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Math Chapter 7
Intgrals

Chapter 7: Integrals

Exercise – 7.1

Question 1:

By the method of inspection obtain an integral (or anti – derivative) of the sin 3x.

Answer:

As the derivative is sin 3x and x is the function of the anti – derivative of sin 3x.

ddx(cos3x)=–3sin3xsin3x=–13ddx(cos3x)sin3x=ddx(−13cos3x)Hence,theanti–derivativeofsin3xis(−13cos3x)

Question 2:

By the method of inspection obtain an integral (or anti – derivative) of the cos 2x.

Answer:

As the derivative is cos 2x and x is the function of the anti – derivative of cos 2x

ddx(sin2x)=–2cos2xcos2x=12ddx(sin2x)cos2x=ddx(12(sin2x))Hence,theanti–derivativeofsin2xis(−12cos2x)

Question 3:

By the method of inspection obtain an integral (or anti – derivative) of the e^{5x}.

Answer:

As the derivative is e^{5x} and x is the function of the anti – derivative of e^{5x}

ddx(e5x)=5e5xe5x=15ddx(e5x)e5x=ddx(15e5x)Hence,theanti–derivativeofe5xis15e5x

Question 4:

By the method of inspection obtain an integral (or anti – derivative) of the $(mx + n)^2$.

Answer:

As the derivative is $(mx + n)^2$ and x is the function of the anti – derivative of $(mx + n)^2$

$\frac{d}{dx}(mx+n)^3 = 3m(mx+n)^2$
$(mx+n)^2 = \frac{1}{3m}\frac{d}{dx}(mx+n)^3$
$(mx+n)^2 = \frac{d}{dx}\left(\frac{1}{3m}(mx+n)^3\right)$
Hence, the anti – derivative of $(mx+n)^2$ is $\frac{1}{3m}(mx+n)^3$

Question 5:

By the method of inspection obtain an integral (or anti – derivative) of the $\sin 3x – 5e^{2x}$

Answer:

As the derivative is $(\sin 3x – 5e^{2x})$ and x is the function of the anti – derivative of $(\sin 3x – 5e^{2x})$

$\frac{d}{dx}\left(-\frac{1}{3}\cos 3x - \frac{5}{2}e^{2x}\right) = \sin 3x - 5e^{2x}$
Hence, the anti – derivative of $\sin 3x - 5e^{2x}$ is $\left(-\frac{1}{3}\cos 3x - \frac{5}{2}e^{2x}\right)$

Question 6:

By the method of inspection obtain an integral of the $\int(4e^{2u}+1)du$

Answer:

Integral of $(4e^{2u}+1)$ and u is the function of the integral $(4e^{2u}+1)$.
$\int(4e^{2u}+1)du$
$4\int e^{2u}du+\int 1du$
$4\left(\frac{e^{2u}}{2}\right)+u+c$
$2e^{2u}+u+c$
Where c is the constant.

Question 7:

By the method of inspection obtain an integral of the ∫u2(1–1u2)du

Answer:

Integral of u2(1–1u2) and u is the function of the integral u2(1–1u2)

∫u2(1–1u2)du∫(u2–1)duu33–u+cWherecistheconstant

Question 8:

By the method of inspection obtain an integral of the ∫(au2+bu+c)du

Answer:

Integral of au2+bu+c and u is the function of the integral au2+bu+c

∫(au2+bu+c)dua∫(u2)du+b∫udu+c∫1dua(u33)+b(u22)+cu+CWhereCistheconstant

Question 9:

By the method of inspection obtain an integral of the ∫(au2+eu)du

Answer:

Integral of au2+eu and u is the function of the integral au2+eu

∫(au2+eu)dua∫(u2)du+∫euduua(u33)+eu+CWhereCistheconstant

Question 10:

By the method of inspection obtain an integral of the ∫(u−−√+1u√)2du

Answer:

Integral of (u−−√+1u√)2 and u is the function of the integral (u−−√+1u√)2

(u−−√+1u√)2∫(u+1u–2)du∫udu+∫1udu–2∫1duu22+log|u|–2u+CWhereCistheconstant

Question 11:

By the method of inspection obtain an integral of the ∫u3+4u2+4u2du

Answer:

Integral of and u is the function of the integral u3+4u2+4u2
∫u3+4u2+4u2du∫udu+4∫1du+∫4u2duu22+4u+4x+CWhereCistheconstant

Question 12:

By the method of inspection obtain an integral of the u3+4u+4u√

Answer:

Integral of u3+4u+4u√ and u is the function of the integral u3+4u+4u√
∫u3+4u+4u√du∫(u52+4u12+4u−12)=u7272+4(u32)32+4(u12)12+C=27(u72)+83(u32)+8u12+C=27(u72)+83(u32)+8u−−√+CWhereCistheconstant

Question 13:

By the method of inspection obtain an integral of the u3–u2+u+1u–1

Answer:

Integral of u3–u2+u+1u–1 and u is the function of the integral u3–u2+u+1u–1
∫u3–u2+u+1u–1duOndivinding,weget∫(u2+1)du∫u2du+∫1duu33+u+CWhereCistheconstant

Question 14:

By the method of inspection obtain an integral of the (1–u)u−−√

Answer:

Integral of (1–u)u−−√ and u is the function of the integral (1–u)u−−√

$\int(1+u)\sqrt{u}\,du = \int(\sqrt{u}+u^{\frac{3}{2}})\,du = \int u^{\frac{1}{2}}du+\int u^{\frac{3}{2}}du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}}+\frac{u^{\frac{5}{2}}}{\frac{5}{2}}+C = \frac{2}{3}u^{\frac{3}{2}}+\frac{2}{5}u^{\frac{5}{2}}+C$ Where C is the constant

Question 15:

By the method of inspection obtain an integral of the $\sqrt{u}(3u^2+2u+5)$

Answer:

Integral of $\sqrt{u}(3u^2+2u+5)$ and u is the function of the integral $\sqrt{u}(3u^2+2u+5)$

$\int\sqrt{u}(3u^2+2u+5)\,du = \int(3u^{\frac{5}{2}}+2u^{\frac{3}{2}}+5u^{\frac{1}{2}})\,du = 3\int u^{\frac{5}{2}}du+2\int u^{\frac{3}{2}}du+5\int u^{\frac{1}{2}}du = 3\left(\frac{u^{\frac{7}{2}}}{\frac{7}{2}}\right)+2\left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}}\right)+5\left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)+C = \frac{6}{7}u^{\frac{7}{2}}+\frac{4}{5}u^{\frac{5}{2}}+\frac{10}{3}u^{\frac{3}{2}}+C$ Where C is the constant

Question 16:

By the method of inspection obtain an integral of the $2u-2\cos u+e^u$

Answer:

Integral of $2u-2\cos u+e^u$ and u is the function of the integral $2u-2\cos u+e^u$

$\int(2u-2\cos u+e^u)\,du = 2\int u\,du-2\int\cos u\,du+\int e^u du = 2\frac{u^2}{2}-2(\sin u)+e^u+C = u^2-2\sin u+e^u+C$ Where C is the constant

Question 17:

By the method of inspection obtain an integral of the $(4v^2+2\sin v+6\sqrt{v})$

Answer:

Integral of $(4v^2+2\sin v+6\sqrt{v})$ and v is the function of the integral $(4v^2+2\sin v+6\sqrt{v})$

$\int(4v^2+2\sin v+6\sqrt{v})\,dv = 4\int v^2dv+2\int\sin v\,dv+6\int v^{\frac{1}{2}} = 4\frac{v^3}{3}+2(-\cos v)+6\left(\frac{v^{\frac{3}{2}}}{\frac{3}{2}}\right)+C = \frac{4}{3}v^3-2\cos v+4v^{\frac{3}{2}}+C$ Where C is the constant

Question 18:

By the method of inspection obtain an integral of the $\sec\Theta(\tan\Theta+\sec\Theta)$

Answer:

Integral of $\sec\Theta(\tan\Theta+\sec\Theta)$ and Θ is the function of the integral $\sec\Theta(\tan\Theta+\sec\Theta)$
$\int\sec\Theta(\tan\Theta+\sec\Theta)d\Theta\int(\sec\Theta\tan\Theta+\sec^2\Theta)d\Theta\sec\Theta+\tan\Theta+K Where K is the constant$

Question 19:

By the method of inspection obtain an integral of the $\sec^2\Theta cosec^2\Theta$

Answer:

Integral of $\sec^2\Theta cosec^2\Theta$ and $3–2\sin\Theta\cos^2\Theta c$ is the function of the integral $\sec^2\Theta cosec^2\Theta$
$\int\sec^2\Theta cosec^2\Theta d\Theta\int_{1\cos2\Theta1\sin2\Theta}d\Theta\int\sin^2\Theta\cos^2\Theta d\Theta\int(\tan^2\Theta)d\Theta\int(\sec^2\Theta–1)d\Theta\int\sec^2\Theta d\Theta–$
$\int1d\Theta\tan\Theta–\Theta+K Where K is the constant$

Question 20:

By the method of inspection obtain an integral of the $3–2\sin\Theta\cos^2\Theta$

Answer:

Integral of $3–2\sin\Theta\cos^2\Theta$ and $3–2\sin\Theta\cos^2\Theta$ is the function of the integral $3–2\sin\Theta\cos^2\Theta$
$\int3–2\sin\Theta\cos^2\Theta d\Theta\int(3\cos^2\Theta–2\sin\Theta\cos^2\Theta)d\Theta3\int\sec^2\Theta d\Theta–2\int\tan\Theta\sec\Theta d\Theta3\tan\Theta–$
$2\sec\Theta+K Where K is the constant$

Question 21:

Which of the following below is an integral of $u−−\sqrt{}+1u\sqrt{}$:
$(a)13u^{13}+2u^{12}+C(b)23u^{23}+12u2+C(c)23u^{32}+2u^{12}+C(d)32u^{32}+12u^{12}+C$

Answer:

Integral of u−−√+1u√ and u is the function of the integral u−−√+1u√
∫u−−√+1u√du∫u12du+∫u−12duu3232+u1212+C32u32+2u12Optionciscorrect

Question 22:

Supposeddrf(r)=4r3−3r4,insuchawaythatf(2)=0,thenf(r)is(a)r4+1r3−1298(b)r3+1r4+1298(c)r4+1r3+1298(d)r3+1r4−1298

Answer:

Given,

ddrf(r)=4r3−3r4Integralof4r3−3r4=f(r)f(r)=∫4r3−3r4drf(r)=4∫r3dr−3∫(r−4)drf(r)=4r44−3r−3−3+Kf(r)=r4+1r3+KAnd,f(2)=0f(2)=24+123+K=0 16+18+K=0K=−1298f(r)=r4+1r3−1298Option(a)iscorrect

Exercise 7.2

Question 1:

Obtain an integral (or anti – derivative) of the 2u1+u2

Answer:

Suppose, 1 + u ² = z

2u du = dz

∫2u1+u2=∫1zdzlog|z|+Klog||1+u2||+Klog(1+u2)+K

Question 2:

Obtain an integral (or anti – derivative) of the $\frac{(\log u)^2}{u}$

Answer:

Suppose, $\log|u|=z$

$\log|u|=z \frac{1}{u}du=dz \int\frac{(\log|u|)^2}{u}du=\int z^2 dz=\frac{z^3}{3}+C=\frac{(\log|u|)^3}{3}+C$

Question 3:

Obtain an integral (or anti – derivative) of the $\frac{1}{u+u\log u}$

Answer:

$\frac{1}{u+u\log u}=\frac{1}{u(1+\log u)}$

Suppose, 1 + log u = z

$\frac{1}{u}du=dz \int\frac{1}{u(1+\log u)}du=\int\frac{1}{z}dz=\log|z|+C=\log|1+\log u|+C$

Question 4:

Obtain an integral (or anti – derivative) of the $\sin u.\sin(\cos u)$

Answer:

$\sin u.\sin(\cos u)$

Suppose, cos u = x

– sin u du = dx

$\int\sin u.\sin(\cos u)du=–\int\sin x dx=–[-\cos x]+C=\cos x+C=\cos(\cos u)+C$

Question 5:

Obtain an integral (or anti – derivative) of the sin(mr+n)cos(mr+n)

Answer:

Suppose, sin(mr+n)cos(mr+n)=2sin(mr+n)cos(mr+n)2=sin2(mr+n)2Suppose2(mr+n)=z2m dr=dz∫sin2(mr+n)2dr=12∫sinzdz2m=14m[−cosz]+C=–14mcos2(mr+n)+C

Question 6:

Obtain an integral (or anti – derivative) of the mr+n−−−−−−√

Answer:

Suppose, mr + n = z

m dr = dz

dr=1mdz∫(mr+n)12dr=1m∫z12dz1m(z3232)+C23m(mr+n)32+C

Question 7:

Obtain an integral (or anti – derivative) of the uu+2−−−−√

Answer:

Suppose, u + 2 = z

du = dz

∫uu+2−−−−√du=∫(z–2)z√dz=∫(z32–2z12)dz=∫z32dz–2∫z12)dz=z5252–2z3232+C=25z52–43z32+C=25(x+2)52–43(x+2)32+C

Question 8:

Obtain an integral (or anti – derivative) of the u1+2u2−−−−−−√

Answer:

Suppose, 1 + 2 u^2 = z

$4u\,du = dz$

$$\int u\sqrt{1+2u^2}\,du=\int \frac{\sqrt{z}}{4}dz=\frac{1}{4}\int z^{\frac{1}{2}}dz=\frac{1}{4}\left(\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right)+C=\frac{1}{6}(1+2u^2)^{\frac{3}{2}}+C$$

Question 9:

Obtain an integral (or anti – derivative) of the $(4u+2)\sqrt{u^2+u+1}$

Answer:

Suppose, $u^2 + u + 1 = z$

$(2u + 1)\, du = dz$

$$\int(4u+2)\sqrt{u^2+u+1}\,du=\int 2\sqrt{z}\,dz=2\int\sqrt{z}\,dz=2\left(\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right)+C=\frac{4}{3}(u^2+u+1)^{\frac{3}{2}}+C$$

Question 10:

Obtain an integral (or anti – derivative) of the $\frac{1}{u-\sqrt{u}}$

Answer:

$\frac{1}{u-\sqrt{u}}=\frac{1}{\sqrt{u}(\sqrt{u}-1)}$

Suppose,

$$\sqrt{u}-1=z \qquad \frac{1}{2\sqrt{u}}du=dz \qquad \int\frac{1}{\sqrt{u}(\sqrt{u}-1)}du=\int\frac{2}{z}dz \qquad 2\log|z|+C \qquad 2\log|\sqrt{u}-1|+C$$

Question 11:

Obtain an integral (or anti – derivative) of the $\frac{u}{\sqrt{u+4}}$, $x > 0$

Answer:

Suppose, $u + 4 = r$

$du = dr$

$\int uu+4\surd du=\int(r-4)r\surd dr=\int(r\surd -4r\surd)dr=r3232-4(r1212)+C=23r32-8r12+C=23r.r12-8r12+C=23r12(r-12)+C=23(u+4)12(u+4-12)+C=23(u+4)------\surd(u-8)+C$

Question 12:

Obtain an integral (or anti – derivative) of the $(u3-1)13u5$

Answer:

Suppose, u ^{3} – 1 = r

3 u ^{2} = dr

$\int(u3-1)13u5du=\int(u3-1)13u3.u2du=\int r13(r+1)dr3=13\int(r43+r13)dr=13[r7373+r4343]+C=13[37r73+34r43]+C=17(u3-1)73+14(u3-1)43]+C$

Question 13:

Obtain an integral (or anti – derivative) of the $u2(2+3u3)3$

Answer:

Suppose, $2+3u3=z9u2du=dz\int u2(2+3u3)du=19\int dz(z)3=19\int(z)-3dz=19(z-2-2)+C=-118(1z2)+C=-118(2+3u3)2+C$

Question 14:

Obtain an integral (or anti – derivative) of the $1u(logu)n,x>0$

Answer:

Suppose, $logu=z1udu=dz\int 1u(logu)ndu=\int dzzn=\int z-ndz=z-n+1-n+1+C=z1-n1-n+C=x1-n1-n+C$

Question 15:

Obtain an integral (or anti – derivative) of the $u9-4u2$

Answer:

Suppose, $9-4u^2=r$, $-8u\,du=dr$, $\int\frac{u}{9-4u^2}=-\frac{1}{8}\int\frac{1}{r}dr=-\frac{1}{8}\log|r|+C=-\frac{1}{8}\log\left|\left|9-4u^2\right|\right|+C$

Question 16:

Obtain an integral (or anti – derivative) of the e^{2m+3}

Answer:

Suppose, $2m+3=r$, $2dm=dr$, $\int e^{2m+3}dm=\frac{1}{2}\int e^r dr=\frac{1}{2}(e^r)+C=\frac{1}{2}(e^{2m+3})+C$

Question 17:

Obtain an integral (or anti – derivative) of the $\frac{u}{e^{u^2}}$

Answer:

Suppose, $u^2 = z$

$2u\,du = dz$

$\int\frac{u}{e^{u^2}}du=\frac{1}{2}\int\frac{1}{e^z}dz=\frac{1}{2}\int e^{-z}dz=\frac{1}{2}\frac{e^{-z}}{-1}+C=-\frac{1}{2}e^{-u^2}+C=-\frac{1}{2e^{u^2}}+C$

Question 18:

Obtain an integral (or anti – derivative) of the $\frac{e^{\tan^{-1}\Theta}}{1+\Theta^2}$

Answer:

Suppose, $\tan^{-1}\Theta=z$, $\frac{1}{1+\Theta^2}d\Theta=dz$, $\int\frac{e^{\tan^{-1}\Theta}}{1+\Theta^2}d\Theta=\int e^z dz=e^z+C=e^{\tan^{-1}\Theta}+C$

Question 19:

Obtain an integral (or anti – derivative) of the $\frac{e^{2u}-1}{e^{2u}+1}$

Answer:

$\frac{e^{2u}-1}{e^{2u}+1}$

Dividing the numerator and denominator by e^{u}, we get

$\frac{\frac{e^{2u}-1}{e^u}}{\frac{e^{2u}+1}{e^u}}=\frac{e^u-e^{-u}}{e^u+e^{-u}}$

Suppose,

$e^u+e^{-u}=z \quad (e^u-e^{-u})du=dz \quad \int\frac{e^{2u}-1}{e^{2u}+1}du=\int\frac{e^u-e^{-u}}{e^u+e^{-u}}du=\int\frac{dz}{z}=\log|z|+C=\log|e^u+e^{-u}|+C$

Question 20:

Obtain an integral (or anti – derivative) of the $\frac{e^{2u}-e^{-2u}}{e^{2u}+e^{-2u}}$

Answer:

Suppose, $e^{2u}+e^{-2u}=z \quad (2e^{2u}-2e^{-2u})du=dz \quad 2(e^{2u}-e^{-2u})du=dz \quad \int\frac{e^{2u}-e^{-2u}}{e^{2u}+e^{-2u}}=\int\frac{dz}{2z}dz=\frac{1}{2}\int\frac{1}{z}dz=\frac{1}{2}\log|z|+C=\frac{1}{2}\log||e^{2u}+e^{-2u}||+C$

Question 21:

Obtain an integral (or anti – derivative) of the $\tan^2(2\Theta-3)$

Answer:

$\tan^2(2\Theta-3)=\sec^2(2\Theta-3)-1$

Suppose, $2\Theta-3=z \quad 2d\Theta=dz \quad \int\tan^2(2\Theta-3)d\Theta=\int[\sec^2(2\Theta-3)-1]d\Theta=\frac{1}{2}\int(\sec^2 z)dz-\int 1d\Theta=\frac{1}{2}\tan z-\Theta+C=\frac{1}{2}\tan(2\Theta-3)-\Theta+C$

Question 22:

Obtain an integral (or anti – derivative) of the $\sec^2(7-4\theta)$

Answer:

Suppose, $(7-4\theta)=z \quad -4d\theta=dz \quad \int\sec^2(7-4\Theta)d\theta=-\frac{1}{4}\int\sec^2 z\,dz=-\frac{1}{4}(\tan z)+C=-\frac{1}{4}[\tan(7-4\theta)]+C$

Question 23:

Obtain an integral (or anti – derivative) of the $\frac{\sin^{-1}\theta}{\sqrt{1-\theta^2}}$

Answer:

Suppose, $\sin^{-1}\theta=z$ $\frac{1}{\sqrt{1-\theta^2}}d\theta=dz$ $\int\frac{\sin^{-1}\theta}{\sqrt{1-\theta^2}}d\theta=\int z\,dz=\frac{z^2}{2}+C=\frac{(\sin^{-1}\theta)^2}{2}+C$

Question 24:

Obtain an integral (or anti – derivative) of the $\frac{2\cos\theta-3\sin\theta}{6\cos\theta+4\sin\theta}$

Answer:

$\frac{2\cos\theta-3\sin\theta}{6\cos\theta+4\sin\theta}=\frac{2\cos\theta-3\sin\theta}{2(3\cos\theta+2\sin\theta)}$

Suppose,

$3\cos\theta+2\sin\theta=z$ $(-3\sin\theta+2\cos\theta)d\theta=dz$ $\int\frac{2\cos\theta-3\sin\theta}{6\cos\theta+4\sin\theta}d\theta=\int\frac{dz}{2z}=\frac{1}{2}\int\frac{1}{z}dz=\frac{1}{2}\log|z|+C=\frac{1}{2}\log|3\cos\theta+2\sin\theta|+C$

Question 25:

Obtain an integral (or anti – derivative) of the $\frac{1}{\cos^2\theta(1-\tan\theta)^2}$

Answer:

$\frac{1}{\cos^2\theta(1-\tan\theta)^2}=\frac{\sec^2\theta}{(1-\tan\theta)^2}$

Suppose,

$(1-\tan\theta)=z$ $\sec^2\theta\,d\theta=dz$ $\int\frac{\sec^2\theta}{(1-\tan\theta)^2}d\theta=\int-\frac{dz}{z^2}=-\int z^{-2}dz=\frac{1}{z}+C=\frac{1}{1-\tan\theta}+C$

Question 26:

Obtain an integral (or anti – derivative) of the cosθ√θ√

Answer:

Suppose, θ√=z12θ√dθ=dz∫cosθ√θ√=2∫coszdz=2sinz+C=2sinθ√+C

Question 27:

Obtain an integral (or anti – derivative) of the sin2θ−−−−−√cos2θ

Answer:

Suppose, sin2θ=z2cos2θdθ=dz∫sin2θ−−−−−√cos2θ=12∫z√dz=12(z3232)+C=13z32+C=1
3(sin2θ)32+C

Question 28:

Obtain an integral (or anti – derivative) of the cosθ1+sinθ√

Answer:

Suppose, 1+sinθ=zcosθdθ=dz∫cosθ1+sinθ√dθ=∫dzz√=z1212+C=2z√+C=21+sinθ−−−−−−
−√+C

Question 29:

Obtain an integral (or anti – derivative) of the cotθlogsinθ

Answer:

Suppose, logsinθ=z1sinθ.cosθ=dzcotθdθ=dz∫cotθlogsinθdθ=∫zdz=z22+C=12(logsinθ)
2+C

Question 30:

Obtain an integral (or anti – derivative) of the sinθ1+cosθ

Answer:

Suppose,

$$1+\cos\theta=z \quad -\sin\theta\, d\theta=dz \quad \int \frac{\sin\theta}{1+\cos\theta} d\theta=\int \frac{-dz}{z}=-\int \frac{dz}{z} dz=-\log|z|+C=-\log|1+\cos\theta|+C$$

Question 31:

Obtain an integral (or anti – derivative) of the $\frac{\sin\theta}{(1+\cos\theta)^2}$

Answer:

Suppose, $1+\cos\theta=z \quad -\sin\theta\, d\theta=dz \quad \int \frac{\sin\theta}{1+\cos\theta} d\theta=\int \frac{-dz}{z^2}=-\int \frac{dz}{z^2} dz=-\int z^{-2} dz=\frac{1}{z}+C=\frac{1}{1+\cos\theta}+C$

Question 32:

Obtain an integral (or anti – derivative) of the $\frac{1}{1+\cot\theta}$

Answer:

Suppose, I

$= \int \frac{1}{1+\cot\theta} d\theta=\int \frac{1}{1+\frac{\cos\theta}{\sin\theta}} d\theta=\int \frac{\sin\theta}{\sin\theta+\cos\theta} d\theta=\frac{1}{2}\int \frac{2\sin\theta}{\sin\theta+\cos\theta} d\theta=\frac{1}{2}\int \frac{(\sin\theta+\cos\theta)+(\sin\theta-\cos\theta)}{(\sin\theta+\cos\theta)} d\theta=\frac{1}{2}\int 1 d\theta+\frac{1}{2}\int \frac{(\sin\theta-\cos\theta)}{(\sin\theta+\cos\theta)} d\theta=\frac{1}{2}\theta+\frac{1}{2}\int \frac{(\sin\theta-\cos\theta)}{(\sin\theta+\cos\theta)} d\theta$

Suppose, $(\sin\theta+\cos\theta)=z=(\cos\theta-\sin\theta)d\theta=dz \quad I=\frac{\theta}{2}+\frac{1}{2}\log|z|+C=\frac{\theta}{2}-\frac{1}{2}\log|(\sin\theta+\cos\theta)|+C$

Question 33:

Obtain an integral (or anti – derivative) of the $\frac{1}{1-\tan\theta}$

Answer:

Suppose,

$\int \frac{1}{1-\tan\theta} d\theta=\int \frac{1}{1-\frac{\sin\theta}{\cos\theta}} d\theta=\int \frac{\cos\theta}{\cos\theta-\sin\theta} d\theta=\frac{1}{2}\int \frac{2\cos\theta}{\cos\theta-\sin\theta} d\theta=\frac{1}{2}\int \frac{(\cos\theta-\sin\theta)+(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)} d\theta=\frac{1}{2}\int 1 d\theta+\frac{1}{2}\int \frac{(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)} d\theta=\frac{1}{2}\theta+\frac{1}{2}\int \frac{(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)} d\theta$ Suppose, $(\cos\theta-\sin\theta)=z=(-\sin\theta-\cos\theta)d\theta=dz \quad I=\frac{\theta}{2}-\frac{1}{2}\log|z|+C=\frac{\theta}{2}-\frac{1}{2}\log|(\cos\theta-\sin\theta)|+C$

Question 34:

Obtain an integral (or anti – derivative) of the $\tan\theta\sqrt{}\sin\theta\cos\theta$

Answer:

Suppose, $Suppose, I=\tan\theta\sqrt{}\sin\theta\cos\theta d\theta=\tan\theta\sqrt{}\times\cos\theta\sin\theta\cos\theta\times\cos\theta d\theta=\int\tan\theta\sqrt{}\tan\theta\cos^2\theta d\theta=\int\sec^2\theta\tan\theta\sqrt{} d\theta Suppose, \tan\theta=z \sec^2\theta d\theta=dz I=\int dz z\sqrt{}=2z\sqrt{}+C=2\tan\theta----\sqrt{}+C$

Question 35:

Obtain an integral (or anti – derivative) of the $(1+\log u)^2 u$

Answer:

Suppose, $Suppose, 1+\log u=z 1u du=dz \int(1+\log u)^2 u du=\int z^2 dz=z^3 3+C=(1+\log u)^3 3+C$

Question 36:

Obtain an integral (or anti – derivative) of the $(u+1)(u+\log u)^2 u$

Answer:

$(u+1)(u+\log u)^2 u=(u+1)u(u+\log u)^2=(1+1u)(u+\log u)^2 Suppose, (u+\log u)=z(1+1u)du=dz\int(1+1u)(u+\log u)^2 du=\int z^2 dz=z^3 3+C=13(u+\log u)^3+C$

Question 37:

Obtain an integral (or anti – derivative) of the $u^3\sin(\tan^{-1}u^4)1+u^8$

Answer:

$Suppose, u^4=z 4u^3 du=dz \int u^3\sin(\tan^{-1}u^4)1+u^8 du=14\int\sin(\tan^{-1}z)1+z^2 dz\ldots.(1) Suppose, \tan^{-1}z=s 11+z^2 dz=ds From (1), we get \int u^3\sin(\tan^{-1}u^4)1+u^8 du=14\int\sin s ds=14(-\cos s)+c=-14\cos(\tan^{-1}z)+C=-14\cos(\tan^{-1}u^4)+C$

Question 38:

Which of the following below is the answer for $\int \frac{10u^9+10^u \log_e 10}{u^{10}+10^u} du$:

(a) $10^u - u^{10} + C$ (b) $10^u + u^{10} + C$ (c) $(10^u - u^{10})^{-1} + C$ (d) $\log(10^u + u^{10}) + C$

Answer:

$u^{10}+10^u = z$
$(10u^9 + 10^u \log_e 10) du = dz$
$\int \frac{10u^9+10^u \log_e 10}{u^{10}+10^u} du = \int \frac{dz}{z} = \log z + C = \log(u^{10}+10^u) + C$

Therefore, D is the correct answer

Question 39:

Which of the following below is the answer for $\int \frac{du}{\sin^2 u \cos^2 u}$

(a) $\tan u + \cot u + C$ (b) $\tan u - \cot u + C$ (c) $\tan u \cot u + C$ (d) $\tan u - \cot 2u + C$

Answer:

$I = \int \frac{du}{\sin^2 u \cos^2 u} = \int \frac{1}{\sin^2 u \cos^2 u} du = \int \frac{\sin^2 u + \cos^2 u}{\sin^2 u \cos^2 u} du = \int \frac{\sin^2 u}{\sin^2 u \cos^2 u} du + \int \frac{\cos^2 u}{\sin^2 u \cos^2 u} du = \int \sec^2 u\, du + \int \operatorname{cosec}^2 u\, du = \tan u - \cot u + C$

