#421319

Topic: Direction Cosines and Direction Ratios

If a line makes angles $90^o, 135^o, 45^o$ with x, y, z axes respectively, find its direction cosines.

Solution

Let the direction cosines of the line be l, m, n.

$l = \cos 90^o = 0$

$m = \cos 135^o = -\dfrac{1}{\sqrt{2}}$

$n = \cos 45^o = \dfrac{1}{\sqrt{2}}$

Therefore, the direction cosines of the line are $0, -\dfrac{1}{\sqrt{2}}$ and $\dfrac{1}{\sqrt{2}}$.

#421321

Topic: Direction Cosines and Direction Ratios

Find the direction cosines of a line which makes equal angles with the coordinate axes.

Solution

Let the direction cosines of the line make an angle a with each of the coordinate axes.

$\therefore\ l = \cos a, m = \cos a, n = \cos a$

We know $l^2 + m^2 + n^2 = 1$

$\Rightarrow\ \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\Rightarrow\ 3\cos^2\alpha = 1$

$\Rightarrow\ \cos^2\alpha = \dfrac{1}{3}$

$\Rightarrow\ \cos\alpha = \pm\dfrac{1}{\sqrt{3}}$

Thus, the direction cosines of the line which is equally inclined to the coordinate axes are $\pm\dfrac{1}{\sqrt{3}}, \pm\dfrac{1}{\sqrt{3}}$ and $\pm\dfrac{1}{\sqrt{3}}$.

#421323

Topic: Direction Cosines and Direction Ratios

If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines?

Solution

If a line has direction ratios of $-18, 12$ and -4, then its direction cosines are

$$\frac{-18}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{12}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{-4}{\sqrt{(-4)^2 + (12)^2 + (-4)^2}}$$

i.e., $\dfrac{-18}{22}, \dfrac{12}{22}, \dfrac{-4}{22} \equiv \dfrac{9}{11}, \dfrac{6}{11}, \dfrac{-2}{11}$

Thus, the direction cosines are $-\dfrac{9}{11}, \dfrac{6}{11}$ and $\dfrac{-2}{11}$.

#421325

Topic: Lines

Find the equation of the line which passes through the point $(1, 2, 3)$ and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

Solution

It is given that the line passes through the point $A(1, 2, 3)$.

Therefore, the position vector through A is $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

It is known that the line which passes through point A and parallel to $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a constant.

$\Rightarrow\ \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} + 2\hat{j} - 2\hat{k}\right)$

This is the required equation of the line.

#421328

Topic: Lines

Find the equation of the line in vector and in Cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$

Solution

It is given that the line passes through the point with position vector,

$\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}. . . (1)$

$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}. . . (2)$

It is known that a line through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is given by the equation, $\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow\ \vec{r} = 2\hat{i} - \hat{j} + 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$

This is the required equation of the line in vector form.

$\Rightarrow\quad x\hat{i} - y\hat{j} + z\hat{k} = (\lambda + 2)\hat{i} + (2\lambda - 1)\hat{j} + (-\lambda + 4)\hat{k}$

Eliminating λ, we obtain the Cartesian form equation as

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$

#421333

Topic: Lines

Find the Cartesian equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

Solution

It is given that the line passes through the point $(-2, 4, -5)$ and is parallel to $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ ....(1)

The direction ratios of the line (1) are $3, 5$ and 6.

The required line is parallel to equation (1).

Therefore, its direction ratios are $3k, 5k$ and $6k$, where $k \neq 0$.

It is known that the equation of the line through the point (x_1, y_1, z_1) and with direction ratios, a, b, c is given by $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

Therefore, the equation of the required line is,

$$\frac{x+2}{3k} = \frac{y-4}{5k} = \frac{z+5}{6k}$$

$$\Rightarrow \frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

#421334

Topic: Lines

The Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Write its vector form.

Solution

The Cartesian equation of a line is

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}\ldots\ldots(1)$$

The given line passes through the point $(5,\ -4,6)$. The position vector of this point is $\vec{a}=5\hat{i}-4\hat{j}+6\hat{k}$.

Also, the direction ratios of the given line are $3,7$ and 2.

This means that the line is in the direction of vector $\vec{b}=3\hat{i}+7\hat{j}+2\hat{k}$.

It is known that the line through position vector $\vec{a}$ and in the direction of the vector $\vec{b}$ is given by the equation,

$\vec{r}=\vec{a}+\lambda\vec{b},\lambda\in R$

$\Rightarrow\ \vec{r}=\left(5\hat{i}-4\hat{j}+6\hat{k}\right)+\lambda\left(3\hat{i}+7\hat{j}+2\hat{k}\right)$

This is the required equation of the given line in vector form.

#421337

Topic: Lines

Find the vector and the Cartesian equations of the lines that pass through the origin and $(5,\ -2,3)$

Solution

The required line passes through the origin. Therefore, its position vector is given by,

$\vec{a}=\vec{0}\ldots\ldots(1)$

The direction ratios of the line through origin and $(5,\ -2,3)$ are

$(5-0)=5,(-2-0)=-2,(3-0)=3$

The line is parallel to the vector given by the equation, $\vec{b}=5\hat{i}-2\hat{j}+3\hat{k}$

The equation of the line in vector form through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is , $\vec{r}=\vec{a}+\lambda\vec{b},\lambda\in R$

$\Rightarrow\ \ \Rightarrow\vec{r}=\vec{0}+\lambda\left(5\hat{i}-2\hat{j}+3\hat{k}\right)$

$\Rightarrow\vec{r}=\lambda\left(5\hat{i}-2\hat{j}+3\hat{k}\right)$

The equation of the line through the point (x_1,y_1,z_1) and direction ratios a,b,c is given by, $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

Therefore, the equation of the required line in the Cartesian form is

$$\frac{x-0}{5}=\frac{y-0}{-2}=\frac{z-0}{3}$$

$$\Rightarrow\frac{x}{5}=\frac{y}{-2}=\frac{z}{3}$$

#421343

Topic: Lines

Find the vector and the Cartesian equations of the line that passes through the points $(3,\ -2,\ -5),(3,\ -2,6)$

Solution

Let the line passing through the points $P(3,\ -2,\ -5)$ and $Q(3,\ -2,6)$ be PQ. Since PQ passes through $P(3,\ -2,\ -5)$, its position vector is given by,

$\vec{a}=3\hat{i}-2\hat{j}-5\hat{k}$

The direction ratios of PQ are given by,

$(3-3)=0,(-2+2)=0,(6+5)=11$

The equation of the vector in the direction of PQ is

$\vec{b}=0\hat{i}-0\hat{j}+11\hat{k}=11\hat{k}$

The equation of PQ in vector form is given by $\vec{r}=\vec{a}+\lambda\vec{b},\lambda\in R$

$\Rightarrow\vec{r}=\left(3\hat{i}-2\hat{j}-5\hat{k}\right)+11\lambda\hat{k}$

The equation of PQ in Cartesian form is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$$\Rightarrow\frac{x-3}{0}=\frac{y+2}{0}=\frac{z+5}{11}$$

#421354

Topic: Lines

Find the angle between the following pair of lines:

(i) $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} - 2\hat{j} + 6\hat{k}\right)$ and $\vec{r} = 7\hat{i} - 6\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

(ii) $\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - 56\hat{k} + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

Solution

(i)

If θ be the angle between the given lines.

Then $\cos\theta = \left|\dfrac{\vec{b_1}\cdot\vec{b_2}}{\left|\vec{b_1}\right|\left|\vec{b_2}\right|}\right|$

where $\vec{b_1} = \left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $\vec{b_2} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$\therefore \left|\vec{b_1}\right| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$\left|\vec{b_2}\right| = \sqrt{1^2 + 2^2 + 2^2} = 3$

$\vec{b_1}\cdot\vec{b_2} = \left(3\hat{i} + 2\hat{j} + 6\hat{k}\right).\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$= 3 \times 1 + 2 \times 2 + 6 \times 2 = 3 + 4 + 12 = 19$

$\Rightarrow \cos\theta = \dfrac{19}{7 \times 3}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{19}{21}\right)$

(ii)

The given lines are parallel to the vectors,

$\vec{b_1} = \left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{b_2} = \left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$ respectively.

$\therefore \left|\vec{b_1}\right| = \sqrt{(1)^2 + (-1)^2 + (-2)^2} = \sqrt{6}$

$\left|\vec{b_2}\right| = \sqrt{(3)^2 + (-5)^2 + (-4)^2} = 5\sqrt{2}$

$\vec{b_1}\cdot\vec{b_2} = \left(\hat{i} - \hat{j} - 2\hat{k}\right).\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

$1.3 - 1(-5) - 2(-4) = 3 + 5 + 8 = 16$

If θ is the angle between the given lines then,

$\cos\theta = \left|\dfrac{\vec{b_1}\cdot\vec{b_2}}{\left|\vec{b_1}\right|\left|\vec{b_2}\right|}\right|$

$\Rightarrow \cos\theta = \dfrac{16}{\sqrt{6}.5\sqrt{2}} = \dfrac{16}{\sqrt{2}.\sqrt{3}.5\sqrt{2}} = \dfrac{16}{10\sqrt{3}}$

$\Rightarrow \cos\theta = \dfrac{8}{5\sqrt{3}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{8}{5\sqrt{3}}\right)$

#421364

Topic: Lines

Find the angle between the following pair of lines:

$\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - 56\hat{k} + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

Solution

The given lines are parallel to the vectors, $\vec{b}_1 = \left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{b}_2 = \left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$ respectively.

Therefore, $\left|\vec{b}_1\right| = \sqrt{(1)^2 + (-1)^2 + (-2)^2} = \sqrt{6}$

and $\left|\vec{b}_2\right| = \sqrt{(3)^2 + (-5)^2 + (-4)^2} = 5\sqrt{2}$

Thus $\vec{b}_1 \cdot \vec{b}_2 = \left(\hat{i} - \hat{j} - 2\hat{k}\right) \cdot \left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

$= 1.3 - 1(-5) - 2(-4)$

$= 3 + 5 + 8 = 16$

If θ is the angle between the given lines then,

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}\right|$$

$$\Rightarrow \cos\theta = \frac{16}{\sqrt{6}.5\sqrt{2}} = \frac{16}{\sqrt{2}.\sqrt{3}.5\sqrt{2}} = \frac{16}{10\sqrt{3}}$$

$$\Rightarrow \cos\theta = \frac{8}{5\sqrt{3}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

This the required angle between the given pair of lines.

