tan\left(\frac{x}{2} - 1\right) + c$

$y = 2\tan\left(\frac{x}{2} - 1\right) + c$

2) $\frac{dy}{dx} = \sqrt{4-y^2}(-2 < y < 2)$

$\Rightarrow \frac{dy}{\sqrt{4-y^2}} = dx$

$\Rightarrow \frac{dy}{\sqrt{1-(\frac{y}{2})^2}} = dx$

Let $\frac{y}{2} = t$

Thus $dy = 2dt$

$\Rightarrow \frac{2dt}{\sqrt{1-t^2}}$

$\Rightarrow \int \frac{2dt}{\sqrt{1-t^2}} = \int dx$

$= 2\sin^{-1}(t) = x + c$

$= 2\sin y(\frac{y}{2} = x + c$

3) $\frac{dy}{dx} = 1(y \neq 1)$

$\int dy = \int dx$

$y = x \neq c$

4) $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

$\frac{dy}{dx} = \frac{\sec^2 x \tan y}{\sec^2 y \tan x}$

$\frac{\sec^2 y}{\tan t} dy = -\frac{\sec^2 x}{\tan x} dx$

$\tan y = u, \tan x = u$

$\sec^2 y dy = du, \sec^2 x dx = dQ$

$\Rightarrow \int \frac{du}{u} = \frac{du}{u}$

$\log(u) = n - \log(Q) + \log C$

$\log(u) = \log\left(\frac{c}{Q}\right)$

$\log(uQ) = \log(C)$

$uQ = C$

$\tan y \tan x = C$

5) $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$

$(e^x + e^{-x})dy = (e^x - e^{-x})dx$

$\frac{dy}{dx} = \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

$dy = \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

Integrating both sides.

$\int dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx \quad ...(1)$

$y = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Let $t = e^x + e^{-x}$

$\frac{dt}{dx} = (e^x - e^{-x})dx$

$dx = \frac{dt}{e^x - e^{-x}}$

Substituting values in (1), we get

$\int dy = \int \frac{e^x - e^{-x}}{t} \frac{dt}{e^x - e^{-x}}$

$\int dy = \int \frac{dt}{t}$

$y = \log|t| + C$

Putting back $t = e^x - e^{-x}$

$y = \log|e^x - e^{-x}| + C$

$y = \log(e^x - e^{-x}) + C \qquad (\therefore e^x - e^{-x} > 0)$

6) $\frac{dy}{dx} = (1 + x^2)(1 + y^2) \Rightarrow dy$

$\Rightarrow \int \frac{1}{1+y^2} dy = \int (1 + x^2) dx$

$\Rightarrow \tan^{-1}(y) = x + \frac{x^3}{3} + C$

7) $y \log y\, dx - x\, dy = 0$

$y \log y\, dx = x\, dy$

$\frac{dx}{x} = \frac{dy}{y \log y}$

Intergrating both sides

$\int \frac{dy}{y \log y} = \int \frac{dx}{x}$

Put $t = \log y$

$dt = \frac{1}{y} dy$

$dy = y\, dt$

Hence, our equation becomes

$\int \frac{y\, dt}{y.\, t} = \int \frac{dx}{x}$

$\int \frac{dt}{t} = \int \frac{dx}{x}$

$\log|t| = \log|x| + \log c$

Putting $t = \log y$

$\log(\log y) = \log x + \log c$

$\log(\log y) = \log cx \qquad$ (Using $\log ab = \log a + \log b$)

$\log y = cx$

$y = e^{cx}$

8) $x^2 \frac{dy}{dx} = -y^5$

$\int \frac{dy}{dx} = \int \frac{dx}{xy} \Rightarrow \int y^{-5} dy = -\int dx$

$\Rightarrow \frac{y^{-4}}{-4} = \frac{x^{-3}}{-3} + c$

$\Rightarrow \frac{1}{3x^3} + \frac{1}{4y^4} + C = 0$

$\frac{dy}{dx} = \sin^{-1}(x) \Rightarrow \int dy = \int \sin^{-1}(x) dx$

So $\therefore \int \sin^{-1}(x) dx = x \sin^{-1}(x) + \sqrt{1 - x^2}$

$y = x \sin^{-1}(x) + \sqrt{1 - x^2} + c$

10) $e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$

$e^x \tan y dx = (e^x - 1)\sec^2 y dy$

$\int \frac{\sec^2 y}{\tan y} dy = \int \frac{e^x}{e^x - 1} dx$

$\tan y = u, e^x - 1 = Q$

$\sec^2 y\, dy = du \;\; e^x dx = du$

$\int \frac{du}{u} = \int \frac{du}{u}$

$\log(u) = \log(Q) + \log(C)$

$\tan y = (e^x - 1) = c$

#459779

Topic: Linear Differential Equation

Find the equation of a curve passing through the point $(0, 0)$ and whose differential equation is $y' = e^x \sin x$.

Solution

Equation is given by $y' = e^x \sin x$

$dy = e^x \sin x dx$

Integrating both side we get(use integration by parts)

$y = \frac{e^x \cos x + e^x \sin x}{2} + c$

Now, it has been given that this curve will pass from $(0, 0)$, so putting this in above equation we will get $c = \frac{-1}{2}$

Hence, equation of the curve will be

$y = \frac{e^x \cos x + e^x \sin x}{2} - \frac{1}{2}$

#459780

Topic: Linear Differential Equation

For the differential equation $xy\frac{dy}{dx} = (x + 2)(y + 2)$, find the solution curve passing through the point $(1, -1)$.