Therefore, B is the correct answer

Exercise 7.3

Question 1:

Obtain an integral (or anti – derivative) of the $\sin^2(2u + 5)$

Answer 1:

$$\sin^2(2u+5)=\frac{1-\cos 2(2u+5)}{2}=\frac{1-\cos(4u+10)}{2}$$

$$\int \sin^2(2u+5)\,du=\int \frac{1-\cos(4u+10)}{2}\,du=\frac{1}{2}\int 1\,du-\frac{1}{2}\int \cos(4u+10)\,du=\frac{1}{2}u-\frac{1}{2}\frac{\sin(4u+10)}{4}+C=\frac{1}{2}u-\frac{1}{8}[\sin(4u+10)]+C$$

Question 2:

Obtain an integral (or anti – derivative) of the sin 3u. cos 4u

Answer 2:

As we know, $\sin C\cos D=\frac{1}{2}[\sin(C+D)+\sin(C-D)]$

$$\int \sin 3u.\cos 4u\,du=\int \frac{1}{2}[\sin(3u+4u)+\sin(3u-4u)]=\int \frac{1}{2}[\sin(7u)+\sin(-u)]\,du=\int \frac{1}{2}[\sin(7u)-\sin(u)]\,du$$

$$=\frac{1}{2}\int \sin(7u)\,du-\frac{1}{2}\int \sin(u)\,du=\frac{1}{2}\left(\frac{-\cos 7u}{7}\right)-\frac{1}{2}(-\cos u)+C=\frac{-\cos 7u}{14}+\frac{\cos u}{2}+C$$

Question 3:

Obtain an integral (or anti – derivative) of the cos 2u cos 4u cos 6u

Answer 3:

As we know, $\cos C\cos D=\frac{1}{2}[\cos(C+D)+\cos(C-D)]$

$$\int \cos 2u(\cos 4u\cos 6u)\,du=\int \cos 2u\left[\frac{1}{2}(\cos(4u+6u)+\cos(4u-6u))\right]du$$

$$\frac{1}{2}\int[\cos 2u(\cos(10u)+\cos(-2u))]\,du$$

$$\frac{1}{2}\int[\cos 2u\cos 10u+\cos 2u\cos(-2u)]\,du$$

$$\frac{1}{2}\int[\cos 2u\cos 10u+\cos^2 2u]\,du$$

$$\frac{1}{2}\int\left[\frac{1}{2}(\cos(2u+10u)+\cos(2u-10u))+\left(\frac{1+\cos 4u}{2}\right)\right]du$$

$$\frac{1}{4}\int(\cos 12u+\cos 8u+1+\cos 4u)\,du$$

$$\frac{1}{4}\left[\frac{\sin 12u}{12}+\frac{\sin 8u}{8}+u+\frac{\sin 4u}{4}\right]+C$$

Question 4:

Obtain an integral (or anti – derivative) of the $\sin^3(2u+1)$

Answer 4:

$$I=\int \sin^3(2u+1)\,du$$

$$\int \sin^3(2u+1)\,du=\int \sin^2(2u+1).\sin(2u+1)\,du$$

Supose, $\cos(2u+1)=z$

$$-2\sin(2u+1)\,du=dz$$

$$\sin(2u+1)\,du=\frac{-dz}{2}$$

$$I=\frac{-1}{2}\int(1-z^2)\,dz=\frac{-1}{2}\left\{z-\frac{z^3}{3}\right\}+C=\frac{-1}{2}\left\{\cos(2u+1)-\frac{\cos^3(2u+1)}{3}\right\}+C=\frac{-\cos(2u+1)}{2}+\frac{\cos^3(2u+1)}{6}+C$$

Question 5:

Obtain an integral (or anti – derivative) of the $\sin^3 u \cos^3 u$

Answer 5:

$$I=\int \sin^3 u \cos^3 u\,du=\int \cos^3 u.\sin^2 u \sin u\,du=\int \cos^3 u(1-\cos^2 u).\sin u\,du$$
Suppose, $\cos u=z$, $-\sin u\,du=dz$
$$I=-\int z^3(1-z^2)dz=-\int (z^3-z^5)dz=-\left\{\frac{z^4}{4}-\frac{z^6}{6}\right\}+C=-\left\{\frac{\cos^4}{4}-\frac{\cos^6}{6}\right\}+C=\frac{\cos^6}{6}-\frac{\cos^4}{4}+C$$

Question 6:

Obtain an integral (or anti – derivative) of the sin u sin 2u sin 3u

Answer 6:

$$\sin C\sin D=\frac{1}{2}[\cos(C-D)-\cos(C+D)]$$
$$\int \sin u \sin 2u \sin 3u\,du=\int \sin u.\frac{1}{2}\{\cos(2u-3u)-\cos(2u+3u)\}du=\frac{1}{2}\int(\sin u\cos(-u)-\sin u\cos 5u)du=\frac{1}{2}\int(\sin u\cos u-\sin u\cos 5u)du$$
$$=\frac{1}{2}\int\frac{(2\sin u\cos u)}{2}du-\frac{1}{2}\int \sin u\cos 5u\,du=\frac{1}{2}\int\frac{(\sin 2u)}{2}du-\frac{1}{2}\int \sin u\cos 5u\,du$$
$$=\frac{1}{4}\left[\frac{-\cos 2u}{2}\right]-\frac{1}{2}\int\left\{\frac{1}{2}(\sin(u+5u)+\sin(u-5u))\right\}du=\frac{-\cos 2u}{8}-\frac{1}{4}\int\{(\sin(6u)+\sin(-4u))\}du$$
$$=\frac{-\cos 2u}{8}-\frac{1}{4}\left[\frac{-\cos 6u}{6}+\frac{\cos 4u}{4}\right]+C=\frac{-\cos 2u}{8}-\frac{1}{8}\left[\frac{-\cos 6u}{3}+\frac{\cos 4u}{2}\right]+C=\frac{1}{8}\left[\frac{-\cos 6u}{3}-\frac{\cos 4u}{2}-\cos 2u\right]+C$$

Question 7:

Obtain an integral (or anti – derivative) of the sin 4u sin 8u

Answer 7:

As we know, $\sin C\sin D=\frac{1}{2}[\cos(C-D)-\cos(C+D)]$
$$\int \sin 4u \sin 8u\,du=\int\frac{1}{2}[\cos(4u-8u)-\cos(4u+8u)]du=\frac{1}{2}\int(\cos(-4u)-\cos 12u)du=\frac{1}{2}\int(\cos 4u-\cos 12u)du=\frac{1}{2}\left[\frac{\sin 4u}{4}-\frac{\sin 12u}{12}\right]$$

Question 8:

Obtain an integral (or anti – derivative) of the $\frac{1-\cos u}{1+\cos u}$

Answer 8:

$\frac{1-\cos u}{1+\cos u}=\frac{2\sin^2\frac{u}{2}}{2\cos^2\frac{u}{2}}=\tan^2\frac{u}{2}=\left(\sec^2\frac{u}{2}-1\right)$ $\int\frac{1-\cos u}{1+\cos u}du=\int\left(\sec^2\frac{u}{2}-1\right)du=\left[\frac{\tan\frac{u}{2}}{\frac{1}{2}}-u\right]+C=2\tan\frac{u}{2}-u+C$

Question 10:

Obtain an integral (or anti – derivative) of the $\sin^4 u$.

Answer 10:

$\sin^4 u=\sin^2 u\times\sin^2 u=\left(\frac{1-\cos 2u}{2}\right)\left(\frac{1-\cos 2u}{2}\right)=\frac{1}{4}(1-\cos 2u)^2=\frac{1}{4}(1+\cos^2 2u-2\cos 2u)=\frac{1}{4}\left[1+\left(\frac{1+\cos 4u}{2}\right)-2\cos 2u\right]=\frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4u-2\cos 2u\right]=\frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4u-2\cos 2u\right]$ $\int\sin^4 u\,du=\frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4u-2\cos 2u\right]du=\frac{1}{4}\left[\frac{3}{2}u+\frac{1}{2}\left(\frac{\sin 4u}{4}\right)-\sin 2u\right]+C=\frac{3}{8}u+\left(\frac{\sin 4u}{32}\right)-\frac{\sin 2u}{4}+C$

Question 11:

Obtain an integral (or anti – derivative) of the $\cos^4 2u$

Answer 11:

$\cos^4 2u=(\sin^2 2u)^2=\left(\frac{1+\cos 4u}{2}\right)^2=\frac{1}{4}(1+\cos^2 4u+2\cos 4u)=\frac{1}{4}\left[1+\left(\frac{1+\cos 8u}{2}\right)+2\cos 4u\right]=\frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 8u+2\cos 4u\right]=\frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 8u+2\cos 4u\right]$ $\int\cos^4 u\,du=\frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 8u+2\cos 4u\right]du=\frac{1}{4}\left[\frac{3}{2}u+\frac{1}{2}\left(\frac{\sin 8u}{4}\right)+\sin 4u\right]+C=\frac{3}{8}u+\left(\frac{\sin 8u}{64}\right)+\frac{\sin 4u}{8}+C$

Question 12:

Obtain an integral (or anti – derivative) of the $\frac{\sin^2 u}{1+\cos u}$

Answer 12:

$$\frac{\sin^2 u}{1+\cos u}=\frac{\left(2\sin\frac{u}{2}\cos\frac{u}{2}\right)^2}{2\cos^2\frac{u}{2}}\left[\text{Since}\sin u=2\sin\frac{u}{2}\cos\frac{u}{2};\cos u=2\cos^2\frac{u}{2}-1\right]=\frac{4\sin^2\frac{u}{2}\cos^2\frac{u}{2}}{2\cos^2\frac{u}{2}}=2\sin^2\frac{u}{2}=1-\cos u$$
$$\int\frac{\sin^2 u}{1+\cos u}du=\int(1-\cos u)du=u-\sin u+C$$

Question 13:

Obtain an integral (or anti – derivative) of the $\frac{\cos 2u-\cos 2a}{\cos u-\cos a}$

Answer 13:

$$\frac{\cos 2u-\cos 2a}{\cos u-\cos a}=\frac{-2\sin\frac{2u+2a}{2}\sin\frac{2u-2a}{2}}{-2\sin\frac{u+a}{2}\sin\frac{u-a}{2}}\left[\text{Since},\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}\right]=\frac{\sin(u+a)\sin(u-a)}{\sin\frac{u+a}{2}\sin\frac{u-a}{2}}=\frac{\left[2\sin\frac{u+a}{2}\cos\frac{u+a}{2}\right]\left[2\sin\frac{u-a}{2}\cos\frac{u-a}{2}\right]}{\sin\frac{u+a}{2}\sin\frac{u-a}{2}}=4\cos\frac{u+a}{2}\cos\frac{u-a}{2}=2\left[\cos\left(\frac{u+a}{2}+\frac{u-a}{2}\right)+\cos\left(\frac{u+a}{2}-\frac{u-a}{2}\right)\right]=2[\cos(u)+\cos a]=2\cos u+2\cos a$$

Question 14:

Obtain an integral (or anti – derivative) of the $\frac{\cos u-\sin u}{1+\sin 2u}$

Answer 14:

$$\frac{\cos u-\sin u}{1+\sin 2u}=\frac{\cos u-\sin u}{(\sin^2 u+\cos^2 u)+2\sin u\cos u}\quad\text{Since},\sin^2 u+\cos^2 u=1;\sin 2u=2\sin u\cos u=\frac{\cos u-\sin u}{(\sin u+\cos u)^2}$$
$$\text{Suppose},\sin u+\cos u=z\quad(\cos u-\sin u)du=dz\quad\int\frac{\cos u-\sin u}{1+\sin 2u}du=\int\frac{\cos u-\sin u}{(\sin u+\cos u)^2}du=\int\frac{dz}{z^2}=\int z^{-2}dz=-z^{-1}+C=-\frac{1}{z}+C=-\frac{1}{\sin u+\cos u}+C$$

Question 15:

Obtain an integral (or anti – derivative) of the $\tan^3 2u\sec 2u$

Answer 15:

$$\tan^3 2u\sec 2u=\tan^2 2u\tan 2u\sec 2u=(\sec^2 2u-1)\tan 2u\sec 2u=\sec^2 2u\tan 2u\sec 2u-\tan 2u\sec 2u$$
$$\int\tan^3 2u\sec 2u\,du=\int\sec^2 2u\tan 2u\sec 2u\,du-\int\tan 2u\sec 2u\,du=\int\sec^2 2u\tan 2u\sec 2u\,du-\frac{\sec 2u}{2}+C$$
$$\text{Suppose},\sec 2u=z\quad 2\sec 2u\tan 2u\,du=dz\quad\int\tan^3 2u\sec 2u=\frac{1}{2}\int z^2dz-\frac{\sec 2u}{2}+C=\frac{z^3}{6}-\frac{\sec 2u}{2}+C=\frac{(\sec 2u)^3}{6}-\frac{\sec 2u}{2}+C$$

Question 16:

Obtain an integral (or anti – derivative) of the $\tan^4 u$

Answer 16:

$\tan^4 u = \tan^2 u \times \tan^2 u = (\sec^2 u - 1)\tan^2 u = \sec^2 u \tan^2 u - \tan^2 u = \sec^2 u \tan^2 u - (\sec^2 u - 1) = \sec^2 u \tan^2 u - \sec^2 u + 1$ $\int \tan^4 u\, du = \int \sec^2 u \tan^2 u\, du - \int \sec^2 u\, du + \int 1\, du = \int \sec^2 u \tan^2 u\, du - \tan u + u + C \ldots\ldots(1)$ Now, $\int \sec^2 u \tan^2 u\, du$ Suppose, $\tan u = z$, $\sec^2 u\, du = dz$ $\int \sec^2 u \tan^2 u\, du = \int z^2\, dz = \frac{z^3}{3} + C = \frac{\tan^3 u}{3}$ Therefore, from equation (1) is $\int \tan^4 u\, du = \frac{\tan^3 u}{3} - \tan u + u + C$

Question 17:

Obtain an integral (or anti – derivative) of the $\frac{\sin^3 u + \cos^3 u}{\sin^2 u \cos^2 u}$

Answer 17:

$\frac{\sin^3 u + \cos^3 u}{\sin^2 u \cos^2 u} = \frac{\sin^3 u}{\sin^2 u \cos^2 u} + \frac{\cos^3 u}{\sin^2 u \cos^2 u} = \frac{\sin u}{\cos^2 u} + \frac{\cos u}{\sin^2 u} = \tan u \sec u + \cot u \operatorname{cosec} u$ Therefore, $\int \frac{\sin^3 u + \cos^3 u}{\sin^2 u \cos^2 u} du = \int (\tan u \sec u + \cot u \operatorname{cosec} u)\, du = \sec u - \operatorname{cosec} u + C$

Question 18:

Obtain an integral (or anti – derivative) of the $\frac{\cos 2u + 2\sin^2 u}{\cos^2 u}$

Answer 18:

$\frac{\cos 2u + 2\sin^2 u}{\cos^2 u} = \frac{\cos 2u + (1 - \cos 2u)}{\cos^2 u}$ [Since, $\cos 2u = 1 - 2\sin^2 u$] $= \frac{1}{\cos^2 u} = \sec^2 u$ $\int \frac{\cos 2u + 2\sin^2 u}{\cos^2 u} du = \int \sec^2 u\, du = \tan u + C$

Question 19:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sin u\cos^2 u}$

Answer 19:

$\frac{1}{\sin u\cos^2 u} = \frac{\sin^2 u+\cos^2 u}{\sin u\cos^2 u} = \frac{\sin u}{\cos^2 u} + \frac{1}{\sin u\cos u} = \tan u\sec^2 u + \frac{1}{\sin u\cos u}\times\frac{\cos^2 u}{\cos^2 u} = \tan u\sec^2 u + \frac{\sec^2 u}{\tan u}$

$\int \frac{1}{\sin u\cos^2 u}du = \int \tan u\sec^2 u\,du + \int \frac{\sec^2 u}{\tan u}du$

Suppose, $\tan u = z$, $\sec^2 u\,du = dz$

$\int \frac{1}{\sin u\cos^2 u}du = \int z\,dz + \int \frac{1}{z}dz = \frac{z^2}{2} + \log|z| + C = \frac{1}{2}\tan^2 u + \log|\tan u| + C$

Question 20:

Obtain an integral (or anti – derivative) of the $\frac{\cos 2u}{(\cos u+\sin u)^2}$

Answer 20:

$\frac{\cos 2u}{(\cos u+\sin u)^2} = \frac{\cos 2u}{\cos^2 u+\sin^2 u+2\sin u\cos u} = \frac{\cos 2u}{1+\sin 2u}$

$\int \frac{\cos 2u}{(\cos u+\sin u)^2}du = \int \frac{\cos 2u}{(1+\sin 2u)}du$

Suppose, $1+\sin 2u = Z$, $2\cos 2u\,du = dz$

$\int \frac{\cos 2u}{(\cos u+\sin u)^2}du = \frac{1}{2}\int \frac{1}{z}dz = \frac{1}{2}\log|z| + C = \frac{1}{2}\log|1+\sin 2u| + C = \frac{1}{2}\log\left|(\cos u+\sin u)^2\right| + C = \frac{1}{2}\log|\cos u+\sin u| + C$

Question 21:

Obtain an integral (or anti – derivative) of the $\sin^{-1}(\cos u)$

Answer 21:

$\sin^{-1}(\cos u)$

Suppose, $\cos x = z$

Then, $\sin u = \sqrt{1-u^2}$

$(-\sin u)du = dz$

$du = -\frac{dz}{\sqrt{1-z^2}}$

Therefore, $\int \sin^{-1}(\cos u)du = \int \sin^{-1} z\left(-\frac{dz}{\sqrt{1-z^2}}\right)dz$

Suppose, $\sin^{-1} z = p$

$\frac{1}{\sqrt{1-z^2}}dz = dp$

Therefore, $\int \sin^{-1}(\cos u)du = -\int p\,dp = -\frac{p^2}{2} + C = -\frac{(\sin^{-1} z)^2}{2} + C = -\frac{(\sin^{-1}(\cos u))^2}{2} + C$ …..(1)

As we know, $\sin^{-1} u + \cos^{-1} u = \frac{\pi}{2}$

Therefore, $\sin^{-1}(\cos u) = \frac{\pi}{2} - \cos^{-1}(\cos u) = \left(\frac{\pi}{2} - u\right)$

On substituting in equation (1), we get,

$\int \sin^{-1}(\cos u)du = -\frac{\left[\frac{\pi}{2}-u\right]^2}{2} + C = -\frac{1}{2}\left(\left(\frac{\pi}{2}\right)^2 + u^2 - \pi u\right) + C = -\frac{(\pi)^2}{8} - \frac{u^2}{2} + \frac{1}{2}\pi u + C = \frac{\pi u}{2} - \frac{u^2}{2} + \left(C - \frac{(\pi)^2}{8}\right) = \frac{\pi u}{2} - \frac{u^2}{2} + C_1$

Question 22:

Obtain an integral (or anti – derivative) of the $\frac{1}{\cos(u-m)\cos(u-n)}$

Answer 22:

$$\frac{1}{\cos(u-m)\cos(u-n)}=\frac{1}{\sin(m-n)}\left[\frac{\sin(m-n)}{\cos(u-m)\cos(u-n)}\right]=\frac{1}{\sin(m-n)}\left[\frac{\sin[(u-n)-(u-m)]}{\cos(u-m)\cos(u-n)}\right]=\frac{1}{\sin(m-n)}\frac{\sin(u-n)\cos(u-m)-\cos(u-n)\sin(u-m)}{\cos(u-m)\cos(u-n)}=\frac{1}{\sin(m-n)}[\tan(u-n)-\tan(u-m)]$$

$$\int\frac{1}{\cos(u-m)\cos(u-n)}du=\frac{1}{\sin(m-n)}\int[\tan(u-n)-\tan(u-m)]du=\frac{1}{\sin(m-n)}[-\log|\cos(u-n)|+\log|\cos(u-m)|]=\frac{1}{\sin(m-n)}\log\left[\frac{\cos(u-m)}{\cos(u-n)}\right]+C$$

Question 23:

Which of the following below is the answer for $\frac{\sin^2 u-\cos^2 u}{\sin^2 u\cos^2 u}$

(a) tan u + cot u + C

(b) tan u + cosec u + C

(c) – tan u + cot u + C

(d) tan u + sec u + C

Answer 23:

$$\int\frac{\sin^2 u-\cos^2 u}{\sin^2 u\cos^2 u}du=\int\left(\frac{\sin^2 u}{\sin^2 u\cos^2 u}-\frac{\cos^2 u}{\sin^2 u\cos^2 u}\right)du=\int(\sec^2 u-\operatorname{cosec}^2 u)du=\tan u+\cot u+C$$

Thus, (a) is the correct answer.

Question 24:

Which of the following below is the answer for $\int\frac{e^u(1+u)}{\cos^2(e^u u)}du$

(a) – cot (e u^u) + C

(b) tan (e u^u) + C

(c) tan (e^u) + C

(d) cot (e^u) + C

Answer 24:

$$\int \frac{e^u(1+u)}{\cos^2(e^u u)}du \quad \text{Suppose, } e^u u = z \quad (e^u.u + e^u.1)du = dz \quad \int \frac{dz}{\cos^2 z} = \int \sec^2 z\,dz = \tan z + C = \tan(e^u.u) + C$$

Thus, (b) is the correct answer.

Exercise 7.4

Question 1:

Obtain an integral (or anti – derivative) of the $\frac{3u^2}{u^6+1}$

Answer 1:

$$\text{Suppose, } u^3 = z \quad 3u^2 du = dz \quad \int \frac{3u^2}{u^6+1}du = \int \frac{dz}{z^2+1} = \tan^{-1} z + C = \tan^{-1} u^3 + C$$

Question 2:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{1+4u^2}}$

Answer 2:

$$\text{Suppose, } 2u = z \quad 2du = dz \quad \int \frac{1}{\sqrt{1+4u^2}}du = \frac{1}{2}\int \frac{dz}{\sqrt{1+z^2}} = \frac{1}{2}\left[\log\left|z+\sqrt{1+z^2}\right|\right] + C = \frac{1}{2}\left[\log\left|2u+\sqrt{1+4u^2}\right|\right] + C$$

Question 3:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{(2-u)^2+1}}$

Answer 3:

Suppose, $2-u=z$, $-du=dz$, $\int \frac{1}{\sqrt{(2-u)^2+1}}du=-\int \frac{1}{\sqrt{z^2+1}}dz=-\left[\log\left|z+\sqrt{z^2+1}\right|\right]+C=-\left[\log\left|2-u+\sqrt{(2-u)^2+1}\right|\right]+C=\log\left|\frac{1}{(2-u)+\sqrt{u^2-4u+5}}\right|+C$

Question 4:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{9-25u^2}}$

Answer 4:

Suppose, 5u = z

5 du = dz

$\int \frac{1}{\sqrt{9-25u^2}}du=\frac{1}{5}\int \frac{1}{\sqrt{9-z^2}}dz=\frac{1}{5}\int \frac{1}{\sqrt{3^2-z^2}}dz=\frac{1}{5}\sin^{-1}\frac{z}{3}+C=\frac{1}{5}\sin^{-1}\frac{5u}{3}+C$

Question 5:

Obtain an integral (or anti – derivative) of the $\frac{3u}{1+2u^4}$

Answer 5:

Suppose, $\sqrt{2}u^2=z$, $2\sqrt{2}u\,du=dz$, $\int \frac{3u}{1+2u^4}du=\frac{3}{2\sqrt{2}}\int \frac{dz}{1+z^2}dz=\frac{3}{2\sqrt{2}}\left[\tan^{-1}z\right]+C=\frac{3}{2\sqrt{2}}\left[\tan^{-1}(\sqrt{2}u^2)\right]+C$

Question 6:

Obtain an integral (or anti – derivative) of the $\frac{u^2}{1-u^6}$

Answer 6:

Suppose, $u^3 = z$

$3u^2\,du = dz$

$\int \frac{u^2}{1-u^6}du=\frac{1}{3}\int \frac{dz}{1-z^2}=\frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+z}{1-z}\right|\right]+C=\frac{1}{6}\log\left|\frac{1+u^3}{1-u^3}\right|+C$

Question 7:

Obtain an integral (or anti – derivative) of the $\frac{u-1}{\sqrt{u^2-1}}$

Answer 7:

$\int \frac{u-1}{\sqrt{u^2-1}}du = \int \frac{u}{\sqrt{u^2-1}}du - \int \frac{1}{\sqrt{u^2-1}}du$ For, $\int \frac{u}{\sqrt{u^2-1}}du$ Suppose $u^2-1=z$, $2u\,du=dz$ Therefore, $\int \frac{u}{\sqrt{u^2-1}}du = \frac{1}{2}\int \frac{dz}{\sqrt{z}} = \frac{1}{2}\int z^{-\frac{1}{2}}dz = \frac{1}{2}\left[2z^{\frac{1}{2}}\right]+C = \sqrt{z}+C = \sqrt{u^2-1}+C$ From the above equation we get $\int \frac{u-1}{\sqrt{u^2-1}}du = \int \frac{u}{\sqrt{u^2-1}}du - \frac{1}{\sqrt{u^2-1}}du = \sqrt{u^2-1}+C-\log\left|u+\sqrt{u^2-1}\right|+C_1 = \sqrt{u^2-1}-\log\left|u+\sqrt{u^2-1}\right|+(C+C_1) = \sqrt{u^2-1}-\log\left|u+\sqrt{u^2-1}\right|+C_2$

Question 8:

Obtain an integral (or anti – derivative) of the $\frac{u^2}{\sqrt{u^6+m^6}}$

Answer 8:

Suppose, $u^3 = z$

$3u^2\,du = dz$

$\int \frac{u^2}{\sqrt{u^6+m^6}}du = \frac{1}{3}\int \frac{dz}{\sqrt{z^2+(m^3)^2}} = \frac{1}{3}\log\left|z+\sqrt{z^2+m^6}\right|+C = \frac{1}{3}\log\left|u^3+\sqrt{u^6+m^6}\right|+C$

Question 9:

Obtain an integral (or anti – derivative) of the $\frac{\sec^2 u}{\sqrt{\tan^2 u+4}}$

Answer 9:

Suppose, tan u = z

$\sec^2 u\,du = dz$

$\int \frac{\sec^2 u}{\sqrt{\tan^2 u+4}}du = \int \frac{dz}{\sqrt{z^2+2^2}} = \log\left|z+\sqrt{z^2+4}\right|+C = \log\left|\tan u+\sqrt{\tan^2 u+4}\right|+C$

Question 10:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{u^2+2u+2}}$

Answer 10:

$$\int \frac{1}{\sqrt{u^2+u+2}}du=\int \frac{1}{(u+1)^2+(1)^2} \text{Suppose}, u+1=z \; du=dz => \int \frac{1}{\sqrt{u^2+2u+2}}du=\int \frac{1}{\sqrt{z^2+1}}dz=\log\left|z+\sqrt{z^2+1}\right|+C=\log\left|(u+1)+\sqrt{(u+1)^2+1}\right|+C=\log\left|(u+1)+\sqrt{u^2+2u+1}\right|+C$$

Question 11:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{9u^2+6u+2}}$

Answer 11:

$$\int \frac{1}{\sqrt{9u^2+6u+2}}du=\int \frac{1}{(3u+1)^2+(2)^2} \text{Suppose}, 3u+1=z \; 3du=dz => \int \frac{1}{\sqrt{9u^2+6u+2}}du=\frac{1}{3}\int \frac{1}{\sqrt{t^2+2^2}}dz=\frac{1}{3}\left[\frac{1}{2}\tan^{-1}\left(\frac{z}{2}\right)\right]+C=\frac{1}{3}\left[\frac{1}{2}\tan^{-1}\left(\frac{3u+1}{2}\right)\right]+C$$

Question 12:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{7-6u-u^2}}$

Answer 12:

$7 - 6u - u^2$ = can also be written as $7 - (u^2 + 6u + 9 - 9)$

Therefore,

$7 - (u^2 + 6u + 9 - 9)$

$= 16 - (u^2 + 6u + 9)$

$= 16 - (u + 3)^2$

$= 4^2 - (u + 3)^2$

$$\int \frac{1}{\sqrt{7-6u-u^2}}du=\int \frac{1}{4^2-(u+3)^2}du \; \text{Suppose}, u+3=z \; du=dz \int \frac{1}{4^2-(u+3)^2}du=\int \frac{1}{4^2-(z)^2}dz=\sin^{-1}\left(\frac{z}{4}\right)+C=\sin^{-1}\left(\frac{u+3}{4}\right)+C$$

Question 13:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{(u-1)(u-2)}}$

Answer 13:

(u – 1) (u – 2) can be written as $u^2 - 3u + 2$

Therefore,

$u^2 - 3u + 2$

$= u^2 - 3u + \frac{9}{4} - \frac{9}{4} + 2 = \left(u - \frac{3}{2}\right)^2 - \frac{1}{4} = \left(u - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2$ $\int \frac{1}{\sqrt{(u-1)(u-2)}}du = \int \frac{1}{\sqrt{\left(u-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}du$ Suppose, $u - \frac{3}{2} = z$, $du = dz$ $\int \frac{1}{\sqrt{\left(u-\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}du = \int \frac{1}{\sqrt{z^2 - \left(\frac{1}{2}\right)^2}}dz = \log\left|z + \sqrt{z^2 - \left(\frac{1}{2}\right)^2}\right| + C = \log\left|\left(u - \frac{3}{2}\right) + \sqrt{\left(u - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + C = \log\left|\left(u - \frac{3}{2}\right) + \sqrt{u^2 - 3u + 2}\right| + C$

