

## Chapter 3

### Matrix

#### 1 Mark Questions

**Q1** Write the number of possible matrices which can be made if it has 12 elements.

**Q2** Let  $A = [a_{ij}]$  be a matrix of order  $2 \times 3$  and

$$a_{ij} = \frac{i-j}{i+j}, \text{ write the value of } a_{23}.$$

**Q3** If  $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$  find the relation between a and b.

**Q4** If following information regarding the number of men and women workers in three factories I, II and III is written in the form of  $3 \times 2$  matrix. What does the entry in third row and second column represent?

|             | Men workers | Women workers |
|-------------|-------------|---------------|
| Factory I   | 30          | 25            |
| Factory II  | 25          | 31            |
| Factory III | 27          | 26            |

**Q5** If,  $A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$  and  $B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$

Write the value of (i)  $a_{22} + b_{21}$

(ii)  $a_{11} b_{11} + a_{22} b_{22}$

**Q6** Is it possible to have the product of two matrices to be the null matrix while neither of them is the null matrix? If it is so, give an example.

**Q7** Under what conditions is the matrix equation

$$A^2 - B^2 = (A-B)(A+B) \text{ is true.}$$

**Q8** Write the order of matrix B if A is any matrix of order  $m \times n$  such that

AB and BA both are defined.

Q10

$$\text{If } \mathbf{A} = \begin{bmatrix} -1 & 2 & -5 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix}$$

write the orders of AB and BA.

Q11

Give an example of two non-zero matrices A and B such that

$$\mathbf{AB} = \mathbf{0} \text{ but } \mathbf{BA} \neq \mathbf{0}.$$

Q12

$$\text{If } \mathbf{A} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \text{ find } \mathbf{A}^6.$$

Q13

$$\text{If } \mathbf{A} = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \text{ and } \mathbf{A}^2 = \mathbf{I}, \text{ find the value of } \alpha^2 + \beta\gamma$$

Q14

$$\text{If } \mathbf{A} = \begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix}$$

$$0 < x < \frac{\pi}{2} \text{ and } \mathbf{A} + \mathbf{A}' = \mathbf{I},$$

where I is unit matrix, find value of x.

Q15

If the following matrix is skew symmetric, find the values of a, b, c.

$$\mathbf{A} = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

Q16

If A and B are symmetric matrices and  $\mathbf{AB} = \mathbf{BA}$ , prove that matrix  $\mathbf{X} = \mathbf{AB}$  is also symmetric.

Q17

If A and B are square matrices of same order and B is symmetric, show that  $\mathbf{A}'\mathbf{BA}$  is also symmetric.

Q18

Give an example of a matrix which is both symmetric and skew symmetric

Q19

$$\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} = \mathbf{P} + \mathbf{Q}, \text{ where } \mathbf{P} \text{ is symmetric and } \mathbf{Q} \text{ is}$$

skew symmetric matrix, find the matrices P and Q.

Q20

If A is square matrix then write the value of  $\mathbf{A}(\text{AdjA})$

#### 4 Mark Questions

Q1

For what values of x and y are the following matrices equal

$$\mathbf{A} = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2 - 5y \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

**Q2 Find matrix A such that  $2\mathbf{A}-3\mathbf{B}+5\mathbf{C} = \mathbf{0}$  where,**

$$\mathbf{B} = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

**Q3 Find the values of x and y for which the following matrix equation**

**$\mathbf{A}-3\mathbf{B} = \mathbf{C}$  is satisfied, where**

$$\mathbf{A} = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} x \\ 2y \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

**Q4 Let  $f(x) = x^2 - 5x + 6$ , find  $f(\mathbf{A})$**

$$\text{If, } \mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

**5. If,  $\mathbf{A} = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$  and  $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$  find all those values of  $\alpha$  for which**

$$\mathbf{A}=\mathbf{B}.$$

**Q6 . Using Principle of Mathematical Induction, prove that**

$$\mathbf{A}^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix} \quad \text{Where, } \mathbf{A} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

### 6 Mark Questions

Q1      If  $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$  find x, y such that  $A^2 + xI = yA$

Hence find  $A^{-1}$ .

Q2      . If  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$  Prove that,  $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$  for

every positive integer n.

Q3      The sum of three numbers is -1. If we multiply the second number by 2 , third number by 3 and add them we get 5. If we subtract the third number from the sum of first and second numbers we get -1. Represent it by a system of equations . Find the three numbers using inverse of a matrix .

Q4      If  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  and  $C(x_3, y_3)$  are the vertices of an equilateral triangle with each side equal to 'a' units, prove that,

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = \sqrt{3} a^2$$

### **Answers: Matrix**

#### 1 Mark Questions

Q1      6

Q2      -1/5

Q3       $a=2b$  {  $a=4$ ,  $b=2$  }

Q4      Number of women workers in factory III.

Q5      1, 20

Q6       $A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$   $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$   $AB = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$

Q7       $AB = BA$  ie, if the matrices A & B commute with each other.

Q8       $n \times m$

Q9  $k = 4, a = -4, b = -10, c = 0$

Q10  $1 \times 1, 3 \times 3,$

Q10  $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Q12  $A^6 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Q13  $\alpha^2 + \beta^7 = 1$

Q14  $x = \pi/6$

Q15  $a = -2, b = 0, c = -3$

Q18 Null Matrix

Q19  $P = \begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix} \quad Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Q20  $|A| I$

#### 4 Marks Questions

Q1  $x=2, y=2$

Q2  $A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$

Q3  $x=1, 2 \quad y=3 \pm 3\sqrt{2}$

Q4  $f(A) = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

Q5 No values of  $a$  can be found for which  $A^2 = B$  is true.

Q6 order of  $A=2 \times 3$

$$A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

#### 6 Mark Questions

**Q1**       $x = 8 \quad y = 8$

**Q2**       $A^{-1} = \frac{1}{8} \begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

**Q3**      **Let numbers be x, y, z then**

$$x + y + z = -1$$

$$2y + 3z = 5$$

$$x + y - z = -1$$

**Ans**  $x = -\frac{7}{2}$  ,       $y = \frac{5}{2}$  ,       $z = 0$