Solution

Equation can be written as

$\frac{y}{(y + 2)} dy = \frac{x + 2}{x} dx$

$(1 - \frac{2}{y + 2})dy = \frac{x + 2}{x} dx$

Integrating both the sides, we get

$y - 2\log(y + 2) = x + 2\log(x) + c$

As mentioned, this curve will pass from $(1, -1)$

$-1 - 2\log(-1 + 2) = 1 + 2\log(1) + c$

$\therefore c = -2$

Equation of the curve will be:

$y - 2\log(y + 2) = x + 2\log(x) - 2$

#459781

Topic: Types of Solution of Differential Equation

Find the equation of a curve passing through the point (0,-2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.

Solution

According to question, We have

$y.\dfrac{dy}{dx} = x$ and it is passing through $(0,\ -2)$

$y.dy = x.dx\dfrac{y^2}{2} = \dfrac{x^2}{2} + c$

This curve will pass through $(0,\ -2)$

$\dfrac{(-2)^2}{2} = \dfrac{0^2}{2} + c \therefore c = 2$

Equation of curve is

$\dfrac{y^2}{2} = \dfrac{x^2}{2} + 2$

#459787

Topic: Homogeneous Differential Equation

The general solution of the differential equation $\dfrac{dy}{dx} = e^{x+y}$ is

Solution

This equation can be written as

$\dfrac{dy}{dx} = e^x e^y$

$e^{-y}dy = e^x dx$

Integrating both the sides, we get

$-e^{-y} = e^x + c$

$e^{-y} + e^x = C$

#459793

Topic: Homogeneous Differential Equation

In the following, show that the given differential equation is homogeneous and solve each of them.

1. $(x^2 + xy)dy = (x^2 + y^2)dx$

2. $y' = \dfrac{x+y}{x}$

3. $(xy)dy(x + y)dx = 0$

4. $(x^2 + y^2)dx + 2xydy = 0$

5. $x^2\dfrac{dy}{dx} = x^2 - 2y^2 + xy$

6. $x\,dy.\,y\,dx = \sqrt{x^2 + y^2}dx$

7. $\{x\cos\left(\dfrac{y}{x}\right) + y\sin\left(\dfrac{y}{x}\right)\}y\,dx = \{y\sin\left(\dfrac{y}{x}\right) - x\cos\left(\dfrac{y}{x}\right)\}x\,dy$

8. $x\dfrac{dy}{dx} - y + x\sin\left(\dfrac{y}{x}\right) = 0$

9. $y\,dx + x\log\left(\dfrac{y}{x}\right)dy - 2x\,dy = 0$

Solution

1) $(x^2 + xy)dy – (x^2 + y^2)dx$

$\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{x^2 + xy} = \dfrac{1 + (\frac{y}{x})^2}{1 + (\frac{y}{x})}$

$u = \dfrac{y}{x}$

$\dfrac{dy}{dx} = \dfrac{1}{x}\left(\dfrac{dy}{dx}\right) - \dfrac{y}{x^2} = \dfrac{1}{x}\dfrac{dy}{dx} - \left(\dfrac{u}{x}\right)$

$\left(\frac{du}{dx}+\frac{u}{x}\right)x=\frac{dy}{dx}$

$\left(\frac{xdu}{dx}+4\right)=\frac{1+4^2}{1+4}$

$\frac{xdu}{dx}=\frac{1+4^2}{1+4}-u=\frac{1+4^2-4-4^2}{1+4}$

$\frac{xdu}{dx}=\left(\frac{1-4}{1+4}\right)$

$\Rightarrow\left(\frac{1+4}{1-4}\right)du=\frac{dx}{x}$

$\int\frac{-4-1}{1-4}dx=-\int\frac{dx}{x}$

$=\int\frac{-4+1-1-1}{1-4}du=-\int\frac{dx}{x}$

$=\int\left(1-\frac{2}{1-4}\right)du=-\int\frac{dx}{x}$

$u+2ln(1-y)=-ln(x)+c$

$\Rightarrow u+2ln(1-4)=-ln(x)+c$

$\Rightarrow u=\frac{y}{x}$

$\Rightarrow\frac{y}{x}=2ln\left(1-\frac{y}{x}\right)=-ln(x)+c$

$\Rightarrow y+2xln\left(1-\frac{y}{x}\right)=-xln(x)+xc$

2) $y^1=\frac{x+y}{x}$

$y^1=1+\frac{y}{x}$

$\frac{y}{x}=u\Rightarrow\left(\frac{dy}{dx}\right)=4+x\frac{du}{dx}$

$\frac{xdu}{dx}=1\Rightarrow\int du=\int\frac{dx}{x}$

$u=ln(x)+c$

$\frac{y}{x}=ln(x)+c\Rightarrow y=x\,ln(x)+cx$

3) wrong equation

4) $(x^2+y^2)dx+2xydy=0$

$\frac{dy}{dx}=\frac{-(x^2+y^2)}{2xy}$

$\frac{dy}{dx}=\frac{-\left(1+\left(\frac{y}{x}\right)^2\right)}{2\left(\frac{y}{x}\right)}$