#421370

Topic: Lines

Find the angle between the following pair of lines:

(i) $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

(ii) $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

Solution

(i)

Let $\vec{b_1}$ and $\vec{b_2}$ be the vectors parallel to the pair of lines,

$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and

$\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ respectively,

then $\vec{b_1} = \left(2\hat{i} + 5\hat{j} - 3\hat{k}\right)$ and $\vec{b_2} = \left(-\hat{i} + 8\hat{j} + 4\hat{k}\right)$

$\left|\vec{b_1}\right| = \sqrt{(2)^2 + (5)^2 + (-3)^2} = \sqrt{38}$

$\left|\vec{b_2}\right| = \sqrt{(-1)^2 + (8)^2 + (4)^2} = \sqrt{81} = 9$

$\vec{b_1} \cdot \vec{b_2} = \left(2\hat{i} + 5\hat{j} - 3\hat{k}\right) \cdot \left(-\hat{i} + 8\hat{j} + 4\hat{k}\right)$

$= 2(-1) + 5 \times 8 + (-3).4 = -2 + 40 - 12 = 26$

The angle θ between the given pair of lines is given by the relation,

$\Rightarrow \cos\theta = \left|\frac{\vec{b_1} \cdot \vec{b_2}}{\left|\vec{b_1}\right|\left|\vec{b_2}\right|}\right|$

$\Rightarrow \cos\theta = \frac{26}{9\sqrt{38}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$

(ii)

We have,

$\vec{b_1} = \left(2\hat{i} + 2\hat{j} + \hat{k}\right)$

$\vec{b_2} = \left(4\hat{i} + \hat{j} + 8\hat{k}\right)$

$\therefore \left|\vec{b_1}\right| = \sqrt{(2)^2 + (2)^2 + (1)^2} = \sqrt{9} = 3$

$\left|\vec{b_2}\right| = \sqrt{(4)^2 + (1)^2 + (8)^2} = \sqrt{81} = 9$

$\vec{b_1} \cdot \vec{b_2} = \left(2\hat{i} + 2\hat{j} + \hat{k}\right) \cdot \left(4\hat{i} + \hat{j} + 8\hat{k}\right)$

$= 2 \times 4 + 2 \times 1 + 1 \times 8 = 8 + 2 + 8 = 18$

If θ is the angle between the given pair of lines, then $\cos\theta = \left|\frac{\vec{b_1} \cdot \vec{b_2}}{\left|\vec{b_1}\right|\left|\vec{b_2}\right|}\right|$

$\Rightarrow \cos\theta = \frac{18}{3 \times 9} = \frac{2}{3}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$

#421372

Topic: Lines

The angle between the following pair of lines:

$\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$ is $\theta = \cos^{-1}\frac{a}{3}$ then $a =$

Answer: 2

Solution

The drs of given two lines are $2, 2, 1$ and $4, 1, 8$

Let the angle between those two lines be θ

We have $cos\theta = \frac{2\times4+2\times1+1\times8}{3\times9} = \frac{18}{27} = \frac{2}{3}$

Therefore $\theta = cos^{-1}\frac{2}{3}$

#421409

Topic: Lines

Find the values of p so the line $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

Solution

The given equations can be written in the standard form as

$\frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2}$ and $\frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$

The direction ratios of the lines are $-3, \frac{-3p}{7}, 2$ and $\frac{-3p}{7}, 1, -5$ respectively.

Two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2, are perpendicular to each other, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

$\therefore\ (-3)\left(\frac{-3p}{7}\right) + \left(\frac{-3p}{7}\right).(1) + (2).(-5) = 0$

$\Rightarrow\ \frac{9p}{7} + \frac{2p}{7} = 10$

$\Rightarrow\ 11p = 70$

$\Rightarrow\ p = \frac{70}{11}$

#421468

Topic: Lines

Find the shortest distance between the lines

$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and

$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$

Solution

The equations of the given lines are

$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and

$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$

It is known that the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by,

$$d = \left|\frac{\left(\vec{b_1} \times \vec{b_2}\right)\left(\vec{a_1} - \vec{a_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \ldots\ldots (1)$$

Comparing the given equations, we obtain

$\vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{b_1} = \hat{i} - \hat{j} + \hat{k}$

$\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

$\vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\vec{a_2} - \vec{a_1} = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + \hat{k}\right) = \left(\hat{i} - 3\hat{j} - 2\hat{k}\right)$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$\vec{b_1} \times \vec{b_2} = (-2-1)\hat{i} - (2-2)\hat{j} + (1+2)\hat{k} = -3\hat{i} + 3\hat{k}$

$\Rightarrow \left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-3)^2 + (3)^2} = \sqrt{18} = 3\sqrt{2}$

Substituting all the values in equation (1), we obtain

$$d = \left|\frac{\left(3\hat{i} + 3\hat{k}\right).\left(\hat{i} - 3\hat{j} - 2\hat{k}\right)}{3\sqrt{2}}\right|$$

$$\Rightarrow d = \left|\frac{-3.1 + 3(-2)}{3\sqrt{2}}\right|$$

$$\Rightarrow d = \left|\frac{-9}{3\sqrt{2}}\right|$$

$$\Rightarrow d = \frac{3}{\sqrt{2}} = \frac{3 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{2}}{2}$$

Therefore, the shortest distance between the two lines is $\frac{3\sqrt{2}}{2}$ units.

#421639

Topic: Lines

Find the shortest distance between lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Solution

It is known that the shortest distance between the two lines,

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ and $\frac{x-x_2}{a}=\frac{y-y_2}{b}=\frac{z-z_2}{c}$ is given by,

$$d=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\left(b_1c_2-b_2c_1\right)^2+\left(c_1a_2-c_2a_1\right)^2+\left(a_1b_2-a_2b_1\right)^2}}\ldots(1)$$

Comparing the given equations, we obtain

$x_1=-1, y_1=-1, z_1=-1$

$a_1=7, b_1=-6, c_1=1$

$x_2=3, y_2=5, z_2=7$

$a_2=1, b_2=-2, c_2=1$

Then, $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$

$=4(-6+2)-6(7-1)+8(-14+6)$

$=-16-36-64=-116$

$\Rightarrow \sqrt{\left(b_1c_2-b_2c_1\right)^2+\left(c_1a_2-c_2a_1\right)^2+\left(a_1b_2-a_2b_1\right)^2}=\sqrt{(-6+2)^2+(1+7)^2+(-14+6)^2}=\sqrt{16+36+64}=\sqrt{116}=2\sqrt{29}$

Substituting all the values in equation (1), we obtain

$d=\frac{116}{2\sqrt{29}}=\frac{-58}{\sqrt{29}}=\frac{-2\times 29}{\sqrt{29}}=-2\sqrt{29}$

Since distance is always non-negative, the distance between the given lines is $2\sqrt{29}$ units.

#421786

Topic: Lines

Find the shortest distance between the lines whose vector equations are

$\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and

$\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$

Solution

The given lines are $\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$ and

$\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$

It is known that the shortest distance between the lines, $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by,

$$d = \left| \frac{\left(\vec{b_1} \times \vec{b_2}\right)\left(\vec{a_1} - \vec{a_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|} \right| \ldots\ldots (1)$$

Comparing the given equations with $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_1} + \mu\vec{b_1}$, we obtain

$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{b_2} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{a_2} - \vec{a_1} = (4\hat{i} + 5\hat{j} + 6\hat{k}) - (\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{b_1} \times \vec{b_2} = \sqrt{(-9)^2 + (3)^2 + (9)^2} = \sqrt{18 + 9 + 81} = \sqrt{171} = 3\sqrt{19}$

$\left(\vec{b_1} \times \vec{b_2}\right).\left(\vec{a_1} - \vec{a_2}\right) = \left(-9\hat{i} + 3\hat{j} + 9\hat{k}\right).\left(3\hat{i} + 3\hat{j} + 3\hat{k}\right)$

$= -9 \times 3 + 3 \times 3 + 9 \times 3 = 9$

Substituting all the values in equation (1), we obtain

$$d = \left|\frac{9}{3\sqrt{19}}\right| = \frac{3}{\sqrt{19}}$$

Therefore, the shortest distance between the two given lines is $\frac{3}{\sqrt{19}}$ units.

#421811

Topic: Lines

Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and

$\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$

Solution

The given lines are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$

$\Rightarrow \vec{r} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) + t\left(-\hat{i} + \hat{j} - 2\hat{k}\right) \ldots\ldots (1)$

$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

$\Rightarrow \vec{r} = \left(\hat{i} - \hat{j} + 3\hat{k}\right) + s\left(\hat{i} + 2\hat{j} - 2\hat{k}\right) \ldots\ldots\ldots (2)$

It is known that the shortest distance between the lines,

$\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$, is given by

$$d = \left|\frac{\left(\vec{b_1} \times \vec{b_2}\right)\left(\vec{a_1} - \vec{a_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \ldots\ldots (1)$$

For the given equations,

$\vec{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{b_2} = -\hat{i} + \hat{j} - 2\hat{k}$

$\vec{a_2} = \hat{i} - \hat{j} - \hat{k}$

$\vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a_2} - \vec{a_1} = \left(\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) = \hat{j} - 4\hat{k}$

$$\left(\vec{b_1} \times \vec{b_2}\right)\left(\vec{a_1} - \vec{a_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$= (-2+4)\hat{i} - (2+2)\hat{j} + (2-1)\hat{k}$

$\Rightarrow \left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(2)^2 + (-4)^2 + (-3)^2} = \sqrt{4+16+9} = \sqrt{29}$

$\therefore \left(\vec{b_1} \times \vec{b_2}\right).\left(\vec{a_1} - \vec{a_2}\right) = \left(2\hat{i} - 4\hat{j} - 3\hat{k}\right).\left(\hat{j} - 4\hat{k}\right) = -4 + 12 = 8$

Substituting all the values in equation (3), we obtain,

$$d = \left|\frac{8}{\sqrt{29}}\right| = \frac{8}{\sqrt{29}}$$

Therefore, the shortest distance between the lines is $\frac{8}{\sqrt{29}}$ units.

#421823

Topic: Plane

In the following case, determine the direction cosines of the normal to the plane and the distance from the origin.

(i) $z = 2$

(ii) $x + y + z = 1$

(iii) $2x + 3y - 5 = 0$

(iv) $5y + 8 = 0$

Solution

(i) The equation of the plane is $z = 2$ or $0x + 0y + z = 2$....(1)

The direction ratios of normal are $0, 0$ and 1

$\therefore \sqrt{0^2 + 0^2 + 1^2} = 1$

Dividing both sides of equation (1) by 1, we obtain

$0.x + 0.y + 1.z = 2$

This is of the form $lx + my + nz = d$, where l, m, n are the direction cosines of normal to the plane and d is the distance of the perpendicular drawn from the origin.

Therefore, the direction cosines are $0, 0, 1$ and the distance of the from the origin is 2 units.

(ii) $x + y + z = 1$......(1)

The direction ratios of normal are $1, 1$ and 1.

$\therefore \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

Dividing both sides of equation (1) by $\sqrt{3}$, we obtain

$\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}y + \frac{1}{\sqrt{3}}z = \frac{1}{\sqrt{3}} \ldots\ldots (2)$

This equation is of the form $lx + my + nz = d$, where l, m, n are the direction cosines of normal to the plane and d is the distance of normal from the origin.