Question 14:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{8+3u-u^2}}$

Answer 14:

$\frac{1}{\sqrt{8+3u-u^2}}$ can also be written as $8 - \left(u^2 - 3u + \frac{9}{4} - \frac{9}{4}\right)$ Therefore, $\frac{41}{4} - \left(u - \frac{3}{2}\right)^2$ $\int \frac{1}{\sqrt{8+3u-u^2}}du = \int \frac{1}{\sqrt{\frac{41}{4} - \left(u-\frac{3}{2}\right)^2}}du$ Suppose $u - \frac{3}{2} = z$, $du = dz$ $\int \frac{1}{\sqrt{\frac{41}{4} - \left(u-\frac{3}{2}\right)^2}}du = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - z^2}}dz = \sin^{-1}\left(\frac{z}{\frac{\sqrt{41}}{2}}\right) + C = \sin^{-1}\left(\frac{u-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right) + C = \sin^{-1}\left(\frac{2u-3}{\sqrt{41}}\right) + C$

Question 15:

Obtain an integral (or anti – derivative) of the $\frac{1}{\sqrt{(u-m)(u-n)}}$

Answer 15:

$(u-m)(u-n)$ can also be written as $u^2 - (m+n)u + mn$ Therefore, $u^2 - (m+n)u + mn = u^2 - (m+n)u + \frac{(m+n)^2}{4} - \frac{(m+n)^2}{4} + mn = \left[u - \frac{(m+n)}{2}\right]^2 - \frac{(m-n)^2}{4}$ $\int \frac{1}{\sqrt{(u-m)(u-n)}}du = \int \frac{1}{\sqrt{\left\{u - \frac{(m+n)}{2}\right\}^2 - \left(\frac{(m-}{}\right.}}$

n)2)2√du Suppose,u–((m+n)2)=zdu=dz∫1{u–(m+n)2}2–((m–n)2)2√du=∫1z2–(m–n2)2√dz log||z+z2–(m–n2)2−−−−−−−−−√||+Clog||{u–(m+n2)}+(u–m)(u–n)−−−−−−−−−−√||+C

Question 16:

Obtain an integral (or anti – derivative) of the 4u+12u2+u–3√

Answer 16:

Suppose,4u+1=Addx(2u2+u–3)+B….(1)4u+1=A(4u+1)+B4u+1=4Au+A+B

Equate the coefficients of u and the constants on both the sides, we get

4 A = 4 => A = 1

A + B = 1 => B = 0

From (1), we get

Suppose, 2 u^2 + u – 3 = z

(4 u + 1) du = dz

∫4u+12u2+u–3√du=∫1z√dz=2z√+C=22u2+u–3−−−−−−−−√+C

Question 17:

Obtain an integral (or anti – derivative) of the u+2u2–1√

Answer 17:

Suppose,u+2=Addu(u2–1)+B….(1)u+2=A(2u)+B

Equate the coefficients of u and the constants on both the sides, we get

2 A = 1 => A = (1 / 2)

B = 2

From (1), we get

From(1),weget,(u+2)=12(2u)+2Then,∫u+2u2–1√du=∫12(2u)+2u2–1√=12∫2uu2–1√du+∫2u2–1√du…..(2) In12∫2uu2–1√du=12∫dzz√=12[2z√]+C=z√+C=u2–1−−−−√Then,∫2u2–1√du=2∫1u2–1√duInequation(2),weget,∫u+2u2–1√du=u2–1−−−−√+2log||u+u2–1−−−−√||+C

Question 18:

Obtain an integral (or anti – derivative) of the 5u–21+2u+3u2

Answer 18:

Suppose, 5u–2=Addu(1+2u+3u2)+B5u–2=A(2+6u)+B

Equate the coefficients of u and the constants on both the sides, we get

5=6A=>A=562A+B=–2=>B=–1135u–2=56(2+6u)+–113∫5u–21+2u+3u2du=∫56(2+6u)–1131+2u+3u2du=56∫2+6u1+2u+3u2du–113∫11+2u+3u2du Suppose,I1=∫2+6u1+2u+3u2duandI2=∫11+2u+3u2du∫5u–21+2u+3u2du=56I1–113I2….(1)I1=∫2+6u1+2u+3u2duandI2=∫11+2u+3u2duSuppose,1+2u+3u2=z(2+6u)du=dz I1=∫dzzI1=log|z|+CI1=log||1+2u+3u2||+C…..(2)I2=∫11+2u+3u2du 1+2u+3u2canalsobewrittenas1+3(u2+23u)Therefore,1+3(u2+23u)=1+3(u2+23u+19–19)=1+3(u+13)2–13=23+3(u+13)2=3[(u+13)2+29]=3[(u+13)2+(2√3)2] I2=13∫[1[(u+13)2+(2√3)2]]du=13[12√3tan−1(u+132√3)]=13[32√tan−1(3u+12√)]+C=12√tan−1(3u+12√)+C…..(3)

Substituting equations (2) and (3) in equation (1), we get,

∫5u–21+2u+3u2du=56∫2+6u1+2u+3u2du–113∫11+2u+3u2du∫5u–21+2u+3u2du=56[log||1+2u+3u2||]–113[12√tan−1(3u+12√)]+C=56log||1+2u+3u2]–1132√tan−1(3u+12√)+C

Question 19:

Obtain an integral (or anti – derivative) of the 6u+7(u–5)(u–4)√

Answer 19:

Suppose, 6u+7(u–5)(u–4)√=6u+7u2–9u+20√Suppose,6u+7=Addu(u2–9u+20)+B6u+7=A(2u–9)+B

Equate the coefficients of u and the constants on both the sides, we get,

2 A = 6 => A = 3

– 9 + B = 7 => B = 34

6 u + 7 = 3 (2 u – 9) + 34

∫6u+7u2–9u+20√du=∫3(2u–9)+34u2–9u+20√du=3∫2u–9u2–9u+20√du+34∫1u2–9u+20√du Suppose,I1=∫2u–9u2–9u+20√duandI2=∫1u2–9u+20√du∫6u+7u2–9u+20√du=3I1+34I2I1=∫2u–9u2–9u+20√duSuppose,u2–9u+20=z(2u–9)du=dzI1=dzz√I1=2z√I1=2u2–9u+20−−−−−−−−−−√andI2=∫1u2–9u+20√duu2–9u+20canalsobewrittenasu2–9u+20+814–814 Therefore,u2–9u+20+814–814=(u–92)2–14=(u–92)2–(12)2I2=∫1(u–92)2–(12)2√duI2=log||(u–92)+u2–9u+20−−−−−−−−−−√||+C…..(3)

Substituting equations (2) and (3) in equation (1), we get,

∫6u+7u2–9u+20√du=3[2u2–9u+20−−−−−−−−−−√]+34log[((u–92)+u2–9u+20−−−−−−−−−−√)]+C=6u2–9u+20−−−−−−−−−−√+34log[((u–92)+u2–9u+20−−−−−−−−−−√)]+C

Question 20:

Obtain an integral (or anti – derivative) of the u+24u–u2√

Answer 20:

Suppose, u+2=Addu(4u–u2)+B(u+2)=A(4–2u)+B

Equate the coefficients of u and the constants on both the sides, we get,

–2A=1=>A=–124A+B=2=>B=4(u+2)=–12(4–2u)+4∫u+24u–u2√du=∫–12(4–2u)+44u–u2√du=–12∫4–2u4u–u2√du+4∫14u–u2√du Suppose,I1=∫4–2u4u–u2√duandI2=∫14u–u2√du∫u+24u–u2√du=–12I1+4I2…..(1)Then,I1=∫4–2u4u–u2√duSuppose,4u–u2=z(4–2u)du=dzI1=dzz√=2z√=24u–u2−−−−−−√…..(2)I2=∫14u–u2√duSuppose,4u–u2=–(−4u+u2)(4–2u)du=–(−4u+u2+4–4)=4–(u–2)2I2=∫1(2)2–(u–2)2√du=sin–1(u–22)…..(3)

Substituting equations (2) and (3) in equation (1), we get,

∫u+24u–u2√du=–12(24u–u2−−−−−√)+4sin–1(u–22)+C=–4u–u2−−−−−√+4sin–1(u–22)+C

Question 21:

Obtain an integral (or anti – derivative) of the u+24u2+2u+3√

Answer 21:

∫u+2u2+2u+3√du=12∫2(u+2)u2+2u+3√du=12∫2u+4u2+2u+3√du=12∫2u+2u2+2u+3√du+12∫2u2+2u+3√du=12∫2u+2u2+2u+3√du+∫1u2+2u+3√duSuppose,I1=∫2u+2u2+2u+3√duandI2=∫1u2+2u+3√du∫u+2u2+2u+3√du=12I1+I2 I1=∫2u+2u2+2u+3√duSuppose,u2+2u+3=z(2u+2)du=dzI1=∫dzz√=2z√=2u2+2u+3−−−−−−−−−√…..(2)I2=∫1u2+2u+3√duu2+2u+3=u2+2u+1+2=(u+1)2+(2–√)2I2=∫1(u+1)2+(2√)2√du=log||(u+1)+u2+2u+3−−−−−−−−−√||…..(3)

Substituting equations (2) and (3) in equation (1), we get,

∫u+2u2+2u+3√du=12(2u2+2u+3−−−−−−−−−√)+log||(u+1)+u2+2u+3−−−−−−−−−√||+C=u2+2u+3−−−−−−−−−√+log||(u+1)+u2+2u+3−−−−−−−−−√||+C

Question 22:

Obtain an integral (or anti – derivative) of the u+2u2–2u–5√

Answer 22:

Suppose,(u+3)=Addu(u2–2u–5)(u+3)=A(2u–2)+B

Equate the coefficients of u and the constants on both the sides, we get,

2A=1=>A=12–2A+B=3=>B=4(u+3)=12(2u–2)+4∫u+3u2–2u–5√du=∫12(2u–2)+4u2–2u–5du=12∫(2u–2)+4u2–2u–5du+4∫1u2–2u–5du Suppose,I1=∫(2u–2)+4u2–2u–5duandI2=∫1u2–2u–5du∫u+3u2–2u–5√du=12I1+4I2….(1)Then,I1=∫(2u–2)+4u2–2u–5duSuppose,u2–2u–5=Z(2u–2)du=dzI1=∫dzz=log|z|+C=log||u2–2u–5||+C….(2) I2=∫1u2–2u–5du=∫1(u2–2u+1)–6du=∫1(u–1)2+(6√)2du=126√log(u–1–6√u–1+6√)……(3)

Using equations (2) and (3) in equation (1), we get,

$\int \frac{u+3}{\sqrt{u^2-2u-5}} du = \frac{1}{2}\log\left|u^2-2u-5\right| + 4\left[\frac{1}{2\sqrt{6}}\log\left(\frac{u-1-\sqrt{6}}{u-1+\sqrt{6}}\right)\right] + C = \frac{1}{2}\log\left|u^2-2u-5\right| + \frac{2}{\sqrt{6}}\log\left(\frac{u-1-\sqrt{6}}{u-1+\sqrt{6}}\right)$

Question 23:

Obtain an integral (or anti – derivative) of the $\frac{5u+3}{\sqrt{u^2+4u+10}}$

Answer 23:

$$\text{Suppose, } 5u+3 = A\frac{d}{du}(u^2+4u+10)+B$$
$$5u+3 = A(2u+4)+B$$

Equate the coefficients of u and the constants on both the sides, we get,

$$2A=5 \Rightarrow A=\frac{5}{2}$$
$$4A+B=3 \Rightarrow B=-7$$
$$5u+3=\frac{5}{2}(2u+4)-7$$
$$\int \frac{5u+3}{\sqrt{u^2+4u+10}}du = \int \frac{\frac{5}{2}(2u+4)-7}{\sqrt{u^2+4u+10}}du = \frac{5}{2}\int \frac{2u+4}{\sqrt{u^2+4u+10}}du - 7\int \frac{1}{\sqrt{u^2+4u+10}}du$$
$$\text{Suppose, } I_1=\int \frac{2u+4}{\sqrt{u^2+4u+10}}du \text{ and } I_2=\int \frac{1}{\sqrt{u^2+4u+10}}du$$
$$\int \frac{5u+3}{\sqrt{u^2+4u+10}}du = \frac{5}{2}I_1 - 7I_2 \quad \ldots..(1)$$
$$I_1=\int \frac{2u+4}{\sqrt{u^2+4u+10}}du$$
$$\text{Suppose, } u^2+4u+10=z$$
$$(2u+4)du=dz$$
$$I_1=\int \frac{dz}{\sqrt{z}} = 2\sqrt{z} = 2\sqrt{u^2+4u+10} \quad \ldots..(2)$$
$$I_2=\int \frac{1}{\sqrt{u^2+4u+10}}du = \int \frac{1}{\sqrt{(u^2+4u+4)+6}}du = \int \frac{1}{\sqrt{(u+2)^2+(\sqrt{6})^2}}du = \log\left|(u+2)\sqrt{u^2+4u+10}\right| \quad \ldots..(3)$$

Substituting equations (2) and (3) in equation (1), we get,

$$\int \frac{5u+3}{\sqrt{u^2+4u+10}}du = \frac{5}{2}\left[2\sqrt{u^2+4u+10}\right] - 7\log\left|(u+2)+\sqrt{u^2+4u+10}\right| + C = 5\sqrt{u^2+4u+10} - 7\log\left|(u+2)+\sqrt{u^2+4u+10}\right| + C$$

Question 24: Which of the following below is the answer for $\int \frac{du}{u^2+2u+2}du$

$$(a)\ u\tan^{-1}(u+1)+C \quad (b)\ \tan^{-1}(u+1)+C \quad (c)\ (u+1)\tan^{-1}(u)+C \quad (d)\ \tan^{-1}(u)+C$$

Answer 24:

$$\int \frac{du}{u^2+2u+2}du = \int \frac{du}{(u^2+2u+1)+1} = \int \frac{1}{(u+1)^2+(1)^2}du = \left[\tan^{-1}(u+1)\right]+C$$

Thus, (b) is the correct answer.

Question 25: Which of the following below is the answer for ∫du9u–4u2√du
(a)19sin−19u–88+C(b)12sin−18u–99+C(c)13sin−19u–88+C(d)12sin−19u–88+C

Answer 25:

∫du9u–4u2√du=∫du−4(u2–94u)√=∫du−4(u2–94u+8164–8164)√=∫1−4√[(u–98)2–(98)2]du=12∫1(98)2–(u–98)2√du=12[sin−1(u–9898)]+C=12sin−1(8u–99)+C

Thus, (b) is the correct answer.