$\frac{y}{x}=u\Rightarrow\frac{dy}{dx}=u=x\frac{du}{dx}$

$$u = \frac{xdu}{dx} = \frac{-(1+4^2}{24}$$

$$\frac{xdu}{dx} = \frac{-(1+4^2)}{24} - 4 = \frac{-1-4^2-24^2}{24}$$

$$\frac{xdu}{dx} = \frac{-1-34^2}{24}$$

$$\int \frac{24}{1+34^2} du = -\int \frac{dx}{x}$$

$$\int \frac{dt}{1+3t} = -\int \frac{dx}{x}$$

$$\frac{1}{3} ln(1+3t) = -ln(x) + ln(c)$$

$$\frac{1}{3} ln(1+3t) = +ln(\frac{C}{x})$$

$$\frac{1}{3} ln(1+3x^2) = ln(\frac{c}{x})$$

$$(1+3x^2)^{1/3} = \left(\frac{C}{x}\right)$$

5) $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$

$$\frac{dy}{dx} = 1 - 2\left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right)$$

$$\frac{y}{x} = 4 \Rightarrow \frac{dy}{dx} = 4 + x\frac{dy}{dx}$$

$$4 + x\frac{dy}{dx} = 1 - 2y^2 + 4$$

$$x\frac{dy}{dx} = 1 - 24^2 + 0$$

$$\int \frac{dy}{1-24^2} = \int \frac{dx}{x}$$

$$\int \frac{du}{1-(\sqrt{24})^2} = \int \frac{dx}{x}$$

$$\sqrt{24} = +7\sqrt{2}du = d+$$

$$\Rightarrow \frac{1}{\sqrt{2}} \int \frac{d+}{1- +2} = \int \frac{dx}{x}$$

$$= \frac{1}{\sqrt{2}}\left[\int\left(\frac{1/2}{1- +} + \frac{(1/2)}{1+1}\right)dt\right] = ln(x) + c$$

$$\frac{1}{12}\left[\frac{-1}{2} ln(1- +) + \frac{1}{2} ln(1+1)\right] = ln(x) + c$$

$$\frac{1}{12}\left[\frac{-1}{2} ln(1-4^2) + \frac{1}{2} ln(1+4^2)\right] = ln(x) + c$$

$$\Rightarrow \frac{1}{12} \times \frac{1}{2} ln\left(\frac{1+4^2}{1-4^2}\right) - ln + c$$

$$\frac{1}{2\sqrt{x}} ln\left(\frac{x^2+y^2}{x^2-y^2}\right) = ln\, x + c$$

6) wrong question

7) $\left[x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right]ydx = \left[y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right]x\,dy$

$$\left[\cos\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)\sin\left(\frac{y}{x}\right)\right](\frac{y}{x})dx = \left[\frac{y}{x}\sin\left(\frac{y}{x}\right) - \cos\left(\frac{y}{x}\right)\right]dy$$

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)\left[\frac{\cos(\frac{y}{x}) + (\frac{y}{x})\sin(\frac{y}{x})}{-\cos(\frac{y}{x}) + (\frac{y}{x})\sin(\frac{y}{x})}\right]$$

$$\frac{y}{x} = u = \frac{dy}{dx} = u + x\frac{du}{dx}$$

$$y + x\frac{dy}{dx} = 4\left[\frac{\cos u + u\sin u}{-\cos u + u\sin u}\right]$$

$$s\frac{dy}{dx} = 4\left[\frac{\cos u + 4\sin u}{u\sin u - \cos u} - 1\right]$$

$$= 4\left[\frac{\cos u + u\sin u - u\sin u + \cos 4}{u\sin u - \cos u}\right]$$

$$x\frac{dy}{dx} = \frac{2u\cos u}{u\sin u - \cos u}$$

$$\left(\frac{u\sin u - \cos u}{u\cos u}\right)du = 2dx$$

$$\int\left(\tan u - \frac{1}{4}\right)du = \int 2dx$$

$$\Rightarrow ln|\sec u| - ln(u) = 2x + c$$

$$\Rightarrow ln\left(\frac{\sec u}{u}\right) = 2x + c$$

$$\Rightarrow\left(\frac{\sec(\frac{y}{x})}{(\frac{y}{x})}\right) = 2x + c$$

$$\Rightarrow ln\left(\frac{x\sec(\frac{y}{x})}{y}\right) = 2x + c$$

8) $x\frac{dy}{dx} - y + x\sin(\frac{y}{x}) = 0$

$$\frac{dy}{dx} - \frac{y}{x} + \sin(\frac{y}{x}) = 0$$

$$u = \frac{y}{x} \Rightarrow \frac{dy}{dx} = u + x\frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{y}{x} - \sin(\frac{y}{x})$$

$$u + x\frac{dy}{dx} = u - \sin(u)$$

$$\Rightarrow x\frac{dy}{dx} = -\sin u \Rightarrow \int\frac{dy}{dx} = -\int\frac{dx}{x}$$

$$\int cosec\,udu = -\int\frac{dx}{x}$$

$$\Rightarrow \log\int cosecu - \cot u = -ln(x) + ln(c)$$

$$\log ln|\cos u - \cot u| = ln|\frac{x}{c}|$$

$$(cosec\,u - \cot u).\,x = c$$

$$x\left[cose(\frac{y}{x}) - \cot(\frac{y}{x})\right] = c$$

9) $ydx + x\log(\frac{y}{x})dy - 2xdy = 0$

$$(\frac{y}{x}dx + \left[\log(\frac{y}{x} - 2\right]dy = 0$$

$$\frac{dy}{dx} = \frac{(\frac{y}{x}}{2 - \log\ (\frac{y}{x})}$$

$$\frac{y}{x} = u \Rightarrow \frac{dy}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{dy}{dx} = \frac{u}{2 - \log\ (u)}$$

