#423196

Topic: Graphical Method

A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs.4 per unit food and F_2s costs Rs.6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the mineral nutritional requirements?

Solution

Let the diet contain x units of food F_1 and y units of food F_2.

Therefore, $x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows

	Vitamin A(units)	Mineral (units) Cost Per unit	(Rs)
Food $F_1(x)$	3	4 4	
Food $F_2(y)$	6	3 6	
Requirement	80	100	

The cost of food F_1 is Rs.4 per unit and of food F_2 is Rs.6 per unit. Therefore, the constraints are

$3x + 6y \geq 80$

$4x + 3y \geq 100$

$x, y \geq 0$

Total cost of the diet, $Z = 4x + 6y$

The mathematical formulation of the given problem is

Minimise $Z = 4x + 6y......(1)$

subject the constraints

$3x + 6y \geq 80......(2)$

$4x + 3y \geq 100.....(3)$

$x, y \geq 0.......(4)$

The feasible region determined by the constraints is as given.

It can be seen that the feasible region is unbounded

The corner points are $A\left(\dfrac{80}{3}, 0\right)$, $B\left(24, \dfrac{4}{3}\right)$ and $C\left(0, \dfrac{100}{3}\right)$

The values of Z at these corner points are as follows.

Corner point	$Z = 4x + 6y$	
$A\left(\dfrac{80}{3}, 0\right)$	$\dfrac{320}{3} = 106.67$	
$B\left(24, \dfrac{4}{3}\right)$	104	$\rightarrow$ Minimum
$C\left(0, \dfrac{100}{3}\right)$	200	

As the feasible region is unbounded, therefore, 104 may or may not be the minimum value Z

For this, we draw a graph of the inequality, $4x + 6y < 104$or $2x + 3y < 52$ and check whether the resulting half plane has points in common with the feasible region or not. It can be seen that the feasible region has no common point with $2x + 3y < 52$

Therefore, the minimum cost of the mixture will be Rs.104.