Therefore, the direction cosines of the normal are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$ and the distance of normal from the origin is $\frac{1}{\sqrt{3}}$ units.

(iii) $2x + 3y - z = 5$.........(1)

The direction ratios of normal are $2, 3$ and -1

$\therefore \sqrt{(2)^2 + (3)^2 + (-1)^2} = \sqrt{14}$

Dividing both sides by $\sqrt{14}$, we obtain

$\frac{2}{\sqrt{14}}x + \frac{3}{\sqrt{14}}y - \frac{1}{\sqrt{14}}z = \frac{5}{\sqrt{14}}$

This equation of the form $lx + my + nz = d$, where l, m, n are the direction cosines of normal to the plane and d is the distance of normal from the origin.

Therefore, the direction cosines of the normal to the plane are $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ and $\frac{-1}{\sqrt{14}}$ and the distance of normal from the origin is $\frac{5}{\sqrt{14}}$ units

(iv) $5y + 8 = 0$

$\Rightarrow 0x - 5y + 0z = 8$.....(1)

The direction ratios of normal are $0, -5$ and 0.

$\therefore \sqrt{0 + (-5)^2 + 0} = 5$

Dividing both sides of equation (1) by 5, we obtain $-y = \frac{8}{5}$

This equation is of the form $lx + my + nz = d$, where l, m, n are the direction cosines of normal to the plane and d is the distance of normal from the origin.

Therefore, the direction cosines of the normal to the plane are $0, -1$ and 0 and the distance of normal from the origin is $\frac{8}{5}$ units.

#421828

Topic: Direction Cosines and Direction Ratios

Passage

In the following case, determine the direction cosines of the normal to the plane and the distance from the origin.

$x + y + z = 1$

Solution

Given, $x + y + z = 1$......(1)

The direction ratios of normal are $1, 1$ and 1.

Now $\sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

Therefore, the direction cosines of the normal are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$

and the distance of plane from the origin is $= \left|\frac{0 + 0 + 0 - 1}{\sqrt{1^2 + 1^2 + 1^2}}\right| = \frac{1}{\sqrt{3}}$

#421832

Topic: Plane

Passage

In the following case, determine the direction cosines of the normal to the plane and the distance from the origin.

$2x + 3y - z = 5$

Solution

Given, $2x + 3y - z = 5$.........(1)

The direction ratios of normal are $2, 3$ and -1.

$\therefore \sqrt{(2)^2 + (3)^2 + (-1)^2} = \sqrt{14}$

Dividing both sides by $\sqrt{14}$, we obtain

$\frac{2}{\sqrt{14}}x + \frac{3}{\sqrt{14}}y - \frac{1}{\sqrt{14}}z = \frac{5}{\sqrt{14}}$

This equation of the form $lx + my + nz = d$, where l, m, n are the direction cosines of normal to the plane and d is the distance of normal from the origin.

Therefore, the direction cosines of the normal to the plane are $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ and $\frac{-1}{\sqrt{14}}$ and the distance of normal from the origin is $\frac{5}{\sqrt{14}}$ units.

#421835

Topic: Plane

Passage

In the following case, determine the direction cosines of the normal to the plane and the distance from the origin.

$5y + 8 = 0$

Solution

Given, $5y + 8 = 0$

$\Rightarrow 0x - 5y + 0z = 8$.....(1)

The direction ratios of normal are $0, -5$ and 0.

Now $\sqrt{0 + (-5)^2 + 0} = 5$

Therefore, the direction cosines of the normal to the plane are $0, -1$ and 0

and the distance of plane from the origin is $= \left|\frac{5(0) + 8}{\sqrt{0^2 + 5^2 + 0^2}}\right| = \frac{8}{5}$

#421866

Topic: Plane

Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i} + 5\hat{j} - 6\hat{k}$

Solution

The normal vector is $\vec{n} = 3\hat{i} + 5\hat{j} - 6\hat{k}$

$$\therefore \hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{(3)^2 + (5)^2 + (6)^2}} = \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}}$$

It is known that the equation of the plane with position vector $\vec{r}$ is given by, $\vec{r}.\hat{n} = d$

$$\Rightarrow \vec{r} \cdot \left(\frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}}\right) = 7$$

This is the vector equation of the required plane.

#421869

Topic: Plane

Passage

Find the Cartesian equation of the following planes:

$\vec{r}.\left(\hat{i} + \hat{j} - \hat{k}\right) = 2$

Solution

It is given that equation of the plane is

$\vec{r}.\left(\hat{i} + \hat{j} - \hat{k}\right) = 2$.....(1)

For any arbitrary point $P(x, y, z)$ on the plane, position vector $\vec{r}$ is given by,

$\vec{r} = x\hat{i} + y\hat{j} - z\hat{k}$

Substituting the value of $\vec{r}$ in equation (1), we obtain

$\left(x\hat{i} + y\hat{j} - z\hat{k}\right).\left(x\hat{i} + y\hat{j} - z\hat{k}\right) = 2$

$\Rightarrow x + y - z = 2$

This is the Cartesian equation of the plane.

#421871

Topic: Plane

Passage

Find the Cartesian equation of the following planes:

$\vec{r} \cdot \left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = 1$

Solution

Given, $\vec{r} \cdot \left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = 1$........(1)

For any arbitrary point $P(x, y, z)$ on the plane, position vector $\vec{r}$ is given by

$\vec{r} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

Substituting the value of $\vec{r}$ in equation (1), we obtain

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = 1 \Rightarrow 2x + 3y - 4z = 1$

This is the Cartesian equation of the plane.

#421876

Topic: Plane

Passage

Find the Cartesian equation of the following planes:

$\vec{r}.\left[(s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k}\right] = 15$

Solution

$\vec{r}\cdot\left[(s-2t)\hat{i}+(3-t)\hat{j}+(2s+t)\hat{k}\right]=15$.....(1)

For any arbitrary point $P(x, y, z)$ on the plane, position vector $\vec{r}$ is given by,

$\vec{r}=\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$

Substituting the value of $\vec{r}$ in equation (1), we obtain

$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left[(s-2t)\hat{i}+(3-t)\hat{j}+(2s+t)\hat{k}\right]=15$

$\Rightarrow\ (s-2t)x+(3-t)y+(2s+t)z=15$

This is the Cartesian equation of the given plane.

#421882

Topic: Direction Cosines and Direction Ratios

In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

$2x+3y+4z-12=0$

A $\left(\frac{24}{29},\frac{36}{49},\frac{48}{29}\right)$

B $\left(\frac{24}{49},\frac{36}{49},\frac{48}{49}\right)$

C $\left(\frac{24}{29},\frac{36}{29},\frac{48}{29}\right)$

D $\left(\frac{24}{49},\frac{36}{29},\frac{48}{49}\right)$

Solution

Let the coordinates of the foot of perpendicular O from the origin to the plane be (x_1, y_1, z_1)

$2x+3y+4z-12=0$

$\Rightarrow\ 2x+3y+4z=12$......(1)

The direction ratios of normal are $2, 3$ and 4

$\therefore\ \sqrt{(2)^2+(3)^2+(4)^2}=\sqrt{29}$

Dividing both sides of equation (1) by $\sqrt{29}$, we obtain

$\frac{2}{\sqrt{29}}x+\frac{3}{\sqrt{29}}y+\frac{4}{\sqrt{29}}z=\frac{12}{\sqrt{29}}$

This equation is of the form $lx+my+nz=d$, where l, m, n are the direction cosines of normal to the plane and d is the distance of normal from the origin.

The coordinates of the foot of the perpendicular are given by (ld, md, nd)

Therefore, the coordinates of the foot of the perpendicular are

$\left(\frac{2}{\sqrt{29}}\cdot\frac{12}{\sqrt{29}},\frac{3}{\sqrt{29}}\cdot\frac{12}{\sqrt{29}},\frac{4}{\sqrt{29}}\cdot\frac{12}{\sqrt{29}}\right)$ i.e., $\left(\frac{24}{29},\frac{36}{29},\frac{48}{29}\right)$

#421886

Topic: Direction Cosines and Direction Ratios

In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

$3y+4z-6=0$

A $\left(0,\frac{24}{25},\frac{18}{25}\right)$

B $\left(0, \frac{24}{25}, \frac{24}{25}\right)$

C $\left(0, \frac{18}{25}, \frac{24}{25}\right)$

D None of these

Solution

Let the coordinates of the foot of perpendicular P from the origin to the plane be (x_1, y_1, z_1)

$3y + 4z - 6 = 0$

$\Rightarrow 0x + 3y + 4z = 6 = 6$....(1)

The direction ratios of the normal are $0, 3$ and 4

$\therefore \sqrt{0 + (3)^2 + (4)^2} = 5$

Dividing both sides of equation (1) by 5, we obtain

$0x + \frac{3}{5}y + \frac{4}{5}z = \frac{6}{5}$

This equation is of the form $lx + my + nz = d$, where l, m, n are the direction cosines of normal to the plane and d is the distance of normal from the origin.

The coordinates of the foot of the perpendicular are

$\left(0, \frac{3}{5} \cdot \frac{6}{5}, \frac{4}{5} \cdot \frac{6}{5}\right)$ i.e., $\left(0, \frac{18}{25}, \frac{24}{25}\right)$

#421887

Topic: Direction Cosines and Direction Ratios

In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

$x + y + z = 1$

Solution

Let the coordinates of the foot of perpendicular P from the origin to the plane be (x_1, y_1, z_1)

$x + y + z = 1$........(1)

The direction ratios of the normal are $1, 1$ and 1

$\therefore \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$

Dividing both sides of equation (1) by $\sqrt{3}$, we obtain

$\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}y + \frac{1}{\sqrt{3}}z = \frac{1}{\sqrt{3}}$

This equation is of the form $lx + my + mz = d$, where l, m, n are the direction cosines of normal to the plane and d is the distance of normal from the origin.

The coordinates of the foot of the perpendicular are given by (ld, md, nd)

Therefore, the coordinates of the foot of the perpendicular are

$\left(\frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}\right)$ i.e., $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

#421888

Topic: Direction Cosines and Direction Ratios

Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $5y + 8 = 0$.

Solution

Let the coordinates of the foot of perpendicular P from the origin to the given plane be (x_1, y_1, z_1).

We have, $5y + 8 = 0$

$\Rightarrow\ 0x - 5y + 0z = 8$......(1)

The direction ratios of the normal are $0,\ -5$ and 0

$\therefore\ \sqrt{0 + (5)^2 + 0} = 5$

Dividing both sides of equation (1) by 5, we obtain

$-y = \frac{8}{5}$

This equation is of the form $lx + my + nz = d$, where $l.\ m.\ n$ are the direction cosines of normal to the plane and d is the distance of normal from the origin.