$$x\frac{dy}{dx} = \frac{4 - 24 + 4\log u}{2 - \log u}$$

$= \dfrac{-4 + u\log u}{2 - \log u}$

$\frac{2 - \log\ u}{4\log\ u - 4} du = \dfrac{dx}{x}$

$\dfrac{2 - \log u}{u(\log u - 1)} du = \frac{dx}{x}$

$\log u = \ +$

$\frac{1}{4} du = dt$

$\frac{2-1}{4-1} dt = \frac{dx}{x}$

$\left(\frac{+ -1-1}{4-1}\right) dt = \frac{-dx}{x}$

$\int (\frac{1}{1-1}) dt = \int - \frac{dx}{x}$

$4 - ln(4 - 1) = -ln(x) + c$

$\log u = \log(\log u - 1) = -ln(x) + c$

$\log(\frac{y}{x}) - \log(\log(\frac{y}{x} - 1) = -ln(x) + c$

#459797

Topic: Homogeneous Differential Equation

A homogeneous differential equation of the from $\dfrac{dx}{dy} = h\left(\dfrac{x}{y}\right)$ can be solved by making the substitution:

A $y = vx$

B $v = yx$

[✓ **C**] $x = vy$

D $x = v$

Solution

A homogeneous differential equation of the form

$\dfrac{dx}{dy} = h(\dfrac{x}{y})$ can be solved by substituting

$x = vy$

Then

$\dfrac{dx}{dy} = v + y\dfrac{dv}{dy}$

Hence

$\dfrac{dx}{dy} = h(\dfrac{x}{y})$

$\rightarrow v + y\dfrac{dv}{dx} = h(v)$

$y\dfrac{dv}{dy} = h(v) - v$

$\dfrac{dv}{h(v) - v} = \dfrac{dy}{y}$

Let $f(v) = h(v) - v$ Then

$\dfrac{dv}{f(v)} = \dfrac{dy}{y}$

Integrating both sides give us

$\int \dfrac{dv}{f(v)} = \int \dfrac{dy}{y}$

$\int \dfrac{dv}{f(v)} = ln(y) + c$

#459798

Topic: Homogeneous Differential Equation

Which of the following is a homogeneous differential equation?

A $(4x + 6y + 5)dy + (3y + 2x + 4)dx = 0$

B $(xy)dx + (x^3 + y^3)dy = 0$

C $(x^3 + 2y^2)dx + 2xydy = 0$

D $y^2dx + (x^2 + xy + y^2)dy = 0$

Solution

A) $(4x + 6y + 5)dy + (3y + 2x + 4)dx = 0$

$M(x) = 4x + 6y + 5$

$N(x) = 3y + 2x + 4$

If we put $x = 2x,\ y = 2y$

$M(2x, 2y) = z(4x + 6y) + 5$

$N(2x, 2y) = z(3y + 2x) + 4$

$M(2x, 2y) \neq z^nM(x, y)$

$N(2x, 2y) \neq z^nN(x, y)$

So, it is not a homogeneous.

B) $(xy)dx + (x^2 + y^3)dy = 0$

$M(x, y) = x^3 + y^3$

$N(x, y) = xy$

$M(2x, 2y) = z^2(x^3 + y^3) = z^3(M(x, y)$

$N(2x, 2y) = z^2(xy) = z^2N(x, y)$

but degree is not same

So, it is not homogeneous equation.

C) $(x^3 + 2y^2)dx + 2xydy = 0$

$M(x, y) = 2xy$

$M(2x, 2y) = 2z^2xy = z^2M(x, y)$

$N(x, y) = x^3 + 2y^2$

$N((2x, 2y) = z^3x^3 + 2z^2y^2 \neq z^3, z^nN(x, y)$

So, it is not a homogenous.

D) $y^2dx + (x^2 + xy + y^2)dy = 0$

$m(x, y) = X^2 + xy + y^2$

$n(x, y) = Y^2$

$m(2x, 2y) = z^2(x^2 + xy + y62) = z^2m(x, y)$

$N(2x + 2y) = z^2y^2 = z^2N(x, y)$

So, it is a homogeneous equation.

#459802

Topic: Types of Solution of Differential Equation

For each of the differential equations, find the general solution:

1. $\frac{dy}{dx} + 2y = \sin x$

2. $\frac{dy}{dx} + 3y = e^{-2x}$

3. $\frac{dy}{dx} + \frac{y}{x} = x^2$

4. $\frac{dy}{dx} + (\sec x)y = \tan x \quad \ldots\left(0 \le x < \frac{\pi}{2}\right)$

5. $\cos^2 x\frac{dy}{dx} + y = \tan x \quad \ldots\left(0 \le x < \frac{\pi}{2}\right)$

6. $x\frac{dy}{dx} + 2y = x^2 \log x$

7. $x \log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

8. $(1 + x^2)dy + 2xydx = \cot x dx \quad \ldots(x \ne 0)$

9. $x\frac{dy}{dx} + y - x + xy\cot x = 0 \quad \ldots(x \ne 0)$

10. $(x + y)\frac{dx}{dy} = 1$

11. $y\,dx + (x - y^2)dy = 0$

12. $(x + 3y^2)\frac{dy}{dx} = y \quad \ldots(y > 0)$

#459803

Topic: Types of Solution of Differential Equation

For each of the differential equations given, find a particular solution satisfying the given condition.