Exercise 7.4

Question 1:

Obtain an integral (or anti – derivative) of the following rational number u(u+1)(u+2)
Answer 1:

Suppose, u(u+1)(u+2)=Au+1+Bu+2=>u=A(u+2)+B(u+1)
Equate the coefficients of u and the constants on both the sides, we get,

A + B = 1

2 A + B = 0

On solving, we get,

A = – 1 and B = 2

u(u+1)(u+2)=−1u+1+2u+2=>∫u(u+1)(u+2)du=−1u+1+2u+2du=–log|u+1|+2log|u+2|+C=log(u+2)2–log|u+1|+C=log(u+1)2(u+1)+C

Question 2:

Obtain an integral (or anti – derivative) of the following rational number $\frac{1}{u^2-9}$

Answer 2:

Suppose, $\frac{1}{(u+3)(u-3)} = \frac{A}{u+3} + \frac{B}{u-3}$ $1 = A(u-3) + B(u+3)$

Equate the coefficients of u and the constants on both the sides, we get,

A + B = 0

– 3 A + 3 B = 1

On solving, we get

$A = -\frac{1}{6} \text{ and } B = \frac{1}{6}$ $\frac{1}{(u+3)(u-3)} = \frac{-1}{6(u+3)} + \frac{1}{6(u-3)} \Rightarrow \int \frac{1}{u^2-9} du = \int \left(\frac{-1}{6(u+3)} + \frac{1}{6(u-3)} \right) du = -\frac{1}{6} \log|u+3| + \frac{1}{6} \log|u-3| + C = \frac{1}{6} \log\left| \frac{(u-3)}{(u+3)} \right| + C$

Question 3:

Obtain an integral (or anti – derivative) of the following rational number $\frac{3u-1}{(u-1)(u-2)(u-3)}$

Answer 3:

Suppose, $\frac{3u-1}{(u-1)(u-2)(u-3)} = \frac{A}{(u-1)} + \frac{B}{(u-2)} + \frac{C}{(u-3)}$ $3u - 1 = A(u-2)(u-3) + B(u-1)(u-3) + C(u-1)(u-2) \ldots..(1)$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B + C = 0

– 5 A – 4 B – 3 C = 1

6 A + 3 B + 2 C = – 1

On solving, we get,

A = 1, B = – 5, and C = 4

$\frac{3u-1}{(u-1)(u-2)(u-3)} = \frac{1}{(u-1)} - \frac{5}{(u-2)} + \frac{4}{(u-3)}$ $\int \frac{3u-1}{(u-1)(u-2)(u-3)} du = \int \left\{ \frac{1}{(u-1)} - \frac{5}{(u-2)} + \frac{4}{(u-3)} \right\} du = \log|u-1| - 5\log|u-2| + 4\log|u-3| + C$

Question 4:

Obtain an integral (or anti – derivative) of the following rational number $u(u–1)(u–2)(u–3)$

Answer 4:

Suppose, $u(u–1)(u–2)(u–3)=A(u–1)+B(u–2+C(u–3)u=A(u–2)(u–3)+B(u–1)(u–3)+C(u–1)(u–2)\ldots..(1)$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B + C = 0

– 5 A – 4 B – 3 C = 1

6 A + 3 B + 2 C = 0

On solving, we get,

$A=12,B=–2andC=32u(u–1)(u–2)(u–3)=12(u–1)–2(u–2+32(u–3)\int u(u–1)(u–2)(u–3)du=\int\{12(u–1)–2(u–2)+32(u–3)\}du=12\log|u–1|–2\log|u–2|+32\log|u–3|+C$

Question 5:

Obtain an integral (or anti – derivative) of the following rational number $2uu2+3u+2$

Answer 5:

Suppose, $2uu2+3u+2=A(u+1)+B(u+2)2u=A(u+2)+B(u+1)\ldots.(1)$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B = 2

2 A + B = 0

On solving, we get,

A = – 2 and B = 4

$2u(u+1)(u+2)=–2(u+1)+4(u+1)\int 2u(u+1)(u+2)du=\int\{4(u+1)–2(u+1)\}du=4\log|u+2|–2\log|u+1|+C$

Question 6:

Obtain an integral (or anti – derivative) of the following rational number 1–u2u(1–2u)

Answer 6:

1–u2u(1–2u) is not a proper fraction.

Dividing (1 – u ^{2}) by u (1 – 2 u), we get,

1–u2u(1–2u)=12+12(2–uu(1–2u))Suppose,2–uu(1–2u)=Au+B(1–2u)(2–u)=A(1–2u)+Bu…….(1)

Equate the coefficients of u^{2}, u and the constants on both the sides, we get,

– 2 A + B = – 1

And A = 2

On solving, we get,

A = 2 and B = 3

2–uu(1–2u)=2u+3(1–2u)Usinginequation(1),weget,1–u2u(1–2u)=12+12{2u+3(1–2u)}=u2+log|u|+32(–2)log|1–2u|+C=u2+log|u|–34log|1–2u|+C

Question 7:

Obtain an integral (or anti – derivative) of the following rational number u(u2+1)(u–1)

Answer 7:

Suppose, u(u2+1)(u–1)=Au+B(u2+1)+Cu–1u=(Au+B)(u–1)+C(u2+1)u=Au2–Au+Bu–B+Cu2+C

Equate the coefficients of u^{2}, u and the constants on both the sides, we get,

A + C = 0

– A + B = 1

– B + C = 0

On solving, we get,

A=–12,B=12,andC=12Usingequation(1),wegetu(u2+1)(u–1)=(–12u+12)(u2+1)+12(u–1) ∫u(u2+1)(u–1)du=–12∫u(u2+1)du+12∫1(u2+1)+12∫12∫1u–1du=–14∫2u(u2+1)+12tan–1u+12log|u–1|+C∫2u(u2+1)du,let(u2+1)=z=>2udu=dz∫2u(u2+1)du=∫dzz=log|z|=log||u2+1||∫u(u2+1)(u–

$1)du=-\frac{1}{4}\log|(u^2+1)|+\frac{1}{2}\tan^{-1}u+\frac{1}{2}\log|u-1|+C=\frac{1}{2}\log|u-1|-\frac{1}{4}\log|(u^2+1)|+\frac{1}{2}\tan^{-1}u+C$

Question 8:

Obtain an integral (or anti – derivative) of the following rational number $\frac{u}{(u-1)^2(u+2)}$

Answer 8:

$\frac{u}{(u-1)^2(u+2)}=\frac{A}{(u-1)}+\frac{B}{(u-1)^2}+\frac{C}{u+2}$ $u=A(u-1)(u+2)+B(u+2)+C(u-1)^2$

Putting u = 1, we get,

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + C = 0

A + B – 2 C = 1

– 2 A + 2 B + C = 0

On solving, we get,

$A=\frac{2}{9}, B=\frac{1}{3}$ and $C=-\frac{2}{9}$ $\frac{u}{(u-1)^2(u+2)}=\frac{2}{9(u-1)}+\frac{1}{3(u-1)^2}-\frac{2}{9(u+2)}$ $\int\frac{u}{(u-1)^2(u+2)}du=\frac{2}{9}\int\frac{1}{(u-1)}du+\frac{1}{3}\int\frac{1}{(u-1)^2}du-\frac{2}{9}\int\frac{1}{(u+2)}du=\frac{2}{9}\log|u-1|+\frac{1}{3}\left(\frac{-1}{u-1}\right)-\frac{2}{9}\log|u+2|+C=\frac{2}{9}\log\left|\frac{u-1}{u+2}\right|-\frac{1}{3(u-1)}+C$

Question 9:

Obtain an integral (or anti – derivative) of the following rational number $\frac{3u+5}{u^3-u^2-u+1}$

Answer 9:

$\frac{3u+5}{u^3-u^2-u+1}=\frac{3u+5}{(u-1)^2(u+1)}$ Suppose, $\frac{3u+5}{(u-1)^2(u+1)}=\frac{A}{(u-1)}+\frac{B}{(u-1)^2}+\frac{C}{(u+1)}$ $3u+5=A(u-1)(u+1)+B(u+1)+C(u-1)^2$ $3u+5=A(u^2-1)+B(u+1)+C(u^2+1-2u)$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + C = 0

B – 2 C = 3

– A + B + C = 5

On solving, we get,

B = 4

$$A=-\frac{1}{2} \text{ and } C=\frac{1}{2}$$
$$\frac{3u+5}{(u-1)^2(u+1)}=-\frac{1}{2(u-1)}+\frac{4}{(u-1)^2}+\frac{1}{2(u+1)}$$
$$\int\frac{3u+5}{(u-1)^2(u+1)}du=-\frac{1}{2}\int\frac{1}{(u-1)}du+4\int\frac{1}{(u-1)^2}du+\frac{1}{2}\int\frac{1}{(u+1)}du=-\frac{1}{2}\log|u-1|+4\left(-\frac{1}{u-1}\right)+\frac{1}{2}\log|u+1|+C=\frac{1}{2}\log\left|\frac{u+1}{u-1}\right|-\frac{4}{(u-1)}+C$$

Question 10:

Obtain an integral (or anti – derivative) of the following rational number $\frac{2u-3}{(u^2-1)(2u+3)}$

Answer 10:

$$\frac{2u-3}{(u^2-1)(2u+3)}=\frac{2u-3}{(u+1)(u-1)(2u+3)}$$

Suppose, $\frac{2u-3}{(u^2-1)(2u+3)}=\frac{A}{(u+1)}+\frac{B}{(u-1)}+\frac{C}{(2u+3)}$

$$(2u-3)=A(u-1)(2u+3)+B(u+1)(2u+3)+C(u+1)(u-1)$$
$$(2u-3)=A(2u^2+u-3)+B(2u^2+5u+3)+C(u^2-1)$$
$$(2u-3)=(2A+2B+C)u^2+(A+5B)u+(-3A+3B-C)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

2A + 2B + C = 1

A + 5B = 2

– 3A + 3B – C = – 3

On solving, we get,

$$\frac{2u-3}{(u+1)(u-1)(2u+3)}=\frac{5}{2(u+1)}-\frac{1}{10(u-1)}-\frac{24}{5(2u+3)}$$
$$\int\frac{2u-3}{(u+1)(u-1)(2u+3)}=\frac{5}{2}\int\frac{1}{(u+1)}du-\frac{1}{10}\int\frac{1}{(u-1)}du-\frac{24}{5}\int\frac{1}{(2u+3)}du=\frac{5}{2}\log|u+1|-\frac{1}{10}\log|u-1|-\frac{24}{5\times2}\log|2u+3|+C=\frac{5}{2}\log|u+1|-\frac{1}{10}\log|u-1|-\frac{12}{5}\log|2u+3|+C$$

Question 11:

Obtain an integral (or anti – derivative) of the following rational number $\frac{5u}{(u+1)(u^2-4)}$

Answer 11:

$$\frac{5u}{(u+1)(u^2-4)}=\frac{5u}{(u+1)(u+2)(u-2)}$$ Suppose, $$\frac{5u}{(u+1)(u+2)(u-2)}=\frac{A}{(u+1)}+\frac{B}{(u+2)}+\frac{C}{(u-2)}$$ $$5u=A(u+2)(u-2)+B(u+1)(u-2)+C(u+1)(u+2)\ldots.(1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B + C = 0

– B + 3C = 5 and

– 4A – 2B + 2C = 0

On solving, we get,

$$A=\frac{5}{3}, B=-\frac{5}{2} \text{ and } C=\frac{5}{6}$$ $$\frac{5u}{(u+1)(u+2)(u-2)}=\frac{5}{3(u+1)}-\frac{5}{2(u+2)}+\frac{5}{6(u-2)}$$ $$\int\frac{5u}{(u+1)(u+2)(u-2)}du=\frac{5}{3}\int\frac{1}{(u+1)}du-\frac{5}{2}\int\frac{1}{(u+2)}du+\frac{5}{6}\int\frac{1}{(u-2)}du=\frac{5}{3}\log|u+1|-\frac{5}{2}\log|u+2|+\frac{5}{6}\log|u-2|+C$$

Question 12:

Obtain an integral (or anti – derivative) of the following rational number $\frac{u^3+u+1}{u^2-1}$

Answer 12:

$\frac{u^3+u+1}{u^2-1}$ is not a proper fraction.
So, dividing (u^3 + u + 1) by u^2 – 1, we get,

$$\frac{u^2+u+1}{u^2-1}=u+\frac{2u+1}{u^2-2}$$ Suppose, $$\frac{2u+1}{u^2-2}=\frac{A}{(u+1)}+\frac{B}{(u-1)}$$ $$2u+1=A(u-1)+B(u+1)\ldots..(1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B = 2

– A + B = 1

On solving, we get,

$$A=\frac{1}{2} \text{ and } B=\frac{3}{2}$$ $$\frac{u^2+u+1}{u^2-1}=u+\frac{1}{2(u+1)}+\frac{3}{2(u-1)}$$ Integrating on both the sides, we get, $$\int\frac{u^2+u+1}{u^2-1}du=\int u\,du+\frac{1}{2}\int\frac{1}{u+1}du+\frac{3}{2}\int\frac{1}{(u-1)}du=\frac{u^2}{2}+\frac{1}{2}\log|u+1|-\frac{3}{2}\log|u-1|+C$$

Question 13:

Obtain an integral (or anti – derivative) of the following rational number $\frac{2}{(1-u)(1+u^2)}$

Answer 13:

Suppose, $\frac{2}{(1-u)(1+u^2)}=\frac{A}{1-u}+\frac{Bu+C}{1+u^2}$ $2=A(1+u^2)+(Bu+C)(1-u)$ $2=A+Au^2+Bu-Bu^2+C-Cu$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A – B = 0

B – C = 0

A + C = 2

On solving, we get,

A = 1, B = 1 and C = 1

$\frac{2}{(1-u)(1+u^2)}=\frac{1}{1-u}+\frac{u+1}{1+u^2}$ $\int\frac{2}{(1-u)(1+u^2)}du=\int\frac{1}{1-u}du+\int\int\frac{u}{1+u^2}du+\int\frac{1}{1+u^2}du=-\log|u-1|+\frac{1}{2}\log\left|1+u^2\right|+\tan^{-1}u+C$

Question 14:

Obtain an integral (or anti – derivative) of the following rational number $\frac{3u-1}{(u+2)^2}$

Answer 14:

Suppose, $\frac{3u-1}{(u+2)^2}=\frac{A}{(u+2)}+\frac{B}{(u+2)^2}$ $3u-1=A(u+2)+B$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A = 3

2A + B = – 1

B = – 7

$\frac{3u-1}{(u+2)^2}=\frac{3}{(u+2)}-\frac{7}{(u+2)^2}$ $3u-1=A(u+2)+B$ $\frac{3u-1}{(u+2)^2}=3\int\frac{1}{(u+2)}du-7\int\frac{u}{(u+2)^2}du=3\log|u+2|-7\left(\frac{-1}{(u+2)}\right)+C=3\log|u+2|+\left(\frac{7}{(u+2)}\right)+C$

Question 15:

Obtain an integral (or anti – derivative) of the following rational number 1u4–1

Answer 15:

1u4–1=1(u2–1)(u4+1)=1(u+1)(u–1)(1+u2)Suppose,1(u+1)(u–1)(1+u2)=A(u+1)+B(u–1)+Cu+D(u2+1)1=A(u+1)(u2+1)+B(u+1)(u2+1)+(Cu+D)(u2–1)1=A(u3+u–u2–1)+B(u3+u+u2+1)+Cu2+Du2–Cu–D1=(A+B+C)u3+(−A+B+D)u2+(A+B–C)u+(−A+B–D)

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B + C = 0

– A + B + D = 0

A + B – C = 0

– A + B – D = 1

On solving, we get,

A=–14,B=14,C=0andD=–121u4–1=–14(u+1)+14(u–1)–12(u2+1)∫1u4–1du=–14∫1(u+1)du+14∫1(u–1)du+12∫1(u2+1)=–14log|u+1|+14log|u–1|–12tan–1u+C=14log||u–1u+1||–12tan–1u+C

Question 16:

Obtain an integral (or anti – derivative) of the following rational number 1u(um+1)
[Hint: multiply denominator and numerator by u^{n-1} and put u^n = z]

Answer 16:

1u(um+1)

Multiplying denominator and numerator by u^{n-1}, we get,

1u(um+1)=um–1um–1.u(um+1)=um–1um(um+1)Suppose,um=z=>um–1du=dz∫1u(um+1)du=∫um–1um(um+1)du=1m∫1z(z+z)duSuppose,1z(z+z)=Az+B(z+1)1=A(1+z)+Bz

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A = 1 and B = – 1

$1z(z+z)=1z–1(z+1)∫1u(um+1)du=1m∫\{1z–1(z+1)\}+C=1m[log|um|–log|um+1|]+C=1mlog||umum+1||$

Question 17:

Obtain an integral (or anti – derivative) of the following rational number $cosu(1–sinu)(2–sinu)$
[Hint: Put sin u = z]

Answer 17:

$cosu(1–sinu)(2–sinu)$

Suppose, sin u = z => cos u du = dz

$∫cosu(1–sinu)(2–sinu)du=∫dz(1–z)(2–z)$Suppose,$1(1–z)(2–z)=A(1–z)+B(2–z)1=A(2–z)+B(1–z)$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

– 2A – B = 0

2A + B = 1

On solving, we get,

A = 1 and B = – 1

$1(1–z)(2–z)=1(1–z)–1(2–z)∫cosu(1–sinu)(2–sinu)du=∫\{1(1–z)–1(2–z)\}dz=–log|1–z|+log|2–z|+C=log||2–z1–z||+C=log||2–sinu1–sinu||+C$

Question 18:

Obtain an integral (or anti – derivative) of the following rational number $(u2+1)(u2+2)(u2+3)(u2+4)$

Answer 18:

$(u2+1)(u2+2)(u2+3)(u2+4)=1–(4u2+10)(u2+3)(u2+4)$Suppose,$(4u2+10)(u2+3)(u2+4)=Au+B(u2+3)+Cu+D(u2+4)(4u2+10)=(Au+B)(u2+4)+(Cu+D)(u2+3)(4u2+10)=Au3+4Au+Bu2+4B+Cu3+3Cu+Du2+3D(4u2+10)=(A+C)u3+(B+D)u2+(4A+3C)u+(4B+3D)$

Equate the coefficients of u^3, u^2, u and the constants on both the sides, we get,

A + C = 0

B + D = 4

4 A + 3 C = 0

4 B + 3 D = 10

On solving, we get,

A = 0, B = – 2, C = 0 and D = 6

$$\frac{(4u^2+10)}{(u^2+3)(u^2+4)} = \frac{-2}{(u^2+3)} + \frac{6}{(u^2+4)}$$
$$\frac{(u^2+1)(u^2+2)}{(u^2+3)(u^2+4)} = 1 - \left(\frac{-2}{(u^2+3)} + \frac{6}{(u^2+4)}\right)$$
$$\int \frac{(u^2+1)(u^2+2)}{(u^2+3)(u^2+4)}du = \int\left\{1 + \frac{2}{(u^2+3)} + \frac{6}{(u^2+4)}\right\} = \int\left\{1 + \frac{2}{u^2+(\sqrt{3})^2} - \frac{6}{u^2+2^2}\right\}du = u + 2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{u}{\sqrt{3}}\right) - 6\left(\frac{1}{2}\tan^{-1}\frac{u}{2}\right) + C = u + \frac{2}{\sqrt{3}}\tan^{-1}\frac{u}{\sqrt{3}} - 3\tan^{-1}\frac{u}{2} + C$$

Question 19:

Obtain an integral (or anti – derivative) of the following rational number $\frac{2u}{(u^2+1)(u^2+3)}$

Answer 19:

$$\frac{2u}{(u^2+1)(u^2+3)}$$

Suppose, u^2 = z

2u du = dz

$$\int \frac{2u}{(u^2+1)(u^2+3)}du = \int \frac{dz}{(z+1)(z+3)} \ldots.(1)$$
Suppose, $$\frac{1}{(z+1)(z+3)} = \frac{A}{(z+1)} + \frac{B}{(z+3)}$$
$$1 = A(z+3) + B(z+1) \ldots.(1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B = 0

3 A + B = 1

On solving, we get,

$$A = \frac{1}{2} \text{ and } B = -\frac{1}{2}$$
$$\frac{1}{(z+1)(z+3)} = \frac{1}{2(z+1)} - \frac{1}{2(z+3)}$$
$$\int \frac{2u}{(u^2+1)(u^2+3)}du = \int\left\{\frac{1}{2(z+1)} - \frac{1}{2(z+3)}\right\}dz = \frac{1}{2}\log|(z+1)| - \frac{1}{2}\log|(z+3)| + C = \frac{1}{2}\log\left|\frac{z+1}{z+3}\right| + C = \frac{1}{2}\log\left|\frac{u^2+1}{u^2+3}\right| + C$$

Question 20:

Obtain an integral (or anti – derivative) of the following rational number $\frac{1}{u(u^4-1)}$

Answer 20:

$\frac{1}{u(u^4-1)}$

Multiplying denominator and numerator by u^3, we get,

$$\frac{1}{u(u^4-1)} = \frac{u^3}{u^4(u^4-1)}$$
$$\int \frac{1}{u(u^4-1)}du = \int \frac{u^3}{u^4(u^4-1)}du$$
Suppose, $u^4 = t \Rightarrow 4u^3 du = dz$
$$\int \frac{1}{u(u^4-1)}du = \frac{1}{4}\int \frac{dz}{z(z-1)}$$
Suppose, $\frac{1}{z(z-1)} = \frac{A}{z} + \frac{B}{z-1}$
$$1 = A(z-1) + Bz \quad \ldots..(1)$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A = – 1 and B = 1

$$\frac{1}{z(z-1)} = \frac{-1}{z} + \frac{1}{z-1}$$
$$\int \frac{1}{u(u^4-1)}du = \frac{1}{4}\int\left(\frac{-1}{z} + \frac{1}{z-1}\right)dz = \frac{1}{4}\left[-\log|z| + \log|z-1|\right] + C = \frac{1}{4}\log\left|\frac{z-1}{z}\right| + C = \frac{1}{4}\log\left|\frac{u^4-1}{u^4}\right| + C$$

Question 21:

Obtain an integral (or anti – derivative) of the following rational number $\frac{1}{(e^u-1)}$

Answer 21:

$\frac{1}{(e^u-1)}$

Suppose, $e^u = z$

$e^u\, du = dz$

$$\int \frac{1}{(e^u-1)}du = \int \frac{1}{z-1} \times \frac{dz}{z} = \int \frac{1}{z(z-1)}dz$$
Suppose, $\frac{1}{z(z-1)} = \frac{A}{z} + \frac{B}{z-1}$
$$1 = A(z-1) + Bz$$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A = – 1 and B = 1

$$\frac{1}{z(z-1)} = \frac{-1}{z} + \frac{1}{z-1}$$
$$\int \frac{1}{z(z-1)}du = \log\left|\frac{z-1}{z}\right| + C = \log\left|\frac{e^u-1}{e^u}\right| + C$$

Question 22: Which of the following below is an integral of $\frac{u\,du}{(u-1)(u-2)}$

$(a)\log\left|\frac{(u-1)^2}{u-2}\right|+C \quad (b)\log\left|\frac{(u-2)^2}{u-2}\right|+C \quad (c)\log\left|\left(\frac{u-1}{u-2}\right)^2\right|+C \quad (d)\log|(u-1)(u-2)|+C$

Answer 22:

Suppose, $\frac{u\,du}{(u-1)(u-2)}=\frac{A}{(u-1)}+\frac{B}{u-2}$

$u=A(u-2)+B(u-1)\ldots.(1)$

Equate the coefficients of u and the constants on both the sides, we get,

A = – 1 and B = 2

$\frac{u\,du}{(u-1)(u-2)}=\frac{-1}{(u-1)}+\frac{2}{u-2}$

$\int\frac{u\,du}{(u-1)(u-2)}d=\left\{\frac{-1}{(u-1)}+\frac{2}{u-2}\right\}du=-\log|u-1|+2\log|u-2|+C=\log\left|\frac{(u-2)^2}{u-1}\right|+C$

Hence, option (b) is the correct answer.

Question 23: Which of the following below is an integral of $\int\frac{du}{u(u^2+1)}du$

$(a)\log|u|-\frac{1}{2}\log(u^2+1)+C \quad (b)\log|u|+\frac{1}{2}\log(u^2+1)+C \quad (c)-\log|u|+\frac{1}{2}\log(u^2+1)+C \quad (d)\log|u|+\frac{1}{2}\log(u^2+1)+C$

Answer 23:

Suppose, $\frac{1}{u(u^2+1)}=\frac{A}{u}+\frac{Bu+C}{u^2+1}$

$1=A(u^2+1)+(Bu+C)u$

Equate the coefficients of u^2, u and the constants on both the sides, we get,

A + B = 0

C = 0

A = 1

On solving, we get,

A =1, B = – 1, and C = 0

$\frac{1}{u(u^2+1)}=\frac{1}{u}+\frac{-U}{u^2+1}$

$\int\frac{1}{u(u^2+1)}du=\int\left\{\frac{1}{u}-\frac{u}{u^2+1}\right\}du=\log|u|-\frac{1}{2}\log\left|u^2+1\right|+C$

Hence, option (a) is the correct answer.

Exercise 7.6

Question 1:

Obtain an integral of u sin u.

Answer 1:

Suppose, I = $\int u \sin u\,du$

Integrating the equation by parts by taking u as first function and sin u as second function, we get,

$$I = u\int \sin u\,du - \int\left\{\left(\frac{d}{du}u\right)\int \sin u\,du\right\}du = u(-\cos u) - \int 1.(-\cos u)du = -u\cos u + \sin u + C$$

Question 2:

Obtain an integral of u sin 3u.

Answer 2:

Suppose, I = $\int u \sin u\,du$

Integrating the equation by parts by taking u as first function and sin 3u as second function, we get,

$$I = u\int \sin 3u\,du - \int\left\{\left(\frac{d}{du}u\right)\int \sin 3u\,du\right\}du = u\left(\frac{-\cos 3u}{3}\right) - \int 1.\left(\frac{-\cos 3u}{3}\right)du = \frac{-u\cos 3u}{3} + \frac{1}{9}\sin 3u + C$$

Question 3:

Obtain an integral of $u^2.e^u$

Answer 3:

Suppose, I = $\int u^2.e^u du$

Integrating the equation by parts by taking u^2 as first function and e^u as second function, we get,

$I=u^2\int e^u du - \int\{(\frac{d}{du}u^2)\int e^u du\}du = u^2e^u - \int 2ue^u du = u^2e^u - 2\int ue^u du$ Integrating by parts, we get, $= u^2e^u - 2[u\int e^u du - \int\{(\frac{d}{du}u).\int e^u du\}]du = u^2e^u - 2[ue^u - \int e^u du] = u^2e^u - 2[ue^u - e^u] = u^2e^u - 2ue^u - 2e^u + C = e^u(u^2 - 2u + 2) + C$

Question 4:

Obtain an integral of u log u.

Answer 4:

Suppose, I = $\int u \log u du$

Integrating the equation by parts by taking log u as first function and u as second function, we get,

$I = \log u\int u du - \int\{(\frac{d}{du}\log u)\int u du\}du = \log u.\frac{u^2}{2} - \int\frac{1}{u}.\frac{u^2}{2}du = \frac{u^2\log u}{2} - \int\frac{u}{2}du = \frac{u^2\log u}{2} = \frac{u^2}{4} + C$

Question 5:

Obtain an integral of u log 2u.

Answer 5:

Suppose, I = $\int u \log 2u du$

Integrating the equation by parts by taking log 2 u as first function and u as second function, we get,

$I = \log 2u\int u du - \int\{(\frac{d}{du}2\log u)\int u du\}du = \log 2u.\frac{u^2}{2} - \int\frac{2}{2u}.\frac{u^2}{2}du = \frac{u^2\log 2u}{2} - \int\frac{u}{2}du = \frac{u^2\log 2u}{2} - \frac{u^2}{4} + C$

Question 6:

Obtain an integral of u^2 log u.

Answer 6:

Suppose, I = $\int u^2 \log u\, du$

Integrating the equation by parts by taking log u as first function and u^2 as second function, we get,

$$I=\log u\int u^2 du-\int\left\{\left(\frac{d}{du}\log u\right)\int u^2 du\right\}du=\log u.\frac{u^3}{3}-\int\frac{1}{u}.\frac{u^2}{3}du=\frac{u^3\log u}{3}-\int\frac{u^2}{3}du=\frac{u^3\log u}{2}-\frac{u^3}{9}+C$$

Question 7:

Obtain an integral of u $\sin^{-1}$u.