The coordinates of the foot of the perpendicular are given by $(ld.\ md,\ nd)$

Therefore, the given coordinates of the foot of the perpendicular are

$\left(0,\ -1\left(\frac{8}{5}\right), 0\right)$ i.e., $\left(0,\ -\frac{8}{5}, 0\right)$

#421908
Topic: Plane

Passage

Find the vector and Catesian equation of the planes

that passes through the point $(1, 0,\ -2)$ and the normal to the plane is $\hat{i} + \hat{j} - \hat{k}$

Solution

The position vector of point $(1, 0,\ -2)$ is $\vec{N}$ perpendicular to the plane is $\vec{N} = \hat{i} + \hat{j} - \hat{k}$.

The vector equation of the plane is given by, $(\vec{r} - \vec{a}).\ \vec{N} = 0$

$\Rightarrow\ \left[\vec{r} - (\hat{i} - 2\hat{k})\right].\left(\hat{i} + \hat{j} - \hat{k}\right) = 0$.......(1)

The position vector of any point $P(x, y, z)$ in the plane is given by,

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Therefore, equation (1) becomes

$\left[\left(x\hat{i} + y\hat{j} - z\hat{k}\right) - \left(\hat{i} - 2\hat{k}\right)\right].\left(\hat{i} + \hat{j} - \hat{k}\right) = 0$

$\Rightarrow\ \left[(x-1)\hat{i} + y\hat{j} + (z+2)\hat{k}\right].\left(\hat{i} + \hat{j} - \hat{k}\right) = 0$

$\Rightarrow\ (x - 1) + y - (z + 2) = 0$

$\Rightarrow\ x + y - z - 3 = 0$

$\Rightarrow\ x + y - z = 3$

#421919
Topic: Plane

Passage

Find the vector and Catesian equation of the planes

that passes through the point $(1, 4, 6)$ and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$

Solution

The position vector of point $(1, 4, 6)$ is $\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

The normal vector $\vec{N}$ perpendicular to the plane is $\vec{N} = \hat{i} - 2\hat{j} + \hat{k}$

The vector equation of the plane is given by $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$\Rightarrow \left[\vec{r} - (\hat{i} + 4\hat{j} + 6\hat{k})\right] \cdot \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0$.......(1)

Where $\vec{r}$ is the position vector of any point $P(x, y, z)$ in the plane.

And given by, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Therefore, the equation(1) becomes

$\Rightarrow \left[(x-1)\hat{i} + (y-4)\hat{j} + (z-6)\hat{k}\right] \cdot \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0$

$\Rightarrow (x-1) - 2(y-4) + (z-6) = 0$

$\Rightarrow x - 2y + z + 1 = 0$

#421923
Topic: Plane

Passage

Find the equations of the planes that passes through three points.

$(1, 1, -1), (6, 4, -5), (-4, -2, 3)$

Solution

The given points are $A(1, 1, -1)$, $B(6, 4, -5)$ and $C(-4, -2, 3)$.

$$\begin{vmatrix} 1 & 1 & -1 \\ 6 & 4 & -5 \\ -4 & -2 & 3 \end{vmatrix} = (12 - 10) - (18 - 20) - (12 + 16) = 2 + 2 - 4 = 0$$

$\Rightarrow$ Points are collinear.

Since A, B, C are collinear points, there will be infinite number of planes passing through the given points.

#421954
Topic: Plane

Passage

Find the equations of the planes that passes through three points.

$(1, 1, 0), (1, 2, 1), (-2, 2, -1)$

Solution

It is known that the equation of the plane through the points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) is,

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow (-2)(x-1) - 3(y-1) + 3z = 0$

$\Rightarrow -2x - 3y + 3z + 2 + 3 = 0$

$\Rightarrow -2x - 3y + 3z = -5$

$\Rightarrow 2x + 3y - 3z = 5$

This is the Cartesian equation of the required plane.

#421969
Topic: Plane

Find the intercepts cut off by the plane $2x + y - z = 5$.

Solution

Given, $2x + y - z = 5$

$\Rightarrow \frac{2}{5}x + \frac{y}{5} - \frac{z}{5} = 1$

$\Rightarrow \frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$

It is known that the equation of a plane in intercept from is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, where a, b, c are the intercepts cut off by the plane at x, y and z axes respectively.

Therefore, for the given equation,

$a = \frac{5}{2}, b = 5$ and $c = -5$

Thus, the intercepts cut off by the given plane are $\frac{5}{2}, 5$ and -5.

#421971
Topic: Plane

Find the equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane.

Solution

The equation of the plane ZOX is $y = 0$.

Any plane parallel to it is of the form $y = a$.

Since the y-intercept of the plane is 3,

$\therefore \ a = 3$

Thus, the equation of the required plane is $y = 3$.

#422067
Topic: Plane

Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4z = 0$ and $x + y + z - 2 = 0$ and passes through the point $(2, 2, 1)$

Solution

Equation of plane passing through line of intersection of given planes is given by,

$(3x - y + 2z - 4) + \alpha(x + y + z - 2) = 0$, where $\alpha \in R$. . . . . . . (1)

The plane passes through the point $(2, 2, 1)$.

Therefore, this point will satisfy equation (1),

$\therefore \ (3 \times 2 - 2 + 2 \times 1 - 4) + \alpha(2 + 2 + 1 - 2) = 0$

$\Rightarrow \ 2 + 3\alpha = 0$

$\Rightarrow \ \alpha = -\frac{2}{3}$

Substituting $\alpha = -\frac{2}{3}$ in equation (1), we obtain

$\Rightarrow \ (3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$

$\Rightarrow \ (9x - 3y + 6z - 12) - 2(x + y + z - 2) = 0$

$\Rightarrow \ 7x - 5y + 4z - 8 = 0$

This is the required equation of the plane.

#422108
Topic: Plane

Find the vector equation of the plane passing through the intersection of the planes

$\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=7$, $\vec{r}\cdot\left(2\hat{i}+5\hat{j}+3\hat{k}\right)=9$ and through the point $(2,1,3)$

Solution

The equations of the planes are

$\Rightarrow\ \vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)-7=0$..........(1)

$\vec{r}\cdot\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9=0$..........(2)

The equations of the planes through the intersection of the planes given in equations (1) and (2) is given by,

$\left[\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)-7\right]+\lambda\left[\vec{r}\cdot\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9\right]=0$, where $\lambda\in R$

$\vec{r}\cdot[\left(2\hat{i}+2\hat{j}-3\hat{k}\right)+\lambda\left(2\hat{i}+5\hat{j}+3\hat{k}\right)]=9\lambda+7$

$\vec{r}\cdot\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(3\lambda-3)\hat{k}\right]=9\lambda+7$......(3)

The plane passes through the point $(2,1,3)$. Therefore, its poisition vector is given by,

$\vec{r}=2\hat{i}+\hat{j}+3\hat{k}$

Substituting in equation (3), we obtain

$\left(2\hat{i}+\hat{j}-3\hat{k}\right)\cdot\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(3\lambda-3)\hat{k}\right]=9\lambda+7$

$\Rightarrow\ 2(2+2\lambda)+(2+5\lambda)-3(3\lambda-3)=9\lambda+7$

$\Rightarrow\ 9\lambda=8$

$\Rightarrow\ 9\lambda=8\Rightarrow\lambda=\frac{8}{9}$

Substituting $\lambda=\frac{8}{9}$ in equation (3), we obtain

$\vec{r}\cdot\left(\frac{34}{9}\hat{i}+\frac{58}{9}\hat{j}-\frac{3}{9}\hat{k}\right)=15$

#422156

Topic: Plane

Find the equation of the plane through the line of intersection of the planes

$x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$

Solution

The equation of the plane through the intersection of the plane $x+y+z=1$ and $2x+3y+4z=5$ is

$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$

$\Rightarrow\ (2\lambda+1)x+(3\lambda+1)y+(4\lambda+1)z-(5\lambda+1)=0$......(1)

The direction ratios a_1, b_1, c_1 of this plane are $(2\lambda+1)$, $(3\lambda+1)$ and $(4\lambda+1)$

The plane in equation (1) is perpendicular to $x-y+z=0$

Its direction ratios a_2, b_2, c_2 are $1,\ -1$ and 1

Since the planes are perpendicular,

$a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow\ (2\lambda+1)-(3\lambda+1)+(4\lambda+1)=0$

$\Rightarrow\ 3\lambda+1=0$

$\Rightarrow\ \lambda=-\frac{1}{3}$

Substituting $\lambda=-\frac{1}{3}$ in equation (1), we obtain

$\frac{1}{3}x-\frac{1}{3}z+\frac{2}{3}=0$

$\Rightarrow\ x-z+2=0$

This is the required equation of the plane.

#422211

Topic: Plane

Find the angle between the planes whose vector equations are

$\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=5$ and $\vec{r}\cdot\left(3\hat{i}-3\hat{j}+5\hat{k}\right)=3$

Solution

The equations of the given planes are $\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=5$ and $\vec{r}\cdot\left(3\hat{i}-3\hat{j}+5\hat{k}\right)$

It is known that if $\vec{n_1}$ and $\vec{n_2}$ are normal to the planes $\vec{r}\cdot\vec{n_1}=d_1$ and $\vec{r}\cdot\vec{n_2}=d_2$ then the angle between them, θ is given by

$$\cos\theta=\left|\frac{\vec{n_1}\cdot\vec{n_2}}{\left|\vec{n_1}\right|\left|\vec{n_2}\right|}\right|\ldots.(1)$$

Here $\vec{n_1}=2\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{n_2}=3\hat{i}-3\hat{j}+5\hat{k}$

$\therefore\ \vec{n_1}\cdot\vec{n_2}=\left(2\hat{i}+2\hat{j}-3\hat{k}\right)\left(3\hat{i}-3\hat{j}+5\hat{k}\right)=2.3+2.(-3)+(-3).5=-15$

$\left|\vec{n_1}\right|=\sqrt{(2)^2+(2)^2+(-3)^2}=\sqrt{17}$

$\left|\vec{n_2}\right|=\sqrt{(3)^2+(-3)^2+(5)^2}=\sqrt{43}$

Substituting the value of $\vec{n_1}\cdot\vec{n_2}$ and $\left|\vec{n_1}\right|\left|\vec{n_2}\right|$ in equation (1), we obtian

$$\cos\theta=\left|\frac{-15}{\sqrt{17}.\sqrt{43}}\right|$$

$$\Rightarrow\ \cos\theta=\frac{15}{\sqrt{731}}$$

$$\Rightarrow\theta=\cos^{-1}\frac{15}{\sqrt{731}}$$

#422387

Topic: Plane

Passage

In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

$7x+5y+6z+30=0$ and $3x-y-10z+4=0$

Solution

The direction ratios of normal to the plane, $L_1: a_1x+b_1y+c_1z=0$ are a_1, b_1, c_1 and $L_2: a_1x+b_2y+c_2z=0$ are a_2, b_2, c_2

$L_1 \parallel L_2$, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$L_1 \perp L_2$, if $a_1a_2+b_1b_2+c_1c_2=0$

The angle between L_1 and L_2 is given by,

$$\theta=\cos^{-1}\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_1^2}}\right|$$

The equations of the planes are $7x+5y+6z+30=0$ and $3x-y-10z+4=0$.