1. $\frac{dy}{dx} + 2y\tan x = \sin x; y = 0$ when $x = \frac{\pi}{3}$

2. $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}; y = 0$ when $x = 1$

3. $\frac{dy}{dx} - 3\,y\cot x = \sin 2x; y = 2$ when $x = \frac{\pi}{2}$

Solution

1) $\frac{dy}{dx} + 2y\tan x = \sin x;\ y = 0,$ when $x = \frac{\pi}{3}$

$4(x) = \int_e 2\tan x.\ dx = 2ln(x) = \sec^2 x$

$\sec^2 x(y^1 + 2\tan xy) = \sin x.\ sec^2 x$

$(\sec^2 x)^1 = \sin x.\ \sec^2 x$

$\sec 62x.\ y = \int x.\ \sec^2 x.\ dx$

$\int \sin x.\ \frac{1}{\cos^2 x}.\ dx$

$\cos x = \ +$

$\sin x.\ dx = \ - d +$

$= \int \frac{-dt}{+2} = \frac{1}{+} = \frac{1}{\cos x} + c$

$y = \cos x + c\cot^2 x$

$y(x = \frac{\pi}{3}) = 0$

$o = \cos(\frac{\pi}{3}) + c\cos^2(\frac{\pi}{3})$

$o = \frac{1}{2} + c. \frac{1}{4}$

$c = 2$

$y(x) = \cos(x) + \cos^2 x$

2) $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1 + x^2}; y(x = 1) = 0$

$\frac{dy}{dx} = \frac{2x}{1 + x62}y = \frac{1}{(1 + x^2)^2}$

$4(x) = \int_e \frac{2x}{1 + x^2}. dx$

$= \int_e \frac{dt}{1 + 1} = eln(1 + 1)$

$= (1 + x^2)$

$(1 + x^2)(y^1 + \frac{2x}{1 + x^2}y) = \frac{1}{(1 + x^2)^2}. (1 + x^2)$

$((1 + x^2)y)^1 = \frac{1}{1 + x^2}$

$(1 + x^2)y = \int \frac{1}{1 + x^2}. dx$

$(1 + x^2)y = \tan(x) + c$

$y = \frac{\tan(x)}{1 + n^2} + \frac{c}{1 + x^2}$

$y(n = 1) = o \Rightarrow 0 = \frac{\frac{\pi}{2}}{2} + \frac{c}{2}$

$c = -\frac{\pi}{4}$

$y = \frac{\tan(x)}{1 + x^2} - \frac{\pi}{4(1 + x^2)}$

3) $\frac{dy}{dx} - 3y\cot x = \sin x; (y(x = \frac{x}{2}) = 2$

$4(x) = \int_e - 3\cot x. dx = -3\log|\sin x| = \frac{1}{\sin^3 x}$

$\frac{1}{\sin^2 x}(y^2 - 3y\cot x) = \frac{\sin\$2x}{\sin^3 x}$

$\left[\frac{y}{\sin^3 z}\right]^1 = \frac{\sin x^2 x}{\sin^3 x}$

$\frac{y}{\sin^3 x} = \int \frac{2\cos x}{\sin x}. dx$

$\sin x = + \Rightarrow \cos x. dx = d+$

$\int \frac{2dt}{+2}$

$\frac{-2}{+} = \frac{-2}{\sin x} = c$

$y = -2\sin^2 x + c\sin^3 x$

$y(x = \frac{\pi}{2}) = 2$

$2 = -2 + c$

$c = 4$

$y(x) = -2\sin^2 x + 4\sin^3 x$

#459806

Topic: Linear Differential Equation

The Integrating Factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is:

A e^{-x}

B e^{-y}

$\boxed{\text{C}}$ $\frac{1}{x}$

D x

Solution

For Liner equation $\frac{dy}{dx} + Py = Q$

Integrating factor is given by $I = e^{\int Pdx}$

So here, $P = \frac{-1}{x}$

$I = e^{\int \frac{-1}{x} dx}$

$I = e^{-\ln\,(x)}$

$I = e^{\ln\,(\frac{1}{x})}$

$\Rightarrow I = \frac{1}{x}$

#459807

Topic: Linear Differential Equation

The Integrating factor of the differential equation

$(1 - y^2)\frac{dx}{dy} + yx = ay$ is

A $\frac{1}{y^2 - 1}$

B $\frac{1}{\sqrt{y^2 - 1}}$

C $\frac{1}{1 - y^2}$

$\boxed{\text{D}}$ $\frac{1}{\sqrt{1 - y^2}}$

Solution

Given DE is $(1 - y^2)\frac{dx}{dy} + yx = ay$

$\Rightarrow \frac{dx}{dy} + \frac{y}{1 - y^2}x = a\frac{y}{1 - y^2}$

which is an exact DE of the form $\frac{dx}{dy} + P(y)x = Q(y)$

Integrating factor $IF = e^{\int P(y)dy} = e^{\int \frac{y}{1 - y^2}dy}$

Let $t = 1 - y^2$

$\Rightarrow dt = -2ydy$

i.e. $IF = e^{\int \frac{-1}{2t}dt} = e^{-\frac{1}{2}\ln\,t} = \frac{1}{\sqrt{t}} = \frac{1}{\sqrt{1 - y^2}}$

#459809

Topic: Introduction

For each of the differential equation given below, indicate its order and degree (if defined).

(i) $\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$

(ii) $\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x$

(iii) $\frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) = 0$

Solution

Differential equations are often classified with respect to order.

The order of a differential equation is the order of the highest order derivative present in the equation.

The degree of a differential equation is the power of the highest order derivative in the equation.

i) order is 2 and degree is 1.

ii) order is 1 and degree is 3.

iii) order is 4 but degree is not defined because it is not polynomial.

#459812

Topic: Types of Solution of Differential Equation

Verify that the given function (implicit or explicit) is a solution of the corresponding differential equation.