Answer 7:

Suppose, I = $\int u\sin^{-1}u\, du$

Integrating the equation by parts by taking $\sin^{-1}$u as first function and u as second function, we get,

$$I=\sin^{-1}u\int u\,du-\int\left\{\left(\frac{d}{du}\sin^{-1}u\right)\int u\,du\right\}du=\sin^{-1}u\frac{u^2}{2}-\int\frac{1}{\sqrt{1-u^2}}.\frac{u^2}{2}du=\frac{u^2\sin^{-1}u}{2}+\frac{1}{2}\int\frac{-u^2}{\sqrt{1-u^2}}du=\frac{u^2\sin^{-1}u}{2}+\frac{1}{2}\int\left\{\frac{1-u^2}{\sqrt{1-u^2}}-\frac{1}{\sqrt{1-u^2}}\right\}du=\frac{u^2\sin^{-1}u}{2}+\frac{1}{2}\int\left\{\sqrt{1-u^2}-\frac{1}{\sqrt{1-u^2}}\right\}du=\frac{u^2\sin^{-1}u}{2}+\frac{1}{2}\left\{\int\sqrt{1-u^2}du-\int\frac{1}{\sqrt{1-u^2}}du\right\}=\frac{u^2\sin^{-1}u}{2}+\frac{1}{2}\left\{\frac{u}{2}\sqrt{1-u^2}+\frac{1}{2}\sin^{-1}u-\sin^{-1}u\right\}+C=\frac{u^2\sin^{-1}u}{2}+\frac{u}{4}\sqrt{1-u^2}+\frac{1}{4}\sin^{-1}u-\frac{1}{2}\sin^{-1}u+C=\frac{1}{4}(2u^2-1)\sin^{-1}u+\frac{u}{4}\sqrt{1-u^2}+C$$

Question 8:

Obtain an integral of u $\tan^{-1}$ u

Answer 8:

Suppose, I = $\int u\tan^{-1}u\, du$

Integrating the equation by parts by taking $\tan^{-1}$ u as first function and u as second function, we get,

$$I=\tan^{-1}u\int u\,du-\int\left\{\left(\frac{d}{du}\tan^{-1}u\right)\int u\,du\right\}du=\tan^{-1}u\frac{u^2}{2}-\int\frac{1}{1+u^2}.\frac{u^2}{2}du=\frac{u^2\tan^{-1}u}{2}-\frac{1}{2}\int\frac{u^2}{1+u^2}du=\frac{u^2\tan^{-1}u}{2}-\frac{1}{2}\int\left\{\frac{u^2+1}{1+u^2}-\frac{1}{1+u^2}\right\}du=\frac{u^2\tan^{-1}u}{2}-\frac{1}{2}\int\left\{1-\frac{1}{1+u^2}\right\}du=\frac{u^2\tan^{-1}u}{2}-\frac{1}{2}(u-\tan^{-1}u)+C=\frac{u^2\tan^{-1}u}{2}-\frac{u}{2}+\frac{1}{2}\tan^{-1}u+C$$

Question 9:

Obtain an integral of u $\cos^{-1}$ u

Answer 9:

Suppose, I = $\int u\cos^{-1}u\,du$

Integrating the equation by parts by taking $\cos^{-1}$ u as first function and u as second function, we get,

$$I=\cos^{-1}u\int u\,du-\int\left\{\left(\frac{d}{du}\cos^{-1}u\right)\int u\,du\right\}du=\cos^{-1}u\frac{u^2}{2}-\int\frac{-1}{\sqrt{1-u^2}}.\frac{u^2}{2}du=\frac{u^2\cos^{-1}u}{2}-\frac{1}{2}\int\frac{1-u^2+1}{\sqrt{1-u^2}}du=\frac{u^2\cos^{-1}u}{2}+\frac{1}{2}\int\left\{\sqrt{1-u^2}+\frac{-1}{\sqrt{1-u^2}}\right\}du=\frac{u^2\cos^{-1}u}{2}-\frac{1}{2}I_1-\frac{1}{2}\cos^{-1}u\ldots..(1)$$

$$\text{where}\ I_1=\int\sqrt{1-u^2}\,du$$

$$I_1=u\sqrt{1-u^2}-\int\frac{d}{du}\sqrt{1-u^2}\int u\,du$$

$$I_1=u\sqrt{1-u^2}-\int\frac{-2u}{2\sqrt{1-u^2}}.u\,du$$

$$I_1=u\sqrt{1-u^2}-\int\frac{-u^2}{\sqrt{1-u^2}}du$$

$$I_1=u\sqrt{1-u^2}-\int\frac{1-u^2-1}{\sqrt{1-u^2}}du$$

$$I_1=u\sqrt{1-u^2}-\left\{\int\sqrt{1-u^2}\,du+\int\frac{-du}{\sqrt{1-u^2}}\right\}$$

$$I_1=u\sqrt{1-u^2}-\{I_1+\cos^{-1}u\}$$

$$2I_1=u\sqrt{1-u^2}-\cos^{-1}u$$

$$I_1=\frac{u}{2}\sqrt{1-u^2}-\frac{1}{2}\cos^{-1}u$$

Using in equation (1), we get,

$$I=\frac{u^2\sin^{-1}u}{2}-\frac{1}{2}\left(\frac{u}{2}\sqrt{1-u^2}-\frac{1}{2}\cos^{-1}u\right)-\frac{1}{2}\cos^{-1}u=\frac{(2u^2-1)}{4}\cos^{-1}u-\frac{u}{4}\sqrt{1-u^2}+C$$

Question 10:

Obtain an integral of $(\sin^{-1}u)^2$

Answer 10:

Suppose, I = $\int(\sin^{-1}u)^2.1\,du$

Integrating the equation by parts by taking $(\sin^{-1}u)^2$ as first function and 1 as second function, we get,

$$\sin^{-1}u\int 1\,du-\int\left\{\frac{d}{du}(\sin^{-1}u)^2.\int 1.du\right\}du=(\sin^{-1}u)^2.u-\int\frac{2(\sin^{-1}u)}{\sqrt{1-u^2}}.u\,du=u.(\sin^{-1}u)^2+\int\sin^{-1}u.\left(\frac{-2u}{\sqrt{1-u^2}}\right)du=u.(\sin^{-1}u)^2+\left[\sin^{-1}u\int\frac{-2u}{\sqrt{1-u^2}}du-\int\left\{\left(\frac{d}{du}\sin^{-1}u\right)\int\frac{-2u}{\sqrt{1-u^2}}du\right\}du\right]=u.(\sin^{-1}u)^2+\left[\sin^{-1}u.2\sqrt{1-u^2}-\int\frac{1}{\sqrt{1-u^2}}.2\sqrt{1-u^2}\,du\right]=u.(\sin^{-1}u)^2+2\sqrt{1-u^2}\sin^{-1}u-\int 2\,du=u.(\sin^{-1}u)^2+2\sqrt{1-u^2}\sin^{-1}u-2u+C$$

Question 11:

Obtain an integral of $\frac{u\cos^{-1}u}{\sqrt{1-u^2}}$

Answer 11:

Suppose, I = $\int \frac{u\cos^{-1}u}{\sqrt{1-u^2}}du \quad I = -\frac{1}{2}\int \frac{-2u}{\sqrt{1-u^2}}.\cos^{-1}u\,du$

Integrating the equation by parts by taking cos $^{-1}$ u as first function and $-\frac{2u}{\sqrt{1-u^2}}$ as second function, we get,

$$I = -\frac{1}{2}\left[\cos^{-1}u\int \frac{-2u}{\sqrt{1-u^2}}du - \int\left\{\left(\frac{d}{du}\cos^{-1}u\right)\int \frac{-2u}{\sqrt{1-u^2}}du\right\}du\right] = -\frac{1}{2}\left[\cos^{-1}u.2\sqrt{1-u^2} - \int \frac{-1}{\sqrt{1-u^2}}.2\sqrt{1-u^2}\,du\right] = -\frac{1}{2}\left[2\sqrt{1-u^2}\cos^{-1}u + \int 2du\right] = -\frac{1}{2}\left[2\sqrt{1-u^2}\cos^{-1}u + 2u\right] + C = -\left[\sqrt{1-u^2}\cos^{-1}u + u\right] + C$$

Question 12:

Obtain an integral of u sec ^{2} u

Answer 12:

Suppose, I = $\int u\sec^2 u\,du$

Integrating the equation by parts by taking u as first function and sec ^{2} u as second function, we get,

$$u\int \sec^2 u\,du - \int\left\{\left\{\frac{d}{du}.u\right\}\int \sec^2 u\,du\right\}du = u\tan u - \int 1.\tan u\,du = u\tan u - \log|\cos u| + C$$

Question 13:

Obtain an integral of tan $^{-1}$ u

Answer 13:

Suppose, I = $\int \tan^{-1}u\,du$

Integrating the equation by parts by taking tan $^{-1}$ u as first function and 1 as second function, we get,

$$\tan^{-1}u\int 1du-\int\left\{\left(\frac{d}{du}\tan^{-1}u\right)\int 1du\right\}du=\tan^{-1}u.u-\int\frac{1}{1+u^2}.udu=\tan^{-1}u.u-\frac{1}{2}\int\frac{2u}{1+u^2}.du=u\tan^{-1}u-\frac{1}{2}\log\left|1+u^2\right|+C=u\tan^{-1}u-\frac{1}{2}\log(1+u^2)+C$$

Question 14:

Obtain an integral of u (log u) ^{2}.

Answer 14:

Suppose, I =

Integrating the equation by parts by taking (log u) ^{2} as first function and 1 as second function, we get,

$$I=(\log u)^2\int udu-\int\left[\left\{\left(\frac{d}{du}\log u\right)^2\right\}\int udu\right]du=\frac{u^2}{2}(\log u)^2-\left[\int 2\log u.\frac{1}{u}.\frac{u^2}{2}du\right]=\frac{u^2}{2}(\log u)^2-\int u\log udu$$

Integrating the equation again by parts, we get,

$$I=\frac{u^2}{2}(\log u)^2\int udu-\left[\log u\int udu-\left\{\left(\frac{d}{du}\log u\right)\int udu\right\}du\right]=\frac{u^2}{2}(\log u)^2-\left[\frac{u^2}{2}-\log u-\int\frac{1}{u}.\frac{u^2}{2}du\right]=\frac{u^2}{2}(\log u)^2-\frac{u^2}{2}(\log u)+\frac{1}{2}\int udu=\frac{u^2}{2}(\log u)^2-\frac{u^2}{2}(\log u)+\frac{u^2}{4}+C$$

Question 15:

Obtain an integral of (u ^{2} + 1) log u

Answer 15:

Suppose, I

$$=\int(u^2+1)\log udu=\int u^2\log udu+\int\log udu \quad \text{Suppose, } I=I_1+I_2+\ldots\ldots(1) \quad \text{Where, } I_1=\int u^2\log udu \text{ and } I_2=\int\log udu \quad I_1=\int u^2\log udu$$

Integrating the equation by parts by taking u as first function and u ^{2} as second function, we get,

$$I_1=(\log u)-\int u^2du-\int\left\{\left(\frac{d}{du}\log u\right)\int u^2du\right\}du=\log u.\frac{u^3}{3}-\int\frac{1}{u}.\frac{u^3}{3}du=\frac{u^3}{3}\log u-\frac{1}{3}\left(\int u^2du\right)du=\frac{u^3}{3}\log u-\frac{u^3}{9}+C_1\ldots.(2) \quad I_2=\int\log udu$$

Integrating the equation by parts by taking u as first function and u ^{2} as second function, we get,

$I_2=(\log u)-\int 1du-\int\{(\frac{d}{du}\log u)\int 1du\}=\log u.u-\int\frac{1}{u}.udu=u\log u-\int 1du=u\log u-u+C_2$.....(3)

Substituting equations (2) and (3) in equation (1), we get,

$I=\frac{u^3}{3}\log u-\frac{u^3}{9}+C_1+u\log u-u+C_2=\frac{u^3}{3}\log u-\frac{u^3}{9}+u\log u-u+(C_1+C_2)=(\frac{u^3}{3}+u)\log u-\frac{u^3}{9}-C$

Question 16:

Obtain an integral of e^u (sin u + cos u)

Answer 16:

Suppose, I

$=\int e^u(\sin u+\cos u)du$ Suppose, $f(u)=\sin u$ $f'(u)=\cos u$ $I=\int e^u\{f(u)+f'(u)\}du$ As we know, $\int e^u\{f(u)+f'(u)\}du=e^uf(u)+C$ $I=e^u\sin u+C$

Question 17:

Obtain an integral of $\frac{e^u}{(1+u)^2}$

Answer 17:

Suppose, I = $\int\frac{ue^u}{(1+u)^2}du=\int e^u\{\frac{u}{(1+u)^2}\}du=\int e^u\{\frac{1+u-1}{(1+u)^2}\}du=\int e^u\{\frac{1}{1+u}-\frac{1}{(1+u)^2}\}du$ Suppose, $f(u)=\frac{1}{1+u}, f'(u)=\frac{-1}{(1+u)^2}$ $\int\frac{ue^u}{(1+u)^2}du=\int e^u\{f(u)+f'(u)\}du$ As we know, $\int e^u\{f(u)+f'(u)\}du=e^uf(u)+C$ $\int\frac{ue^u}{(1+u)^2}du=\frac{e^u}{1+u}+C$

Question 18:

Obtain an integral of $e^u(\frac{1+\sin u}{1+\cos u})$

Answer 18:

$e^u(\frac{1+\sin u}{1+\cos u})=e^u(\frac{\sin^2\frac{u}{2}+\cos^2\frac{u}{2}+2\sin\frac{u}{2}\cos\frac{u}{2}}{2\cos^2\frac{u}{2}})=\frac{e^u(\sin\frac{u}{2}+\cos\frac{u}{2})^2}{2\cos^2\frac{u}{2}}=\frac{1}{2}e^u(\frac{\sin\frac{u}{2}+\cos\frac{u}{2}}{\cos\frac{u}{2}})^2$ $=\frac{1}{2}e^u[\tan\frac{u}{2}+1]^2=\frac{1}{2}e^u[1+\tan\frac{u}{2}]^2=\frac{1}{2}e^u[1+\tan^2\frac{u}{2}+2\tan\frac{u}{2}]=\frac{1}{2}e^u[\sec^2\frac{u}{2}+2\tan\frac{u}{2}]$ $\frac{e^u(1+\sin u)du}{(1+\cos u)}=[\frac{1}{2}\sec^2\frac{u}{2}+2\tan\frac{u}{2}]$.....(1) Suppose, $\tan\frac{u}{2}=f(u)$ so $f'(u)=\frac{1}{2}\sec^2\frac{u}{2}$

As we know,

$$\int e^u\{f(u)+f'(u)\}du = e^u f(u)+C$$

Considering equation (1), we get,

$$\int \frac{e^u(1+\sin u)du}{(1+\cos u)} = e^u \tan\frac{u}{2}+C$$

Question 19:

Obtain an integral of $e^u\left(\frac{1}{u}-\frac{1}{u^2}\right)$

Answer 19:

Suppose, I = $\int e^u\left(\frac{1}{u}-\frac{1}{u^2}\right)du$

Suppose, $\frac{1}{u}=f(u)$, $f'(u)=-\frac{1}{u^2}$

As we know, $\int e^u\{f(u)+f'(u)\}du=e^u f(u)+C$

$I=\frac{e^u}{u}+C$

Question 20:

Obtain an integral of $\frac{(u-3)e^u}{(u-1)^3}$

Answer 20:

$$\int e^u\left\{\frac{(u-3)}{(u-1)^3}\right\}du=\int e^u\left\{\frac{(u-1-2)}{(u-1)^3}\right\}du=\int e^u\left\{\frac{1}{(u-1)^2}-\frac{2}{(u-1)^3}\right\}du$$

$f(u)=\frac{1}{(u-1)^2}$, $f'(u)=-\frac{2}{(u-1)^3}$

As we know, $\int e^u\{f(u)+f'(u)\}du=e^u f(u)+C$

$$\int e^u\left\{\frac{(u-3)}{(u-1)^3}\right\}du=\frac{e^u}{(u-1)^2}+C$$