Here, $a_1=7, b_1=5, c_1=6$

$a_2, b_2=-1, c_2=-10$

$a_1a_2+b_1b_2+c_1c_2=7\times3+5\times(-1)+6\times(-10)=-44\neq0$

Therefore, the given planes are not perpendicular

$\frac{a_1}{a_2}=\frac{7}{3}, \frac{b_1}{b_2}=\frac{5}{-1}=-5, \frac{c_1}{c_2}=\frac{6}{-10}=\frac{-3}{5}$

It can be seen that $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Therefore, the given planes are not parallel.

The angle between them is given by,

$$\theta=\cos^{-1}\left|\frac{7\times3+5(-1)+6\times(-10)}{\sqrt{(7)^2+(5)^2+(6)^2}\times\sqrt{(3)^2+(-1)^2+(-10)^2}}\right|$$

$$=\cos^{-1}\left|\frac{21-5-60}{\sqrt{110}\times\sqrt{110}}\right|$$

$$=\cos^{-1}\left|\frac{44}{110}\right|=\cos^{-1}\frac{2}{5}$$

#422404

Topic: Plane

Passage

In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

$2x+y+3z-2=0$ and $x-2y+5=0$

Solution

The direction ratios of normal to the plane, $L_1: a_1x+b_1y+c_1z=0$ are a_1, b_1, c_1 and $L_2: a_1x+b_2y+c_2z=0$ are a_2, b_2, c_2

$L_1 \parallel L_2$, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$L_1 \perp L_2$, if $a_1a_2+b_1b_2+c_1c_2=0$

The angle between L_1 and L_2 is given by,

$$\theta=\cos^{-1}\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_1^2}}\right|$$

Here the equations of the planse are $2x+y+3z-2=0$ and $x-2y+5=0$

$\Rightarrow a_1=2, b_1=-2, c_1=3$ and $a_2=1, b_2=-2, c_2=0$

Now $a_1a_2+b_1b_2+c_1c_2=2\times1+1\times(-2)+3\times0=0$

Thus, the given planes are perpendicular to each other.

#422405

Topic: Plane

Passage

In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

$2x-2y+4z+5=0$ and $3x-3y+6z-1=0$

Solution

The direction ratios of normal to the plane, $L_1: a_1x+b_1y+c_1z=0$ are a_1, b_1, c_1 and $L_2: a_1x+b_2y+c_2z=0$ are a_2, b_2, c_2

$L_1 \parallel L_2$, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$L_1 \perp L_2$, if $a_1a_2+b_1b_2+c_1c_2=0$

The angle between L_1 and L_2 is given by,

$$\theta=\cos^{-1}\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}.\sqrt{a_2^2+b_2^2+c_1^2}}\right|$$

The equations of the planse are $2x-2y+4z+5=0$ and $3x-3y+6z-1=0$.

Here $a_1=2, b_1=-2, c_1=4$ and

$a_2=3, b_2=-3, c_2=6$

$a_1a_2+b_1b_2+c_1c_2=2\times3+(-2)(-3)+4\times6=6+6+24=36\neq0$

Thus, the given planes are nor perpendicular to each other.

Now $\frac{a_1}{a_2}=\frac{2}{3}, \frac{b_1}{b_2}=\frac{-2}{-3}=\frac{2}{3}$ and $\frac{c_1}{c_2}=\frac{4}{6}=\frac{2}{3}$

$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Thus, the given planes are parallel to each other

#422431

Topic: Plane

Passage

In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

$2x-y+3z-1=0$ and $2x-y+3z+3=0$

Solution

The direction ratios of normal to the plane, $L_1: a_1x+b_1y+c_1z=0$ are a_1, b_1, c_1 and $L_2: a_1x+b_2y+c_2z=0$ are a_2, b_2, c_2

$L_1 \parallel L_2$, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$L_1 \perp L_2$, if $a_1a_2+b_1b_2+c_1c_2=0$

The angle between L_1 and L_2 is given by,

$$\theta=\cos^{-1}\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}.\sqrt{a_2^2+b_2^2+c_1^2}}\right|$$

The equations of the planes are $2x-y+3z-1=0$ and $2x-y+3z+3=0$

Here, $a_1=2, b_1=-1, c_1=3$ and $a_2=2, b_2=-1, c_2=3$

$\frac{a_1}{a_2}=\frac{2}{2}=1, \frac{b_1}{b_2}=\frac{-1}{-1}=1$ and \cfrac { { c }_{ 1 } }{ { c }_{ 2 } } =\cfrac{3}{3}=1

\therefore \cfrac { { a }_{ 1 } }{ { a }_{ 2 } } =\cfrac { { b }_{ 1 } }{ { b }_{ 2 } } =\cfrac { { c }_{ 1 } }{ { c }_{ 2 } }

Thus, the given lines are parallel to each other.

#422484

Topic: Plane

Passage

In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

$4x+8y+z-8=0$ and $y+z-4=0$

Solution

The direction ratios of normal to the plane, ${L}_{1}:{a}_{1}x+{b}_{1}y+{c}_{1}z=0$ are ${a}_{1}, {b}_{1}, {c}_{1}$ and ${L}_{2}:{a}_{1}x+{b}_{2}y+{c}_{2}z=0$ are ${a}_{2}, {b}_{2}, {c}_{2}$

${L}_{1}\parallel {L}_{2}$, if $\cfrac{{a}_{1}}{{a}_{2}}=\cfrac{{b}_{1}}{{b}_{2}}=\cfrac{{c}_{1}}{{c}_{2}}$

${L}_{1}\bot {L}_{2}$, if ${a}_{1}{a}_{2}+{b}_{1}{b}_{2}+{c}_{1}{c}_{2}=0$

The angle between ${L}_{1}$ and ${L}_{2}$ is given by,

$\theta =\cos ^{ -1 }{ \left| \cfrac { { a }_{ 1 }{ a }_{ 2 }+{ b }_{ 1 }{ b }_{ 2 }+{ c }_{ 1 }{ c }_{ 2 } }{ \sqrt { { a }_{ 1 }^{ 2 }+{ b }_{ 1 }^{ 2 }+{ c }_{ 1 }^{ 2 }.\sqrt { { a }_{ 2 }^{ 2 }+{ b }_{ 2 }^{ 2 }+{ c }_{ 1 }^{ 2 } } } } \right| }$

The equations of the given planes are $4x+8y+z-8=0$ and $y+z-4=0$.

Here ${a}_{1}=4,{b}_{1}=8, {c}_{1}=1$ and ${a}_{2}=0,{b}_{2}=1, {c}_{1}=1$

${a}_{1}{a}_{2}+{b}_{1}{b}_{2}+{c}_{1}{c}_{2}=4\times 0+8\times 1+1=9\ne 0$

Therefore, the given lines are not perpendicular to each other.

$\cfrac { { a }_{ 1 } }{ { a }_{ 2 } } =\cfrac{4}{0}, \cfrac { { b }_{ 1 } }{ { b }_{ 2 } } =\cfrac{8}{1}=8, \cfrac { { c }_{ 1 } }{ { c }_{ 2 } } =\cfrac{1}{1}=1$

$\therefore \cfrac { { a }_{ 1 } }{ { a }_{ 2 } } \neq \cfrac { { b }_{ 1 } }{ { b }_{ 2 } } \neq \cfrac { { c }_{ 1 } }{ { c }_{ 2 } }$

Therefore, the given lines are not parallel to each other.

The angle between the planes is given by,

$\theta =\cos ^{ -1 }{ \left| \cfrac { 4 \times 0+8\times 1+1\times 1 }{ \sqrt { { \left(4 \right) }^{ 2 }+{ \left(8 \right) }^{ 2 }+{ \left(1 \right) }^{ 2 } } \times \sqrt { { \left(0 \right) }^{ 2 }+{ \left(1 \right) }^{ 2 }+{ \left(1 \right) }^{ 2 } } } \right| }$

$=\cos ^{ -1 }{ \left| \cfrac { 9 }{ 9\sqrt { 2 } } \right| }$

$=\cos ^{ -1 }{ \cfrac { 1 }{ \sqrt { 2 } } } ={45}^{o}$

#422500

Topic: Plane

Passage

In the following case, find the distance of each of the given points from the corresponding given plane

Point, Plane: $(0,0,0)$, $3x-4y+12z=3$

Solution

We know that the distance between a point $p({x}_{1}, {y}_{1}, {z}_{1})$ and a plane $Ax+By+Cz=D$ is given by,

$d=\left| \cfrac { A{ x }_{ 1 }+B{ y }_{ 1 }+C{ z }_{ 1 }-D }{ \sqrt { { A }^{ 2 }+{ B }^{ 2 }+{ C }^{ 2 } } } \right|$ ....(1)

Thus distance of point $(0,0,0)$ from the plane $3x-4y+12z=3$ is

$d=\left| \cfrac { 3\times 0-4\times 0+12\times 0-3 }{ \sqrt { { \left(3 \right) }^{ 2 }+{ \left(-4 \right) }^{ 2 }+{ \left(12 \right) }^{ 2 } } } \right| =\cfrac { 3 }{ \sqrt { 169 } } =\cfrac { 3 }{ 13 }$

#422506

Topic: Plane

Passage

In the following case, find the distance of each of the given points from the corresponding given plane

Point, Plane: $(3,-2,1)$, $2x-y+2z+3=0$

Solution

We know that the distance between a point $p(x_1, y_1, z_1)$ and a plane $Ax+By+Cz=D$ is given by,

$$d=\left| \cfrac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}} \right| ....(1)$$

Thus distance of point $(3,-2,1)$ from the plane $2x-y+2z+3=0$ is

$$d=\left| \cfrac{2\times 3-(-2)+2\times 1+3}{\sqrt{(2)^2+(-1)^2+(2)^2}} \right| =\cfrac{13}{3}=\cfrac{13}{3}$$

#422514
Topic: Plane

Passage

In the following case, find the distance of each of the given points from the corresponding given plane

Point $(2,3,-5)$, plane $x+2y-2z=9$

Solution

We know that the distance between a point $p(x_1, y_1, z_1)$ and a plane $Ax+By+Cz=D$ is given by,

$$d=\left| \cfrac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}} \right| ....(1)$$

Thus distance of point $(2,3,-5)$ from the plane $x+2y-2z=9$ is

$$d=\left| \cfrac{2+2\times 3-2(-5)-9}{\sqrt{(1)^2+(2)^2+(-2)^2}} \right| =\cfrac{9}{3}=3$$

#422522
Topic: Plane

Passage

In the following case, find the distance of each of the given points from the corresponding given plane

Point, Plane: $(-6,0,0)$, $2x-3y+6z-2=0$

Solution

We know that the distance between a point $p(x_1, y_1, z_1)$ and a plane $Ax+By+Cz=D$ is given by,

$$d=\left| \cfrac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}} \right| ....(1)$$

Thus distance of point $(-6,0,0)$ from plane $2x-3y+6z-2=0$ is

$$d=\left| \cfrac{2(-6)-3\times 0+6\times 0-2}{\sqrt{(1)^2+(2)^2+(-2)^2}} \right| =\left| \cfrac{-14}{\sqrt{49}} \right| =\cfrac{14}{7}=2$$

#422539
Topic: Lines

Find the angle between the lines whose direction ratios are a,b,c and $b-c$, $c-a$, $a-b$

Solution

The angle θ between the lines with direction cosines a,b,c and $b-c$, $c-a$, $a-b$ is given by,

$$\cos\theta =\left| \cfrac{a(b-c)+b(c-a)+c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}} \right|$$

$$\Rightarrow \cos\theta =0$$

$$\Rightarrow \theta=\cos^{-1} 0$$

$$\Rightarrow \theta ={90}^{o}$$

Thus, the angle between the lines is ${90}^{o}$.