(i) $y = e^x(a\cos x + b\,sin\,x)$: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

(ii) y = x \,\sin 3x : \cfrac{d^2y}{dx^2}+9y-6\,\cos\,3x=0

(iii) x^2 = 2y^2 \,\log\, y : (x^2+ y^2) \cfrac{dy}{dx}-xy= 0

Solution

(i) y=e^x(a\cos x+b \sin x)

Differenttiating Both sides w.r.t.x

\dfrac{dy}{dx} = \dfrac{d}{dx}\left[e^x(a\cos x+b\sin x)\right]

y' = \dfrac{d(e^x)}{dx}.[a\cos x+b\sin x]+e^x\dfrac{d}{dx}[a\cos x+b\sin x]

y'=e^x[a\cos x+b\sin x]=e^x[-a\sin x+b\cos x]

y'=y+e^x[-a\sin x+b\cos x] ...(1)

Again Differentiating both sides w.r.t.x

y''=y'+ \dfrac{d(e^x)}{dx}[-a\sin x+b\cos x]+e^x\dfrac{d}{dx}[-a\sin x+b\cos x]

y''=y'+e^x[-a\sin x+b \cos x]=e^x[-a\cos x+b(-\sin x)]

y''=y'+(y'-y)+e^x[-a\cos x-b\sin x] (From (1)]

y''=2y'-y-e^x[a\cos x+b\sin x]

y''=2y'-y-y (Using y=e^x(a\cos x+b\sin x)]

y''=2y'-2y

y''-2y'+2y=0

Which is the required differential equation

(ii) y=x\sin 3x ; \cfrac{d^2y}{dx^2}+9y-6\,\cos\,3x=0

\dfrac{dy}{dx} = \sin 3x+3x\cos 3x

\dfrac{d^2y}{dx^2}=3\cos 3x+3\cos 3x-9x\sin3x

or \dfrac{d^2y}{dx} = 6 \cos 3x-9x\sin 3x

where x\sin 3x=y

\dfrac{d^2y}{dx^2}=9y-6\cos 3x=0

This is the proof that y=x\sin 3x is the solution of the given differential equation.

(iii) x^2=2y^2\log y; (x^2+ y^2) \cfrac{dy}{dx}-xy= 0

Differentiating both the sides we get;

2x=4y\log y\dfrac{dy}{dx} + \dfrac{2y^2}{y} \dfrac{dy}{dx}

2x=\dfrac{dy}{dx} (4y\log y + 2y)

\therefore x = \dfrac{dy}{dx} (2y\log y+y)

Multiplying both the sides by y, we get,

xy=\dfrac{dy}{dx} (2y^2\log y+y^2)

Therefore (x^2+y^2)\dfrac{dy}{dx} - xy=0

Hence proved.

#459813

Topic: Formation of Differential Equation

Form the differential equation representing the family of curves given by (x a)^2 + 2y^2 = a^2, where a is an arbitrary constant

Solution

Given, (xa)^2+2y^2=a^2

Differentiating it once: 2x{a}^{2}+4y{y}^{'}=0

{a}^{2}=\dfrac{-2y{y}^{'}}{x}

Putting this in main equation:

-2xy{y}^{'}+2{y}^{2}=\dfrac{-2y{y}^{'}}{x}

#459818

Topic: Types of Solution of Differential Equation

Prove that x^2 y^2 = c (x^2 + y^2)^2 is the general solution of differential equation (x^{ 3 }3x\, y^{ 2 })dx=(y^{ 3 }3x^{ 2 }y)dy where c is a parameter.

#459819

Topic: Formation of Differential Equation

Form the differential equation of the family of circles in the first quadrant, which touches the coordinate axes.

Solution

If a circle touches coordinate axis then the center will be at (a,a) and will have a radius of a

Equation of circle is {(x-a)}^{2}+{(y-a)}^{2}={a}^{2}

Differentiating this for one time, we get

2(x-a)+2(y-a){y}^{'}=0

From here, we find a which is \dfrac{2x+2y{y}^{'}}{2(1+{y}^{'})}

This can be plugged in main equation and we can get required differential equation

Equation of circle is {(x-\dfrac{2x+2y{y}^{'}}{2(1+{y}^{'})})}^{2}+{(y-\dfrac{2x+2y{y}^{'}}{2(1+{y}^{'})})}^{2}={(\dfrac{2x+2y{y}^{'}}{2(1+{y}^{'})})}^{2}

#459821

Topic: Homogeneous Differential Equation

Find the general solution of the differential equation \dfrac{dy}{dx}+\sqrt{\dfrac{1-y^2}{1-x^2}}=0.

Solution

Given, \cfrac {dy}{dx}+\sqrt {\cfrac {1-y^2}{1-x^2}}=0

\cfrac{dy}{\sqrt{1-{y}^{2}}}=\cfrac{-dx}{\sqrt{1-{x}^{2}}}

Integrating both sides, we get

{\sin}^{-1}y={\cos}^{-1}x+c

#459822

Topic: Types of Solution of Differential Equation

Show that the general solution of the differential equation \dfrac{dy}{dx}+\dfrac{y^2+y+1}{x^2+x+1}=0 is given by (x+y+1)=A(1-x-y-2xy), where A is parameter.

Solution

$x+y+1 = A(1-x-y-2xy)$

Differentiate both sides w.r.t. x

$\Rightarrow 1+y' = A(-1-y'-2y-2xy')$

$\Rightarrow (1+A+2Ax)y' = -1+ A(-1-2y)$

$\Rightarrow y' = -\cfrac{1+A(1+2y)}{1+A(1+2x)}$ .....(1)

Now, from the given equation, $A = \cfrac{x+y+1}{1-x-y-2xy}$.

Substitute this value of x in equation (1)

$\Rightarrow y' = -\cfrac{1+\dfrac{(x+y+1)(1+2y)}{1-x-y-2xy}}{1+\dfrac{(x+y+1)(1+2x)}{1-x-y-2xy}}\\=-\cfrac{1-x-y-2xy+x+y+1+2xy+2y^2+2y}{1-x-y-2xy+x+y+1+2x^2+2xy+2x}\\=-\cfrac{2y^2+2y+2}{2x^2+2x+2}$

$\Rightarrow \dfrac{dy}{dx} + \dfrac{y^2+y+1}{x^2+x+1} = 0$

Hence, $x+y+1=A(1-x-y-2xy)$ is the general solution of the given differential equation as it satisfies the equation.

#459825

Topic: Formation of Differential Equation

Find the equation of the curve passing through the point $(0, \cfrac{\pi}{4})$ whose differential equation is $\sin x \cos y .dx + \cos x \sin y .dy = 0$.

Solution

$\sin x\cos y dx + \cos x \sin y dy = 0$

$\Rightarrow \cos x \sin y dy = -\sin x\cos y dx$

$\Rightarrow \cfrac{\sin y}{\cos y} dy = \cfrac{-\sin x}{\cos x} dx$

Integrate both sides,

$\displaystyle \int \cfrac{\sin y}{\cos y}dy = \int \cfrac{-\sin x}{\cos x} dx$

Let $\cos y = u$ and $\cos x = v$. Then, $-\sin y dy = du$ and $-\sin x dx = dv$

$\Rightarrow \displaystyle \int\cfrac{-du}u = \int\cfrac{dv}v$

$\Rightarrow -\ln u = \ln v + C$

$\Rightarrow \ln u + \ln v = c$, where c is the constant of integration.

$\Rightarrow \ln (uv) = c$

$\Rightarrow uv = k$, where $k = e^c$ = constant.

Resubstitute the values for u and v,

$\cos x\cos y = k$

This is the general solution of the given differential equation.

This curve passes through $\left(0,\dfrac{\pi}4\right)$

$\Rightarrow \cos 0 \cos \dfrac{\pi}4 = k$

$\Rightarrow k = \dfrac{1}{\sqrt2}$

Hence, the equation of the curve is

$\cos x\cos y = \cfrac1{\sqrt2}$

#459828

Topic: Types of Solution of Differential Equation

Find the particular solution of the differential equation

$(1+e^{2x})dy+(1+y^2)e^xdx=0$, given that $y=1$ when $x=0$.

Solution

(1+e^{2x})dy+(1+y^2)e^xdx=0\\ \dfrac{dy}{(1+y^2)}=- \dfrac{e^x dx}{(1+e^{2x})}\\ \tan^{-1}y=- \dfrac{e^x dx}{(1+e^{2x})}

Let t=e^x\;\; dt=e^xdx\\ \tan^{-1}y=- \dfrac{dt}{(1+t^2)}\\ \tan^{-1}y=- \tan^{-1}t+c\\ \tan^{-1}y=- \tan^{-1}(e^x)+c

When x=0 and y=1,

\tan^{-1}1=- \tan^{-1}(e^0)+c\\ \dfrac{\pi}{4}=- \dfrac{\pi}{4}+c\\ c=\dfrac{\pi}{2}

\tan^{-1}y=- \tan^{-1}(e^x)+ \dfrac{\pi}{2}

#459833

Topic: Types of Solution of Differential Equation

Find a particular solution of the differential equation (x-y)(dx+dy)=dx-dy, given that y=-1 when x=0.

Solution

let t=(x-y)\\ dt=dx-dy

\Rightarrow dx+dy = dx-dy+2dy = dt+2dy

Put this in given equation

t(dt+2dy)=dt\\ dt+2dy=\dfrac{dt}{t}\\ 2dy=\dfrac{dt}{t}-dt

Integrate both sides

2y=\ln t-t+c\\ 2y=\ln(x-y)-(x-y)+c

Now y=-1 \;\; x=0

-2=\ln(0+1)-(0+1)+c\\ \Rightarrow c=-1\\

Equation is 2y=\ln(x-y)-(x-y)-1

#459844

Topic: Linear Differential Equation

Solve the differential equation y\,e^{\frac{x}{y}}dx=\left(x\,e^{\frac{x}{y}}+y^2\right)dy(y\neq 0).

Solution

The differential equation is

ye^{\frac{x}{y}} dx = (xe^{\frac{x}{y}}+y^{2}) dy

let's take p as p = e^{\frac{x}{y}}

so differential equation will become

ypdx = (xp+y^{2})dy

differentiating p with respect to x

dp = e^{\frac{x}{y}}(\frac{dx}{y}-x\frac{dy}{y^{2}})

\implies y^{2}\frac{dp}{dx} = p(y-x\frac{dy}{dx})

\implies pydx= y^{2}dp+pxdy

apply this on above differential equation

xpdy+y^{2}dp=xpdy+y^{2}dy

\implies dy=dp,

\implies y= p+C,

hence the solution is :

y= e^{\frac{x}{y}}+C

#459846

Topic: Linear Differential Equation

Solve the differential equation $\left[\cfrac{e^{-2\sqrt{x}}}{\sqrt{x}}-\cfrac{y}{\sqrt{x}}\right]\cfrac{dx}{dy}=1$ …$(x\neq 0)$.