#422545
Topic: Lines

Find the equation of a line parallel to x-axis and passing through the origin

Solution

The line parallel to x-axis and passing through the origin is x-axis itself.

Let A be a point on x-axis.

Therefore, the coordinates of A are given by $(a,0,0)$, where $a\in R$.

Direction ratios of OA are $(a-0)=a,0,0$.

The equation of OA is given by,

$\cfrac{x-0}{a}=\cfrac{y-0}{0}=\cfrac{z-0}{0}$

$\cfrac{x}{1}=\cfrac{y}{0}=\cfrac{z}{0}=a$

Thus, the equation of line parallel to x-axis and passing through origin is $\cfrac{x}{1}=\cfrac{y}{0}=\cfrac{z}{0}$.

#422559

Topic: Lines

If the coordinates of the points A,B,C,D be (1,2,3), (4,5,7), (-4,3,-6) and (2,9,2) respectively, then find the angle between the lines AB and CD

Solution

The coordinates of A,B,C and D are (1,2,3), (4,5,7), (-4,3,-6) and (2,9,2) respectively.

The direction ratios of AB are $(4-1)=3$, $(5-2)=3$ and $(7-3)=4$

The direction ratios of CD are $(2-(-4))=6$, $(9-3)=6$ and $(2-(-6))=8$

It can be seen that $\cfrac { { a }_{ 1 } }{ { a }_{ 2 } } =\cfrac { { b }_{ 1 } }{ { b }_{ 2 } } =\cfrac { { c }_{ 1 } }{ { c }_{ 2 } } =\cfrac { 1 }{ 2 }$

Therefore, AB is parallel to CD.

Thus, the angle between AB and CD is either ${0}^{o}$ or ${180}^{o}$.

#422576

Topic: Lines

If the lines $\cfrac{x-1}{-3}=\cfrac{y-2}{2k}=\cfrac{z-3}{2}$ and $\cfrac{x-1}{3k}=\cfrac{y-1}{1}=\cfrac{z-6}{-5}$ are perpendicular, find the value of k

Solution

The direction of ratios of the lines $\cfrac{x-1}{-3}=\cfrac{y-2}{2k}=\cfrac{z-3}{2}$ and $\cfrac{x-1}{3k}=\cfrac{y-1}{1}=\cfrac{z-6}{-5}$ are -3, 2k, 2 and 3k, 1, -5 respectively.

It is known that two lines with direction ratios ${a}_{1}, {b}_{1}, {c}_{1}$ and ${a}_{2}, {b}_{2}, {c}_{2}$ are perpendicular, if ${a}_{1}{a}_{2}+{b}_{1}{b}_{2}+{c}_{1}{c}_{2}=0$

$\therefore -3(3k)+2k\times 1+2(-5)=0$

$\Rightarrow -9k+2k-10=0$

$\Rightarrow 7k=-10$

$\Rightarrow k=\cfrac{-10}{7}$

Therefore for $k=-\cfrac{10}{7}$, the given lines are perpendicular to each other.

#422590

Topic: Plane

Find the vector equation of the plane passing through (1,2,3) and perpendicular to the plane $\vec { r } .\left(\hat { i } +2\hat { j } -5\hat { k } \right) +9=0$

Solution

The position vector of the point (1,2,3) is $\vec { { r }_{ 1 } } =\hat { i } +2\hat { j } +3\hat { k }$

The direction ratios of the normal to the plane $\vec { r } .\left(\hat { i } +2\hat { j } -5\hat { k } \right) +9=0$, are 1,2 and -5 and the normal vector is $\vec { N } =\hat { i } +2\hat { j } -5\hat { k }$

The equation of a line passing through a point and perpendicular to the given plane is given by, $\vec { r } =\vec { r_1 } +\lambda \vec { N }$,

$\lambda \in R$

$\Rightarrow \vec { r } =(\hat { i } +2\hat { j } +3\hat { k })+\lambda(\hat { i } +2\hat { j } -5\hat { k })$

#422614

Topic: Plane

Find the equation of the plane passing through (a,b,c) and parallel to the plane $\vec { r } .\left(\hat { i } +\hat { j } +\hat { k } \right) =2$

Solution

Any plane to parallel to the plane $\vec { r } .\left(\hat { i } +\hat { j } +\hat { k } \right) =2$ is for the form

$\vec { r } .\left(\hat { i } +\hat { j } +\hat { k } \right) =\lambda$ .....(1)

The plane passes through the point (a,b,c). Therefore, the position vector $\vec { r }$ of this point is $a\hat { i } +b\hat { j } +c\hat { k }$.

Therefore, equation (1) becomes

$(a\hat { i } +b\hat { j } +c\hat { k }).(\hat { i } +\hat { j } +\hat { k })=\lambda$

$\Rightarrow a+b+c=\lambda$

Substituting $\lambda=a+b+c$ in equation (1) we obtain

$\vec { r } .\left(\hat { i } +\hat { j } +\hat { k } \right) =a+b+c$......(2)

This is the vector equation of the required plane.

Substituting $\vec { r } =x\hat { i } +y\hat { j } +z\hat { k }$ in equation (2), we obtain

$(x\hat { i } +y\hat { j } +z\hat { k }).(\hat { i } +\hat { j } +\hat { k })=a+b+c$

$\Rightarrow x+y+z=a+b+c$

#422667

Topic: Lines

Find the shortest distance between lines $\vec { r } =6\hat { i } +2\hat { j } +2\hat { k } +\lambda \left(\hat { i } -2\hat { j } +2\hat { k } \right)$ and $\vec { r } =-4\hat { i } -\hat { k } +\mu \left(3 \hat { i } -2\hat { j } -2\hat { k } \right)$

Solution

The given lines are

$\vec { r } =6\hat { i } +2\hat { j } +2\hat { k } +\lambda \left(\hat { i } -2\hat { j } +2\hat { k } \right)$ ....(1)

$\vec { r } =-4\hat { i } -\hat { k } +\mu \left(3 \hat { i } -2\hat { j } -2\hat { k } \right)$ .....(2)

It is known that the shortest distance between two lines, $\vec { r } =\vec { { a }_{ 1 } } +\lambda \vec { { b }_{ 1 } }$ and $\vec { r } =\vec { { a }_{ 2 } } +\lambda \vec { { b }_{ 2 } }$ is given by,

$d=\left| \cfrac { \left(\vec { { b }_{ 1 } } \times \vec { { b }_{ 2 } } \right) .\left(\vec { { a }_{ 2 } } -\vec { { a }_{ 1 } } \right) }{ \left| \vec { { b }_{ 1 } } \times \vec { { b }_{ 2 } } \right| } \right|$ ....(3)

Comparing $\vec { r } =\vec { { a }_{ 1 } } +\lambda \vec { { b }_{ 1 } }$ and $\vec { r } =\vec { { a }_{ 2 } } +\lambda \vec { { b }_{ 2 } }$ to equations (1) and (2), we obtain

$\vec { { a }_{ 1 } } =6\hat { i } +2\hat { j } +2\hat { k }$

$\vec { { b }_{ 1 } } =\hat { i } -2\hat { j } +2\hat { k }$

$\vec { { a }_{ 2 } } =-4\hat { i } -\hat { k }$

$\vec { { b }_{ 2 } } =3\hat { i } -2\hat { j } -2\hat { k }$

$\Rightarrow \vec { { a }_{ 2 } } -\vec { { a }_{ 1 } } =(-4\hat { i } -\hat { k })-(6\hat { i } +2\hat { j } +2\hat { k })=10\hat { i } -2\hat { j } -3\hat { k }$

$\Rightarrow \left| \vec { { b }_{ 1 } } \times \vec { { b }_{ 2 } } \right| =\begin{vmatrix} \hat { i } & \hat { j } & \hat { k } \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}=(4+4)\hat { i } -(-2-6)\hat { j } +(-2+6)\hat { k } =8\hat { i } +8\hat { j } +4\hat { k }$

$\therefore \left| \vec { { b }_{ 1 } } \times \vec { { b }_{ 2 } } \right| =\sqrt { { \left(8 \right) }^{ 2 }+{ \left(8 \right) }^{ 2 }+{ \left(4 \right) }^{ 2 } } =12$

$\left(\vec { { b }_{ 1 } } \times \vec { { b }_{ 2 } } \right) .\left(\vec { { a }_{ 2 } } -\vec { { a }_{ 1 } } \right) =(\left(8\hat { i } +8\hat { j } +4\hat { k } \right) .\left(-10\hat { i } -2\hat { j } -3\hat { k } \right) =-80-16-12=-108$

Substituting all the values in equation (1), we obtain

$d=\left| \cfrac { -108 }{ 12 } \right| =9$

Therefore, the shortest distance between the two given lines is 9 units.

#422804

Topic: Plane

Find the coordinates of the point where the line through (5,1,6) and (3,4,1) crosses the ZX-plane

Solution

The line passing through the points $(5,1,6)$ and $(3,4,1)$ is given by,

$\cfrac{x-5}{3-5}=\cfrac{y-1}{4-1}=\cfrac{z-6}{1-6}$

$\Rightarrow \cfrac{x-5}{-2}=\cfrac{y-1}{3}=\cfrac{z-6}{-5}=k$ (say)

$\Rightarrow x=5-2k, y=3k+1, z=6-5k$

Any point on the line is of the form $P(5-2k, 3k+1, 6-5k)$.