Solution

differential equation is

$(\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}})\dfrac{dx}{dy}=1$

$\implies \dfrac{e^{-2\sqrt{x}}}{\sqrt{x}}=\dfrac{dy}{dx}+\dfrac{y}{\sqrt{x}}$

this first order equation of form

$y'+P(x)y=Q(x)$

integrating factor would be

$I.F=e^{\int{P(x)dx}}$

so integrating factor would be

$I.F=e^{\int{\frac{dx}{\sqrt{x}}}}=e^{2\sqrt{x}}$

so solution would be

$y\times I.F=\int{I.F\times Q(x)dx}$

solving using above method

$y\ e^{2\sqrt{x}}=\int{e^{2\sqrt{x}}\frac{e^{-2\sqrt{x}}}{\sqrt{x}}dx}$

$\implies y\ e^{2\sqrt{x}}=\int{\frac{dx}{\sqrt{x}}}$

$\implies y\ e^{2\sqrt{x}}=2\sqrt{x}+C$

hence the solution is

$y=\dfrac{2\sqrt{x}+C}{e^{2\sqrt{x}}}$

#459852

Topic: Types of Solution of Differential Equation

Find a particular solution of the differential equation $\dfrac{dy}{dx}+y\, \cot\,x=4x\, \text{cosec } x(x\neq 0)$, given that y = 0 when $x=\dfrac{\pi}{2}$

Solution

$\dfrac{dy}{dx}+y\cot x = 4x\text{cosec }x$

$IF = e^{ \int {\cot xdx}}= \sin x$

$y\times \sin x = \displaystyle \int {4x\sin x.\text{cosec }xdx}$

$y\times \sin x = \displaystyle \int {4x}dx$

(since $\sin x.\text{cosec }x=1$)

$y\times \sin x = 2.x^2 +c$

Now $y=0\;\; x=\dfrac{\pi}{2}$

$0=2.\cfrac{\pi^2}{4}+c$

$c=-\cfrac{\pi^2}{2}$

Particular solution is

$y\times \sin x = 2x^2 -\dfrac{\pi^2}{2}$

#459856

Topic: Types of Solution of Differential Equation

Find a particular solution of the differential equation $(x+1)\cfrac{dy}{dx}=2\,e^{-y}-1$, given that $y=0$ when $x=0$.

Solution

(x+1)\cfrac{dy}{dx}=2e^{-y}-1\\ \cfrac{dy}{2e^{-y}-1}= \cfrac{dx}{x+1}\\ \cfrac{dy}{2e^{-y}-1}=\ln(x+1)+c

Let t=2e^{-y}-1

So, dt=-2e^{-y}dy

Putting these in obtained equation

\cfrac{-dt}{t(t+1)}=\ln(x+1)+c\\ \ln(t)-\ln(t+1)=\ln(x+1)+c\\ \ln(2e^{-y}-1)-\ln(2e^{-y})=\ln(x+1)+c

Putting x=0,\;\;y=0

\ln(1)-\ln2=\ln1+c\\ c=-\ln2

Particular solution is

\ln(2e^{-y}-1)-\ln(2e^{-y})=\ln(x+1)-\ln2

#459858

Topic: Homogeneous Differential Equation

The general solution of the differential equation \dfrac{y\, dx-x\, dy}{y}=0 is:

A xy = C

B x = Cy^2

[✓ **C**] y=Cx

D y=Cx^2

Solution

\dfrac{ydx-xdy}{y}=0\\ dx-\dfrac{x}{y}dy=0\\ dx=\dfrac{x}{y}dy\\ \dfrac{dx}{x}=\dfrac{dy}{y}\\ lnx=lny+lnc\\ x=yc

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#459863

Topic: Linear Differential Equation

The general solution of a differential equation of the type \dfrac{dx}{dy}+P_1x=Q_1 is

A y\, e^{\int {P_1dy}} = \int{(Q_1e^{\int {P_1dy}})}dy+C

B y\, e^{\int {P_1dx}} = \int{(Q_1e^{\int {P_1dx}})}dx+C

[✓ **C**] x\, e^{\int {P_1dy}} = \int{(Q_1e^{\int {P_1dy}})}dy+C

D x\, e^{\int {P_1dx}} = \int{(Q_1e^{\int {P_1dx}})}dx+C

Solution

Given DE is \dfrac{dx}{dy}+P_1x=Q_1

which is an exact DE, whose integrating factor is given by

IF= e^{\int{P_1\ dy}}

General solution of this DE is given by

x\ e^{\int{P_1dy}}=\displaystyle \int{(Q_{1}\ e^{\int{P_1dy}})dy}+C

#459866

Topic: Linear Differential Equation

The general solution of the differential equation e^x dy + (y\,e^x + 2x) dx = 0 is

A x\, e^y+x^2=C

B x\, e^y+y^2=C

C y\, e^x-x^2=C

[✓ **D**] y\, e^x+x^2=C

Solution

e^xdy+(ye^x+2x)dx=0

\Rightarrow e^xdy=-(ye^x+2x)dx

\Rightarrow \dfrac{dy}{dx}+y=-2x.e^{-x} which is an exact DE

\therefore IF = e^{\displaystyle \int{1dx}}=e^x

General solution is given by

y\ IF=\int IF\ . Q(x)\ dx+C

y e^x= \displaystyle \int{e^x(-2xe^{-x}dx)}+C

ye^x=-x^2+C

\Rightarrow ye^x+x^2=C