Since the line passes through ZX-plane,

$\Rightarrow$ Y-coordinate of point P will be 0 $\Rightarrow 3k+1=0\Rightarrow k=-\cfrac{1}{3}$

Therefore, the required point is $\left(\cfrac { 17 }{ 3 } ,0,\cfrac { 23 }{ 3 } \right)$.

#422817

Topic: Plane

Find the coordinates of the point where the line through $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane $2x+y+z=7$

Solution

Equation of line passes through the points $(3,-4,-5)$ and $(2,-3,1)$ is given by

$\cfrac{x-3}{2-3}=\cfrac{y+4}{-3+4}=\cfrac{z+5}{1+5}$

$\Rightarrow \cfrac{x-3}{-1}=\cfrac{y+4}{1}=\cfrac{z+5}{6}=k$ (say)

$\Rightarrow x=3-k, y=k, z=6k-5$

Therefore, any point on the line is of the form $(3-k, k-4, 6k-5)$

This point lies on the plane $2x+y+z=7$

$\therefore 2(3-k)+(k-4)+(6k-5)=7$

$\Rightarrow 5k-3=7\Rightarrow k=2$

Hence, the coordinates of the required point are $(3-2, 2-4, 6\times 2-5)$ i.e,$(1,-2,7)$

#422829

Topic: Plane

Find the equation of the plane passing through the point $(-1,2,3)$ and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$

Solution

2bThe equation of the plane passing through $(-1,2,3)$ will be given by $a(x+1) + b(y-2) + c(z-3) = 0$, where a,b and c are the direction ratios of the normal to the plane.

Also, this plane is perpendicular to the planes $x+2y+3z = 5$ and $3x+3y+z = 0$. According to the perpendicularity condition of planes, the dot product of the direction ratios of the normals to the two planes should be 0.

$\Rightarrow a+2b+3c = 0$ and $3a+3b+c=0$

$\Rightarrow c = -\cfrac{a+2b}3$

Substituting this in the second equation,

$3a+3b - \dfrac{a+2b}3=0$

$\Rightarrow 9a+9b - a - 2b = 0$

$\Rightarrow 8a+7b = 0$

$\Rightarrow b = -\cfrac{8a}7$

$\Rightarrow c = -\cfrac{a+2b}3\\=-\cfrac{a-\dfrac{16a}7}3\\=\cfrac{9a}{7\times 3}\\=\cfrac{3a}7$

Substituting these values of b and c in the equation of the plane,

$a(x+1) -\dfrac{8a}7(y-2)+\dfrac{3a}7 (z-3) = 0$

$\Rightarrow 7(x+1) - 8(y-2) + 3(z-3) = 0$

$\Rightarrow 7x-8y+3z + 7 +16 - 9 = 0$

$\Rightarrow 7x-8y+3z +14=0$

Hence, the equation of the required plane is $7x-8y+3z + 14 = 0$

#422837

Topic: Plane

If the points (1,1,p) and (-3, 0, 1) be equidistant from the plane

\vec { r } .\left(3\hat { i } +4\hat { j } -12\hat { k } \right) +13=0, then find the value of p.

Solution

The position vector through the point (1,1,p) is \vec { { a }_{ 1 } } =\hat { i } +\hat { j } +p\hat { k }

Similarly, the position vector through the point (-3,0,1) is \vec { { a }_{ 2 } } =-3\hat { i } +\hat { k }

The equation of the given plane is \vec { r } .\left(3\hat { i } +4\hat { j } -12\hat { k } \right) +13=0

It is known that the perpendicular distance between a point whose position vector is \vec { a } and the plane \vec { r } .\vec { N } =d, is given by

D=\cfrac { \leftl \vec { a } .\vec { N } -d \rightl }{ \leftl \vec { N } \rightl }

Here, \vec { N } =\left(3\hat { i } +4\hat { j } -12\hat { k } \right) and d=-13

Therefore, the distance between the point (1,1,p) and the given plane is

{ D }_{ 1 }=\cfrac { \leftl \left(\hat { i } +\hat { j } +p\hat { k } \right) .\left(3\hat { i } +4\hat { j } -12\hat { k } \right) +13 \rightl }{ \leftl 3\hat { i } +4\hat { j } -12\hat { k } \rightl }

\Rightarrow \cfrac { \leftl 3+4-12p+13 \rightl }{ \sqrt { { \left(3 \right) }^{ 2 }+{ \left(4 \right) }^{ 2 }+{ \left(-12 \right) }^{ 2 } } }

\Rightarrow \cfrac { \leftl 20-12p \rightl }{ 13 } ......(1)

Similarly, the distance between the point (-3, 0, 1) and the given plane is

{ D }_{ 2 }=\cfrac { \leftl \left(-3\hat { i } +\hat { k } \right) .\left(3\hat { i } +4\hat { j } -12\hat { k } \right) +13 \rightl }{ \leftl 3\hat { i } +4\hat { j } -12\hat { k } \rightl }

\Rightarrow \cfrac { \leftl -9-12+13 \rightl }{ \sqrt { { \left(3 \right) }^{ 2 }+{ \left(4 \right) }^{ 2 }+{ \left(-12 \right) }^{ 2 } } }

\Rightarrow {D}_{2}=\cfrac{8}{13}.......(2)

It is given that, the difference between the required plane and the points (1,1,p) and (-3,0,1) is equal

\therefore {D}_{1}={D}_{2}

\Rightarrow \cfrac { \leftl 20-12p \rightl }{ 13 } =\cfrac{8}{13}

\Rightarrow 20-12p=8 or -(20-12p)=8

\Rightarrow 12p=12 or 12p=28

\Rightarrow p=1 or p=\cfrac{7}{3}

#422846

Topic: Plane

Find the equation of the plane passing through the line of intersection of the planes \vec { r } .\left(\hat { i } +\hat { j } +\hat { k } \right) =1 and \vec { r } .\left(2\hat { i } +3\hat { j } -\hat { k } \right) +4=0 and parallel to x-axis.

Solution

The given planes are

\vec { r } .\left(\hat { i } +\hat { j } +\hat { k } \right) =1

\Rightarrow \vec { r } .\left(\hat { i } +\hat { j } +\hat { k } \right) -1=0

& \vec { r } .\left(2\hat { i } +3\hat { j } -\hat { k } \right) +4=0

The equation of any plane passing through the line of intersection of these planes is

\left[\vec { r } .\left(\hat { i } +\hat { j } +\hat { k } \right) -7 \right] +\lambda \left[\vec { r } .\left(2\hat { i } +3\hat { j } -\hat { k } \right) +4 \right] =0

\vec { r } .\left[(2\lambda +1)\hat { i } +(3\lambda +1)\hat { j } +(1-\lambda)\hat { k } \right] +\left(4\lambda +1 \right) ......(1)

Its direction ratios are (2\lambda+1), (3\lambda+1) and (1-\lambda)

The required plane is parallel to x-axis. Therefore, its normal is perpendicular to x-axis.

The direction ratios of x-axis are 1,0 and 0.

\therefore 1.(2\lambda+1)+0(3\lambda+1)+0(1-\lambda)=0

\Rightarrow 2\lambda+1=0\Rightarrow \lambda=-\cfrac{1}{2}

Substituting \lambda=-\cfrac{1}{2} in equation (1), we obtain

\Rightarrow \vec { r } .\left[-\cfrac { 1 }{ 2 } \hat { j } +\cfrac { 3 }{ 2 } \hat { k } \right] +(-3)=0

\Rightarrow \vec { r }\cdot (\hat { j } -3\hat { k })+6=0

Therefore, its cartesian equation is y-3z+6=0

#422860

Topic: Plane

If O be the origin and the coordinates of P be $(1,2,-3)$, then find the equation of the plane passing through P and perpendicular to OP.

Solution

The coordinates of the points O and P are $(0,0,0)$ and $(1,2,-3)$ respectively.

Therefore, the direction ratios of OP are

$(1-0)=1,(2-0)=2$ and $(-3-0)=-3$

Thus the direction ratios of normal are $1,2$ and -3 and the point P is $(1,2,-3)$.

Thus, the equation of the required plane is

$1(x-1)+2(y-2)-3(z+3)=0$

$\Rightarrow x+2y-3z-14=0$

#422861

Topic: Plane

Find the equation of the plane which contains the line of intersection of the planes

$\vec { r } .\left(\hat { i } +2\hat { j } +3\hat { k } \right) -4=0$, $\vec { r } .\left(2\hat { i } +\hat { j } -\hat { k } \right) +5=0$ and which is perpendicular to the plane $\vec { r } .\left(5\hat { i } +3\hat { j } -6\hat { k } \right) +8=0$.

Solution

$\vec { r } .\left(\hat { i } +2\hat { j } +3\hat { k } \right) -4=0$.....(1)

$\vec { r } .\left(2\hat { i } +\hat { j } -\hat { k } \right) +5=0$......(2)

The equations of the plane passing through the line intersection of the plane given in equation (1) and equation (2) is

$\left[\vec { r } .\left(\hat { i } +2\hat { j } +3\hat { k } \right) -4 \right] +\lambda \left[\vec { r } .\left(2\hat { i } +\hat { j } -\hat { k } \right) +5 \right] =0$

$\vec { r } .\left[(2\lambda +1)\hat { i } +(\lambda +2)\hat { j } +(3-\lambda)\hat { k } \right] +\left(5\lambda -4 \right) =0$.....(3)

The plane in equation (3) is perpendicular to the plane

$\therefore 5(2\lambda+1)+3(\lambda+2)-6(3-\lambda)=0$

$\Rightarrow 19\lambda-7=0$

$\Rightarrow \lambda=\cfrac{7}{19}$

Substituting $\lambda=\cfrac{7}{19}$ in equation (3) we obtain

$\Rightarrow \vec { r } .\left[\cfrac { 33 }{ 19 } \hat { i } +\cfrac { 45 }{ 19 } \hat { j } +\cfrac { 50 }{ 19 } \hat { k } \right] -\cfrac { 41 }{ 19 } =0$

$\Rightarrow \vec { r } .(\left(33\hat { i } +45\hat { j } +50\hat { k } \right) -41)=0$.......(4)

This is the vector equation of the required plane.

The cartesian equation of this plane can be obtained by substituting $\vec { r } =x\hat { i } +y\hat { j } +z\hat { k }$ in equation (3).

$(x\hat { i } +y\hat { j } +z\hat { k }).\left(33\hat { i } +45\hat { j } +50\hat { k } \right) -41=0$

$\Rightarrow 33x+45y+50z-41=0$

#422862

Topic: Plane

Find the distance of the point $(-1,-5,-10)$ from the point of intersection of the line $\vec { r } =\left(2\hat { i } -\hat { j } +2\hat { k } \right) +\lambda \left(3\hat { i } +4\hat { j } +2\hat { k } \right)$ and the plane $\vec { r } .\left(\hat { i } -\hat { j } +\hat { k } \right) =5$

Solution

The equation of the given line is

$\vec { r } =\left(2\hat { i } -\hat { j } +2\hat { k } \right) +\lambda \left(3\hat { i } +4\hat { j } +2\hat { k } \right)$ .....(1)

The equation of the given plane is

$\vec { r } .\left(\hat { i } -\hat { j } +\hat { k } \right) =5$........(2)

Substituting the value of $\vec { r }$ in equation (2), we get,

$\left[\left(2\hat { i } -\hat { j } +2\hat { k } \right) +\lambda \left(3\hat { i } +4\hat { j } +2\hat { k }\right) \right][\hat{i}-\hat{j}+\hat{k}]=5$

$\implies \left[(2+3\lambda)\hat { i } +(4\lambda-1)\hat { j } +(2+2\lambda)\hat { k } \right][\hat{i}-\hat{j}+\hat{k}]=5$

$\implies (2+3\lambda)-(4\lambda-1)+(2\lambda+2)=5$

$\implies \lambda =0$

Thus the point of intersection of the given line and the plane is,

$\vec { r } =2\hat { i } -\hat { j } +2\hat { k } =(2,-1,2)$

This means that the position vector of the point of intersection of the line and plane is given by the coordinates (2,-1,2). The point is (-1,-5,-10).

Hence, distance d between the points, (2,-1,2) and (-1,-5,-10) is

$d=\sqrt { { \left(-1-2 \right) }^{ 2 }+{ \left(-5+1 \right) }^{ 2 }+{ \left(-10-2 \right) }^{ 2 } } =\sqrt { 9+16+144 } =\sqrt { 169 } =13$

#422863

Topic: Plane

Find the vector equation of the line passing through (1,2,3) and parallel to the planes $\vec { r } .\left(\hat { i } -\hat { j } +2\hat { k } \right) =5$ and $\vec { r } .\left(3\hat { i } +\hat { j } +\hat { k } \right) =6$.

Solution

Let the required line be parallel to vector $\vec { b }$ is given by,

$\vec { r } ={b}_{1}\hat { i } +{b}_{2}\hat { j } +{b}_{3}\hat { k }$

The position vector of the point (1,2,3) is $\vec { a } =\hat { i } +2\hat { j } +3\hat { k }$.

The equation of line passing through (1,2,3) and parallel to $\vec { b }$ is given by,

$\vec { r } =\vec { a } +\lambda \vec { b }$

$\Rightarrow \vec { r } =\left(\hat { i } +2\hat { j } +3\hat { k } \right) +\lambda\left({b}_{1}\hat { i } +{b}_{2}\hat { j } +{b}_{3}\hat { k } \right)$.....(1)

The equations of the given planes are

$\vec { r } .\left(\hat { i } -\hat { j } +2\hat { k } \right) =5$.......(2)

$\vec { r } .\left(3\hat { i } +\hat { j } +\hat { k } \right) =6$.......(3)

The line in equation (1) and plane in equation (2) are parallel. Therefore, the normal to the plane of equation (2) and the given line are perpendicular.

$\Rightarrow \left(\hat { i } -\hat { j } +2\hat { k } \right) .\lambda \left({b}_{1}\hat { i } +{b}_{2}\hat { j } +{b}_{3}\hat { k } \right) =0$

$\Rightarrow \lambda({b}_{1}-{b}_{2}+2{b}_{3})=0$

$\Rightarrow {b}_{1}-{b}_{2}+2{b}_{3}=0$.......(4)

Similarly, $\left(3\hat { i } +\hat { j } +\hat { k } \right) .\lambda \left({b}_{1}\hat { i } +{b}_{2}\hat { j } +{b}_{3}\hat { k } \right) =0$

$\Rightarrow \lambda(3{b}_{1}+{b}_{2}+{b}_{3})=0$

$\Rightarrow 3{b}_{1}+{b}_{2}+{b}_{3}=0$........(5)

From equations (4) and (5), we obtain

$\cfrac{{b}_{1}}{(-1)\times 1-1\times 2}=\cfrac{{b}_{2}}{2\times 3-1\times 1}=\cfrac{{b}_{3}}{1\times 1-3(-1)}$

$\Rightarrow \cfrac{{b}_{1}}{-3}=\cfrac{{b}_{2}}{5}=\cfrac{{b}_{3}}{4}$

Therefore, the direction ratios of $\vec { b }$ are -3,5 and 4.

$\therefore \vec { b }={b}_{1}\hat { i } +{b}_{2}\hat { j } +{b}_{3}\hat { k } =-3\hat { i } +5\hat { j } +4\hat { k }$

Substituting the value of $\vec { b }$ in equation (1), we obtain

$\vec { r } =\left(\hat { i } +2\hat { j } +3\hat { k } \right) +\lambda \left(-3\hat { i } +5\hat { j } +4\hat { k } \right)$

This is the equation of the required line.

#422865

Topic: Lines

Find the vector equation of the line passing through the point (1,2,-4) and perpendicular to the two lines: $\cfrac{x-8}{3}=\cfrac{y+19}{-16}=\cfrac{z-10}{7}$ and $\cfrac{x-15}{3}=\cfrac{y-29}{8}=\cfrac{z-5}{-5}$.

Solution

Let the required line be parallel to the vector $\vec { b }$ given by $\vec { b } ={b}_{1}\hat { i } +{b}_{2}\hat { j } +{b}_{3}\hat { k }$

The position vector of the point (1,2,-4) is $\vec { a } =\hat { i } +2\hat { j } -4\hat { k }$.

The equation of the line passing through (1,2,-4) and parallel to vector $\vec { b }$ is $\vec { r } =\vec { a } +\lambda \vec { b }$.

$\Rightarrow \vec { r } =\left(\hat { i } +2\hat { j } -4\hat { k } \right) +\lambda ({b}_{1}\hat { i } +{b}_{2}\hat { j } +{b}_{3}\hat { k })$......(1)

The equations of the lines are,

$\cfrac{x-8}{3}=\cfrac{y+19}{-16}=\cfrac{z-10}{7}$.....(2)

$\cfrac{x-15}{3}=\cfrac{y-29}{8}=\cfrac{z-5}{-5}$ ....(3)

Line (1) and line (2) are perpendiciuar to each other.

$\therefore 3{b}_{1}-16{b}_{2}+7{b}_{3}=0$........(4)

Also, line (1) and line (3) are perpendicular to each other.

$\therefore 3{b}_{1}+8{b}_{2}-5{b}_{3}=0$........(5)

From equations (4) and (5), we obtain

$\cfrac{{b}_{1}}{(-16)(-5)-8\times 7}=\cfrac{{b}_{2}}{7\times 3-3(-5)}=\cfrac{{b}_{3}}{3\times 8-3(-16)}$

$\Rightarrow \cfrac{{b}_{1}}{24}=\cfrac{{b}_{2}}{36}=\cfrac{{b}_{3}}{72}$

$\Rightarrow \cfrac{{b}_{1}}{2}=\cfrac{{b}_{2}}{3}=\cfrac{{b}_{3}}{6}$

$\therefore$ direction of $\vec { b }$ are 2,3 and 6.

$\therefore \vec { b } =2\hat { i } +3\hat { j } +6\hat { k }$

Substituting $\vec { b } =2\hat { i } +3\hat { j } +6\hat { k }$ in equation (1), we obtain

$\vec { r } =\left(\hat { i } +2\hat { j } -4\hat { k } \right) +\lambda \left(2\hat { i } +3\hat { j } +6\hat { k } \right)$

This is the equation of the required line.

#422867

Topic: Lines

Distance between the two planes : 2x+3y+4z=4 and 4x+6y+8z=12 is

A 2 units

B 4 units

C 8 units

D $\cfrac{2}{\sqrt {29}}$ units

Solution

The equations of the planes are

2x+3y+4z=4....(1)

4x+6y+8z=12

$\Rightarrow$ 2x+3y+4z=6.......(2)

It can be seen that the given planes are parallel.

Thus distance (D) between them is given by.

$D=\left| \cfrac { { d }_{ 2 }-{ d }_{ 1 } }{ \sqrt { { a }^{ 2 }+{ b }^{ 2 }+{ c }^{ 2 } } } \right|$

$\Rightarrow D=\left| \cfrac { 6-4 }{ \sqrt { { 2 }^{ 2 }+{ \left(3 \right) }^{ 2 }+{ \left(4 \right) }^{ 2 } } } \right| =\cfrac{2}{\sqrt {29}}$

#428867

Topic: Direction Cosines and Direction Ratios

Find the direction cosines of the vector $\hat {i}+2\hat {j}+3\hat {k}$.

Solution

Let $\vec {a}=\hat {i}+2\hat {j}+3\hat {k}$

$\therefore |\vec {a}|=\sqrt {1^2+2^2+3^2}=\sqrt {1+4+9}=\sqrt {14}$

Hence, the direction cosines of $\vec {a}$ are $\left (\dfrac {1}{\sqrt {14}}, \dfrac {2}{\sqrt {14}}, \dfrac {3}{\sqrt {14}}\right)$.

#428871

Topic: Direction Cosines and Direction Ratios

Find the direction cosines of the vector joining the points A (1, 2, -3) and B (-1, -2, 1) directed from A to B.

Solution

The given points are A (1, 2, -3) and B (-1, -2, 1).

$\therefore \vec {AB}=(-1-1)\hat {i}+(-2-2)\hat {j}+\left \{1-(-3)\right \}\hat {k}$

$\therefore \vec {AB}=-2\hat {i}-4\hat {j}+4\hat {k}$

$\therefore |\vec {AB}|=\sqrt {(-2)^2+(-4)^2+4^2}=\sqrt {4+16+16}=\sqrt {36}=6$

Hence, the direction cosines of $\vec {AB}$ are $\left (-\dfrac {2}{6}, -\dfrac {4}{6}, \dfrac {4}{6}\right)=\left (-\dfrac {1}{3}, -\dfrac {2}{3}, \dfrac {2}{3}\right)$.

#428872

Topic: Direction Cosines and Direction Ratios

Show that the vector $\hat {i}+\hat {j}+\hat {k}$ is equally inclined to the axes OX, OY and OZ.

Solution

Let $\vec {a}=\hat {i}+\hat {j}+\hat {k}$

Then, $|\vec {a}|=\sqrt {1^2+1^2+1^2}=\sqrt 3$

Therefore, the direction cosines of $\vec {a}$ are $\left (\dfrac {1}{\sqrt {3}}, \dfrac {1}{\sqrt {3}}, \dfrac {1}{\sqrt {3}}\right)$.

Now, let α, β, and γ be the angles formed by $\vec {a}$ with the positive direction directions of x, y, and z axes.

Then, we have $\cos\alpha =\dfrac {1}{\sqrt {3}}, \cos\beta =\dfrac {1}{\sqrt {3}}, \cos\gamma =\dfrac {1}{\sqrt {3}}$.

Hence, the given vector is equally inclined to axes OX, OY, and OZ.